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Robust Image Registration using Improved Local Descriptors and Support Vector Machines

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Abstract

This thesis presents a detailed study and improvement to local descriptor processes for registering images for the purpose of three-dimensional reconstruction, using four Māori artefacts as case studies. The motivation for the research came from the issues which still exist in image registration when dealing with large magnitudes of image transformations.

Four major pieces of work were carried out in the course of this research. First, an evaluation was carried out to study the performance of local descriptor processes and based on the results, the local descriptor process was divided into three stages, of which two were closely analysed. Second, the local descriptor formation stage was studied, and two methods, colour and hybrid local descriptor methods, were developed using colour images instead of greyscale images to improve the uniqueness of local descriptors. Third, the local descriptor matching stage was studied, and a new method based on support vector machines was developed. Fourth, an assisted image registration programme was developed and is a semi-automatic approach for registering images.

Extensive amount of experiments were carried out to validate these work. It was found that the colour and hybrid local descriptor methods had gains in matching accuracy of up to 10% over existing methods, and the support vector machine matching method had increased matching performance of up to 20%. When the two methods were combined, it was found that performance gains of up to 25% could be achieved. For the assisted image registration programme, up to 50% improvement was achieved, and the advantage was more significant as the magnitude of image transformation increased, highlighting the need for such programme.

These results show that the proposed work in this research are significant contributions to literature. In addition, these results show that the proposed methods can be used successfully for registering images for three-dimensional reconstruction, where the image transformation between images are often large. As there is currently a need to reconstruct Māori artefacts, this research has provided a new approach for registering images of these artefacts, which could then be used to construct three-dimensional models of the artefacts.

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List of Abbreviations

1D	One-dimensional
2D	Two-dimensional
3D	Three-dimensional
ALH	Adaptive Local Hyperplane
CLD	Colour Local Descriptors
CSIFT	Colour Scale Invariant Feature Transform
CT	Computed Tomography
DoG	Difference of Gaussian
DoH	Difference of Hessian
EPICS	Engineering Projects In Community Services
GLOH	Gradient Location Orientation Histogram
HLD	Hybrid Local Descriptors
ISDA	Iterative Single Data Algorithm
KHNN	k -local Hyperplane Nearest Neighbour
LESH	Local Energy-Based Shape Histogram
LoG	Laplacian of Gaussian
LOOCV	Leave-One-Out Cross-Validation
MATLAB	MATrix LABoratory
MRI	Magnetic Resonance Imaging
NN	Nearest Neighbour
NNR	Nearest Neighbour Ratio
PC	Principal Component
PCA	Principal Component Analysis
RANSAC	RANdom SAmple Consensus
RFE-SVMs	Recursive Feature Elimination with Support Vector Machines
SIFT	Scale Invariant Feature Transform
SVM	Support Vector Machines

List of Symbols

Due to the vast number of symbols present in this thesis, the symbols are listed in the chapter they first appear.

List of symbols from Chapter 2: Literature Review.

c_x, c_y	Principal point
f_x, f_y	Focal length
$O(n)$	Big O notation
\mathbf{H}^{12}	Homography matrix
\mathbf{K}	Camera/intrinsic matrix
$(\mathbf{R}^1, \mathbf{T}^1),$ $(\mathbf{R}^2, \mathbf{T}^2)$	Extrinsic parameters consisting of the rotation and translation matrices for the reference and sensed images, respectively
s	Distortion factor
T_n	Triangular number
(x^1, y^1)	Location of a single pixel in the reference image
$\hat{x}^1, \hat{y}^1, \hat{l}^1$	Components of \mathbf{x}^1
$\mathbf{x}^1, \mathbf{x}^2$	Location of pixels in the reference and sensed images, respectively
\mathbf{X}	Location of 3D points which correspond to \mathbf{x}

List of symbols from Chapter 3: Performance Evaluation of Local Descriptor Methods.

C_o^1, C_o^2	Camera centre of the reference and sensed images, respectively
d^1, d^2	Distance from the reference and sensed image planes to the object, respectively
h_o^1, h_o^2	Height of the object in the reference image and sensed images, respectively
I^1, I^2	Reference and sensed image planes, respectively
O	Object
S	Scale
x	Location of a pixel along the x-axis
x_c	Radial distortion centre along the x-axis
\hat{x}	Corrected pixel location along the x-axis taking into account radial distortion
\hat{y}	Corrected pixel location along the y-axis taking into account radial distortion
ΔT	Translation change from C_o^1 to C_o^2
θ	Angle of change when the camera is moved from C_o^1 to C_o^2

List of symbols from Chapter 4: Colour and Hybrid Local Descriptors Methods.

a, b, c, d	Real numbers of quaternions
ang^{O_1}, ang^{O_2}	Opponent angles
ang^{S_1}, ang^{S_2}	Spherical angles
c_1, c_2, c_3	Channels of the $c_1c_2c_3$ colour model
C	Covariance matrix
d_{m_1}	Difference of two neighbouring pixels in a given orientation for the m_1 colour channel
d_x, d_y	Wavelet responses along the horizontal and vertical axes
D	Diagonal matrix of eigenvalues
$H(R, G, B)$	Hue image
i, j, k	Imaginary components of quaternions
$I(R, G, B)$	Intensity image
l_1, l_2, l_3	Channels of the $l_1l_2l_3$ colour model
L^2	Euclidean distance measure
m	Number of local descriptor pairs
m_1, m_2, m_3	Channels of the $m_1m_2m_3$ colour model
M	Maximum value for R, G and B
MNCC	Normalised cross-correlation
p	Number of pixels along the horizontal or vertical axis in an interest region
p^1, p^2	Number of pixels along the horizontal or vertical axis in an interest region for the reference and sensed images, respectively
q	Quaternion
r	Number of interest regions
$r(R, G, B),$ $g(R, G, B),$ $b(R, G, B)$	Normalised R, G and B
$\mathbf{r}^1, \mathbf{r}^2$	Interest regions of the reference and sensed images, respectively
$\overline{\mathbf{r}^1}, \overline{\mathbf{r}^2}$	Mean of the interest regions of the reference and sensed images, respectively
$\widehat{\mathbf{r}^2}$	Mean subtracted region from the sensed image
R, G, B	Red, green and blue pixels
$S(R, G, B)$	Saturation image
v	Size of the local descriptor concerned
$v_{m_1}, v_{m_2}, v_{m_3}$	Number of vectors for each of the three colour channels of the $m_1m_2m_3$ colour model
V	Eigenvectors

w_{L^2}, w_{MNCC}	Weights for the Euclidean distance measure and modified normalised cross-correlation values, respectively
$w_{m_1}, w_{m_2}, w_{m_3}$	Weights for the three colour channels of the $m_1m_2m_3$ colour model
$(x, y), (u, v)$	Indices for the pixels in an image
x_i^1, x_i^2	i^{th} vector of local descriptors from the reference image and sensed image, respectively
$\tilde{x}(m_1), \tilde{x}(m_2), \tilde{x}(m_3)$	Median for the three colour channels of the $m_1m_2m_3$ colour model
$\mathbf{x}_1, \mathbf{x}_2$	Image locations of two neighbouring pixels
\mathbf{X}	Input data matrix
Y	Greyscale pixel
δang^O	Opponent angle with error analysis
δang^S	Spherical angle with error analysis
λ_i	i^{th} eigenvalue
τ_{HLD}	Threshold for the hybrid local descriptor method
τ_{L^2}	Threshold for the threshold matching method using the Euclidean distance measure

List of symbols from Chapter 5: Local Descriptor Matching with Support Vector Machines.

b	Bias term for SVM
C, σ	Penalty and threshold for SVM with a Gaussian kernel
c_i	Square of the weights for RFE-SVMs
d	Order of polynomial for SVM with a polynomial kernel
$G(\mathbf{x}, c_i)$	Gaussian kernel
k	Number of iterations for cross-validation
L^1	Rectilinear distance measure
L^p	p -norm distance measure
n	Number of available classes for SVM
$n(\text{LD}^1), n(\text{LD}^2)$	Number of local descriptors in the reference and sensed images, respectively
$n(\text{LD}_{\text{correct}}), n(\text{LD}_{\text{incorrect}})$	Number of correctly and incorrectly matched local descriptor pairs, respectively
$n(\text{LD}_{\text{total}})$	Total number of local descriptors
p	Number of features to be removed at each iteration by RFE-SVMs
\mathbf{r}	Ranking vector for RFE-SVMs
\mathbf{s}	Indices of vectors of local descriptors to be emptied and ranked in \mathbf{r}

w_i	Weight of the i^{th} value in the difference vector for SVM with a Gaussian kernel
w_i	Weights of each input vector for RFE-SVMs
\mathbf{x}, \mathbf{x}_i	Input matrix for SVM and the i^{th} input vector
\mathbf{X}^0	Input matrix for RFE-SVMs
$\mathbf{y} \ y_i$	Output vector for SVM and the i^{th} output value
\Re^n	n -dimension real number
τ_T	Threshold for the threshold matching method
τ_{NN}	Threshold for the nearest neighbour method
τ_{NNR}	Threshold for the nearest neighbour ratio method
τ_{SVM}	Threshold for SVM output in the range of [-1, 1]