Upstream Financial Flows, Intangible Investment, and Allocative Efficiency

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Abstract

The recent decades have witnessed upstream financial flows and rising intangible investment. Given heterogeneous pledgeability, we find that financial inflows have opposite short-run and long-run effects on intangible-tangible investment composition and the efficiency of capital formation. By reducing the investment elasticity along the extensive margin, rising wealth inequality undermines the efficiency gains from financial inflows. Similarly, market frictions and policy distortions that hinder entrepreneurship may also reduce the investment elasticity. Thus, our mechanism offers a new perspective for understanding cross-country differences in intangible-tangible investment composition.

JEL Classification: E22, E25, F41

Keywords: financial frictions, heterogeneous pledgeability, intangible capital, investment elasticity, wealth inequality

1. Introduction

The world economy has witnessed two prominent phenomena in the recent decades. First, financial capital flows are "upstream" from poor to rich countries

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(Prasad et al., 2006; Ju and Wei, 2010), which is regarded as one of the causes for the declining interest rates in major advanced countries.[1] Second, many advanced countries are moving towards the knowledge-based economy where intangible capital (e.g., computerized information, patents and brands, and organizational capital) becomes increasingly important.[2] As shown in figure 1, intangible investment exceeds the tangibles in Finland, France, the Netherlands, Sweden, the United Kingdom and the United States between 2000 and 2013, while the opposite applies to other European countries (Corrado et al., 2018).

How would upstream financial flows and declining interest rates affect intangible-tangible investment composition and the efficiency of capital formation? We build an overlapping generations (OLG, hereafter) model and get three major findings.

- First, given heterogeneous pledgeability, tangible and intangible investments differ in the unit cost, which distorts within-firm investment composition.

[1] The literature features two major hypotheses for the global trends in interest rates. The global savings glut hypothesis (Bernanke, 2005) emphasizes the excessive supply of savings from fast-growing emerging markets, while the secular stagnation hypothesis (Summers, 2014) highlights the paucity of investment opportunities in advanced economies. Rachel and Smith (2017) and Crews et al. (2016) compare the empirical plausibility of the two hypotheses. See CEA (2015) for the literature survey on alternative causes. Upstream financial flows can be an equilibrium outcome, if rich countries are more financially developed than poor countries (Caballero et al., 2008; Mendoza et al., 2009; Wang et al., 2017; von Hagen and Zhang, 2014).

[2] Haskel and Westlake (2018) propose five reasons for the rise of intangible investment, including the rising costs of labor-intensive service, advances in information and social technologies, changing industrial structure, product and labor market deregulation, globalization and growing market sizes. Taxes and subsidies that affect firms expectations on future capital returns may also influence intangible investment. Taking these trends as given, we analyze how financial inflows affect intangible-tangible investment composition.

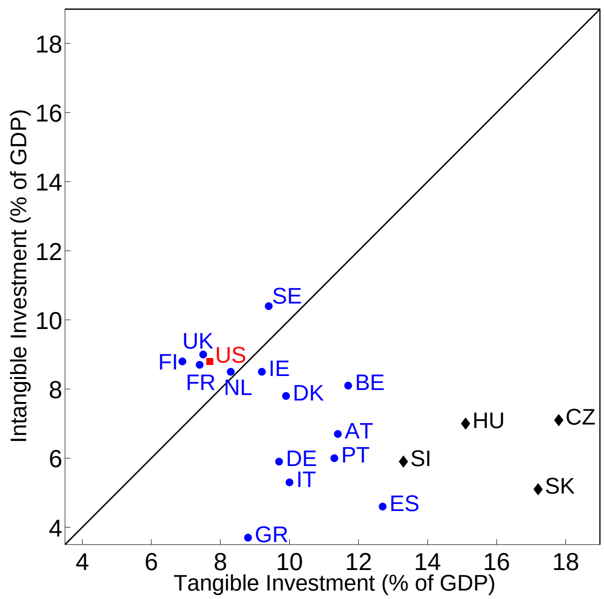

Figure 1: Intangible and Tangible Investment in the U.S. and Europe: 2000-2013

By affecting the unit-cost differential, financial inflows have opposite short-run and long-run effects on the efficiency of capital formation.

- Second, whether financial inflows improve allocative efficiency in the long run depends on the elasticity of aggregate investment demand. Rising wealth inequality tends to reduce the investment elasticity along the extensive margin, which undermines the efficiency gains from financial inflows.

- Third, financial integration may lead to multiple steady states in our model. If it occurs, changes in the world interest rate may have asymmetric and disproportionate effects on allocative efficiency and on the income level.

Our findings hinge upon three key assumptions. First, both tangible and intangible investments are essential for capital formation, while only the tangibles can be pledged as collateral. As entrepreneurs have to finance the intangibles fully with their own funds, the unit cost of intangible is constant at one. In contrast, the unit cost of tangibles is one minus the collateral value per unit of tangibles. The unit-cost differential induces entrepreneurs to invest inefficiently more (less) in the tangibles (intangibles), which distorts capital formation. Allocative efficiency is measured by the input-output ratio of capital formation. For simplicity, we call it the productivity.

Financial inflows affect the unit cost of tangibles in two ways. First, the inflows of cheap funds lower the domestic interest rate. Second, by augmenting aggregate investment, financial inflows lead to the fall in the marginal product of capital (MPK, hereafter) in the next period. The interest rate effect raises the collateral value and reduces the unit cost of tangibles in the current period, while the MPK effect works in the opposite direction.

Upon financial integration, the interest rate effect dominates the MPK effect so that the unit cost of tangibles falls immediately. It induces entrepreneurs to shift the investment towards the tangibles, which amplifies the initial distortion and reduces the productivity. Along the convergence path, the income growth raises the aggregate investment demand, which further attracts financial inflows and stimulates capital formation. As the domestic interest rate is aligned with the world rate, the interest rate effect is mute. The MPK effect raises the unit cost of tangibles, which induces entrepreneurs to shift investment towards the intangibles. Thus, the investment composition improves and the productivity rises over time. *By affecting the unit-cost differential, financial inflows lead to the immediate fall and the subsequent rises in productivity.* This is our first finding.

Can the productivity eventually exceed its initial level? Given the international interest rate differential, the more elastic the aggregate investment demand, the larger the financial inflows and the investment expansion, the stronger the MPK effect, the smaller the immediate fall and the larger the subsequent rises in productivity, the more likely the productivity eventually exceeds its initial level.

In order to characterize the investment elasticity analytically, we introduce two additional assumptions, i.e., agents differ in net wealth, and capital formation is subject to a minimum investment requirement (MIR, hereafter).[3] Given the binding borrowing constraints, only those with sufficiently high net wealth can meet the MIR and invest, while the others just lend out their net wealth. This way, the borrowing constraints and the MIR jointly act as a barrier to entrepreneurship, which endogenizes the mass of entrepreneurs. A fall in the interest rate and/or a rise in the current income allow more agents to meet the MIR and invest. The higher the wealth inequality, the less responsive the mass of entrepreneurs with respect to these changes, the less elastic the aggregate investment demand along the extensive margin, the less likely the productivity exceeds its initial level in the long run. This is our second finding.

We derive the first two findings in the scenario where financial inflows *marginally* reduce the domestic interest rate and the economy converges to a new steady state *in the neighborhood of the initial one*. By moving from the marginal to the global analysis, we then study the model dynamics over the entire state space. As shown in Matsuyama (2004), the extensive margin of investment is a channel through which financial integration amplifies income changes; if the extensive margin effect is strong enough, multiple steady states arise.[4] By endogenizing the produc-

[3]Zhang (2017) discusses the literature on the implications of the MIR versus fixed cost.

[4]A rise in the current income allows more agents to be entrepreneurs, which raises (reduces) the credit demand (supply) along the extensive margin. Under autarky, the interest rate is an en-

tivity in Matsuyama's framework, we show that *the impacts of financial inflows on allocative efficiency depend on the initial income level in the host country, the world interest rate changes may have asymmetric effects, and a large change in the world interest rate may have disproportionately larger impacts than a moderate interest rate change*. This is our third finding. It suggests that a country should seriously consider the structural characteristics of its economy and the dynamic patterns of world interest rates, when it plans to liberalize its capital account or to adopt capital flow management policies (IMF, 2012, 2020).

Related Literature The literature shows that financial integration may improve allocative efficiency via the cross-firm and/or cross-project resource allocation (Obstfeld, 1994; Acemoglu and Zilibotti, 1997; Varela, 2018; Alessandria and Qian, 2005). We propose the *within-firm*, intangible-tangible investment composition as a novel channel through which financial inflows have opposite short-run and long-run efficiency effects.

Although the literature proposes various channels through which financial integration fosters productivity growth, the empirical evidence is mixed (Kose et al., 2009a). Bonfiglioli (2008) and Bekaert et al. (2011) find the positive productivity effect of financial integration. Kose et al. (2009b) argue that debt flows are neg-

dogenous stabilizer and it rises to eliminate the excess credit demand. Thus, aggregate investment is financed entirely by domestic saving, and the diminishing MPK dampens the income change. Under financial integration, the domestic interest rate is aligned with the world rate and it does not act as the endogenous stabilizer. Given a rise in the current income, the excess credit demand is covered by financial inflows so that aggregate investment exceeds domestic saving. If the extensive margin effect is strong enough, financial inflows are so large that the investment expansion dominates the diminishing MPK and the future income rises more than proportionately. In this case, the initial steady state becomes unstable and multiple steady states arise.

atively correlated with TFP growth, while this negative relationship is partially attenuated in the economies with better financial markets and institutional quality. Kose et al. (2011) show that an economy needs to attain certain "threshold" levels of institutional quality and financial development so as to gain from financial integration.

In our model, debt flows trigger the *non-monotonic productivity dynamics*, while the long-run effect depends on the investment elasticity. Besides the MIR and the borrowing constraints, other market frictions and policy distortions that hinder entrepreneurship would dampen the growth of incumbent firms (Klapper et al., 2006; Gutierrez et al., 2021) and reduce the investment elasticity, while innovations and reforms that mitigate these frictions and distortions may raise the investment elasticity, stimulate intangible investment and enhance the productivity gains from financial inflows. Consistent with the spirit of Kose et al. (2009b) and Kose et al. (2011), our findings suggest that identifying the institutional factors relevant for the investment elasticity may help improve the empirical estimates on the productivity effects of financial flows.

Heterogeneous pledgeability between tangible and intangible assets is well documented in the literature. Almeida and Campello (2007) find that firms with more tangible assets are less likely to be financially constrained. Bates et al. (2009) show that high-tech industries have to use internal funds to finance their R&D, and Gatchev et al. (2009) show that marketing expenses and product development are financed mostly out of retained earnings and equity. Falato et al. (forthcoming) show that only 3% of U.S. secured syndicated loans use patents or brands as collateral. Benmelech et al. (2020) find that U.S. non-financial firms have witnessed a secular decline of secured debts, and Dell'Ariccia et al. (2021)

estimate that rising intangible capital explains around 30% of the secular decline in the share of U.S. commercial lending in banks' loan portfolios. The literature has explored the implications of rising intangible capital in the closed-economy setting (Giglio and Severo, 2012; Lopez and Moppett, 2018; Wang, 2017), while we study this issue in the open-economy setting.

Antunes and Cavalcanti (2013) and Jaumotte et al. (2013) show that financial globalization affects wealth inequality and welfare, while the impacts of inequality on the consequences of financial globalization have not been well studied in the literature. We propose a novel mechanism through which rising wealth inequality reduces the investment elasticity, dampens financial inflows, and worsens allocative efficiency.

The rest of the paper is structured as follows. Section 2 sets up the model and section 3 analyzes the autarkic equilibrium. Section 4 explores the productivity dynamics under financial integration, and section 5 studies the implications of multiple steady states. Section 6 discusses two scenarios of world interest rate hikes. Section 7 concludes. Appendices include some robustness checks and model extensions.

2. The Model Setting

Our model features three key assumptions in the two-period OLG framework, i.e., heterogeneous pledgeability between tangible and intangible investment, the MIR, and wealth inequality. Each serves a particular purpose as explained below.

Consider country N in the world economy. Every period, a continuum of agents are born and they live for two periods. The population size of each generation is normalized at unity. Agents are endowed with labor when young and they only consume when old. When young, agent $j \in [0,1]$ supplies its labor endowment $l_j = (1-\theta)\varepsilon_j$ to the market, where $\varepsilon_j \in (1,\infty)$ follows the Pareto distribution.[5] Let $G(\varepsilon_j) = 1 - \varepsilon_j^{-\frac{1}{\theta}}$ denote the cumulative distribution function, where $\theta \in (0,1)$ denotes the inverse of the shape parameter. The aggregate labor supply is constant at unity, $L \equiv \int_1^\infty l_j dG(\varepsilon_j) = 1$.

In period t, labor and capital are hired for the production of final goods, and capital fully depreciates during production. The final good is used for consumption and investment. The markets for final good, capital, and labor are competitive. Y_t denotes aggregate output, $L = 1$ and K_t denote the aggregate inputs of labor and capital, w_t and q_t denote the wage rate and the rental price of capital.

$$Y_t = \left(\frac{K_t}{\alpha}\right)^{\alpha}\left(\frac{L}{1-\alpha}\right)^{1-\alpha}, \quad q_t K_t = \alpha Y_t \quad \text{and} \quad w_t L = (1-\alpha)Y_t. \tag{1}$$

By investing $m_{j,t}$ units of final goods in period t, agent j gets $k_{j,T,t+1}$ units of tangible capital and $k_{j,I,t+1}$ units of intangible capital which jointly offer $k_{j,t+1} = \left(\frac{k_{j,I,t+1}}{\eta}\right)^{\eta}\left(\frac{k_{j,T,t+1}}{1-\eta}\right)^{1-\eta}$ units of capital service in period $t+1$,[6] if the project meets the MIR, $m_{j,t} = k_{j,I,t+1} + k_{j,T,t+1} \geq \mathfrak{m}$. Otherwise, the project has zero output.

If the project's rate of return exceeds the interest rate r_t, the agent prefers to finance the project with loans. In the case of default, intangible capital is lost, while lenders can seize tangible capital in period $t+1$. After deducting the liquidation

[5]Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson et al., 2011; Gabaix, 2009). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003).

[6]Giglio and Severo (2012) and McGrattan (2020) use the Cobb-Douglas form for capital formation, while Caggese and Pérez-Orive (2022) assume that tangible and intangible capital are perfect complements. Appendix A.1 shows that our results hold if capital formation takes the CES form (Falato et al., forthcoming).

costs, the lenders get $\lambda p_{t+1}k_{j,T,t+1}$, where p_{t+1} denotes the price of tangible capital and $\lambda \in (0,1]$ measures the level of financial development. In equilibrium, the agent borrows up to the pledgeable value and uses its net wealth (i.e., the labor income) to cover the gap,

$$b_{j,t} \geq \frac{\lambda p_{t+1}k_{j,T,t+1}}{r_t}, \text{ and } k_{j,I,t+1}+k_{j,T,t+1}-b_{j,t} \leq n_{j,t} = w_t l_j. \tag{2}$$

The following subsections explain the roles of the three key assumptions.

2.1. Heterogeneous Pledgeability and Investment Distortion

As intangible capital cannot serve as collateral, the agent has to use its own funds to finance it. The unit cost of intangibles is $u_{I,t}=1$, while the unit return is the marginal revenue of intangibles. As the agent can borrow up to $\frac{\lambda p_{t+1}}{r_t}$ per unit of tangibles, the unit cost of tangibles is $u_{T,t}=1-\frac{\lambda p_{t+1}}{r_t}$ and the unit return is the marginal revenue minus the debt payment per unit of tangibles. The agent equalizes the internal rate of return on the two types of investment,[7] which gives

$$u_{T,t}=1-\frac{\lambda p_{t+1}}{r_t}=\frac{a_{j,t}}{1-a_{j,t}}\frac{1-\eta}{\eta}\left[1-\frac{\lambda p_{t+1}}{q_{t+1}}\left(\frac{a_{j,t}}{1-a_{j,t}}\frac{1-\eta}{\eta}\right)^{-\eta}\right], \tag{3}$$

where $a_{j,t} \equiv \frac{k_{j,I,t+1}}{m_{j,t}}$ denotes the agent's intangible fraction of investment. Equation (3) specifies $a_{j,t}$ as a function of parameters $\{\lambda,\eta\}$ and prices $\{r_t, p_{t+1}, q_{t+1}\}$. It implies that the agents who meet the MIR choose the same value for $a_{j,t}$. Thus, we drop subscript j and use a_t instead. Rewrite the capital formation function as

$$k_{j,t+1}=\Phi_t m_{j,t}, \quad \text{where} \quad \Phi_t \equiv \left(\frac{a_t}{\eta}\right)^{\eta}\left(\frac{1-a_t}{1-\eta}\right)^{1-\eta} \tag{4}$$

[7] See the proof of Proposition 1 in the online appendix for details. Appendix A.2 confirms our findings in the setting where the intangibles also serve as collateral, but with a lower pledgeability.

denotes the input-output ratio of capital formation. Φ_t reflects allocative efficiency and, for simplicity, we call it the productivity. For $a_t = \eta$, the investment composition is efficient and Φ_t is maximized at unity. For $a_t < \eta$, $\Phi_t < 1$ and $\frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1 - a_t} > 0$; the higher the a_t, the smaller the investment distortion, the higher the productivity.

The agent equalizes its marginal revenue of tangibles to the market price,

$$q_{t+1} \frac{\partial k_{j,t+1}}{\partial k_{j,T,t+1}} = q_{t+1} \Phi_t \frac{1-\eta}{1-a_t} = p_{t+1}. \tag{5}$$

As long as the project's rate of return exceeds the interest rate $q_{t+1}\Phi_t > r_t$, the agent borrows to the limit. Combine equations (3)-(5) to get

$$u_{T,t} = 1 - \frac{1-\eta}{1-a_t} \frac{\lambda}{\psi_t} = \frac{a_t}{1-a_t} \frac{1-\eta}{\eta} (1-\lambda), \text{ where } \psi_t \equiv \frac{r_t}{q_{t+1}\Phi_t} < 1 \tag{6}$$

is the normalized interest rate. Given the unit-cost differential $u_{I,t} - u_{T,t} = 1 - u_{T,t} > 0$, the agent invests inefficiently more (less) in the tangibles (intangibles), $a_t < \eta$. Given λ, the lower the ψ_t, the higher the pledgeable value of tangibles $\frac{1-\eta}{1-a_t} \frac{\lambda}{\psi_t}$, the lower the unit cost of tangibles $u_{T,t}$, the larger the unit-cost differential, the larger the investment distortion, the lower the a_t and the Φ_t.

Remark 1: by creating the unit-cost differential, heterogeneous pledgeability leads to the investment distortion. The intangible fraction of investment moves positively with the normalized interest rate, reflecting the agent's optimal decision.

2.2. MIR and Borrowing Constraints: the Barrier to Entrepreneurship

Given $\psi_t < 1$, the borrowing constraints are binding. Define the unit cost of investment as the weighted average of the two unit costs. Use equation (6) to get

$$u_t \equiv a_t u_{I,t} + (1-a_t) u_{T,t} = 1 - \frac{\lambda(1-\eta)}{\psi_t} = \frac{a_t}{\eta} [1 - \lambda(1-\eta)]. \tag{7}$$

Let $\underline{\varepsilon}_t$ denote the cutoff value associated with the agents who just meet the MIR,

$$\frac{w_t(1-\theta)\underline{\varepsilon}_t}{u_t} = \mathfrak{m}, \Rightarrow \underline{\varepsilon}_t = \frac{u_t}{w_t}\frac{\mathfrak{m}}{1-\theta}. \tag{8}$$

Those with $\varepsilon_j \geq \underline{\varepsilon}_t$ can meet the MIR and are called *entrepreneurs*, with the mass $\tau_t = \underline{\varepsilon}_t^{-\frac{1}{\theta}}$. When young, they put the labor income in the project and borrow to the limit; when old, they get the project revenue, repay the debt, and consume the rest, $c^e_{j,t+1}$. Those with $\varepsilon_j \in [1, \underline{\varepsilon}_t)$ cannot meet the MIR and are called *households*. They lend out the labor income when young and consume the savings return when old, $c^h_{j,t+1}$.

$$\tau_t = \underline{\varepsilon}_t^{-\frac{1}{\theta}}, \;\; n_{j,t} = w_t l_j, \;\; c^e_{j,t+1} = n_{j,t}\left(\frac{q_{t+1}\Phi_t - r_t}{u_t} + r_t\right), \;\; c^h_{j,t+1} = n_{j,t} r_t. \tag{9}$$

Remark 2: the MIR and the borrowing constraints act as a barrier to entrepreneurship, which endogenizes the mass of entrepreneurs.

2.3. Wealth Inequality and Elasticities of Aggregate Investment Demand

Given the inelastic labor supply,[8] the wage rate is proportional to aggregate income and determined by the investment made one period earlier, $w_t = \frac{(1-\alpha)Y_t}{L} = \left(\frac{1-\alpha}{\alpha}\frac{K_t}{L}\right)^{\alpha}$. We use the wage rate as a proxy for the income level in our analysis.

Given the binding borrowing constraints, the aggregate investment demand is

$$M_t \equiv \int_{\underline{\varepsilon}_t}^{\infty} m_{j,t} dG(\varepsilon_j) = \frac{\delta_t w_t L}{u_t}, \text{where } \delta_t \equiv \frac{\int_{\underline{\varepsilon}_t}^{\infty} w_t l_j dG(\varepsilon_j)}{w_t L} = \underline{\varepsilon}_t^{-\frac{1-\theta}{\theta}} = \tau_t^{1-\theta} \tag{10}$$

denotes the net wealth share of entrepreneurs. A rise in the current income raises the wage rate and individual net wealth, and a fall in the unit cost of investment

[8] Appendix A.5 analyzes the case of the endogenous labor supply. If the individual labor supply responds positively to the wage rate, our mechanism is reinforced and our findings hold.

raises the leverage multiplier. In either case, entrepreneurs borrow and invest more, while more agents meet the MIR and become entrepreneurs. The former is the intensive margin effect and the latter is the extensive margin effect. Use equations (8)-(10) to derive the *partial elasticities* of M_t with respect to the income level and the unit cost of investment,

$$\frac{\partial \ln M_t}{\partial \ln w_t} = \underbrace{\frac{\partial \ln w_t L}{\partial \ln w_t}}_{\text{Intensive margin effect } = 1} + \underbrace{\frac{\partial \ln \delta_t}{\partial \ln \tau_t}\frac{\partial \ln \tau_t}{\partial \ln w_t}}_{\text{Extensive margin effect } = \frac{1}{\theta} - 1} = \frac{1}{\theta}, \tag{11}$$

$$\frac{\partial \ln M_t}{\partial \ln u_t} = \underbrace{\frac{\partial \ln \frac{1}{u_t}}{\partial \ln u_t}}_{\text{Intensive margin effect } = -1} + \underbrace{\frac{\partial \ln \delta_t}{\partial \ln \tau_t}\frac{\partial \ln \tau_t}{\partial \ln u_t}}_{\text{Extensive margin effect } = -(\frac{1}{\theta} - 1)} = -\frac{1}{\theta}. \tag{12}$$

For $\theta \in (0,1)$, the partial elasticities (in absolute value) exceed unity, due to the extensive margin effect. The larger the θ, the higher the wealth inequality, the less elastic the mass of entrepreneurs $\frac{\partial \ln \tau_t}{\partial \ln w_t} = -\frac{\partial \ln \tau_t}{\partial \ln u_t} = \frac{1}{\theta}$, the weaker the extensive margin effect $\frac{\partial \ln \delta_t}{\partial \ln w_t} = -\frac{\partial \ln \delta_t}{\partial \ln u_t} = \frac{1}{\theta} - 1$, the less elastic the investment demand.[9]

Remark 3: given the binding borrowing constraints, the partial elasticities of the aggregate investment demand are inversely related to wealth inequality.

[9] We distinguish explicitly the aggregate investment demand and aggregate investment. Equation (10) specifies the *aggregate investment demand* as a log-linear, increasing (decreasing) function of w_t (u_t), independent of whether country N is under autarky or under financial integration. *Aggregate investment* refers to the equilibrium value determined jointly by the aggregate investment demand and the total funds available for country N, which differs fundamentally under autarky versus under financial integration.

2.4. Market Equilibrium

Under autarky, the markets for credit and capital clear domestically,[10]

$$\int_{\underline{\varepsilon}_t}^{\infty}(m_{j,t}-n_{j,t})dG(\varepsilon_j)=\int_{1}^{\underline{\varepsilon}_t}n_{j,t}dG(\varepsilon_j),\ \Rightarrow\ M_t=w_tL, \tag{13}$$

$$K_{t+1}=\int_{\underline{\varepsilon}_t}^{\infty}\Phi_{j,t}m_{j,t}dG(\varepsilon_j)=\Phi_tM_t. \tag{14}$$

So far, we have focused on the case of $q_{t+1}\Phi_t>r_t$ where the borrowing constraints are binding. If $q_{t+1}\Phi_t=r_t$, the agents who meet the MIR do not have strong incentives to borrow to the limit. In equilibrium, they choose $a_{j,t}=\eta$ and the productivity is constant at $\Phi_t=1$. Under autarky, aggregate investment is financed by domestic saving.

Definition 1. *Under autarky, the market equilibrium in country N is a set of individual choices, $\{n_{j,t},m_{j,t},a_t,\Phi_t,c^e_{j,t},c^h_{j,t},u_t\}$, the threshold value $\{\underline{\varepsilon}_t\}$, aggregate quantities $\{Y_t,K_t,M_t\}$, and the prices $\{q_t,w_t,r_t\}$ satisfying equations* (1)*,* (5)*,* (7)*-*(9)*, and* (13)*-*(14)*.*

Country N and the rest of the world are inherently identical, except that the former is more financially developed and its population share in the world is negligible. Under financial integration, the interest rate in country N is aligned with the world rate $r_t=r^*$, and the domestic investment-savings gap is covered by financial flows.

Definition 2. *Under financial integration, the market equilibrium in country N is a set of individual choices, $\{n_{j,t},m_{j,t},a_t,\Phi_t,c^e_{j,t},c^h_{j,t},u_t\}$, the threshold value $\{\underline{\varepsilon}_t\}$,*

[10]The no-default equilibrium does not incur any liquidation costs. According to the Walras' law, the final good market clears, $M_t+\int_{\underline{\varepsilon}_t}^{\infty}c^e_{j,t}dG(\varepsilon_j)+\int_1^{\underline{\varepsilon}_t}c^h_{j,t}dG(\varepsilon_j)=Y_t$.

aggregate quantities $\{Y_t, K_t, M_t\}$*, and the prices* $\{q_t, w_t\}$ *satisfying equations* (1), (5), (7)-(9)*, and* (14)*, given* $r_t = r^*$.

M_t units of funds invested in period t yields $K_{t+1} = \Phi_t M_t$ units of capital service in period $t+1$. The social rate of return is $\frac{q_{t+1}K_{t+1}}{M_t} = q_{t+1}\Phi_t$.

In the next sections, we use the law of motion for wage to analyze the model dynamics and the steady-state properties. Combine equations (14) and (1) to get

$$w_{t+1} = \left(\frac{K_{t+1}}{L\rho}\right)^{\alpha} = \left(\frac{M_t\Phi_t}{L\rho}\right)^{\alpha}, \text{ where } \rho \equiv \frac{\alpha}{1-\alpha}, \tag{15}$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = (\underbrace{\frac{\partial \ln M_t}{\partial \ln w_t}}_{\text{Investment size effect}} + \underbrace{\frac{\partial \ln \Phi_t}{\partial \ln w_t}}_{\text{Productivity effect}})[1 - \underbrace{(1-\alpha)}_{\text{Neoclassical effect}}]. \tag{16}$$

A rise in the current income affects the future income in three ways. First, it raises the size of aggregate investment $\frac{\partial \ln M_t}{\partial \ln w_t}$. Second, it may improve the intangible-tangible investment composition and raise the productivity $\frac{\partial \ln \Phi_t}{\partial \ln w_t}$. Both effects stimulate capital formation. Third, given the *neoclassical* production function, capital formation affects the future income *less than proportionately*, $\frac{\partial \ln w_{t+1}}{\partial \ln K_{t+1}} = 1-(1-\alpha) < 1$. As shown later, the steady-state properties depend on the relative size of the three effects.

3. The Autarkic Equilibrium

Under autarky, aggregate investment is funded by domestic saving $M_t = S_t = w_t L$ so that the investment size effect is constant at one. If the productivity effect is dominated by the neoclassical effect at any autarkic steady state, the law of motion for wage has a slope less than unity there, $\frac{\partial \ln w_{t+1}}{\partial \ln w_t}|_{w_A} < 1$, so that the autarkic steady state is stable and unique. Thus, we first explore the determinants of the productivity effect.

For $w_t \geq \bar{w}_A \equiv [1-\lambda(1-\eta)]^{\frac{1}{1-\theta}} \frac{m}{1-\theta}$, the income level is sufficiently high and so is the mass of entrepreneurs. As a result, the domestic credit demand aligns the interest rate with the social rate of return and the borrowing constraints are slack. In this case, the investment allocation is efficient, $a_t = \eta$ and $\Phi_t = 1$, so that the productivity effect is mute $\frac{\partial \ln \Phi_t}{\partial \ln w_t} = 0$. Then, the neoclassical effect is the convergence force that drives country N towards a steady state.

For $w_t < \bar{w}_A$, the mass of entrepreneurs is inefficiently low and so is the domestic credit demand. As a result, the interest rate is below the social rate of return, $r_t < q_{t+1}\Phi_t$. In this case, the borrowing constraints are binding, and the unit-cost differential distorts the investment composition, $a_t < \eta$ and $\Phi_F < 1$. We use the domestic investment-saving diagram to show the productivity effect intuitively.

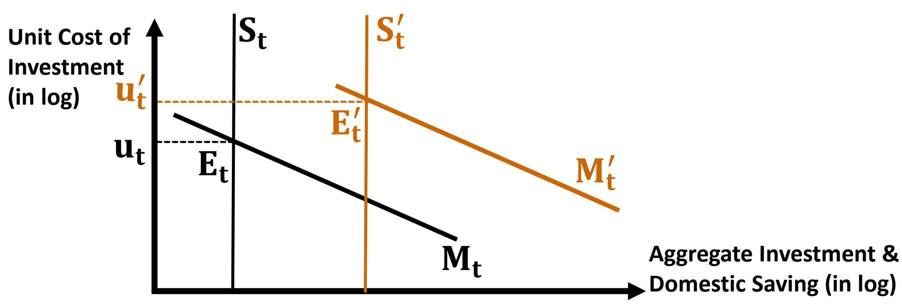

Figure 2: Aggregate Investment-Saving Balance: A Rise in the Current Income

According to equations (12) and (7), the aggregate investment demand M_t is log-linear in u_t and decreases with the normalized interest rate ψ_t. The domestic saving $S_t = w_t L$ is inelastic to u_t and ψ_t. Given the current income, point E_t in figure 2 denotes the domestic investment-saving balance. A rise in the current income shifts the domestic saving rightwards proportionately to S_t', while it shifts the aggregate investment demand rightwards more than proportionately to M_t', $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta} > \frac{\partial \ln S_t}{\partial \ln w_t} = 1$, due to the extensive margin effect. The unit cost of investment rises to restore the domestic investment-saving balance. Equations (17) shows that the endogenous variables rise with the income level,[11] and equation (18) specifies the productivity effect.

$$\tau_t = \frac{w_t(1-\theta)}{\mathfrak{m}},\ u_t = \delta_t = \tau_t^{1-\theta},\ \psi_t = \frac{\lambda(1-\eta)}{1-\delta_t},\ \text{and}\ a_t = \frac{u_t\eta}{1-\lambda(1-\eta)}, \tag{17}$$

$$\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \frac{\partial \ln \Phi_t}{\partial \ln a_t}\frac{\partial \ln a_t}{\partial \ln w_t} = \left(1 - \frac{1-\eta}{1-a_t}\right)(1-\theta). \tag{18}$$

Given the binding borrowing constraints, $a_t < \eta$. *The income rise triggers a positive productivity effect via the extensive margin of investment.*[12] Given the investment size effect constant at one, the steady-state properties depend on the relative size of the neoclassical effect and the productivity effect. The former is negatively related to the capital share (α), while the latter is negatively related to the degree of wealth inequality (θ) and the intangible fraction of investment (a_t).

Proposition 1. *Let* $\underline{\theta} \equiv \max\{1 - \frac{\frac{1}{\alpha}-1}{\eta}, 0\} < \alpha$, $Z \equiv \frac{1-\theta}{\rho^{\rho}\mathfrak{m}}$, *and* $\tilde{\lambda}_A \equiv \frac{1-Z^{1-\theta}}{1-\eta}$.

For $\theta \in [\underline{\theta}, 1)$*, there exists a unique, stable steady state under autarky.*

Given $\theta \in [\underline{\theta}, 1)$ *and* $\lambda < \min\{\tilde{\lambda}_A, 1\}$*, the borrowing constraints are binding, the interest rate is below the social rate of return, the intangible fraction of investment is inefficiently low in the autarkic steady state, i.e.,* $\psi_A < 1$*,* $a_A < \eta$ *and* $\Phi_A < 1$*, where* X_A *denotes the steady-state value of variable* X_t *under autarky.*

[11]The income rise allows more agents to become entrepreneurs, which raises (reduces) the domestic credit demand (supply) along the *extensive margin*. The normalized interest rate ψ_t rises to clear the credit market.

[12]The online appendix E analyzes a model with the exogenous mass of entrepreneurs. Under autarky, the income change affects domestic saving and investment demand in equal proportions, $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{\partial \ln S_t}{\partial \ln w_t} = 1$. Thus, u_t stays put and so do ψ_t and a_t. In that case, the productivity effect is mute $\frac{\partial \ln \Phi_t}{\partial \ln w_t} = 0$.

The intuitions behind proposition 1 are as follows. For a given income change, the higher the wealth inequality (θ), the smaller the extensive margin effect, the less elastic the aggregate investment demand, the smaller the response of the normalized interest rate and the change in the unit-cost differential, the smaller the productivity effect, the more likely it is dominated by the neoclassical effect so that there is a unique, stable steady state under autarky.[13]

The lower λ, the lower the domestic credit demand, the lower the normalized interest rate, the larger the unit-cost differential and investment distortion, the lower the a_t and the Φ_t, the lower the income level in the steady state.

Assumption 1. $\theta \in [\underline{\theta}, 1)$, $0 < \lambda^* < \lambda < \min\{\tilde{\lambda}_A, 1\}$, *and* $\frac{L}{L+L^*} \to 0$.

Under assumption 1, there is a unique, autarkic steady state where the borrowing constraints are binding in country N and in the rest of the world. Compared to the rest of the world, country N has a higher interest rate, a higher intangible-tangible investment ratio, a higher productivity, and a higher income per capita in the autarkic steady state .

4. Productivity Dynamics under Financial Integration

Country N is initially in the steady state under autarky, with $r_A > r^*$. From period 0 on, agents can freely borrow and lend abroad, and country N eventually converges to a new steady state.[14]

[13]The lower the capital share α, the less the income change gets amplified $\frac{\partial \ln Y_{t+1}}{\partial \ln K_{t+1}} = \alpha$, the stronger the neoclassical effect, the more likely it dominates the productivity effect so that $\theta > \underline{\theta}$ holds. Figure F.11 in the online appendix shows the threshold conditions.

[14]As shown in section 5 and in the online appendix B, financial integration may lead to multiple steady states. If so, country N still converges *along a unique path* from the autarkic to the new

4.1. Immediate Impacts in Period 0

Given $r_A > r^*$, country N receives financial inflows, which aligns the interest rate with the world rate. By augmenting aggregate investment, financial inflows trigger a fall in the MPK in the next period. Whether the normalized interest rate rises or falls depends on the relative size of the interest rate effect and the MPK effect. We use the domestic investment-saving diagram to show their net effect.

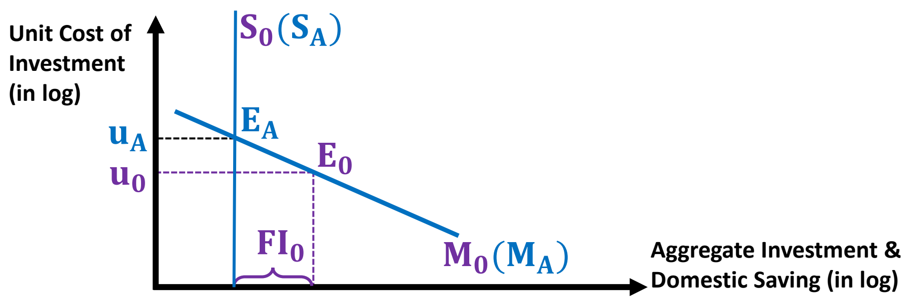

Figure 3: Financial Inflows and Domestic Investment-Saving Imbalance in Period 0

In figure 3, point E_A shows the domestic investment-saving balance in the autarkic steady state. As the current income is determined by aggregate investment made in the previous period, $w_0 = w_A$ holds. Thus, the aggregate investment demand curve and the domestic saving curve stay put in period 0. Given $\theta > 0$, the M_0 curve is downward sloping. In order to justify financial inflows, the unit cost of investment falls $u_0 < u_A$. According to equation (7), the normalized interest rate must also fall, $\psi_0 < \psi_A$, implying that the interest rate effect dominates the

steady state so that our findings still hold. Our findings also hold in the scenario where country N is initially in a stable steady state under financial integration, with the binding borrowing constraints; from period 0 on, the world interest rate declines and country N converges to a new steady state.

MPK effect.[15]

$$\frac{\partial \ln \psi_0}{\partial \ln r_0} = \underbrace{\frac{\partial \ln r_0}{\partial \ln r_0}}_{\text{Interest rate effect (=1)}} - \underbrace{\frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0}}_{\text{MPK effect}} = \frac{1}{1 + \frac{1-u_0}{u_0}\left(\frac{1-\alpha}{\theta} + \alpha\frac{\eta - a_0}{1-a_0}\right)} > 0. \quad (19)$$

The fall in ψ_0 leads to the fall in a_0, which amplifies the initial investment distortion and lowers the productivity. Given the interest rate differential, the larger the wealth inequality θ, the smaller the extensive margin effect and the investment elasticity, the steeper the M_0, the smaller the financial inflows and investment expansion, the smaller the MPK effect, the larger the falls in ψ_0 and in the productivity Φ_0. This way, *the investment elasticity is key to the size of financial inflows and the magnitude of productivity loss in period 0.*

4.2. Subsequent Impacts along the Convergence Path

In period 0, financial inflows raise the size but worsen the composition of aggregate investment. As the former dominates the latter, financial inflows stimulate capital formation, which raises the income level in period 1. As shown in figure 2, the income rise shifts the saving curve rightwards proportionately, while it shifts the investment demand curve rightwards more than proportionately, due to the extensive margin effect. The rising gap between the aggregate investment demand and the domestic saving attracts more financial inflows in period 1, which further stimulates capital formation and lowers the MPK in period 2.

$$\frac{\partial \ln \psi_t}{\partial \ln w_t} = \underbrace{\frac{\partial \ln r_t}{\partial \ln w_t}}_{\text{Interest rate effect (=0)}} - \underbrace{\frac{\partial \ln q_{t+1} \Phi_t}{\partial \ln w_t}}_{\text{MPK effect (-)}} = \frac{\frac{u_t}{1-u_t}}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t} + \alpha\frac{\eta - a_t}{1-a_t}\right) + 1} > 0. \quad (20)$$

[15] Given that the interest rate falls in period 0, $\frac{\partial \ln x_0}{\partial \ln r_0} > 0$ implies that variable x_t falls in period 0.

As the interest rate is aligned with the world rate $r_t = r^*$, the interest rate effect is mute. The fall in the MPK leads to a higher normalized interest rate, which triggers the investment shift towards the intangibles and improves allocative efficiency. This process goes on until country N reaches a new steady state.

For a given income rise, the larger the wealth inequality θ, the smaller the extensive margin effect and the investment elasticity, the smaller the rightward shift of M_t, the smaller the rise in the investment-saving gap, the smaller the financial inflows and investment expansions, the smaller the MPK effect, the smaller the rises in ψ_t and in the productivity Φ_t. Again, *the investment elasticity is key to the size of financial inflows and the magnitude of productivity gains over time.*

Proposition 2. *The productivity falls upon financial inflows, $\Phi_0 < \Phi_A$, while it rises over time, $\Phi_{t+1} > \Phi_t$. If the aggregate investment elasticity exceeds a threshold value $\frac{1}{\theta} > \frac{1}{\alpha}$, the productivity eventually exceeds its initial level, $\Phi_F > \Phi_A$.*

Proposition 2 shows that the long-run productivity effect of financial inflows depends on the relative size of θ and α. The intuition is as follows. Start from the autarkic steady state. For a given decline in the interest rate,

- the smaller the wealth inequality θ, the more elastic the aggregate investment demand, the larger the financial inflows and the MPK effect, the more likely the productivity exceeds its initial level;

- the larger the capital share α, the weaker the neoclassical effect, the larger the contribution of capital formation to income growth $\frac{\partial \ln Y_t}{\partial \ln K_t} = \alpha$, the higher the long-run income level, the larger the rise in the mass of entrepreneurs $\tau_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1}{\theta}}$ and the investment expansion along the extensive margin

$\delta_t = \tau_t^{1-\theta}$, the larger the financial inflows and the MPK effect, the more likely the productivity exceeds its initial level.

Figure 4 shows the productivity dynamics in three cases,[16] with the time $t = 0, 1, 2, \ldots$ on the horizontal axis. It confirms the predictions of proposition 2.

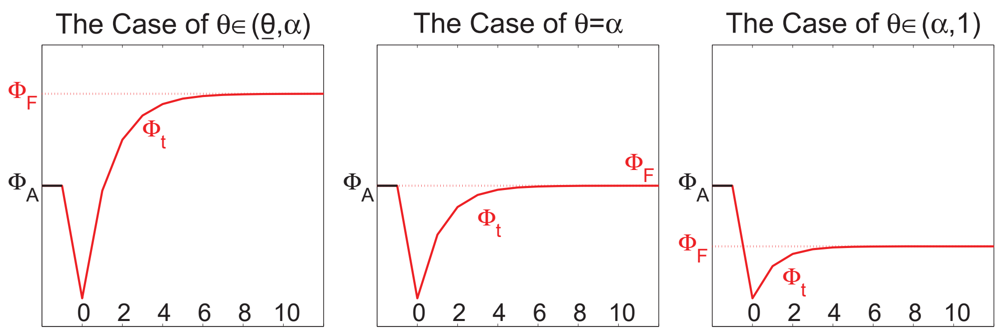

Figure 4: Financial Integration and the Productivity Dynamics

Our findings suggest that a country may witness the productivity gains from debt inflows, if *the elasticity of aggregate investment demand exceeds a threshold value.* Besides financial frictions and the MIR, other market frictions and policy distortions that hinder entrepreneurship may also reduce the investment elasticity and undermine the productivity gains from financial inflows. These findings call for empirical research on the determinants of investment elasticity and the related threshold conditions for productivity gains.

Appendix A checks the robustness of our findings under alternative settings. See our working paper version (Zhang, 2021) for more details.

Our model features the extensive margin of aggregate investment as a novel channel through which financial inflows affect the productivity. For comparison

[16]The steady-state value of Φ_t is a function of θ. For a better visibility, we re-scale the vertical axes of figure 4. Thus, the levels of Φ_t in the three cases should not be compared literally.

purpose, the online appendix E analyzes a model with the *exogenous* mass of entrepreneurs. Similar as predicted by our model, the productivity Φ_t falls upon financial inflows, while it rises over time. However, as the extensive margin effect is absent, financial inflows and investment expansions are damped so that the cumulative MPK effects are dominated by the interest rate effect. Thus, the normalized interest rate is strictly lower than its initial level and so is the productivity.

In our model, the labor income is *the only source* of individual net wealth relevant for aggregate investment, $n_{j,t} = l_j w_t$, and the dispersion of individual net wealth closely resembles that of labor endowment which is exogenous by assumption. One can embed our core mechanism into a continuous-time, perpetual youth framework (Blanchard, 1985) where agents can accumulate wealth over a longer time horizon. Given $q_{t+1}\Phi_t > r_t$, the borrowing constraints are binding and, due to the leverage effect, entrepreneurs earn a higher rate of return than households. The rate-of-return differential allows entrepreneurs to accumulate wealth at a rate faster than households, which endogenizes the wealth distribution. By widening the rate-of-return differential, financial inflows raise the entrepreneurial wealth share and amplify the wealth inequality in the next period. It further stimulates financial inflows in the next period and makes the productivity gains more likely. Thus, the relationship between wealth inequality and the productivity effect may depend on the way we model the wealth distribution.

Given $q_{t+1}\Phi_t > r_t$, entrepreneurs earn a higher rate of return on their net wealth than households, $\frac{q_{t+1}\Phi_t - r_t}{u_t} + r_t > r_t$. As shown in the online appendix C, by affecting the rate-of-return differential, financial inflows have redistributive effects among the agents in the same generation; by fostering capital formation and income growth, financial inflows have redistributive effects among the agents in different generations.

5. Equilibrium Shifts and Allocative Efficiency

In this section, we first derive the threshold conditions under which multiple steady states arise under financial integration, given $r^* = r_A$. Then, we show that the existence of multiple steady states depends on the interest rate differential.

Given $r^* = r_A$, the autarkic steady state is still a steady state under financial integration. A marginal rise in the current income from there allows more agents to become entrepreneurs, leading to the excess domestic credit demand. As the domestic interest rate is aligned with the world rate, it does not serve as the endogenous stabilizer.[17] The excess credit demand is covered by financial inflows, which augments aggregate investment and triggers a rise in the productivity.

$$\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\frac{1-\alpha}{\left(\frac{u_t}{1-u_t} + \alpha \frac{\eta - a_t}{1-a_t}\right)} + \theta} > 0 \quad \text{and} \quad \frac{\partial \ln \Phi_t}{\partial \ln w_t} = \frac{\frac{\eta - a_t}{1-a_t}}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t} + \alpha \frac{\eta - a_t}{1-a_t}\right) + 1} > 0,$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \Big(\underbrace{\frac{\partial \ln M_t}{\partial \ln w_t}}_{\text{Investment size effect}} + \underbrace{\frac{\partial \ln \Phi_t}{\partial \ln w_t}}_{\text{Productivity effect}} \Big)\Big[1 - \underbrace{(1-\alpha)}_{\text{neoclassical effect}}\Big] = \frac{\frac{\alpha}{1-\alpha}\left(\frac{u_t}{1-u_t} + \frac{\eta - a_t}{1-a_t}\right)}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t} + \alpha \frac{\eta - a_t}{1-a_t}\right) + 1}.$$

The larger the α, the smaller the neoclassical effect; *the lower the* θ, the larger the extensive margin effect, the larger the investment size effect and the productivity effect; *the lower the* λ, the tighter the borrowing constraints, the lower the mass of entrepreneurs τ_t and the domestic credit demand, the more responsive the aggregate investment and the productivity to income changes, the larger the investment

[17] Under autarky, the domestic interest rate serves as an endogenous stabilizer and rises to eliminate the excess credit demand and then, aggregate investment is funded by domestic saving.

size effect and the productivity effect. If the investment size effect and the productivity effect jointly dominate the neoclassical effect around the initial steady state, $\frac{\partial \ln w_{t+1}}{\partial \ln w_t}|_{w_A} > 1$ holds so that the initial steady state becomes unstable and multiple steady states arise. Proposition 3 describes the relevant conditions.[18]

Proposition 3. *Given $r^* = r_A$, financial integration may lead to multiple steady states, if $\theta < \alpha$ and $\lambda < \tilde{\lambda}_F \equiv \min\{\frac{1-\frac{1-\alpha}{1-\theta}}{1-\eta}, 1\}$.*

In the following, we show that the existence of multiple steady states depends on the interest rate differential, $|r^* - r_A|$. Start from the autarkic steady state. The larger the interest rate differential, the larger the financial flows, the larger the income change in the next period. For a sufficiently large interest rate differential, the law of motion for wage is shifted away from the autarkic steady state to such an extent that multiple steady states vanish and there exists a unique steady state.

Corollary 1. *Given $\theta \in (\underline{\theta}, \alpha)$, $\lambda < \tilde{\lambda}_F$, and $Z \in (\underline{Z}_F, \bar{Z}_F)$, there are two threshold values around the autarkic interest rate, $\hat{r}^* < r_A < \tilde{r}^*$. Under financial integration, there exists a unique steady state, if the initial interest rate differential is sufficiently large, i.e., $r^* < \hat{r}^* < r_A$ or $r^* > \tilde{r}^* > r_A$; there are multiple steady states if the initial interest rate differential is small, i.e., $r^* \in [\hat{r}^*, \tilde{r}^*]$.*

In the rest of this section, we discuss the implications of corollary 1 in the case of $r^* < r_A$, while the case of $r^* > r_A$ is analyzed in section 6.

Let us start with the autarkic equilibrium. Given $\theta \in (\underline{\theta}, \alpha)$, $\lambda < \tilde{\lambda}_F$, and $Z \in (\underline{Z}_F, \bar{Z}_F)$, the dashed curve in the left panels of figure 5 shows the law of motion for wage under autarky, and point A shows the autarkic steady state.

[18] Figure B.7 in the online appendix shows the threshold conditions for multiple steady states.

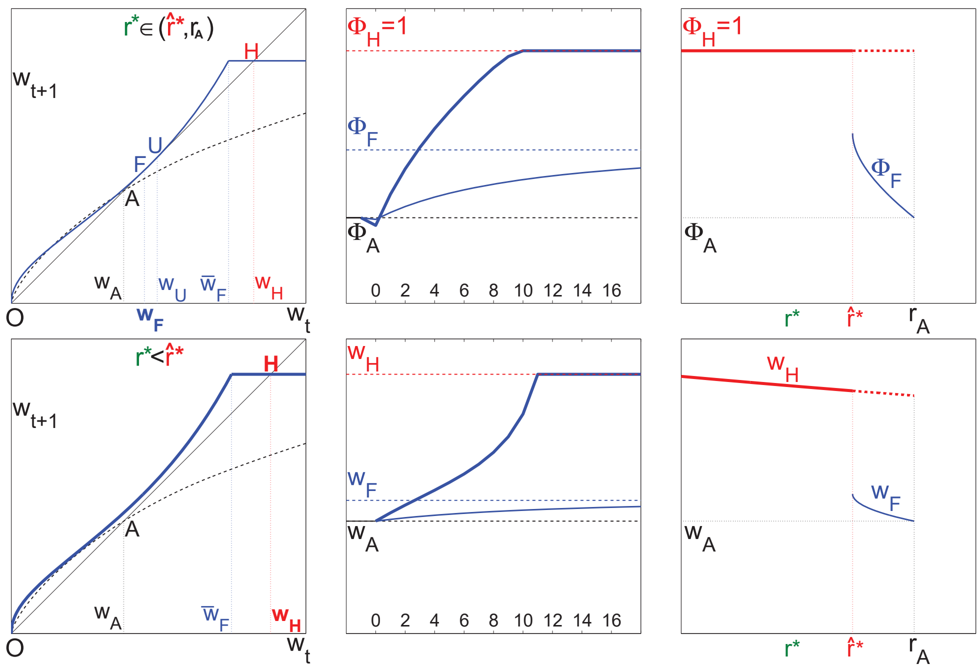

Figure 5: Short-Run and Long-Run Impacts of Financial Integration

5.1. *The initial income level matters.*

We first revisit the case analyzed in section 4, i.e., country N is initially in the autarkic steady state, while the world interest rate is *slightly* below the domestic rate, $r^* \in (\hat{r}^*, r_A)$. Given the small interest rate differential, financial inflows are small in period 0 so that the investment size effect is dominated by the neoclassical effect. The thin, solid curve in the upper-left panel of figure 5 shows the law of motion for wage under financial integration;[19] there are two stable steady states

[19] Under financial integration, for $w_t \geq \bar{w}_F \equiv [1-\lambda(1-\eta)]\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^{\theta}(r^*)^{-\frac{\theta}{1-\alpha}}$, aggregate investment is so high that the social rate of return is equal to the world interest rate $q_{t+1}\Phi_t = r^*$, the borrowing constraints are slack, the investment composition is efficient, i.e., $a_t = \eta$ and $\Phi_t = 1$, and the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$; for $w_t < \bar{w}_F$, the social rate of return is

(H and F) and an unstable steady state (U), with $w_H > w_U > w_F > w_A$. Start from the autarkic steady state (A). Financial integration allows country N to converges to steady state F. The thin, solid curve in the upper-middle (lower-middle) panel shows the productivity (income) dynamics, with the time $t = 0, 1, 2, ...$ on the horizontal axis. Given $\theta \in (\underline{\theta}, \alpha)$, the productivity falls in period 0, while it eventually exceeds the initial level, $\Phi_F > \Phi_A > \Phi_0$.

Given the interest rate differential, the higher the initial income level, the larger the mass of entrepreneurs and their investment demand, the larger the financial inflows, the more likely the investment size effect dominates the neoclassical effect, the faster the income growth. If country N's initial income level is higher than that in steady state U, $w_0 > w_U$, the mass of entrepreneurs is so large that their investment demand attracts large financial inflows. The resulting income amplification drives country N to steady state H with $w_H > \bar{w}_F$, as shown in the upper-left panel of figure 5. In steady state H, the borrowing constraints are slack; the income and the productivity are much higher than the initial levels, $w_H \gg w_U$ and $\Phi_H = 1 \gg \Phi_U$. Here, *the initial income level matters for the long-run equilibrium and the convergence path.*

5.2. The initial interest rate differential matters.

Consider the second case where country N is initially in the autarkic steady state, and the world interest rate is *much lower than* the domestic rate, $r^* < \hat{r}^* < r_A$. Compared to the case analyzed in subsection 5.1, the larger interest rate differential triggers the larger financial inflows, which shifts the law of motion for wage

higher than the world interest rate $q_{t+1}\Phi_t > r^*$, the borrowing constraints are binding, and the investment composition is inefficient, i.e., $a_t < \eta$ and $\Phi_t < 1$.

upwards to such an extent that multiple steady states vanish, as shown by the thick, solid curve in the lower-left panel of figure 5. Country N converges to the unique steady state H where the borrowing constraints are slack, the income and the productivity are much higher than the initial levels, $w_H \gg w_A$ and $\Phi_H = 1 \gg \Phi_A$. The thick, solid curve in the upper-middle (lower-middle) panel shows the productivity (income) dynamics.

We have studied two cases where country N is initially at the autarkic steady state. In the first case, given a moderate interest rate differential $r^* \in (\hat{r}^*, r_A)$, multiple steady states exist under financial integration. Country N converges to the stable steady state close to the initial one. The long-run levels of productivity and income are the continuous, decreasing functions of the interest rate differential in the interval of $r^* \in (\hat{r}^*, r_A)$, as shown by the solid, thin, downward-sloping curves in the right panels of figure 5, respectively. In the second case, given a large interest rate differential $r^* < \hat{r}^* < r_A$, there is a unique steady state under financial integration where the productivity is constant at $\Phi_H = 1$ and the income level is a continuous, decreasing function of the world interest rate $w_H = (r^*)^{-\rho}$ in the interval of $r^* < \hat{r}^*$, as shown by the thick, solid curves in the two right panels.

If the world interest rate falls marginally from above to below the threshold value $\hat{r}^*$, country N shifts from an equilibrium with multiple steady states to the one with a unique steady state, which causes the discontinuous jump in the long-run levels of productivity and income.[20] Thus, *a large interest rate differential*

[20] The threshold value $\hat{r}^*$ is defined as the world interest rate that makes the law of motion for wage tangent with the 45° line in the convex part and cross the 45° line in the flat part. A marginal decline in the world interest rate shifts the law of motion for wage upwards, which eliminates the multiple steady states.

has a disproportionately larger impact than a moderate one, if the former triggers the equilibrium shift.

5.3. The direction of equilibrium shift matters.

Consider the third case where the world interest rate is slightly above the threshold value $r^* > \hat{r}^*$ and country N is initially in steady state F under financial integration. From period 0 on, the world interest rate falls marginally below the threshold value $r^* < \hat{r}^*$. The thin (thick), solid curve in the upper-left (lower-left) panel of figure 5 shows the law of motion for wage before (after) period 0. Country N converges from the initial steady state F (as shown in the upper-left panel) to the new steady state H (as shown in the lower-left panel). The long-run levels of productivity and income rise significantly, as shown by the discontinuous jumps around $\hat{r}^*$ in the right panels.

Consider the opposite case where the world interest rate is slightly below the threshold value $r^* < \hat{r}^*$ and country N is initially in steady state H under financial integration (as shown in the lower-left panel). From period 0 on, the world interest rate rises marginally above the threshold value $r^* > \hat{r}^*$. Country N converges to the new steady state H (as shown in the upper-left panel) where the productivity remains unchanged $\Phi_H = 1$ and the income level falls marginally $w_H = (r^*)^{-\rho}$, as shown by the thick, dashed lines in the right panels of figure 5. Thus, *a fall and a rise in the world interest rate have asymmetric effects*. The discontinuity occurs if the world interest rate change shifts country N from the equilibrium with multiple steady states to the one with a unique steady state. The equilibrium shift in the opposite direction does not cause such a discontinuity. Here, *the direction of equilibrium shift matters for the continuity/discontinuity in the long-run levels of productivity and income.*

6. Impacts of World Interest Rate Hikes

Recently, the United States has prepared for interest-rate hikes if inflation persists. This section analyzes the impacts of world interest rate hikes in two cases.

Suppose that country N is under financial integration and the world interest rate slightly below the threshold value $\hat{r}^*$. The dashed curve in the left panels of figure 6 shows the law of motion for wage. Country N is initially in steady state H where the borrowing constraints are slack and $\Phi_H = 1$.

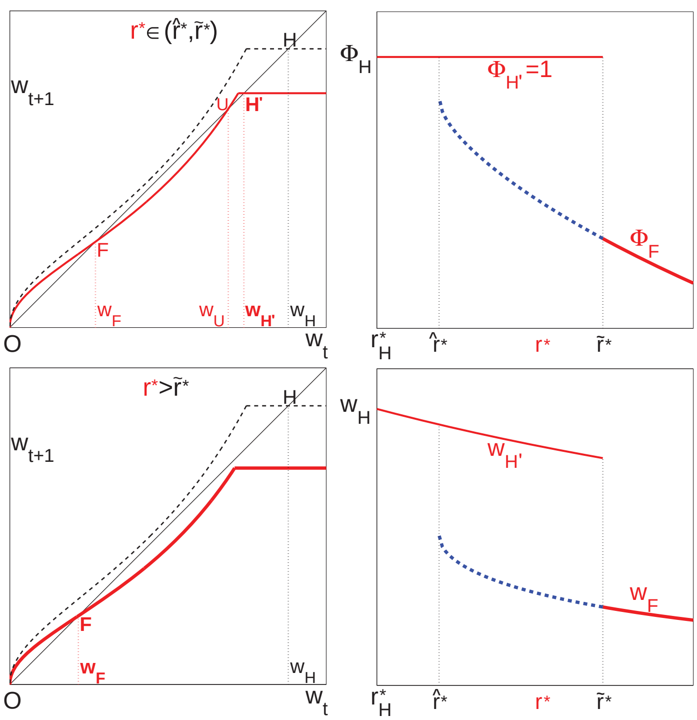

Figure 6: Impacts of World Interest Rate Hikes

From period 0 on, the world interest rate hike reduces financial inflows and weakens capital formation in country N, which shifts the law of motion for wage downwards. In figure 6, the solid curve in the upper-left panel shows the law of motion for wage under a moderate rate hike $r^* \in (\hat{r}^*, \tilde{r}_t)$, while the solid curve in the lower-left panel shows that under a large rate hike $r^* > \tilde{r}^*$.

- For the moderate rate hike, country N converges from steady state H to H′ where the income level is proportional to the interest rate change $w_{H'} = (r^*)^{-\rho}$ and the productivity stays unchanged $\Phi_{H'} = \Phi_H = 1$.
- For the large rate hike, country N converges from steady state H to F with the disproportionately lower levels of income and productivity, $w_F \ll w_H$ and $\Phi_F \ll \Phi_H = 1$.

Given $w_0 = w_H$, if the world interest rate rises marginally from below to above the threshold value $\tilde{r}^*$, the model economy moves from the equilibrium with multiple steady states to the one with a unique steady state. The equilibrium shift causes the discontinuous fall in the long-run levels of productivity (income), as shown by the piecewise, solid curve in the upper-right (lower-right) panel of figure 6.[21]

To sum up, a minor change in the world interest rate around the threshold values may trigger the equilibrium shift, leading to disproportionately large changes in endogenous variables. This mechanism sheds some lights on the implications of the U.S. interest rate hikes. Given that the world interest rate has stayed at the record low level for over a decade, if the U.S. interest rate hikes lead to a moderate rise in the world interest rate, it may not have significant impacts on income

[21]The horizontal axis denotes the world interest rate r^*. r_H^* refers to the initial world interest rate, while $\hat{r}^*$ and $\tilde{r}^*$ are the two threshold values specified in Corollary 1. In the reversed case where the world interest rate falls from above to below the threshold value $\tilde{r}^*$, the long-run income level and the productivity are the continuous function of $r^* > \hat{r}^*$, as shown by the dashed curve in the right panels of figure 6.

and allocative efficiency in small open economies. However, large U.S. interest rate hikes may have disproportionately larger impacts, which may induce some countries to consider capital controls.

7. Conclusion

This paper proposes the *within-firm, intangible-tangible investment composition* as a novel channel through which financial inflows have opposite short-run and long-run effects on the efficiency of capital formation. Given financial frictions and the MIR, rising wealth inequality makes the aggregate investment demand less elastic along the extensive margin, which dampens financial inflows and undermines the productivity gains. Other market frictions and policy distortions that hinder entrepreneurship may also reduce the investment elasticity and undermine the productivity gains.

Groth and Khan (2010) find that the U.S. manufacturing sectors differ substantially in the investment elasticity with respect to the shadow value of capital. Our findings suggest that financial inflows are more likely to stimulate intangible investment and trigger productivity gains in the sectors with higher investment elasticity. Besides, our mechanism offers a new perspective for understanding the impacts of monetary expansions. By lowering the borrowing cost, monetary expansions induces firms to invest relatively more in the tangibles, which worsens allocative efficiency in the short run. Whether monetary expansions improves the investment composition and leads to productivity gains in the long run depends on the investment elasticity.

Although the empirical literature has documented that intangible capital has a lower pledgeability than the tangibles, the rising importance of intangible assets has incentivized various innovations in lending practices and contractual arrangements. For example, rather than borrowing purely against their assets, large U.S. firms borrow predominantly against cash flows from their operations and against the going-concern value of their business (Lian and Ma, 2021). Firms with low liquidation values obtain loans with more monitoring and tighter performance covenants (Kermani and Ma, 2020). In order to retain key employees for intangible investment, firms (particularly in IT industries) offer stock options and wage contracts promising higher future compensations, while lowering current compensation to labor frees up internal cash flows that can be used to finance intangible investment (Sun and Xiaolan, 2019; Döttling et al., 2020; Kiyotaki and Zhang, 2018). The endogenous interactions between intangible investment and financial innovations deserve further research.

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Appendix A. Robustness Check and Model Extensions

Appendix A.1. Elasticity of Substitution between the Tangibles and the Intangibles

In our model, tangible and intangible investments are used for capital formation in the Cobb-Douglas fashion. Following Falato et al. (forthcoming), one can assume that capital formation takes the CES form,

$$k_{j,t+1}=\left[\eta\left(\frac{k_{j,I,t+1}}{\eta}\right)^{\frac{\sigma-1}{\sigma}}+(1-\eta)\left(\frac{k_{j,T,t+1}}{1-\eta}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Then, the allocative efficiency and the unit cost of investment become,

$$\Phi_t=\left[\eta\left(\frac{a_t}{\eta}\right)^{\frac{\sigma-1}{\sigma}}+(1-\eta)\left(\frac{1-a_t}{1-\eta}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \text{ and}$$

$$u_t=\frac{a_t}{\eta}\left[\eta+(1-\lambda)(1-\eta)\mathbb{A}_t^{1-\sigma}\right], \text{ where } \mathbb{A}_t\equiv\left(\frac{1-\eta}{\eta}\frac{a_t}{1-a_t}\right)^{\frac{1}{\sigma}} \tag{A.1}$$

$$\frac{\partial u_t}{\partial a_t}=1-(1-\lambda)\mathbb{A}_t\left(1-\frac{1}{\sigma a_t}\right)>0;\ \text{ for } a_t\leq\eta,\ \mathbb{A}_t<1.$$

Equation (7) is a special case of equation (A.1) with $\sigma = 1$. In this setting, $\frac{\partial u_t}{\partial a_t} > 0$ holds and so does our core mechanism. The higher the σ, the more substitutable the two types of investments, the larger the within-project investment reallocation triggered by financial inflows. Allowing $\sigma \neq 1$ only affects the magnitude of the productivity dynamics, while the qualitative pattern is same as in our model.

For $a_t < \eta$, $\mathbb{A}_t < 1$ and condition (F.18) in the online appendix becomes

$$\frac{\partial \ln LHS}{\partial \ln a_s} = \frac{1-(1-\lambda)\left(1-\frac{1}{\sigma a_t}\right)\mathbb{A}_t}{1+(1-\lambda)\frac{1-a_t}{a_t}\mathbb{A}_t} - \theta \frac{1-\mathbb{A}_t}{1+\frac{1-a_t}{a_t}\mathbb{A}_t} > 0, \text{ and } \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha-\theta}{\theta}.$$

As $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$ holds in this setting, our proof of proposition 2 also holds. The long-run productivity effect still depends on the relative size of θ and α, as our model predicts.

Appendix A.2. Collateral Constraints Revisited

In our model, only the tangibles serve as collateral for loans, while agents have to finance the intangibles entirely with their own funds. Here, we check the robustness of our findings in a generalized setting where the intangibles also serve as collateral, but they have a lower pledgeability than the tangibles.

If the agent defaults, lenders can seize and liquidate both tangible and intangible capital; after deducting the liquidation costs, the lenders get $\lambda_I p_{I,t+1}$ per unit of intangibles, where $p_{I,t+1}$ and λ_I denote respectively the market price and the pledgeability of intangible capital. Let $\kappa \equiv \frac{\lambda_I}{\lambda} \in [0,1)$ denote the intangible-tangible pledgeability ratio. If the borrowing constraints are binding, the unit cost of intangibles is $u_{I,t} = 1 - \frac{\kappa\lambda p_{I,t+1}}{r_t}$, while the unit return is $q_{t+1}\frac{\partial k_{j,t+1}}{\partial k_{j,I,t+1}} - \kappa\lambda p_{I,t+1}$.

The agent equalizes the internal rate of return on the two types of investment,

$$\frac{q_{t+1}\left(\frac{a_{j,t}}{1-a_{j,t}}\frac{1-\eta}{\eta}\right)^{\eta-1}-\kappa\lambda p_{I,t+1}}{1-\frac{\kappa\lambda p_{I,t+1}}{r_t}}=\frac{q_{t+1}\left(\frac{a_{j,t}}{1-a_{j,t}}\frac{1-\eta}{\eta}\right)^{\eta}-\lambda p_{t+1}}{1-\frac{\lambda p_{t+1}}{r_t}}. \tag{A.2}$$

As $a_{j,t}$ is a function of parameters $\{\lambda,\kappa,\eta\}$ and prices $\{r_t,p_{t+1},q_{t+1}\}$, we drop subscript j and use a_t instead. The agent equalizes its marginal revenue of intangibles to the market price,

$$q_{t+1}\frac{\partial k_{j,t+1}}{\partial k_{j,I,t+1}}=q_{t+1}\Phi_t\frac{\eta}{a_t}=p_{I,t+1}, \tag{A.3}$$

Combine equations (5) and (A.2)-(A.3) to solve for the unit cost of investment,

$$u_t\equiv a_t u_{I,t}+(1-a_t)u_{T,t}=1-[\eta\kappa+(1-\eta)]\frac{\lambda}{\psi_t}, \tag{A.4}$$

$$u_{I,t}=1-\frac{\eta}{a_t}\kappa\frac{\lambda}{\psi_t}=\frac{\eta}{a_t}(1-\kappa\lambda)\frac{u_t}{1-[\kappa\eta+(1-\eta)]\lambda} \tag{A.5}$$

$$u_{T,t}=1-\frac{1-\eta}{1-a_t}\frac{\lambda}{\psi_t}=\frac{1-\eta}{1-a_t}(1-\lambda)\frac{u_t}{1-[\kappa\eta+(1-\eta)]\lambda}, \tag{A.6}$$

$$\frac{a_t}{\eta}\{1-[\kappa\eta+(1-\eta)]\lambda\}=1-\kappa\lambda-\frac{(1-\eta)(1-\kappa)\lambda}{\psi_t}=\frac{(1-\kappa)}{1+\frac{\eta}{1-\eta}\kappa}u_t+\frac{(1-\lambda)+\lambda\eta(1-\kappa)}{\eta+\frac{1-\eta}{\kappa}} \tag{A.7}$$

Our model is a special case of this general setting with $\kappa=0$.[22] Other equilibrium conditions are identical as in our model.

In our model, λ measures the level of financial development. In the generalized setting, κ reflects the heterogeneous pledgeability across asset classes. Given the binding borrowing constraints $\lambda<\tilde{\lambda}_A$, if the intangibles and the tangibles have the same pledgeability $\kappa=1$, the unit costs of tangibles and intangibles equalize so

[22] Plug $\kappa=0$ into equations (A.4)-(A.7) to get $u_{I,t}=1$ and equations (6)-(7) in our model.

that within-project investment composition is efficient, $a_t = \eta$ and $\Phi_t = 1$. Thus, $\kappa < 1$ *is a necessary condition for the within-project investment distortion.*[23]

Given $\kappa \in [0,1)$, the unit-cost differential $u_{I,t} > u_{T,t}$ implies that the unit cost of tangibles is more elastic to the normalized interest rate than that of intangibles,

$$\frac{\partial \ln u_{T,t}}{\partial \ln \psi_t} = \frac{1}{u_{T,t}} - 1 > \frac{\partial \ln u_{I,t}}{\partial \ln \psi_t} = \frac{1}{u_{I,t}} - 1 > 0.$$

In period 0, financial inflows lower ψ_0, which causes $u_{T,0}$ to fall by a larger proportion than the fall in $u_{I,0}$. Thus, the unit-cost differential rises, which induces entrepreneurs to shift investment further towards the tangibles. From period 1 on, the rise in ψ_t causes $u_{T,t}$ to rise by a larger proportion than the rise in $u_{I,t}$. The resulting fall in the unit-cost differential induces entrepreneurs to shift investment towards the intangibles. This way, the productivity responds to financial inflows in the opposite direction over time, the same as in our model. The long-run productivity effect still depends on the relative size of θ and α, as specified in proposition 2. Compared to our model, allowing intangibles to serve as collateral with $\kappa \in [0,1)$ narrows the unit-cost differential and weakens the magnitude of the productivity dynamics, but our findings still hold qualitatively.

Appendix A.3. The MIR on the Tangibles

In our model, the MIR applies to the total project investment. Alternatively, one can assume that the MIR applies to tangible investment only, $k_{j,T,t+1} \geq \mathfrak{m}$. If so, the cutoff value specified by equation (8) becomes

$$\frac{w_t(1-\theta)\underline{\varepsilon}_t}{u_t}(1-a_t) = \mathfrak{m}, \Rightarrow \underline{\varepsilon}_t = \frac{u_t}{(1-a_t)w_t}\frac{\mathfrak{m}}{1-\theta},$$

[23] $\lambda < \tilde{\lambda}_A$ is also a necessary condition for the investment distortion. If the borrowing constraints are slack, agents do not borrow to the limit and the unit cost differential vanishes.

while other equilibrium conditions are identical as specified in section 2. The partial elasticities of M_t are still specified by equations (11)-(12). Thus, the core mechanism of our model holds still in this setting and so does the dynamic pattern of productivity.

In our proof of proposition 2, δ_s and equation (F.17)-(F.18) become

$$\delta_s = \underline{\varepsilon}_s^{-\frac{1-\theta}{\theta}} = \left(\frac{w_s(1-a_s)}{u_s}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1}{\theta}-1}$$

$$\frac{u_s}{\Phi_s^{\theta}(1-a_s)^{1-\theta}}\left(\frac{\mathfrak{m}}{1-\theta}\right)^{1-\theta}\rho^{\theta} = w_s^{\frac{\alpha-\theta}{\alpha}}$$

$$\Rightarrow \frac{\partial \ln LHS}{\partial \ln a_s} = 1+(1-\theta)\frac{a_t}{1-a_t} - \theta\frac{\eta - a_t}{1-a_t} > 0 \text{ and } \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha-\theta}{\alpha}.$$

As $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$ holds here, our proof of proposition 2 also holds. The long-run productivity effect depends on the relative size of θ and α, as in our model.

Assuming the MIR for the tangibles does not change our findings in terms of the dynamic pattern of productivity and the conditions for the long-run productivity effect, while it makes the technical analysis more complicated. For example, the productivity effect specified in equation (18) becomes $\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \left(1-\frac{1-\eta}{1-a_t}\right)\frac{1}{\frac{1}{1-\theta}+\frac{a_t}{1-a_t}}$.

Appendix A.4. Within-Project vs. Cross-Project Investment Reallocation

In our model, financial flows trigger the *within-project* investment reallocation along the intangibles-tangibles margin, which affects the project productivity. Financial inflows may trigger the investment reallocation between the projects with different productivity, which affects aggregate productivity. Suppose that agents are endowed with two projects indexed by $h \in \{1,2\}$. The two projects are subject to the same MIR and have the linear technology, $k_{h,j,t+1} = \phi_h m_{h,j,t}$; the borrowing constraints are project-specific, $b_{h,j,t} \leq \lambda_h \frac{\phi_h q_{t+1}}{r_t} m_{h,j,t}$; project 2 is more productive $\phi_2 > \phi_1$, but it is subject to a tighter borrowing constraint, $\lambda_2 < \lambda_1 \frac{\phi_1}{\phi_2}$.

Agents prefer to invest in the more productive project if they can meet the MIR. Given the binding borrowing constraints for both projects, there are two cutoff values, $\underline{\varepsilon}_{2,t} > \underline{\varepsilon}_{1,t} > 1$, that split agents into three groups. Those with $\varepsilon_j \in (1, \underline{\varepsilon}_{1,t})$ lend out the labor income and are called households; those with $\varepsilon_j \in [\underline{\varepsilon}_{1,t}, \underline{\varepsilon}_{2,t})$ invest in project 1 and are called group-1 entrepreneurs; those with $\varepsilon_j \geq \underline{\varepsilon}_{2,t}$ invest in project 2 and are called group-2 entrepreneurs. Aggregate productivity is the weighted average of the project productivity, $\Phi_t = \varsigma_t \phi_2 + (1 - \varsigma_t)\phi_1$, where $\varsigma_t \equiv \frac{M_{2,t}}{M_{1,t}+M_{2,t}}$ denotes the fraction of aggregate investment allocated in project 2.

Given $r^* < r_A$, upon financial inflows, aggregate investment shifts disproportionately towards project 1, which reduces aggregate productivity in period 0. Along the convergence path, aggregate investment gradually shifts towards project 2, which raises aggregate productivity over time. The higher the wealth inequality, the less elastic the mass of entrepreneurs in each group, the smaller the elasticities of aggregate investment for each type of project, the larger the initial fall and the smaller the subsequent rises in aggregate productivity. This way, by reducing the investment elasticity, rising wealth inequality dampens the productivity dynamics along the within-project margin as well as the between-project margin.

Appendix A.5. Elastic Individual Saving and Labor Supply

In our model, agents are endowed with labor when young and they derive utility from consumption when old. In equilibrium, they supply the labor endowment and save the labor income inelastically to the market. Financial inflows stimulate aggregate investment via two channels. First, upon financial inflows, the fall in the interest rate raises the *pledgeable value* of individual projects, which stimulates aggregate investment. Second, rising capital formation stimulates aggregate demand for labor in the next period. Given the inelastic labor supply, the wage rate rises to clear the market.[24] Higher labor income raises *individual net wealth*, which stimulates investment.

In this subsection, we show that financial inflows may stimulate aggregate investment via two additional channels in a generalized setting with the elastic labor supply and the elastic savings. Suppose that agents derive utility also from leisure and consumption when young. Let $v_{j,t}$ denote agent-j's labor supply. The agent has preferences over leisure, $l_j - v_{j,t}$, consumption when young, $c_{j,y,t}$, and consumption when old, $c_{j,o,t+1}$,

$$U_{j,t} = \zeta \frac{(l_j - v_{j,t})^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + (1-\beta)\frac{c_{j,y,t}^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + \beta \frac{c_{j,o,t+1}^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}},$$

subject to the lifetime budget constraint $c_{j,y,t} + \frac{c_{j,o,t+1}}{r_{j,t}} \le w_t v_{j,t}$. ζ and β denote the respective weight of utility from leisure when young and from consumption when old, σ denotes the elasticity of substitution between leisure and consumption when young as well as that between consumption in the two periods, and $r_{j,t}$ denotes the rate of return on agent-j's saving.[25] The agent's labor supply and consumption choices are

$$v_{j,t} = \frac{l_j}{1 + \frac{1}{\left[\left(\frac{1-\beta}{\zeta}\right)^{\sigma} + \left(\frac{\beta}{\zeta}\right)^{\sigma} r_{j,t}^{\sigma-1}\right] w_t^{\sigma-1}}}, \; c_{j,y,t} = \frac{w_t v_{j,t}}{1 + \left(\frac{\beta}{1-\beta}\right)^{\sigma} r_{j,t}^{\sigma-1}}, \; c_{j,o,t+1} = c_{j,y,t}\left(\frac{r_{j,t}\beta}{1-\beta}\right)^{\sigma}.$$

Let us consider three alternative cases.

[24] As financial inflows in period t raise the capital stock only in period $t+1$, the aggregate labor demand is unaffected in period t and so is the wage rate.

[25] The agents with $\varepsilon_j \in [1, \underline{\varepsilon}_t)$ cannot afford the MIR and they lend their savings $n_{j,t} = w_t v_{j,t} - c_{j,y,t}$ at the market interest rate $r_{j,t} = r_t$. The agents with $\varepsilon_j \ge \underline{\varepsilon}_t$ can afford the MIR and, by putting their savings $n_{j,t} = w_t v_{j,t} - c_{j,y,t}$ in the project, they earn the equity rate $r_{j,t} = \frac{q_{t+1}\Phi_t - r_t}{u_t} + r_t$.

Case 1: Our model is a special case of this general setting with $\beta = 1$ and $\zeta = 0$. In equilibrium, agents supply their entire labor endowment, $v_{j,t} = l_j$, and save their entire labor income when young, $c_{j,y,t} = 0$ and $c_{j,o,t+1} = r_{j,t} w_t l_j$.

Case 2: For $\sigma = 1$, the income effect and the substitution effect cancel out. The agent supplies a constant fraction of its labor endowment to the market $v_{j,t} = \frac{l_j}{1+\zeta}$ and saves a constant fraction of its labor income $w_t v_{j,t} - c_{j,y,t} = \beta w_t v_{j,t}$, regardless of the wage rate and the rate of return. Thus, our findings still hold.

Case 3: For $\sigma > 1$, the substitution effect dominates the income effect.[26] By reducing the borrowing costs, the inflows of cheap foreign funds raise the equity rate, which induces entrepreneurs to put a larger fraction of their labor income in the project. Financial inflows stimulate capital formation, which raises the aggregate labor demand and the wage rate in the next period. The higher equity rate and the higher wage rate induces entrepreneurs to supply more labor so that their labor income rises to a larger extend than in our model.

Here, the elastic labor supply and the elastic saving become two extra channels through which financial inflows stimulate aggregate investment. With a stronger MPK effect, financial inflows are more likely to create the productivity gains.

Appendix A.6. Corrective Tax-Subsidy Scheme

Our model features limited and heterogeneous pledgeability in the credit relationship, which distorts the aggregate allocation along two dimensions. First, due to limited pledgeability, agents cannot borrow against their entire project revenue. The constraint on the aggregate credit demand pushes the interest rate below the

[26] In the case of $\sigma < 1$, the income effect dominates the substitution effect so that the individual labor supply and the individual savings are downward-sloping. This case seems less realistic.

social rate of return, $\psi_t = \frac{r_t}{q_{t+1}\Phi_t} < 1$, which reflects an intertemporal distortion. Second, due to heterogeneous pledgeability, only tangible capital can serve as collateral. The unit-cost differential leads to underinvestment in the intangibles, which reflects an intratemporal distortion.

If the financial sector cannot mitigate the commitment and enforcement problems in the credit relationship, the government may subsidize intangible investment, which reduces the unit-cost differential and improves allocative efficiency. The subsidies can be financed in three ways, i.e., taxing the tangible investment in the current period, taxing the investment revenue in the future period, and taxing the labor income. The first two approaches do not involve income redistribution across individuals in the net term, while the third approach does.

Although restoring allocative efficiency with the tax-subsidy scheme seems appealing in theory, it may trigger moral hazard and rent seeking. Using a novel administrative dataset on corporate tax returns of Chinese firms, Chen et al. (2021) find that firms relabel other expenses as R&D so as to qualify for a special high-tech-firm status which was accompanied by a lower average tax rate of 15% – a large reduction from the statutory rate of 33%. The authors find that one-fourth of the reported R&D increase is due to relabeling. König et al. (forthcoming) find similar results. Despite moral hazard and rent seeking, China's industrial policies have contributed to the rise in intangible investment (especially in the high-tech, solar panel, electric car industries) in the last decade. The America COMPETES Act of 2022 and the United States Innovation and Competition Act show bipartisan support for federal engagement in industrial policy, particularly for the R&D related to semiconductor technology and administrative tasks.

Online Appendices for

"Upstream Financial Flows, Intangible Investment, and Allocative Efficiency"

by Haiping Zhang, University of Auckland

Appendix B. Steady-State Analysis of Financial Integration

We analyze the steady-state property of the model under financial integration.

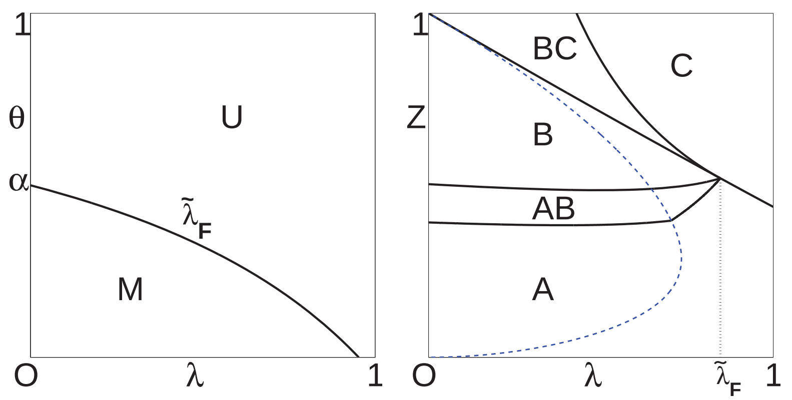

Figure B.7: Steady-State Property under Financial Integration: the Case of $r^* = r_A$

Let $\tilde{\lambda}_F \equiv \min\{\frac{1-\frac{1-\alpha}{1-\theta}}{1-\eta}, 1\}$. In figure B.7, the downward-sloping curve in the left panel shows $\tilde{\lambda}_F$ as a function of θ; for (λ, θ) in region U, the autarkic steady state is still the unique steady state under financial integration; for (λ, θ) in region M, the right panel shows the parameter constellations for five cases in the (λ, Z) spaces, while the solid (dashed) curves in figure B.8 show the laws of motion for wage under financial integration (under autarky) in five cases, respectively.[27] The proof of proposition 3 characterizes the

[27]Given the constant population size in each generation, the cutoff value should be specified

law of motion for wage and specifies the threshold values. Given $r^* = r_A$,[28] multiple steady states arise in three cases.

- In case B, financial integration destabilizes the autarkic steady state (point A), which leads to two stable steady states (point H and point L), with $w_L < w_A < w_H$.

more precisely as $\underline{\varepsilon}_t = \max\left\{1, \frac{u_t}{w_t}\frac{m}{1-\theta}\right\}$, implying the existence of the mass-of-entrepreneurs (MoE, hereafter) constraint, $\tau_t \leq 1$. Under autarky, if the borrowing constraints are binding in equilibrium, there must be some agents who cannot overcome the MIR; hence the MoE constraint is not binding; if the borrowing constraints are slack, those who can overcome the MIR do not have strong incentive to be entrepreneurs; hence the MoE constraint is irrelevant. Under financial integration, the law of motion for wage consists of two or three parts, depending on the bindingness of the borrowing constraints and the MoE constraint. The dashed curve in the right panel of figure B.7 shows a threshold value $\check{\lambda}_F$ in the (λ, Z) space.

- For (λ, Z) to the right of the dashed curve, $\lambda > \check{\lambda}_F$ and the MoE constraint is always slack as long as the borrowing constraints are binding; when the borrowing constraints are slack, agents who can overcome the MIR do not have strong incentive to run the project; hence the MoE constraint is irrelevant. Let $\bar{w}_F \equiv [1-\lambda(1-\eta)]\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^{\theta}(r^*)^{-\frac{\theta}{1-\alpha}}$. For $w_t > \bar{w}_F$, the borrowing constraints are slack; for $w_t < \bar{w}_F$, the borrowing constraints are binding. Hence, the law of motion for wage consists of two parts and the solid (dashed) curves in figure B.8 show the laws of motion for wage under financial integration (autarky).
- For (λ, Z) to the left of the dashed curve, $\lambda < \check{\lambda}_F$. There are two threshold values $\tilde{w}_F < \bar{w}_F$ such that, for $w_t < \tilde{w}_F$, the borrowing constraints are binding and the MoE constraint is slack; for $w_t \in [\tilde{w}_F, \bar{w}_F)$, both the borrowing constraints and the MoE constraint are binding; for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant. Hence, the law of motion for wage consists of three parts and the solid (dashed) curves in figure F.13 show the laws of motion for wage under financial integration (autarky).

[28]One can use the solution approach described in the proof of proposition 3 to analyze the case of $r^* \neq r_A$.

- In case AB, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point H) arise, with $w_A < w_U < w_H$.
- In case BC, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point L) arise, with $w_L < w_U < w_A$.

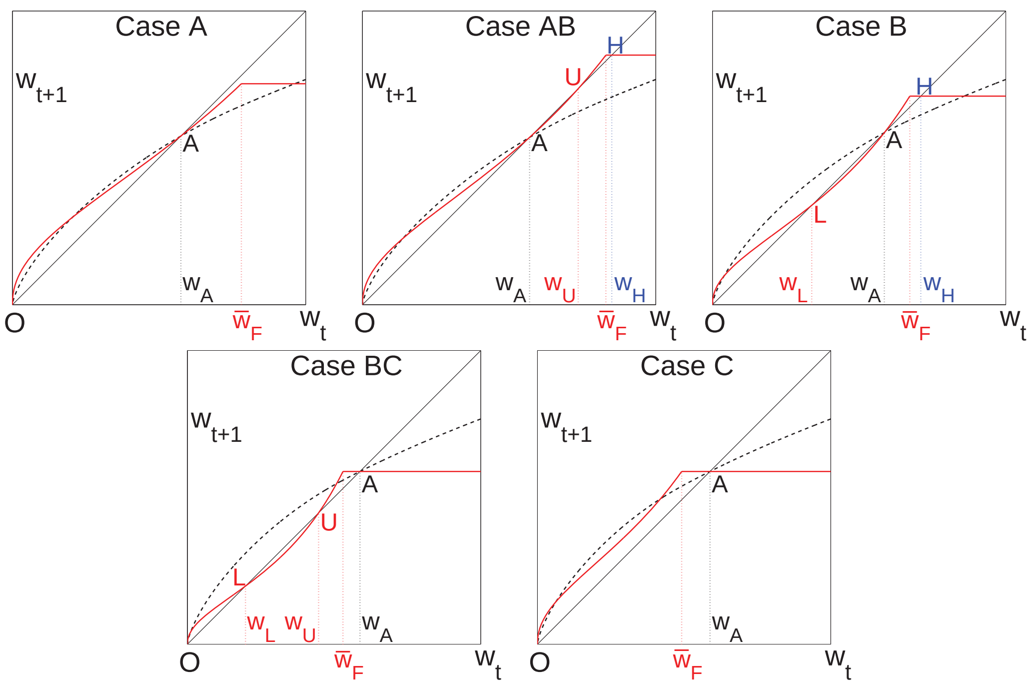

Figure B.8: Laws of Motion for Wage under Financial Integration: $\theta < \alpha$ and $\lambda < \tilde{\lambda}_F$

Appendix C. Distributive Effects of Financial Inflows

In our model, agents consume only when old, and their consumption depends on their labor income when young and the rate of return on their savings. Households earn the interest rate r_t, while entrepreneurs earn the equity rate $\Upsilon_t \equiv \frac{q_{t+1}\Phi_t - r_t}{u_t} + r_t = r_t \frac{1-\lambda(1-\eta)}{\psi_t - \lambda(1-\eta)}$.

Given country N initially in the autarkic steady state, financial integration does not affect the labor income and the savings return of those born before period 0.

Upon financial integration, the interest rate in country N falls to the world level $r_0 = r^* < r_A$; due to the MPK effect, the equity rate also falls $\Upsilon_0 < \Upsilon_A$; the cutoff value also falls $\underline{\varepsilon}_0 < \underline{\varepsilon}_A$. The agents born in period 0 can be categorized into three groups.

- Those with $\varepsilon_j \in (1, \underline{\varepsilon}_0)$ would become households if born before period 0. When they are born in period 0, they still become households and earn an interest rate lower than otherwise in the autarkic steady state $r_0 < r_A$. Given $w_0 = w_A$, the interest rate effect makes them worse off.

- Those with $\varepsilon_j \geq \underline{\varepsilon}_A$ would become entrepreneurs if born before period 0. When they are born in period 0, they still become entrepreneurs and earn an equity rate lower than otherwise in the autarkic steady state $\Upsilon_0 < \Upsilon_A$. Given $w_0 = w_A$, the equity rate effect makes them worse off.

- Those with $\varepsilon_j \in [\underline{\varepsilon}_0, \underline{\varepsilon}_A)$ would become households if born before period 0. When they are born in period 0, they become entrepreneurs and earn a rate of return higher than otherwise in the autarkic steady state $\Upsilon_0 > r_A$. Given $w_0 = w_A$, the rate-of-return effect makes them better off.

Next, consider the generation born in period $t \geq 1$. As mentioned above, aggregate income rises over time, which affects individual welfare in three ways. First, by raising the wage rate, it benefits all agents in equal proportions via the labor income channel. Given the interest rate constant at the world rate, the labor income effect makes the agents with $\varepsilon_j \in (1, \underline{\varepsilon}_t)$ better off than otherwise born one period earlier. Second, by stimulating financial inflows and aggregate investment, the income growth reduces the project rate of return $q_{t+1}\Phi_t$ and lowers the equity rate. If the labor income effect dominates the equity rate effect, the agents with $\varepsilon_j \geq \underline{\varepsilon}_{t-1}$ are better off than otherwise born one period earlier.

Third, by further reducing the cutoff value $\underline{\varepsilon}_t < \underline{\varepsilon}_{t-1}$, the income growth allows the agents with $\varepsilon_j \in [\underline{\varepsilon}_t, \underline{\varepsilon}_{t-1})$ to become entrepreneurs and earn the equity rate rather than the interest rate on their savings. The labor income effect and the rate-of-return effect jointly make them better off than otherwise born one period earlier.

Besides, we can compare the agents born in period t against those born before period 0. Compared to the allocation in the autarkic steady state, financial inflows reduces the interest rate and raises aggregate income. The agents with $\varepsilon_j > \underline{\varepsilon}_A$ are better off, if the labor income effect dominates the equity rate effect. The agents with $\varepsilon_j \in (\underline{\varepsilon}_t, \underline{\varepsilon}_A)$ are better off, due to the labor income effect $w_t > w_A$ and the rate-of-return effect $\Upsilon_t > r_A$. Whether the agents with $\varepsilon_j \in (1, \underline{\varepsilon}_t)$ are better off depends on the relative size of the positive labor income effect and the negative interest rate effect.

Appendix D. The Case of Homogeneous Wealth Distribution

In order to highlight the role of the extensive margin effect, we analyze a special case of $\theta \to 0$ where the distribution of labor endowment degenerates into a unit mass at $l_j = 1$ and agents have the same labor income, $n_t = w_t$. If $w_t < \mathfrak{m}$, an agent has to borrow at least $\mathfrak{m} - w_t$ to run the project. As argued in Matsuyama (2004), the equilibrium allocation involves credit rationing, i.e., the credit is allocated randomly to a fraction of agents who become entrepreneurs, while the rest are denied credit and become households. Due to competition on the credit market, each entrepreneur only demands for the credit of $\mathfrak{m} - w_t$ and invests at the level of the MIR, $m_t = \frac{n_t}{u_t} = \mathfrak{m}$.[29]

In figure D.9, point E_A denotes the domestic investment-saving balance in the autarkic steady state. According to equation (12), $\theta \to 0$ makes the aggregate investment demand perfectly elastic with respect to u_t and the line of M_A is flat. In period 0, given $w_0 =$

[29] According to equations (17), for $\theta \to 0$, $u_t = \tau_t = \frac{w_t}{\mathfrak{m}}$ holds under autarky and so does $m_t = \frac{n_t}{u_t} = \frac{w_t}{u_t} = \mathfrak{m}$. According to equation (F.23), this result also holds under financial integration.

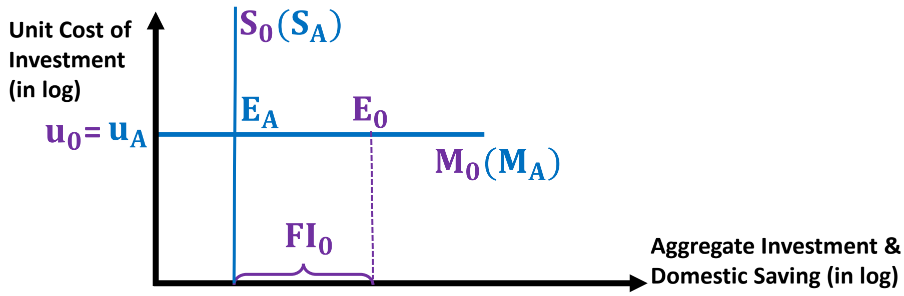

Figure D.9: Financial Inflows and Domestic Investment-Saving Imbalance in Period 0: $\theta \to 0$

w_A and $m_0 = m_A = \mathfrak{m}$, the credit demand of each individual entrepreneur is the same as before $m_0 - w_0 = m_A - w_A$ and so is the unit cost of investment $u_0 = \frac{w_0}{m_0} = \frac{w_A}{m_A} = u_A$. Thus, financial inflows stimulate the aggregate investment demand only along the extensive margin $M_t = \mathfrak{m}\tau_t L$. As a result, the aggregate investment expansion is so large that the social rate of return falls by the same proportion as the change in the interest rate $\frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0} = 1$. Thus, $\psi_0 = \psi_A$ holds and so does $\Phi_0 = \Phi_A$. One can confirm these findings by putting $\theta \to 0$ into equations (19) and (F.12).

Appendix E. A Model with the Exogenous Mass of Entrepreneurs

The endogenous entrepreneur wealth share is key to our findings. For comparison, we build a model which differs from our model in two aspects. First, there is no MIR. Second, only a constant fraction τ of agents in each generation are endowed with the investment project and they are called *entrepreneurs*, while the others do not have the project and are called *households*. Agents are equally endowed with one unit of labor when young[30] and their labor income is homogeneous at w_t. Due to the exogenous mass of entrepreneurs, aggregate investment adjusts only along the **i**ntensive **m**argin. Thus, we call it model IM.

[30] As wealth distribution does not matter for our findings in this model, we assume it away for simplicity.

Assumption 2. $\tau < \eta$ *and* $\lambda \in (0,1]$.

Under assumption 2, the entrepreneurial wealth share is less than the efficient share of intangible investment $\delta_t = \frac{\tau w_t L}{w_t L} = \tau < \eta$. Thus, the borrowing constraints are binding under autarky and the individual optimization is the same as shown in section 2. A rise in current income raises the net wealth of all agents in equal proportions. Due to the fixed masses of entrepreneurs and households, the aggregate credit demand and the aggregate credit supply rise along the intensive margin in equal proportions so that the normalized interest rate stays put and so do the unit cost of investment, the intangible fraction of investment, and the productivity.

$$\begin{aligned} &M_t = \frac{\tau w_t L}{u_t} = w_t L, \ \Rightarrow \ u_t = u_A = \tau, \ \psi_t = \psi_A = \frac{(1-\eta)\lambda}{1-u_t} = \frac{(1-\eta)\lambda}{1-\tau} < 1, \\ &a_t = a_A = \eta \frac{u_t}{1-\lambda(1-\eta)} = \eta \frac{\tau}{1-\lambda(1-\eta)} < \eta, \ \ \Phi_t = \Phi_A < 1. \end{aligned} \tag{E.1}$$

Combine them with equations (1) to get the law of motion for wage,

$$w_{t+1} = \left(\Phi_A \frac{w_t}{\rho}\right)^{\alpha}, \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{neoclassical effect}} < 1. \tag{E.2}$$

The dynamics of aggregate income are purely driven by the neoclassical effect.

Proposition 4. *Under autarky, the borrowing constraints are binding, the normalized interest rate is constant at* $\psi_t = \psi_A < 1$, *the intangible fraction of investment is constant at* $a_t = a_A < \eta$, *and the productivity is constant at* $\Phi_t = \Phi_A < 1$. *Besides,* $\frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_t}{1-a_t} \frac{\partial \ln a_A}{\partial \ln \lambda} > 0$.

There is a unique, autarkic steady state where the social rate of return is $q_A \Phi_A = \rho$ *and the interest rate is* $r_A = \psi_A \rho$, *while* $\frac{\partial r_A}{\partial \lambda} > 0$ *and* $\frac{\partial w_A}{\partial \lambda} > 0$.

Figure E.10 shows the impacts of financial integration on domestic investment-saving imbalances. Point E_A denotes domestic investment-saving balance in the autarkic steady state where the aggregate investment demand M_A and the domestic saving S_A intersect.

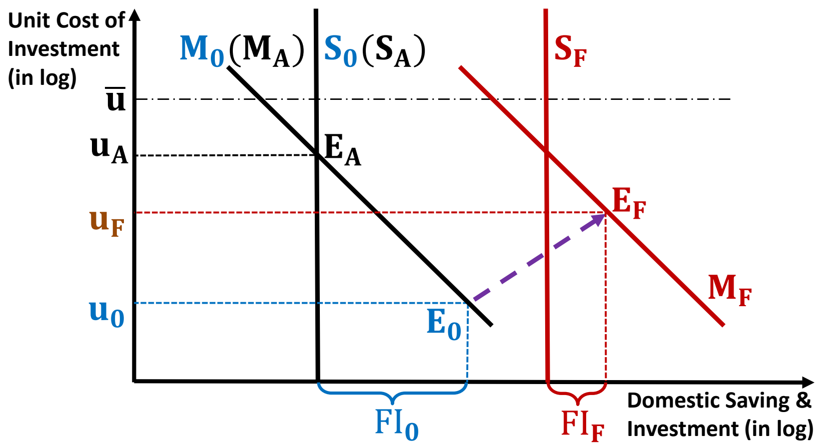

Figure E.10: Financial Integration and Domestic Investment-Saving Imbalance in Model IM

Due to the exogenous mass of entrepreneurs and the homogeneous wealth distribution, the entrepreneurial wealth share is constant at $\delta_t \equiv \frac{\tau w_t L}{w_t L} = \tau$. Given $w_0 = w_A$ and $\delta_t = \tau$, the two lines stay put in period 0. Thus, the normalized interest rate must fall $\psi_0 < \psi_A$ so as to create the excess domestic credit demand and absorb financial inflows. The equilibrium moves downwards from point E_A to E_0. From period $t = 1$ on, the rise in aggregate income shifts the two lines rightwards in equal proportions, while the aggregate investment expansion reduces the social rate of return; as the interest rate is aligned with the world rate $r_t = r^*$, ψ_t rises over time; the dashed arrow shows the path along which country N converges to the new steady state F. Given $\delta_t = \tau$, $u_F < u_A$ must hold so as to justify financial inflows with the excess domestic credit demand in the new steady state.

In model IM, as the mass of entrepreneurs is exogenous, the entrepreneurial wealth share is constant; upon financial integration, the aggregate investment demand and the domestic saving stay put, while the subsequent rises in the income level raise them in equal proportions. In our model, the mass of entrepreneurs is endogenous and so is the entrepreneurial wealth share. Due to the endogenous extensive margin, the aggregate investment demand responds to income rises by a larger proportion than the change in domestic saving; hence financial inflows are larger than in model IM. This way, the endogeneity of the entrepreneurial wealth share is key to the different patterns of the normalized interest rate and the productivity between the two models.

Appendix F. Proofs

Proof of Proposition 1

Proof. The proof consists of four steps.

Step 1: Solve the Individual Optimization Problem and Derive the Unit Costs

The agent chooses tangible and intangible investments as well as loans to maximize its net investment revenue, subject to the budget constraint and the borrowing constraints.

$$\begin{aligned}\mathbb{L}_{j,t+1} \equiv \max_{k_{j,T,t+1},k_{j,I,t+1},b_{j,t}} & q_{t+1}k_{j,t+1} - r_t b_{j,t} - \xi_{j,t}(k_{j,I,t+1} + k_{j,T,t+1} - n_{j,t} - b_{j,t}) \\ & - \zeta_{j,t}\left(b_{j,t} - \lambda \frac{p_{t+1}}{r_t} k_{j,T,t+1}\right).\end{aligned}$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial b_{j,t}} = -r_t + \xi_{j,t} - \zeta_{j,t} = 0, \qquad \Rightarrow \zeta_{j,t} = \xi_{j,t} - r_t, \tag{F.1}$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial k_{j,I,t+1}} = MR_{j,I,t+1} - \xi_{j,t} = 0, \qquad \Rightarrow MR_{j,I,t+1} = \xi_{j,t}, \tag{F.2}$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial k_{j,T,t+1}} = MR_{j,T,t+1} - \xi_{j,t} + \zeta_{j,t}\lambda\frac{p_{t+1}}{r_t} = 0, \quad \Rightarrow MR_{j,T,t+1} = \xi_{j,t} - (\xi_{j,t} - r_t)\lambda\frac{p_{t+1}}{r_t}. \tag{F.3}$$

Let $\mathbb{A}_{j,t} \equiv \frac{a_{j,t}}{1-a_{j,t}}\frac{1-\eta}{\eta}$. The marginal revenues of intangibles and tangibles are respectively,

$$MR_{j,I,t+1} \equiv q_{t+1}\frac{\partial k_{t+1}}{\partial k_{j,I,t+1}} = q_{t+1}\mathbb{A}_{j,t}^{\eta-1}, \text{ and } MR_{j,T,t+1} \equiv q_{t+1}\frac{\partial k_{t+1}}{\partial k_{j,T,t+1}} = q_{t+1}\mathbb{A}_{j,t}^{\eta}.$$

If the borrowing constraint is *slack*, $\zeta_{j,t} = 0$ holds. Use equations (F.1)-(F.3) to get

$$MR_{j,I,t+1} = MR_{j,T,t+1} = \xi_{j,t} = r_t, \Rightarrow a_{j,t} = \eta,\ \Phi_{j,t} = 1, \text{and } q_{t+1} = r_t.$$

As the private and the social rates of return coincide, those who meet the MIR do not have strong incentives to put their entire net wealth in the project or borrow to the limit. Nevertheless, those who run the project choose $a_{j,t} = \eta$. Thus, $a_t = \eta$ and $\Phi_t = 1$ hold.

If the borrowing constraint is *binding*, $\zeta_{j,t} > 0$ holds. Use equations (F.1)-(F.3) to get

$$MR_{j,I,t+1} = \xi_{j,t} = \frac{MR_{j,T,t+1} - \lambda p_{t+1}}{1 - \lambda \frac{p_{t+1}}{r_t}}, \tag{F.4}$$

which gives equation (3). As the right hand side of equation (3) rises with $\mathbb{A}_{j,t}$, there exists a unique $\mathbb{A}_{j,t}$ that solves this equation. As all agents face the same market prices, they choose the same value for $\mathbb{A}_{j,t}$. Thus, we drop subscript j and use $\mathbb{A}_t$ and a_t instead.

As the price of tangibles is equal to the marginal revenue of tangibles at the aggregate level, we simplify the unit cost of tangibles as equation (6). Use it to get equation (7).

Given the binding borrowing constraints, $\zeta_{j,t} > 0$ holds and we combine it with equations (F.1)-(F.3) to get $MR_{j,I,t} = \xi_{j,t} > r_t$ and $MR_{j,T,t} = (\xi_{j,t} - r_t)u_{T,t} + r_t > r_t$. In equilibrium, the social rate of return exceeds the interest rate,

$$q_{t+1}\Phi_t = a_t MR_{j,I,t} + (1 - a_t)MR_{j,T,t} > r_t.$$

Step 2: Derive the Condition for the Binding Borrowing Constraints under Autarky

If the borrowing constraints are slack, $r_t = \Phi_t q_{t+1}$, $a_t = \eta$, and $\Phi_t = 1$. Combine equations (14)-(13) with (1) to get equation (15) as the law of motion for wage. If the borrowing constraints are binding, $r_t < \Phi_t q_{t+1}$ and equations (17) specify some endogenous variables under autarky.

If the borrowing constraints are weakly binding, $\Phi_t = 1$ and $q_{t+1} = r_t$. Use equation (7) to get $u_t = 1 - \lambda(1 - \eta)$. Combine it with equation (17) to get the threshold value $\bar{w}_A = [1 - \lambda(1 - \eta)]^{\frac{1}{1-\theta}} \frac{\mathfrak{m}}{1-\theta} > 0$. If the borrowing constraints are strictly binding, use equations (17), (7), and (5) to prove

$$\frac{\partial \ln a_t}{\partial \ln w_t} = \frac{\partial \ln u_t}{\partial \ln w_t} = 1 - \theta < 1, \text{ and } \frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1 - a_t} > 0, \text{ if } a_t < \eta. \tag{F.5}$$

For $w_t = \bar{w}_A$, $a_t = \eta$; for $w_t < \bar{w}_A$, $a_t < \eta$, $u_t < 1-\lambda(1-\eta)$, and $r_t < q_{t+1}\Phi_t$. Combine equations (14)-(13) with (1) to get (15) as the law of motion for wage.

Step 3: Derive the Condition for the Unique, Stable Steady State under Autarky

Let w_A denote the wage rate at an autarkic steady state. If the slope of the law of motion for wage at any steady state is $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} < 1$, the steady state is stable and unique.

For $w_A > \bar{w}_A$, $\Phi_t = 1$ holds and the productivity effect is mute. Use equation (15) to get $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} = \alpha < 1$ and the stability condition is always satisfied in this case.

For $w_A < \bar{w}_A$, use equation (18) and (15) to get $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} = 1-(1-\alpha)\left[1-\rho\eta(1-\theta)\frac{1-\frac{a_t}{\eta}}{1-a_t}\right]$. The stability condition is satisfied if $\theta \geq \underline{\theta} \equiv \max\{0, 1-\frac{1}{\rho\eta}\}$.

Step 4: Derive the Autarkic Steady-State Patterns of Endogenous Variables

Given $\theta \geq \underline{\theta} \equiv \max\{0, 1-\frac{1}{\rho\eta}\}$, there exists a unique, stable steady state under autarky. Let $\tilde{\lambda}_A$ denote a threshold value such that, for $\lambda = \tilde{\lambda}_A$, the borrowing constraints are weakly binding at the autarkic steady state, with $w_A = \bar{w}_A$, $a_A = \eta$. In this case, $\Phi_A = 1$ and

$$w_A = \rho^{-\rho} = \bar{w}_A = (1-\tilde{\lambda}_A)^{\frac{1}{1-\theta}}\frac{\mathfrak{m}}{1-\theta}, \Rightarrow \tilde{\lambda}_A = 1-Z^{1-\theta}, \text{ where } Z \equiv \frac{1-\theta}{\rho^\rho \mathfrak{m}}. \tag{F.6}$$

For $\lambda > \tilde{\lambda}_A$, the borrowing constraints are slack at the autarkic steady state where the endogenous variables $\{a_A, u_A, r_A, \psi_A, w_A\}$ are constant and independent of λ. For $\lambda < \tilde{\lambda}_A$, the borrowing constraints are binding at the autarkic steady state. Combine equations (5), (15) and (17) to get

$$w_A = u_A^{\frac{1}{1-\theta}}\frac{\mathfrak{m}}{1-\theta}, \Rightarrow \rho\ln\Phi_A = \frac{1}{1-\theta}(\ln a_A + \ln[1-\lambda(1-\eta)] - \ln\eta) - \ln Z$$

$$\rho(1-\theta)\frac{\partial\ln\Phi_A}{\partial\ln a_A}\frac{\partial\ln a_A}{\partial\ln\lambda} = \frac{\partial\ln a_A}{\partial\ln\lambda} - \frac{\lambda(1-\eta)}{1-\lambda(1-\eta)}, \quad \frac{\partial\ln a_A}{\partial\ln\lambda} = \frac{\frac{\lambda(1-\eta)}{1-\lambda(1-\eta)}}{1-\rho(1-\theta)\eta\left(1-\frac{\frac{1}{\eta}-1}{\frac{1}{a_A}-1}\right)}.$$

Given $\theta > \underline{\theta}$ and $\lambda < \tilde{\lambda}_A$, $a_A < \eta$ and $\frac{\partial\ln a_A}{\partial\ln\lambda} > 0$. Use equations (5) and (7) to get

$$\frac{\partial\ln u_A}{\partial\ln\lambda} = \frac{\partial\ln a_A}{\partial\ln\lambda} > 0, \quad \frac{\partial\ln\Phi_A}{\partial\ln\lambda} = \frac{\eta-a_A}{1-a_A}\frac{\partial\ln a_A}{\partial\ln\lambda} > 0, \quad \frac{\partial\ln w_A}{\partial\ln\lambda} = \rho\frac{\partial\ln\Phi_A}{\partial\ln\lambda} > 0.$$

Under autarky, aggregate investment in period t is $M_t = w_t L$, while the revenue of capital goods in period $t+1$ is $q_{t+1}K_{t+1} = \frac{\alpha}{1-\alpha} w_{t+1} L$. Then, the social rate of return is

$$\frac{q_{t+1}K_{t+1}}{M_t} = q_{t+1}\Phi_t = \rho \frac{w_{t+1}}{w_t}. \tag{F.7}$$

In the steady state, $w_{t+1} = w_t$ holds and the social rate of return is constant at $q_A \Phi_A = \rho$; use equation (7) to prove that the interest rate increases strictly with λ.

$$r_A = \rho \psi_A, \ \psi_A = \frac{\lambda}{1-u_A}, \Rightarrow \ \frac{\partial \ln r_A}{\partial \ln \lambda} = \frac{\partial \ln \psi_A}{\partial \ln \lambda} = 1 + \frac{u_A}{1-u_A}\frac{\partial \ln a_A}{\partial \ln \lambda} > 0. \tag{F.8}$$

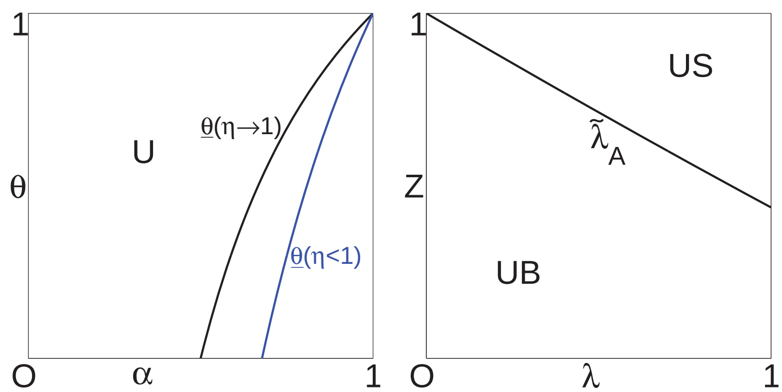

Figure F.11: Threshold Values for the Autarkic Steady State

In figure F.11, the left panel shows the threshold value $\underline{\theta}$ in the $\{\alpha, \theta\}$ space; given $\{\alpha, \theta\}$ in region U, the right panel shows the threshold value $\tilde{\lambda}_A$ in the (λ, Z) space. For (λ, Z) in region UB (US), there is a unique, autarkic steady state where the borrowing constraints are binding (slack), with $w_A < \bar{w}_A$ ($w_A > \bar{w}_A$).

Given $\{\alpha, \theta\}$ in region U of the left panel and (λ, Z) in region UB (US) of the right panel in figure F.11, the solid curve in the left (right) panel of figure F.12 show the law of motion for wage. For comparison purposes, the dashed curves in figure F.12 show the laws of motion for wage in the absence of financial friction.[31] For $w_t \in (0, \bar{w}_A)$, the solid curve lies below the dashed curve and the gap reflects the efficiency losses, $(1-\Phi_t^{\alpha})\left(\frac{w_t}{\rho}\right)^{\alpha}$.

[31] In the absence of financial frictions, $a_t = \eta$ and $\Phi_t = 1$ hold; the law of motion for wage under

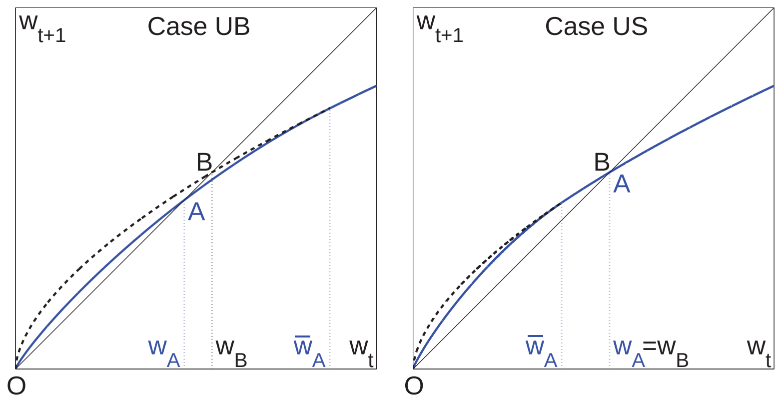

Figure F.12: Laws of Motion for Wage under Autarky: $\theta \geq \underline{\theta}$

$\square$

Proof of Proposition 2

Proof. We focus on the case where r^* is marginally below r_A and financial integration allows country N to converge towards a stable steady state in the neighbourhood of the autarkic one. Appendix B and the proof of proposition 3 give the complete analysis over the entire state space.

Under financial integration, the law of motion for wage is still characterized by equation (15), except that $r_t = r^*$ and $M_t \neq w_t L$. Combine $r_t = r^*$ with equations (1) and (8)-(10) to get

$$w_{t+1}^{\frac{1-\alpha}{\alpha}} = q_{t+1}^{-1} = \frac{\lambda(1-\eta)\rho}{r^*(1-u_t)}\frac{\Phi_t}{\rho} \text{ and } w_{t+1}^{\frac{1}{\alpha}}\frac{\rho}{\Phi_t} = \frac{M_t}{L} = \frac{w_t}{u_t}\delta_t = \left(\frac{w_t}{u_t}\right)^{\frac{1}{\theta}}\left(\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}}, \tag{F.9}$$

$$\Rightarrow \quad \frac{\lambda(1-\eta)\rho}{r^*(1-u_t)}\left(\frac{\Phi_t}{\rho}\right)^{\alpha} = \left(\frac{w_t}{u_t}\right)^{\frac{1-\alpha}{\theta}}\left(\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{(1-\alpha)(1-\theta)}{\theta}} < 0. \tag{F.10}$$

Next, we analyze the impacts of financial integration in three steps.

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autarky is $w_{t+1} = \left(\frac{w_t}{\rho}\right)^{\alpha}$; there is a unique, autarkic steady state, with the wage rate at $w_B = \rho^{-\rho}$.

Step 1: Immediate Impacts of Financial Inflows in Period 0

In period 0, $w_0 = w_A$ and $r_0 = r^* < r_A$. Use equations (4), (7), and (F.9)-(F.10) to derive $\{u_0, \psi_0, a_0, \Phi_0, M_0, w_1\}$ as the implicit functions of r_0. Equations (19) and (F.11)-(F.13) show the responses of ψ_0, M_0, Φ_0, w_1 to the interest rate decline.

$$\frac{\partial \ln M_0}{\partial \ln r_0} = -\frac{1}{(1-\alpha)+\theta\left(\frac{u_0}{1-u_0}+\alpha\frac{\eta-a_0}{1-a_0}\right)} < 0, \tag{F.11}$$

$$\frac{\partial \ln \Phi_0}{\partial \ln r_0} = \frac{\partial \ln \Phi_0}{\partial \ln a_0}\frac{\partial \ln a_0}{\partial \ln u_0}\frac{\partial \ln u_0}{\partial \ln \psi_0}\frac{\partial \ln \psi_0}{\partial \ln r_0} = \frac{\frac{\eta-a_0}{1-a_0}}{\frac{1-\alpha}{\theta}+\left(\frac{u_0}{1-u_0}+\alpha\frac{\eta-a_0}{1-a_0}\right)} > 0, \tag{F.12}$$

$$\frac{\partial \ln w_1}{\partial \ln r_0} = (\underbrace{\frac{\partial \ln M_0}{\partial \ln r_0}}_{\text{Inv. size effect}} + \underbrace{\frac{\partial \ln \Phi_0}{\partial \ln r_0}}_{\text{Prod. effect}})[1-\underbrace{(1-\alpha)}_{\text{neocl. effect}}] = -\frac{\frac{\alpha}{1-\alpha}\left(1-\theta\frac{\eta-a_0}{1-a_0}\right)}{\frac{\theta}{(1-\alpha)}\left(\frac{u_0}{1-u_0}+\alpha\frac{\eta-a_0}{1-a_0}\right)+1} < 0. \tag{F.13}$$

Step 2: Impacts of Financial Inflows in Period $t > 0$

From period $t \geq 1$ on, $r_t = r^*$ and the income level rises over time. Use equations (4), (7), and (F.9)-(F.10) to derive $\{u_t, \psi_t, a_t, \Phi_t, M_t, w_{t+1}\}$ as the implicit functions of w_t. Equations (20) and (F.14)-(F.16) show the responses of $\psi_t, M_t, \Phi_t, w_{t+1}$ to income growth,

$$\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\frac{1-\alpha}{\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)}+\theta} > 0, \tag{F.14}$$

$$\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \frac{\partial \ln \Phi_t}{\partial \ln a_t}\frac{\partial \ln a_t}{\partial \ln u_t}\frac{\partial \ln u_t}{\partial \ln \psi_t}\frac{\partial \ln \psi_t}{\partial \ln w_t} = \frac{\frac{\eta-a_t}{1-a_t}}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)+1} > 0, \tag{F.15}$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = (\underbrace{\frac{\partial \ln M_t}{\partial \ln w_t}}_{\text{Inv. size effect}} + \underbrace{\frac{\partial \ln \Phi_t}{\partial \ln w_t}}_{\text{Prod. effect}})[1-\underbrace{(1-\alpha)}_{\text{neocl. effect}}] = \frac{\frac{\alpha}{1-\alpha}\left(\frac{u_t}{1-u_t}+\frac{\eta-a_t}{1-a_t}\right)}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)+1}. \tag{F.16}$$

Step 3: Long-Run Impacts of Financial Inflows

Let subscript $s \in \{A, F\}$ denote the steady state under autarky versus under financial integration,[32] respectively. Given the binding borrowing constraints, the law of motion

[32] The proof of proposition 3 derives the stability conditions for the steady state.

for wage has the same functional form under the two scenarios, as specified by equations (15) and (10). The steady-state wage rate also takes the same functional form under the two scenarios, $w_s = \left(\frac{\delta_s}{u_s}\frac{\Phi_s}{\rho}\right)^{\frac{\alpha}{1-\alpha}}$. Combine (8) and (10) to get $\delta_s = \left(\frac{w_s}{u_s}\frac{1-\theta}{m}\right)^{\frac{1}{\theta}-1}$. Combine them with (4) and (7)

$$\frac{u_s}{\Phi_s^{\theta}}\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^{\theta} = w_s^{\frac{\alpha-\theta}{\alpha}} \tag{F.17}$$

$$\Rightarrow \frac{\partial \ln LHS}{\partial \ln a_s} = 1-\theta\frac{\eta - a_t}{1-a_t} > 0 \quad \text{and} \quad \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha-\theta}{\alpha}. \tag{F.18}$$

The left-hand side (LHS) of equation (F.17) is an increasing function of a_s, while the relative size of α versus θ determines the monotonicity of the right-hand side (RHS) as a function of w_s.

As financial inflows stimulate the domestic capital formation, the long-run income level is strictly higher than in the autarkic steady state, $w_F > w_A$.

- For $\theta = \alpha$, the wealth-inequality effect and the capital-share effect cancel out so that the RHS of equation (F.17) is constant at unity, $RHS = 1$. Thus, the LHS is also constant at unity in the two steady state, implying that the intangible fraction of investment is identical in the two steady state $a_F = a_A$ and so is the productivity $\Phi_F = \Phi_A$. In this case, the initial fall in Φ_t is exactly offset by the subsequent rises.
- For $\alpha > \theta$, the capital-share effect dominates the wealth-inequality effect so that $\frac{\partial \ln RHS}{\partial \ln w_s} > 0$ holds and so does $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$. Combine them with $w_F > w_A$ to get $a_F > a_A$ and $\Phi_F > \Phi_A$. For $\theta > \alpha$, the opposite applies, $a_F < a_A$ and $\Phi_F < \Phi_A$.

$\square$

Proof of Proposition 3

Proof. The proof consists of three steps.

Step 1: Derive the Law of Motion for Wage under Financial Integration

Combine equations (1) to get the factor price equation,

$$q_t^{\alpha} w_t^{1-\alpha} = 1. \tag{F.19}$$

Iff $q_{t+1}\Phi_t = r^*$, the borrowing constraints are slack, $\Phi_t = 1$. Combine it with (F.19),

$$w_{t+1} = (r^*)^{-\rho}, \tag{F.20}$$

implying that the law of motion for wage is flat in this case.

Iff $q_{t+1}\Phi_t > r^*$, the borrowing constraints are binding and the model dynamics are determined by five equations. First, use $r_t = r^*$ and equation (F.19) to rewrite condition (2) as

$$r_t = \frac{\lambda(1-\eta)q_{t+1}\Phi_t}{1-u_t} = r^*, \;\Rightarrow\; w_{t+1}^{\frac{1}{\rho}} = q_{t+1}^{-1} = \frac{\lambda(1-\eta)\rho}{r^*(1-u_t)}\frac{\Phi_t}{\rho}. \tag{F.21}$$

Second, combine equations (1) with (14) to get (15) as the law of motion for wage. Third, the mass of entrepreneurs cannot exceed the total population in each generation, $\tau_t \leq 1$. Combine it with equations (8) and (10) to get

$$\delta_t = \tau_t^{1-\theta} = \begin{cases} \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}} < 1, & \text{if } w_t < u_t\frac{\mathfrak{m}}{1-\theta}; \\ 1, & \text{if } w_t \geq u_t\frac{\mathfrak{m}}{1-\theta}. \end{cases} \tag{F.22}$$

Given the binding borrowing constraints, equations (7), (10), (15), (F.21), (F.22) specify $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t, which describes the law of motion for wage.

In the case of $\theta = 0$ and $\delta_t < 1$, combine equations (10), (15), and (F.21)-(F.22) to get

$$u_t = \frac{w_t}{\mathfrak{m}}, \tag{F.23}$$

implying that an entrepreneur borrows $\mathfrak{m} - w_t$ and invests $m_t = \mathfrak{m}$ in equilibrium.

Step 2: Derive the Conditions under which the Two Constraints are Binding

For a sufficiently low income level, the mass of entrepreneurs is so low that the borrowing constraints are binding and the MoE constraint is slack. The rise in the current income allows more agents to become entrepreneurs, which may trigger two events: (1) the borrowing constraints become slack; (2) the MoE constraint becomes binding. The law of motion for wage is piecewise and its characterization depends on which event comes first. The MoE constraint matters only when the borrowing constraints are binding.[33]

We first derive the condition for the MoE constraint to be binding, given the binding borrowing constraints. Then, we specify the law of motion for wage in two scenarios.

Define two threshold values,

$$\bar{w}_F \equiv \left[\frac{Z\rho^{\frac{1}{1-\alpha}}}{(r^*)^{\frac{1}{1-\alpha}}}\right]^{\theta} \frac{1-\lambda(1-\eta)}{Z\rho^{\rho}} \quad \text{and} \quad \tilde{w}_F \equiv \frac{\tilde{a}_F}{\eta}\frac{1-\lambda(1-\eta)}{Z\rho^{\rho}},$$

$$\text{where } \tilde{a}_F \text{ is a solution to } \frac{\lambda(1-\eta)\rho}{r^*\left\{1-\frac{\tilde{a}_F}{\eta}[1-\lambda(1-\eta)]\right\}}\left[\left(\frac{\tilde{a}_F}{\eta}\right)^{\eta}\left(\frac{1-\tilde{a}_F}{1-\eta}\right)^{1-\eta}\right]^{\rho} Z = 1. \tag{F.24}$$

Given r^*, solve for the threshold value $\check{\lambda}_F$ as the function of Z such that $\bar{w}_F = \tilde{w}_F$. The dashed curve in the right panel of figure B.7 shows $\check{\lambda}_F$. There are two scenarios.

Scenario 1: for (λ, Z) to the left of the dashed curve, $\lambda < \check{\lambda}_F$ and $\tilde{w}_F < \bar{w}_F$. In this scenario, the MoE constraint becomes binding before the borrowing constraints become slack. By combining $\delta_t = 1$ with equations (4), (7), (10), (15), (F.21), we solve for the threshold value $\tilde{w}_F$ as specified in equation (F.24). By combining $a_t = \eta$ and $\delta_t = 1$ with equations (4), (7), (10), (15), (F.21), we solve for another threshold value $\bar{w}_F \equiv \frac{[1-\lambda(1-\eta)]\rho}{(r^*)^{\frac{1}{1-\alpha}}} > \tilde{w}_F$. The law of motion for wage consists of three parts:

- for $w_t < \tilde{w}_F$, the borrowing constraints are binding and the MoE constraint is slack; the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (4), (7), (10), (15), (F.21) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}}$;

- for $w_t \in (\tilde{w}_F, \bar{w}_F)$, both the borrowing constraints and the MoE constraint are bind-

[33] When the borrowing constraints are slack, the agents who can overcome the MIR do not have strong incentive to be entrepreneurs; hence the MoE constraint is irrelevant.

ing; the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (4), (7), (10), (15), (F.21) and $\delta_t = 1$;

- for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant; the law of motion for wage is characterized by equation (F.20).

Scenario 2: for (λ, Z) to the right of that dashed curve, $\lambda > \check{\lambda}_F$ and $\bar{w}_F < \tilde{w}_F$. In this scenario, the MoE constraint never binds as long as the borrowing constraints are binding. By combining $a_t = \eta$ and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}}$ with equations (4), (7), (10), (15) and (F.21), we solve for the threshold value $\bar{w}_F$ as specified in equation (F.24). The law of motion for wage consists of two parts:

- for $w_t < \bar{w}_F$, the borrowing constraints are binding and the MoE constraint is slack; the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (4), (7), (10), (15), (F.21) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}}$;
- for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant; the law of motion for wage is characterized by equation (F.20).

The solid (dashed) curves in figure B.8 shows the laws of motion for wage under financial integration (under autarky), given $\lambda > \check{\lambda}_F$ and (λ, Z) in the respective region of the right panel in figure B.7. Figure F.13 shows the laws of motion for wage, given $\lambda < \check{\lambda}_F$ and (λ, Z) in the respective region of the right panel of figure B.7.

Step 3: Derive the Threshold Conditions for Multiple Steady States

Under financial integration, multiple steady states arise if there exists an unstable steady state. Let X_U denote the value of variable X_t at the **u**nstable steady state. As shown in equation (F.20), for $w_t \geq \bar{w}_F$, the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$. Thus, if there exists an unstable steady state, $w_U < \bar{w}_F$ must hold and the borrowing constraints must be binding there. How about the MoE constraint at the unstable steady state?

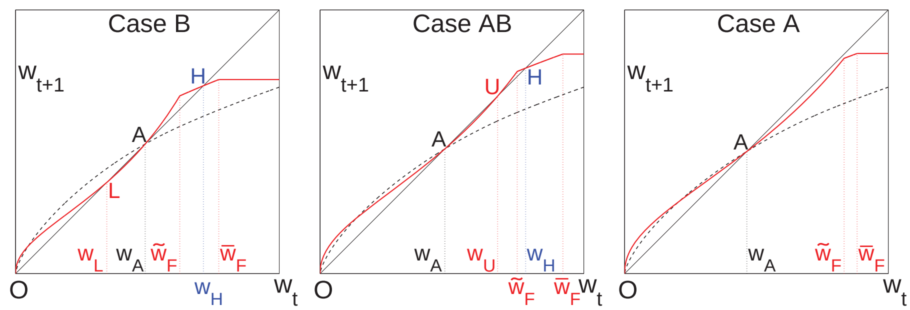

Figure F.13: Laws of Motion for Wage under Financial Integration: $\theta < \alpha$ and $\lambda < \check{\lambda}_F$

If both the MoE constraint and the borrowing constraints are binding, combine equations (4), (7), (10), (15), (F.21) with $\delta_t = 1$,

$$\frac{\partial w_{t+1}}{\partial w_t}\frac{w_t}{w_{t+1}} = \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \frac{1}{1+\rho\left(u_t\frac{1-\eta}{1-a_t} + \frac{\eta - a_t}{1-a_t}\right)} < 1. \tag{F.25}$$

Thus, if there is a steady state where the MoE constraint is binding, the slope of the law of motion for wage is strictly less than unity there. Then, this steady state must be stable.

To sum up, if there exists an unstable steady state, the borrowing constraints must be binding and the MoE constraint must be slack there. Combine equations (4), (7), (10), (15), (F.21) with $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$ to get equation (F.16) specifying the slope of the law of motion for wage in logarithm. In a threshold case where the law of motion for wage has a slope equal to unity at a steady state, combine $\frac{\partial \ln w_{t+1}}{\partial \ln w_t}|_{w_{t+1}=w_t=w_U} = 1$ with (F.16) to get a_U as a function of parameters,

$$\frac{1}{1-\frac{\boldsymbol{a_U}}{\eta}[1-\boldsymbol{\lambda}(1-\eta)]}\frac{1-\frac{\theta}{\alpha}}{1-\theta} - \frac{1-\eta}{1-\boldsymbol{a_U}} = \frac{1-\alpha}{\alpha}. \tag{F.26}$$

Next, we specify the conditions for the existence of multiple steady states, given $r^* = r_A$.

3.1. Parameter Constellation for Multiple Steady States in the (λ, θ) Space

Combine equation (F.26) with $a_U \leq \eta$ and $\lambda \leq 1$ to get $\tilde{\lambda}_F \equiv \min\{\frac{\alpha-\theta}{(1-\theta)(1-\eta)}, 1\}$ as the threshold value shown by the downward-sloping curve in the left panel of figure B.7. For (λ, θ) in region U, equation (F.26) does not have a solution with $a_U \in (0, \eta]$, implying that the autarkic steady state is still the unique, stable steady state under financial integration. For (λ, θ) in region M, multiple steady states may arise as proved below.

3.2. Parameter Constellation for Multiple Steady States in the (λ, Z) Space

Given (λ, θ) in region M of the left panel of figure B.7, multiple steady states arise in three scenarios with (λ, Z) in region BC, B, and AB of the right panel in figure B.7, respectively.

Scenario BC: for (λ, Z) in region BC, $\lambda > \bar{\lambda}_A$ so that the borrowing constraints are slack in the autarkic steady state and $r_A = \rho$, as shown in the right panel of figure F.11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (L) arise, $w_L < w_U < \bar{w}_F$, as shown in the upper-right panel of figure B.8. In the threshold case, the law of motion for wage is tangent with the 45° line at the unstable steady state, which can be characterized by two conditions. First, equation (F.26) specifies a_U as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, combine equations (F.21), (10), (15), (4), (7) with $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathrm{m}}\right)^{\frac{1}{1-\theta}}$, $w_{t+1} = w_t = w_U$, and $r^* = r_A = \rho$ to characterize the unstable steady state

$$\left[\frac{\lambda(1-\eta)}{1-\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\right]^{\frac{\theta-\alpha}{1-\alpha}} \frac{a_U}{\eta}[1-\lambda(1-\eta)] = \left\{\left[\left(\frac{a_U}{\eta}\right)^{\eta}\left(\frac{1-a_U}{1-\eta}\right)^{1-\eta}\right]^{\rho} Z\right\}^{1-\theta}, \tag{F.27}$$

which specifies a_U as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Equations (F.26) and (F.27) specify Z as a function of λ, as shown by the border between region BC and C in the right panel of figure B.7.

Scenario B: for (λ, Z) in region B, $\lambda < \bar{\lambda}_A$ so that the borrowing constraints are binding in the autarkic steady state and $r_A < \rho$, as shown in the right panel of figure F.11. Financial integration destabilizes the autarkic steady state, while two stable steady states (L and H) arise, $w_L < w_A < w_H$, as shown in the upper-left panel of figure B.8 and in the left panel of figure F.13. In the threshold case, the law of motion for wage is tangent with the 45°line at the autarkic steady state, which can be characterized by two conditions. First, $a_A = a_U$ is a function of $\{\lambda, \alpha, \theta, \eta\}$, according to equation (F.26). Second, combine $w_{t+1} = w_t = w_A$ with equations (4), (7), (17), (15) to characterize the autarkic steady state,

$$u_A^{\frac{1}{1-\theta}} = w_A \frac{1-\theta}{\mathfrak{m}} = Z\Phi_A^{\rho}, \Rightarrow \left\{ \frac{a_A}{\eta}[1-\lambda(1-\eta)] \right\}^{\frac{1}{1-\theta}} = Z\left[\left(\frac{a_A}{\eta}\right)^{\eta} \left(\frac{1-a_A}{1-\eta}\right)^{1-\eta} \right]^{\rho} \tag{F.28}$$

which specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. (F.26) and (F.28) specify Z as a function of $\lambda \in (0, \tilde{\lambda}_F)$, shown as the border between region B and AB in the right panel of figure B.7.

Scenario AB: for (λ, Z) in region AB, $\lambda < \bar{\lambda}_A$ so that the borrowing constraints are binding in the autarkic steady state and $r_A < \rho$, as shown in the right panel of figure F.11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (H) arise, as shown in the upper-middle panel of figure B.8 and in the middle panel of figure F.13; as explained in Step 2, the law of motion for wage is piecewise and consists of two or three parts; multiple steady states arise if at least one kink point in the law of motion for wage is above the 45° line.

- **Scenario AB.R:** for (λ, Z) in region AB and to the **r**ight of the dashed curve, $\lambda > \check{\lambda}_F$ and the law of motion for wage has one kink point at $w_t = \bar{w}_F$, where $\bar{w}_F$ is specified in equation (F.24). For $w_t \geq \bar{w}_F$, the borrowing constraints are slack, $\Phi_t = 1$, and the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$. In the threshold case, the kink point is on the 45°line and accordingly, two conditions hold. First, combine $r^* = r_A$ and $w_{t+1} = (r^*)^{-\rho} = w_t = \bar{w}_F$ with equations (F.24) and (F.28) to characterize the kink point

$$\frac{1}{\frac{1-\frac{a_A}{\eta}}{\lambda(1-\eta)} + \frac{a_A}{\eta}} = \left(\frac{a_A}{\eta} \frac{1}{\Phi_A^{(1-\theta)\rho}} \right)^{\frac{1-\alpha}{\alpha-\theta}}, \text{ where } \Phi_A = \left(\frac{a_A}{\eta}\right)^{\eta} \left(\frac{1-a_A}{1-\eta}\right)^{1-\eta}. \tag{F.29}$$

It specifies a_A as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, equation (F.28) specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (F.28) and (F.29) jointly specify Z as a function of λ. It is shown as the part of the border between region AB and A, which is to the right of the dashed curve in the right panel of figure B.7.

- **Scenario AB.L:** for (λ, Z) in region AB and to the **l**eft of the dashed curve, $\lambda < \check{\lambda}_F$ and the law of motion for wage has two kinks at $w_t = \tilde{w}_F$ and $w_t = \bar{w}_F$. In the threshold case, the kink point at $w_t = \tilde{w}_F$ is on the 45°line, where $\tilde{w}_F$ is specified in equation (F.24). In this case, three conditions hold. Let $\tilde{X}_F$ denote the value of variable X_t at that kink point. First, combine equations (F.24) and (F.28) with $r^* = r_A$ to characterize the kink point

$$\frac{1-\frac{a_A}{\eta}[1-\lambda(1-\eta)]}{1-\frac{\tilde{a}_F}{\eta}[1-\lambda(1-\eta)]} \left[\left(\frac{\tilde{a}_F}{a_A}\right)^{\eta} \left(\frac{1-\tilde{a}_F}{1-a_A}\right)^{1-\eta} \right]^{\rho} \left\{ \frac{a_A}{\eta}[1-\lambda(1-\eta)] \right\}^{\frac{1}{1-\theta}} = 1, \tag{F.30}$$

which specifies $\tilde{a}_F$ as a function of a_A and $\{\lambda, \alpha, \eta, \theta\}$. Second, use (4), (7), (10), (15), (F.21) with $\delta_t = 1$ and $w_{t+1} = w_t = \tilde{w}_F$ to get the kink point as a steady state

$$\left(\frac{a_A}{\tilde{a}_F}\right)^{\eta} \left(\frac{1-a_A}{1-\tilde{a}_F}\right)^{1-\eta} = \left\{ \frac{a_A}{\eta}[1-\lambda(1-\eta)] \right\}^{\frac{1-\alpha}{\alpha(1-\theta)}} \left\{ 2-\frac{a_A}{\eta}[1-\lambda(1-\eta)] \right\}^{\frac{1}{\alpha}} \tag{F.31}$$

which specifies $\tilde{a}_F$ as a function of a_A and $\{\lambda, Z, \alpha, \eta, \theta\}$. Third, (F.28) specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, (F.28), (F.30), and (F.31) jointly specify $\{Z, \tilde{a}_F, a_A\}$ as the functions of λ. The relationship between Z and λ is shown as the part of the border between region AB and A in the right panel of figure B.7.

□

Proof of Corollary 1

Proof. $\underline{Z}_F$ and $\bar{Z}_F$ are specified in the proof of proposition 3 as the functions of λ. They are shown as the upper and lower borders of region AB in the right panel of figure B.7.

Define $\hat{r}^*$ as a threshold value such that, for $r^* = \hat{r}^*$, the law of motion for wage under financial integration is tangent with the 45° line where a_U denote the steady-state value of a_t. Use equation (F.26) to solve a_U as a function of λ and other parameters. Combine $w_{t+1} = w_t = w_U$ with equations (5), (7), (10), (15), (F.21) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathrm{m}}\right)^{\frac{1-\theta}{\theta}}$ to get

$$\hat{r}^* \equiv \frac{\lambda(1-\eta)\rho}{1-\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\left[\frac{(\Phi_U^{\rho}Z)^{1-\theta}}{\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\right]^{\frac{1-\alpha}{\alpha-\theta}}, \text{ where } \Phi_U = \left(\frac{a_U}{\eta}\right)^{\eta}\left(\frac{1-a_U}{1-\eta}\right)^{1-\eta}.$$

Define $\tilde{r}^*$ as a threshold value such that, for $r^* = \tilde{r}^*$, the law of motion for wage under financial integration has the kink point on the 45° line. At the kink point, $a_k = \eta$, $\Phi_k = 1$, $u_k = 1-\lambda(1-\eta)$, and $w_{t+1} = w_t = w_k = \bar{w}_F$. Combine them with equations (5), (7), (10), (15), (F.21) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathrm{m}}\right)^{\frac{1-\theta}{\theta}}$ to get $\tilde{r}^* \equiv \rho\left[\frac{Z^{1-\theta}}{1-\lambda(1-\eta)}\right]^{\frac{1-\alpha}{\alpha-\theta}}$. □

Proof of Proposition 4

Proof. Consider the case where the borrowing constraints are binding under autarky. As aggregate investment is funded by domestic saving, the unit cost of investment is constant and so are other endogenous variables, as specified by equations (E.1). Under assumption 2, $\psi_A < 1$ implies that the borrowing constraints are binding. According to equations (E.1),

$$\frac{\partial \psi_A}{\partial \lambda} = \frac{1-\eta}{1-\tau} > 0, \quad \frac{\partial \ln a_A}{\partial \ln \lambda} = \frac{\lambda(1-\eta)}{1-\lambda(1-\eta)} > 0, \quad \frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_A}{1-\eta}\frac{\partial \ln a_A}{\partial \ln \lambda} > 0. \quad \text{(F.32)}$$

As the productivity is constant, the law of motion for wage is log-linear with the slope less than unity, according to equations (E.2). Thus, there exists a unique, autarkic steady state. The social rate of return is defined by equation (F.7). In the autarkic steady state, $w_{t+1} = w_t = w_A$ so that $q_A\Phi_A = \rho$ and $r_A = \psi_A q_A \Phi_A = \psi_A\rho < \rho$. □