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T is Inaccessible

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1. Introduction

This note advances an entirely materialistic and deterministic view of the limits of physical theories.

The following sections cover:

a) A demonstration that any physical theory that is accessible to human intelligence is necessarily an approximation;

b) Recognition that physical theories based on probability and randomness are therefore approximations;

c) A conjecture as to how the randomness and unpredictability that we observe arises from an underlying and inaccessible theory;

d) A conjecture about the origin of unpredictable spontaneous quantum events such as unstable isotope decay.

2. Any Accessible Theory is an Approximation

In this note, we use the term *accessible theory* to mean a theory that can be expressed by one human to another and whose predictions from a given state can be computed in finite time by a Turing machine. (If the state $S(t_0)$ is known, can we compute $S(t)$ in a time less than $t-t_0$?) If a theory does not meet these criteria, it is of no use: either we cannot explain it to each other or we cannot predict results that can be tested experimentally. A theory must be accessible in this sense before it can even be considered falsifiable in the sense of Karl Popper.

There are indications that a quantum computer might be able to solve certain problems that a classical Turing machine cannot solve [6]. This extends but does not change the present argument.

We note that a Turing machine cannot compute real numbers as such, but only fractional numbers to a specific precision, because its representation of numbers is digital. Thus, a Turing machine can never compute exact results involving continuous variables. This line of argument leads to the conclusion that theories involving real numbers are intrinsically problematic [8], but this only strengthens our subsequent argument. For the purpose of predicting experimental results, a specific digital precision is sufficient. Changing the argument to use an analogue computer would not help, because it too has physically limited precision.

The following argument is the logical descendant of one satirised in 1895 by the mathematician Charles Dodgson, writing as Lewis Carroll, in “Sylvie and Bruno Concluded”:

> We actually made a map of the country, on the scale of a mile to the mile! [...] It has never been spread out, yet [...] the farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well.

Let us consider a theory **T**, and stipulate that it can be expressed by at least one human to another, which also implies that it can be expressed in a finite number of symbols. **T** is rather special because it is an exact[1] theory of everything physical. This is broader than a specific "theory of everything" unifying general relativity and quantum mechanics, but it is not concerned with philosophical "why" questions about existence and experience.We will show that **T** cannot be accessible.

For this theory **T** to be accessible, we would need to write the Turing machine program to compute it and make testable predictions in finite time. Specifically, if *S* is the state of the universe, we must compute $S(t) = \mathbf{T}(S(t_0), t)$ in a time less than $t\text{-}t_0$. Since **T** is a theory of *everything*, we will need at a minimum one bit in the program or data to represent the starting condition of every bit in the universe at the instant that we start the computation. Let us call that *N* bits. (If we do not think the universe is finite, we can limit this to the observable universe[2]; this does not change the argument.) We stipulate that this bit string includes no redundancy; it either *is* the state of the universe or it maps 1:1 onto that state. The rest of the program will be another *P* bits, say, and *P* is finite, since it encodes **T**. The Turing machine itself will be made out of something; let us call that another *M* bits.

This assumes that matter is made out of bits, as conjectured by von Weizsäcker, Wheeler and others, sometimes termed "it from bit" [15]. A more extreme option is explored in [7] but this does not exclude the "it from bit" conjecture. For our purposes, this is a safe assumption: a bit has only two states, which is the simplest property a particle of matter could have. The Turing machine itself cannot need less than one bit's worth of matter to store each bit in $N+P$, as well its intrinsic *M* bits.

Therefore, we need at least $N+P+M$ bits to build and program our Turing machine, and the state of the entire universe is *N* bits, which must include the Turing machine (and its presumably finite programmer). Furthermore, the machine must be able to store N' bits, the calculated new state of the universe, after each step of its program, but these bits might overwrite the original *N* bits, and N' might be less than *N*, so we do not consider them further. However, $N+P+M > N$.

Therefore the Turing machine and its data and program are too big to fit into the universe[3], so they cannot exist, so **T** cannot be an accessible theory. *QED*.

There is a possible objection to this argument that the bit string of length $N+P+M$ may be compressible. If our objective was to transmit this bit string from one Turing machine to another, this would be relevant (and each machine would need to contain extra bits to express the compression or decompression algorithm). However, the entire string is needed in order to express the complete state of the universe and start calculation. Alternatively, we could replace *N* in the above argument by N_c+C, where N_c is the compressed string and *C* is a bit string defining the decompression algorithm. Since the universe itself could be similarly compressed, the conclusion is the same.

First corollary: Any accessible theory is necessarily expressed in fewer bits than **T** since it can be stored in an actual Turing machine contained by the universe. Since it has fewer bits that **T** it is intrinsically unable to express all of **T** and can produce less precise predictions than **T**, i.e., it is an approximation. In other words, perfect prediction is impossible; all predictions must be approximate with limited precision.

Second corollary: The history of science is therefore the history of successive approximations. Thus, both quantum mechanics and general relativity are approximations. Any future theory that is accessible in the sense defined here will also necessarily be an approximation.

1 We do not consider whether it is a *complete* theory in Goedel's sense.

2 The observable universe is necessarily finite, since it consists of everything the observer at a given point can detect and of nothing else.

3 There may be a much lower limit to feasible computation than argued here [12].

We can illustrate the point by observing that any theory involving differential equations can only be used to compute predictions by applying numerical analysis, which is an approximate method by construction. In physics, lattice gauge QCD computations are explicitly approximate.

This does not, of course, invalidate all science with one blow. On the contrary, it strongly confirms that the scientific method is the correct one: progress from one theory to another, more precise, by confronting theoretical predictions with experiment. It also creates the context for observations such as the inconsistency between general relativity and quantum mechanics: neither of them is quite right, so inconsistency is unsurprising.

The notion that all useful theories of physics are approximations is not new; see for example the work of Thomas Brody [3].

3. The Notions of Probability and Randomness are Approximations

From the above argument, we see that any theory of physics that depends on probability and/or randomness is necessarily an approximation. In fact, approximation is built into the very definition of probability. If we toss a coin a million times, the tallies of heads and tails are unpredictable, even though we “expect” to get 500,000 heads. However, we will not be surprised if we only get 499,999, or even if one toss ends up with the coin balanced on its edge. It is also commonly accepted in physics that one *might* observe a violation of the Second Law of Thermodynamics. It is, after all, only an approximation.

The philosophical basis of the notion of probability is very unclear. *Observationally* we see systematic patterns in events that are individually unpredictable, and we know that descriptions such as the binomial distribution are generally accurate when we have large numbers of roughly identical events. We know from the success of statistical mechanics that these distributions are even more accurate when trillions of events or particles are involved, and this carries through to the success of quantum mechanics. So the theory of probability as a continuous value $0 \le p \le 1$ (and the concomitant randomness of particle physics events) is one of our most useful theories, despite its strange relationship with determinism [4, 5, 16]. By the argument of the previous section, it is only an approximation, albeit an extremely useful one.

We briefly digress to observe that the idea of probability and the associated notion of randomness seemingly evolved as an adaptive strategy to deal with early animals’ inability to predict complex events. So although it appears fundamental to us, it originated as a useful biological artefact directly linked to unpredictability. The notions of “expect” and “surprise” are fallout from that artefact.

4. How Can a Deterministic Universe Exhibit Randomness?

Our inaccessible theory **T** is supposed to be complete and exact; if it is exact, there is no place for randomness and probability. How could **T** deterministically generate properties that look like randomness in our eyes?

We now conjecture a sufficient explanation. We have mathematical tests for randomness, and one remarkable fact is that although they are determined for all time, the digits of π pass many randomness tests: if you take an arbitrary part of the expansion of π, that string of digits is effectively indistinguishable from a random string of digits, according to such tests. This is not the same as asserting that the digits of π *are* random [1, 2], but it is sufficient for the present argument. To give a simple illustration, if we pick a random ten digit number, the probability that we pick 1415926535 is 10^{-10} regardless of the fact that these are the first ten digits of the fractional part of π in base 10. Conversely, if we observe the values 1415926535 or binary 00100100001111110110101010001000010 in nature, we have no grounds to assume that those digits have been extracted from any particular part of the expansion of π. Of course, another transcendental number than π might serve as well, but π is sufficient for the argument.

Therefore, if **T** involves the digits of π, it is able to describe a universe in which the property we call "randomness" appears in its trillions of predictions of exactly what events occur at which points in space-time. If we have such apparent randomness in nature, an accessible theory of probability will be a useful approximation providing useful predictions. We can never access **T**, but if it involves π, it could produce what we perceive as randomness and probability when trillions of individual events are concerned. The fact that π appears in practically every existing theory of physics is reassuring; we would be very surprised if it did *not* appear in **T**. This is then sufficient to explain how apparent randomness could appear in quantum mechanics and indirectly in the macroscopic world.

It is slightly ironic that one of the most successful computational techniques used in quantum mechanics is the Monte Carlo method, in which pseudo-random numbers are used to extract statistical predictions from a probabilistic theory. This even extends to the estimation of "background" event rates in particle physics experiments [13]. The choice of pseudo-random number generators for Monte Carlo methods must avoid RNGs that *are* predictable, for obvious reasons [11]. If the above argument is correct, such pseudo-random numbers are a deterministic result of **T**. This does not make the whole argument circular, however, because (by our definition) random numbers are not predictable by any accessible theory.

If we accept that apparent randomness emerges deterministically from **T**, and therefore that the theory of probability is no more than a useful approximation, we should consider whether the most extreme deductions and predictions from probability and thermodynamics are to be trusted. Consider for example a prediction that a Boltzmann brain is reasonably likely to form within 10 to the power of 10^{68} years [9]. Can we be sure that the theory of probability is valid for such a value?

5. Explaining Unpredictable Decays

The following text is highly speculative.

Quantum mechanics posits that some physical events, e.g., nuclear decay, are intrinsically random and unpredictable. This is very strongly confirmed by experiment, which is of course deeply disturbing to determinists.

What follows addresses the roles of randomness, probability and unpredictability in quantum mechanics. It does not challenge anything in quantum mechanics, beyond the claim that quantum mechanics is an approximation like every other accessible theory. In particular, nothing here intrinsically challenges quantum entanglement or non-locality. (However, the notion of **T** seems compatible with superdeterminism as an explanation of entanglement [10].)

We observe two types of quantum events: spontaneous events (such as radioactive nuclear decay) and induced events (such as occur in particle physics experiments, where we intentionally collide particles together). To be complete we could add semi-spontaneous events (such as when a spontaneous cosmic ray particle collides with an intentionally placed detector) but for the present argument, we do not need to consider this as a distinct case.

In induced events, we do not consider the *timing* of the event as random – it happens in a pre-defined region of space-time (e.g., the collision point of a collider as the particle beams pass by each other). The proxy for the event's probability is the cross section that applies to the colliding particles in question. (In classical terms, the closer the particles are to exactly the same point in space-time, the higher the event probability. When they are close enough, the event occurs. That is a neo-classical view, but so is the concept of cross section.) Quantum cross sections are in fact more subtle because they also depend on the energy of the particles concerned as well as their proximity. But whether an actual event happens is unpredictable and its probability is effectively measured as a cross section.

In spontaneous decay events, the proxy for the event's probability is the rate (trivially related to the half-life[4]). The interesting fact is that we always see a Poisson distribution, or put more simply: the probability of an event per unit time is constant. If we were not completely used to it, following its initial discovery by Ernest Rutherford in 1900, that would seem like a strange fact in need of explanation. Among others, James Jeans noted its impact on determinism (quoted in [14]).

The deterministic explanation must be that "spontaneous" events are not spontaneous; they are determined by a mechanism that we do not know. Now let us make a conjecture about this mechanism. This is not the only possible conjecture, but it is one that sits well with the above discussion of the origin of randomness. We conjecture that spontaneous decays are the result of collisions with undetected particles which we will call *hypothons*. Assume that hypothons carry just enough energy to move a quantum system from one metastable state to another. Suppose that a constant background of hypothons fills the universe. Presumably they travel very close to the speed of light, in all directions, and their rate (or flux) is locally constant and isotropic. ("Locally" means locally within our observable universe, perhaps.)

Thus, the rate at which hypothons pass through any finite volume is constant. If a hypothon collides with some metastable quantum state, it moves it into another state, with a cross section that is characteristic of the hypothon and the original state. That will *automatically* produce "spontaneous" state changes that fit a Poisson distribution, with the rate proportional to the relevant cross section and the hypothon flux (neither of which we know).

Obviously, we cannot predict the arrival of hypothons any more than we can predict nuclear decay. We will not speculate what hypothons might be. But in induced events, whether a collision produces an event depends on the particle flux and the relevant cross section. In a spontaneous event, whether the event occurs depends on the hypothon flux and the relevant cross section. The distinction between spontaneous and induced events disappears; they are all collisions.

We need not assume any non-deterministic behaviour in this picture. Whether a quantum decay event occurs at a particular point in space-time now always depends on whether a collision event occurs there, and that could be determined by all previous quantum events together. To obtain this property in a way that is experimentally indistinguishable from "randomness", we have simply hypothesised a universal background of undetected hypothons. Assume that their current state is determined by all previous states of the universe: in that case, we may assume that the current state of all quanta, not just the hypothons, is so determined.

So, what determines the current instantaneous state of everything? Let us repeat our conjecture: the theory **T** that determines the state of the universe is, among other things, a function of π as well as other laws of physics. Therefore, what we perceive as unpredictable randomness is in fact derived, by an unthinkably complex calculation of **T** that only the universe itself can perform, from the digits of a transcendental number.

Upon these conjectures, there is no mystical randomness about unpredictability; there is simply incalculable complexity. Probability and randomness are manifestations of unpredictability, which is itself a manifestation of incalculability.

We may review the story of Schrödinger's cat in this light. Whether and when the lethal atom decays inside the box is determined by whether a hypothon arrives at the right time and place. This is determined by **T** but is unpredictable. No magic occurs when an observer opens the box; observing the result does not affect the result, it simply reveals it. (This is distinct from the uncertainty principle, which speaks of an act of observation that directly interacts with the event being observed. Nothing here affects the uncertainty principle, within the approximation where probability is a valid construct.)

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4 For a decay rate k per unit time, half-life = $ln(2)/k$

Some interesting physical questions are raised by the hypothon conjecture:

- Can we learn anything by re-interpreting spontaneous decay rates as being proportional to hypothetical cross sections, and comparing the various unstable quantum states in that light?
- Can we place any limits on the energy density of the hypothon flux?

Finally, we should clarify that this conjecture does not address every probabilistic aspect of quantum theory. For example, the measurement problem, quantum tunnelling, virtual particles and proton decay are not addressed.

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