Uphill Capital Flows, Intangible Investment, and Allocative Efficiency

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Abstract

This paper proposes a novel channel, i.e., *within-firm intangible-tangible investment composition*, through which financial capital flows may have non-trivial implications to allocative efficiency and productivity over time. Consider a model where only the tangibles can be pledged as collateral for loans. Heterogeneous pledgeability creates the unit-cost differential between tangible and intangible investments, which distorts within-firm allocative efficiency. By lowering the interest rate and stimulating the domestic capital formation, the inflows of cheap foreign funds change the unit-cost differential and trigger within-firm investment reallocation. It turns out that allocative efficiency and the productivity of capital formation decline in the short run, while they rise over time. The more elastic the domestic investment, the larger the financial inflows, the more likely the productivity eventually exceeds its initial level. Thus, market frictions and regulatory requirements that hamper entrepreneurial entry may reduce the elasticity of domestic investment, which undermines the productivity gains from financial inflows.

Keywords: financial frictions, financial integration, heterogeneous pledgeability, intangible investment, minimum investment requirements, wealth inequality
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Since the late 1990s, intangible capital (e.g., computerized information, patents and brands, and organizational capital) has become increasingly important for production in advanced economies. In the United States, the United Kingdom, France, Finland, and Sweden, intangible investments exceeded the tangibles between 2000-2013 (Corrado et al., 2018). As the intangibles are hard to liquidate, they usually do not serve as collateral for loans (Döttling et al., 2018; Falato et al., 2018), while the tangibles are commonly pledged as collateral (Eisfeldt and Rampini, 2009). Given heterogeneous pledgeability between the intangibles and the tangibles, a growing literature has explored the implications of rising intangible capital to allocative efficiency in the closed economy (Dell'ariccia et al., 2017; Döttling and Perotti, 2017; Giglio and Severo, 2012; Lopez and Moppett, 2018; Wang, 2017). By introducing heterogeneous pledgeability in *the open-economy setting*, we identify a novel channel, i.e., *within-firm intangible-tangible investment composition*, through which the recent financial globalization may have non-trivial impacts on allocative efficiency and productivity in advanced economies.

The recent financial globalization has a prominent feature, i.e., financial capital flows are "uphill" from poor to rich countries (Prasad et al., 2006). The literature has shown that uphill financial flows can be an equilibrium outcome, if rich countries are more financially developed than poor countries (Caballero et al., 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza et al., 2009; von Hagen and Zhang, 2014). Uphill financial flows are regarded as one of the causes for the declining interest rates in advanced economies[1] (Bernanke, 2011; Caballero and Krishnamurthy, 2009). Consider a model where both tangible and intangible investments are essential for the project of capital formation, while only the tangibles can serve as collateral. The inflows of cheap foreign funds boost up the collateral value of tangibles, which lowers the unit cost of tangibles relative to that of intangibles.[2] Thus, entrepreneurs shift the investment from the intangibles towards the tangibles. This prediction is opposite to the empirical evidence of rising intangible-tangible investment ratio.[3]

We argue that, besides the interest rate effect, financial inflows also stimulate the domestic capital formation and the resulting decline in the marginal product of capital (MPK, hereafter) gradually undermines the collateral value of tangibles over time. Upon financial inflows, the interest rate effect dominates the decreasing MPK effect so that the collateral value of tangibles rises in the net term. Hence, Hence, entrepreneurs shift the investment towards the tangibles, which worsens allocative efficiency in the short run. However, along the convergence path, the interest rate is constant at the world level, while the decreasing MPK effect strictly raises the

[1] The recent literature has two major hypotheses for the global decline of interest rates. The global savings glut hypothesis (Bernanke, 2005) relies on the excessive supply of savings from fast-growing emerging markets, while the secular stagnation hypothesis (Summers, 2014) focuses on the paucity of investment opportunities in advanced economies. Rachel and Smith (2017) and Crews et al. (2016) compare the empirical plausibility of the two hypotheses. See CEA (2015) for a comprehensive literature survey on alternative causes.

[2] As the intangibles are financed fully with internal funds, the unit cost of intangible is constant at one. As entrepreneurs can borrow up to a fraction of the present value of tangibles and only use own funds to cover the rest, the unit cost of tangibles is less than one and negatively related to the collateral value of tangibles. Under autarky, the unit-cost differential induces entrepreneurs to invest inefficiently more (less) in the tangibles (intangibles).

[3] The rise in the intangible-tangible investment ratio can be a natural consequence of technological shifts. Taking that as given, we analyze the impacts of an exogenous interest rate change on this ratio.

unit cost of tangibles. Hence, entrepreneurs shift the investment towards the intangibles, which improves allocative efficiency over time. In this paper, allocative efficiency is measured by the output-input ratio of the individual project for capital formation. For simplicity, we call it the within-project or within-firm productivity, which is more akin to the concept of total factor productivity in growth accounting. Here, *heterogeneous pledgeability gives rise to the unit-cost differential between intangible and tangible investments. By changing the unit-cost differential, financial inflows reduce the within-firm allocative efficiency and productivity in the short run and raise them over time.* This is the first finding of our paper.

Can the productivity eventually exceed its initial level? For a given interest rate decline, the more elastic the domestic investment, the larger the financial inflows and domestic investment expansion, the stronger the decreasing MPK effects, the smaller the initial fall and the larger the subsequent rises in productivity, the more likely the productivity exceed its initial level. This paper highlights the critical role of the endogenous extensive margin in determining the elasticity of domestic investment. Besides the collateral constraints, we make two more assumptions to characterize this mechanism analytically. First, the individual project is subject to a minimum investment requirement (MIR, hereafter). Second, agents differ in net wealth.

If the collateral constraints are binding, only those with sufficiently high net wealth can meet the MIR and run the project, and they are called *entrepreneurs*. The collateral constraints and the MIR jointly act as an entry barrier, which endogenizes the mass of entrepreneurs. The smaller the wealth dispersion, the stronger the mass of entrepreneurs responds to the changes in the interest rate and in aggregate income, the more elastic the domestic investment along the extensive margin. *In the presence of financial inflows, the productivity eventually exceeds its initial level, if the elasticity of domestic investment exceeds the inverse of the capital share in the production function.* This is the second finding of our paper.

Besides financial frictions and the MIR, other market frictions and regulatory requirements that hamper entrepreneurial entry (Klapper et al., 2006) may affect the elasticity of domestic investment and the productivity dynamics via the same mechanism featured in our model. Groth and Khan (2010) show that U.S. manufacturing industries differ substantially in the investment elasticity with respect to the shadow value of capital. The logic of our model can be embedded into a multi-sector setting with sector-specific investment elasticity. Ceteris paribus, in the sector with a higher investment elasticity, financial inflows may have a stronger decreasing MPK effect, which raises the intangibles-tangibles ratio and the productivity to a larger extent in the long run. Thus, the sector-specific investment elasticity may have a potential for explaining the cross-sector differences in the dynamics of intangible investment and productivity.

The first two findings are obtained in the case of a marginal interest rate decline under financial integration. By moving from the marginal to the global analysis, we also study the productivity dynamics over the entire state spaces and obtain the third finding, i.e., *the world interest rate change may have disproportional and asymmetric impacts on productivity, if it shifts the model economy from the equilibrium with multiple steady states to the one with a unique steady state.*[4] This finding offers an alternative perspective for addressing the impacts

[4] As shown in Matsuyama (2004) and Zhang (2017), the endogenous extensive margin amplifies the responses of domestic investment to financial integration. Thus, multiple steady states may arise if the world interest rate is

of the current U.S. interest rate hikes on intangible investment and productivity, given that the world interest rate has stayed at the record low level for nearly a decade.[5]

Related Literature The literature has identified various channels through which financial integration improves allocative efficiency. By facilitating international risk sharing, financial integration induces countries to shift the investment portfolio from safe, low-yield capital towards riskier, high-yield capital (Obstfeld, 1994). By augmenting domestic investment and activating more sectors, capital inflows improve domestic risk-sharing and promote riskier, high-return investment (Acemoglu and Zilibotti, 1997). By improving financial terms, financial integration encourages credit-constrained firms to invest in technology, while the intensified competition induces non-constrained firms to do the same. Here, financial integration raises aggregate productivity via the *cross-firm* reallocation effect and the pro-competition effect (Varela, 2018). By affecting the efficiency of financial intermediaries, financial integration changes the aggregate composition of investment projects and affects aggregate productivity (Alessandria and Qian, 2005; Tressel and Verdier, 2011). By assuming that firms and/or projects differ exogenously in productivity,[6] these articles feature *cross-firm and/or cross-project* resource reallocation as the channel through which financial integration has the *positive effect* on aggregate productivity. In our model, agents have access to the same technology for capital formation and *within-firm* investment reallocation serves as the key channel through which financial integration has the *opposite short-run versus long-run effects* on firm-level productivity.

In Mendoza and Yue (2012), domestic and imported input varieties are imperfect substitutes in the production, while some imported input varieties require working capital financing from foreign creditors. Sovereign defaults keep firms away from world credit markets and the imported varieties have to be replaced by imperfect substitutes. The distortions on the *within-firm* composition of the imported and the domestic input varieties lower the firm's productivity. In our model, the tangibles and the intangibles are imperfect substitutes in capital formation, while only the tangibles can serve as collateral. By affecting the unit-cost differential, financial inflows trigger the *within-firm* investment composition. Essentially, our paper and Mendoza and Yue (2012) share the similar mechanisms through which financial flows affect allocative efficiency. Due to the fixed capital stock, the decreasing MPK effect is mute in Mendoza and Yue (2012), while it is key to the dynamics and the long-run level of productivity in our model.

Although the literature has proposed various channels through which financial integration may foster productivity growth, the empirical evidence is rather mixed (Kose et al., 2009b). Schularick and Steger (2010) find that financial integration was correlated with economic growth before World War I but is no longer today, while Bonfiglioli (2008) and Bekaert et

moderate, while there is a unique steady state if the world interest rate is either sufficiently high or low.

[5] In our model, the dynamic and steady-state patterns are fundamentally different between the two ranges of the world interest rate. Brunnermeier and Sannikov (2014) also conduct the global analysis and find a bimodal stationary distribution over the entire state space, i.e., a stable normal regime and a volatile crisis regime. This global approach differs from the conventional business cycles literature (Bernanke et al., 1999; Carlstrom and Fuerst, 1997; Kiyotaki and Moore, 1997) that takes the local approximation around the deterministic steady state and features the credit multipliers as the amplification mechanism.

[6] The recent trade literature pioneered by Melitz (2003) also assume exogenous productivity distribution among firms and trade liberalization triggers the *cross-firm* reallocation effect.

al. (2011) offer the evidence on the positive productivity effect of financial integration. In particular, Kose et al. (2009a) find that FDI and portfolio equity liabilities boost TFP growth, while external debt is negatively correlated with TFP growth and this negative relationship is partially attenuated in economies with better-developed financial markets and better institutional quality. Kose et al. (2011) point out that, in order to gain from financial integration, an economy needs to attain certain "threshold" levels of institutional quality and financial development. Our paper predicts that debt flows may trigger the non-monotonic productivity responses, while the long-run productivity effect depends critically on the elasticity of domestic investment. These predictions may motivate the empirical research on identifying the fundamental and institutional factors relevant for the elasticity of domestic investment as well as exploring their threshold conditions for long-run productivity gains, in the same spirit as Kose et al. (2011, 2009a).

In the traditional theory of investment (Hayashi, 1982; Lucas and Prescott, 1971; Tobin, 1969), convex capital adjustment costs (CAC, hereafter) are a common assumption for preventing firms from changing their capital stock too quickly. Thus, aggregate investment responds smoothly to exogenous changes, and *its elasticity depends on the convexity of CAC.* Although convex CAC are consistent with the empirical data on aggregate investment, the data on firm-level investment is rather lumpy and volatile. In a RBC model with idiosyncratic shocks to capital formation, Wang and Wen (2012) argue that collateralized borrowing can give rise to convex adjustment costs at the aggregate level yet at the same time generate lumpiness in plant-level investment, and *the elasticity of aggregate investment depends on the distribution of idiosyncratic shocks.* In our model, the collateral constraints and the MIR[7] endogenize the extensive margin so that *the elasticity of aggregate investment is related to wealth inequality.*

Although the intangibles do not serve as collateral for loans, a recent literature shows that they can be financed by equity or labor contracts (Döttling et al., 2018; Kiyotaki and Zhang, 2018; Li, 2019; Sun and Xiaolan, 2019). As shown in subsection 3.3.3, our model predictions still hold, as long as the tangibles have the external financing advantage over the intangibles.

The rest of the paper is structured as follows. Section 1 sets up the model and section 2 analyzes the autarkic equilibrium. Section 3 studies the short-run and the long-run productivity implications of financial inflows as well as checks the robustness of the model predictions under alternative settings. Section 4 analyzes the model dynamics over the entire state space and explores the implications of equilibrium shifts on productivity. Section 5 concludes with some remarks. Appendices include some relevant materials and technical proofs.

1 The Model Setting

This model features three key assumptions in a two-period, overlapping-generation framework: (1) the heterogeneous pledgeability between tangible and intangible investments, (2) the MIR for the project of capital formation, and (3) the heterogeneity in individual net wealth.

Consider a small country N in the world economy. A continuum of agents indexed by $j \in [0,1]$ are born every period and they live for two periods, young and old. The population

[7] See Zhang (2017) for the relevant literature review on the MIR and wealth inequality.

size of each generation is constant and normalized at unity. Agents only consume when old. When young, agent j supplies its labor endowment $l_j = (1-\theta)\varepsilon_j$ inelastically to the market at the wage rate w_t, where $\varepsilon_j \in (1, \infty)$ follows the Pareto distribution,[8] with the cumulative distribution function $G(\varepsilon_j) = 1 - \varepsilon_j^{-\frac{1}{\theta}}$ and $\theta \in (0,1)$. Thus, agents differ in labor income, $n_{j,t} = w_t l_j$. The aggregate labor supply is constant at $L = \int_1^\infty l_j dG(\varepsilon_j) = 1$ every period.

A final good is internationally tradable and chosen as the numeraire. It can be consumed or converted into non-tradable capital goods, which is available for production in the next period. Capital and labor are combined in the Cobb-Douglas fashion for the production of the final good contemporaneously. Capital fully depreciates after the production. The markets for final goods, capital, and labor are perfectly competitive. There is no uncertainty in the model economy. Y_t denotes aggregate output of final goods, $L = 1$ and K_t denote the aggregate inputs of labor and capital, w_t and q_t denote the wage rate and the rental price of capital in period t. To sum up,

$$Y_t = \left(\frac{K_t}{\alpha}\right)^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}, \qquad \text{where} \qquad \alpha \in (0,1), \tag{1}$$

$$q_t K_t = \alpha Y_t \qquad \text{and} \qquad w_t L = (1-\alpha) Y_t. \tag{2}$$

When young, agents have two options to save the labor income $n_{j,t} = w_t l_j$ for future consumption: lending at the gross interest rate r_t and running a project for capital formation. For the second option, agent j spends $m_{j,T,t}$ and $m_{j,I,t}$ units of final goods respectively as **t**angible and **i**ntangible investments in period t, which yields $k_{j,t+1} = \left(\frac{m_{j,I,t}}{\eta}\right)^{\eta} \left(\frac{m_{j,T,t}}{1-\eta}\right)^{1-\eta}$ units of capital in period $t+1$,[9] if its project investment meets the MIR, $m_{j,t} \equiv m_{j,I,t} + m_{j,T,t} \geq \mathfrak{m}$; otherwise, the project yields zero output. The purpose of introducing the MIR is to endogenize the mass of the agents producing capital goods. One could assume alternatively that the MIR applies to tangible investment, $m_{j,T,t} \geq \mathfrak{m}$. It only complicates the analysis, without changing our key findings. See subsection 3.3.2 for further discussion.

Suppose that agent j can meet the MIR.[10] Let $a_{j,t} \equiv \frac{m_{j,I,t}}{m_{j,t}}$ denote the intangible fraction of its investment. For each unit of investment in period t, agent j puts $a_{j,t}$ as the intangibles and $1 - a_{j,t}$ as the tangibles, which yields $\Phi_{j,t} \equiv \left(\frac{a_{j,t}}{\eta}\right)^{\eta} \left(\frac{1-a_{j,t}}{1-\eta}\right)^{1-\eta}$ units of capital in period $t+1$. Thus, the project has a productivity $\Phi_{j,t}$ and a gross rate of return $q_{t+1}\Phi_{j,t}$. The individual capital formation function can be rewritten as $k_{j,t+1} = \Phi_{j,t} m_{j,t}$.

If the project rate of return exceeds the interest rate $q_{t+1}\Phi_{j,t} > r_t$,[11] the agent prefers to borrow as much as possible and gain from leveraged investment. How much can it actually

[8]Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson et al., 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003).

[9]Giglio and Severo (2012) and McGrattan (2017) also take the Cobb-Douglas form for capital formation, while Caggese and Pérez-Orive (2018) assume that tangible and intangible investments are perfect complements. As shown in subsection 3.3.1, our findings still hold qualitatively if capital formation takes a general functional form with the constant elasticity of substitution (CES) as in Falato et al. (2018).

[10]Equation (10) specifies the condition under which this is the case.

[11]As shown in equation (4), the intangible fraction of investment in equilibrium is common for the agents who can meet the MIR and so is the project productivity. One can drop the subscript j. In equilibrium, the project's gross rate of return must be no less than the interest rate, $q_{t+1}\Phi_t \geq r_t$; otherwise, nobody would run the project.

borrow? If the agent defaults in period $t+1$, the intangibles are lost and the project yields zero output.[12] The best that lenders can do is to seize and liquidate the tangibles. After deducting liquidation costs, the lenders get $\lambda p_{t+1} m_{j,T,t}$, where p_{t+1} denotes the price of tangibles and $\lambda \in (0,1]$ measures the level of financial development. In the no-default equilibrium, agent j borrows up to the collateral value of tangibles and cover the gap with own funds $n_{j,t}$,[13]

$$b_{j,t} = \frac{\lambda p_{t+1} m_{j,T,t}}{r_t}, \text{ and } m_{j,I,t} + m_{j,T,t} - b_{j,t} = n_{j,t} = w_t l_j. \tag{3}$$

In the following, we explain the implications of the three key assumptions to individual choices and aggregate variables.

Heterogeneous Pledgeability and Endogenous Intangible-Tangible Investment

As the revenues of intangible investment cannot be pledged for external financing, agent j has to finance the intangibles entirely with own funds. Thus, the unit cost of intangibles is $u_{I,t} = 1$ and the unit return is the marginal revenue of intangibles, $q_{t+1} \frac{\partial k_{j,t+1}}{\partial m_{j,I,t}}$. In contrast, for each unit of tangible investment, the agent can borrow up to $\frac{\lambda p_{t+1}}{r_t}$ and only need to put up $1 - \frac{\lambda p_{t+1}}{r_t}$ units of own funds. Thus, the unit cost of tangibles is $u_{T,t} = 1 - \frac{\lambda p_{t+1}}{r_t}$, while the unit return is the marginal revenue minus the debt repayment per unit of tangibles, $q_{t+1} \frac{\partial k_{j,t+1}}{\partial m_{j,T,t}} - \lambda p_{t+1}$. Agent j allocates its net wealth between the two types of investments optimally so that the internal rates of return equalize, $\frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial m_{j,I,t}}}{u_{I,t}} = \frac{q_{t+1} \frac{\partial k_{j,t+1}}{\partial m_{j,T,t}} - \lambda p_{t+1}}{u_{T,t}}$, or equivalently,[14]

$$u_{T,t} = 1 - \frac{\lambda p_{t+1}}{r_t} = \frac{a_{j,t}}{1-a_{j,t}} \frac{1-\eta}{\eta} \left[1 - \frac{\lambda p_{t+1}}{q_{t+1}} \left(\frac{a_{j,t}}{1-a_{j,t}} \frac{1-\eta}{\eta} \right)^{-\eta} \right]. \tag{4}$$

Thus, the optimal choice of $a_{i,t}$ is a function of the parameters (λ and η) and the market prices (r_t, p_{t+1}, and q_{t+1}). In other words, the agents who meet the MIR optimally choose the same intangible fraction of investment. Hereafter, we drop off subscript j and use a_t to denote it.

Agent j equalizes its marginal revenue of tangibles to the market price,

$$q_{t+1} \frac{\partial k_{j,t+1}}{\partial m_{j,T,t}} = q_{t+1} \Phi_t \frac{1-\eta}{1-a_t} = p_{t+1}, \text{ where } \Phi_t \equiv \left(\frac{a_t}{\eta} \right)^{\eta} \left(\frac{1-a_t}{1-\eta} \right)^{1-\eta} \tag{5}$$

denotes the productivity of capital formation in equilibrium. For $a_t = \eta$, the productivity is maximized at unity, $\Phi_t = \bar{\Phi} = 1$; for $a_t < \eta$, $\frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1-a_t} > 0$ implies that a lower a_t distorts within-firm allocative efficiency and the productivity is also lower.

[12] Excluding the intangibles from the loan contract facilitates an analytical exploration of the model mechanism. As shown in subsection 3.3.3, our findings still hold in the setting where both the tangibles and the intangibles can serve as collateral, but the former has a higher degree of pledgeability than the latter.

[13] Matsuyama (2004) assumes that an agent can borrow against its future project revenue, while we assume that an agent can borrow against the market value of tangibles. As explained in remark A of subsection 3.3.3, these two assumptions are analytically equivalent.

[14] See the proof of Proposition 1 for a formal technical analysis of agent j's optimization.

Plug equation (5) in (4) to substitute away p_{t+1},

$$u_{T,t} = 1 - \frac{1-\eta}{1-a_t}\frac{\lambda}{\psi_t} = \frac{a_t}{1-a_t}\frac{1-\eta}{\eta}(1-\lambda), \text{ where } \psi_t \equiv \frac{r_t}{q_{t+1}\Phi_t} < 1 \tag{6}$$

denotes the normalized interest rate. Given $r_t < q_{t+1}\Phi_t$ or equivalently $\psi_t < 1$, the borrowing constraints are binding and, according to equation (6), $a_t < \eta$ holds.[15] Here, by creating the unit-cost differential $u_{I,t} - u_{T,t} = 1 - u_{T,t} > 0$, heterogeneous pledgeability induces agent j to invest inefficiently more (less) in the tangibles (intangibles), which distorts allocative efficiency and productivity $\Phi_t < \bar{\Phi}$. The lower the ψ_t, the higher the pledgeable value per unit of tangibles $\frac{1-\eta}{1-a_t}\frac{\lambda}{\psi_t}$, the lower the unit cost of tangibles $u_{T,t}$, the larger the unit-cost differential, the more the agents favor tangible investment, the lower the a_t and Φ_t. The positive comovement of a_t and ψ_t reflects the individual optimization of tangibles-intangibles composition.

Remark 1: heterogeneous pledgeability creates the unit-cost differential, which is key to the endogenous composition of tangible and intangible investments.

As shown in section 3, the within-firm allocative efficiency and the productivity decline upon financial inflows, while they rise with aggregate income over time. One can explain the logic behind this result easily by using equation (6). Suppose that country N witnesses financial inflows from period 0 on. In period 0, the interest rate falls to the world level, while the domestic investment expansion lowers the MPK and the rental price of capital in period 1. The responses of ψ_0 and a_0 depend on the relative size of the two forces.

$$\frac{\partial \ln a_0}{\partial \ln r_0} \propto \frac{\partial \ln \psi_0}{\partial \ln r_0} = \underbrace{\frac{\partial \ln r_0}{\partial \ln r_0}}_{\text{interest rate effect (=1)}} - \underbrace{\frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0}}_{\text{MPK effect}} \tag{7}$$

If the interest rate effect dominates the rental-price-of-capital effect, ψ_0 declines and so do a_0 and Φ_0. From period $t = 1$ on, the rise in aggregate income stimulates the domestic investment demand and attract more financial inflows, while the resulting capital formation further reduces the MPK and the rental price of capital $\frac{\partial \ln q_{t+1}\Phi_t}{\partial \ln Y_t} < 0$. Meanwhile, the interest rate is constant at the world level $\frac{\partial \ln r_t}{\partial \ln Y_t} = 0$. Thus, ψ_t strictly rises over time and so do a_t and Φ_t.

$$\frac{\partial \ln a_t}{\partial \ln Y_t} \propto \frac{\partial \ln \psi_t}{\partial \ln Y_t} = \underbrace{\frac{\partial \ln r_t}{\partial \ln Y_t}}_{\text{interest rate effect (0)}} - \underbrace{\frac{\partial \ln q_{t+1}\Phi_t}{\partial \ln Y_t}}_{\text{MPK effect (-)}} > 0 \tag{8}$$

Here, by changing the normalized interest rate and the unit-cost differential, financial inflows have the opposite short-run versus long-run impacts on a_t and Φ_t. Given the interest rate decline, the more elastic the domestic investment demand, the larger the financial inflows, the stronger the MPK effect, the smaller the initial fall and the larger the subsequent rises in ψ_t and a_t, the more likely the productivity eventually exceeds its initial level. Thus, the elasticity of domestic investment demand is key to the long-run productivity effect of financial inflows. In this paper, we highlight explicitly the role of the endogenous extensive margin in determining the elasticity of domestic investment. Next, we endogenize the extensive margin and characterize the elasticity of domestic investment analytically.

[15] If the borrowing constraints are weakly binding, $\psi_t = 1$ and the allocation is efficient with $a_t = \eta$ and $\Phi_t = 1$.

Endogenizing the Mass of Entrepreneurs with the Collateral Constraints and the MIR

Given $q_{t+1}\Phi_t > r_t$, the agent puts its entire labor income in the project and borrows to the limit. Define the unit cost of investment as the weighted average of the two unit costs,

$$u_t \equiv a_t u_{I,t} + (1-a_t)u_{T,t} = 1 - \frac{\lambda(1-\eta)}{\psi_t} = \frac{a_t}{\eta}[1-\lambda(1-\eta)]. \tag{9}$$

For each unit of investment, the agent puts down u_t units of own funds. A cutoff value $\underline{\varepsilon}_t$ is associated with the agents who just meet the MIR,[16]

$$\frac{w_t(1-\theta)\underline{\varepsilon}_t}{u_t} = \mathfrak{m}, \Rightarrow \underline{\varepsilon}_t = \frac{u_t}{w_t}\frac{\mathfrak{m}}{1-\theta}. \tag{10}$$

The agents with $\varepsilon_j \geq \underline{\varepsilon}_t$ can meet the MIR and they are called *entrepreneurs*, with the mass of $\tau_t = \underline{\varepsilon}_t^{-\frac{1}{\theta}}$; when young, they invest their entire labor income $n_{j,t}$ in the project and borrow to the limit $b_{j,t} = n_{j,t}(\frac{1}{u_t}-1)$; when old, they get the investment revenue $q_{t+1}\Phi_t\frac{n_{j,t}}{u_t}$, repay the debt $r_t b_{j,t}$, and consume the rest $c^e_{j,t+1}$. The agents with $\varepsilon_j \in [1, \underline{\varepsilon}_t)$ cannot meet the MIR and they are called *households*; when young, they lend out their labor income $n_{j,t}$; when old, they consume the return to their savings $c^h_{j,t+1}$. To sum up,

$$\tau_t = \underline{\varepsilon}_t^{-\frac{1}{\theta}},\ n_{j,t} = w_t l_j,\ c^e_{j,t+1} = n_{j,t}\left[\frac{q_{t+1}\Phi_t}{u_t} - r_t\left(\frac{1}{u_t}-1\right)\right],\ c^h_{j,t+1} = n_{j,t}r_t. \tag{11}$$

Remark 2: the collateral constraints and the MIR jointly endogenize the mass of entrepreneurs.

Net Wealth Heterogeneity and Partial Elasticities of Domestic Investment Demand

According to equation (2), the wage rate is proportional to aggregate income, given the fixed aggregate labor input $L = 1$. Thus, the wage rate can be taken as a proxy for aggregate income and we use them interchangeably in the following analysis.

In this model, the labor income is the only source of net wealth for young agents. A rise in aggregate income raises the average net wealth of individual agents, while a fall in the unit cost of investment raises the leverage multiplier for entrepreneurs. In either case, individual entrepreneurs can borrow and invest more in the project $m_{j,t} = \frac{w_t l_j}{u_t}$, while more agents can meet the MIR and become entrepreneurs $\tau_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1}{\theta}}$. As a result, the domestic investment demand rises along the intensive and the extensive margins, respectively.

$$M_t \equiv \int_{\underline{\varepsilon}_t}^{\infty} m_{j,t}dG(\varepsilon_j) = \frac{\delta_t w_t L}{u_t},\ \text{ where }\ \delta_t \equiv \frac{\int_{\underline{\varepsilon}_t}^{\infty} w_t l_j dG(\varepsilon_j)}{w_t L} = \underline{\varepsilon}_t^{-\frac{1-\theta}{\theta}} = \tau_t^{1-\theta} \tag{12}$$

[16]Given the constant population size in each generation, the cutoff value should be specified more precisely as $\underline{\varepsilon}_t = \max\left\{1, \frac{u_t}{w_t}\frac{\mathfrak{m}}{1-\theta}\right\}$, implying the existence of the mass-of-entrepreneurs (MoE, hereafter) constraint, $\tau_t \leq 1$. Under autarky, if the borrowing constraints are binding in equilibrium, there must be some agents who cannot overcome the MIR and hence, the MoE constraint is not binding; if the borrowing constraints are slack, those who can overcome the MIR do not have strong incentive to be entrepreneurs and hence, the MoE constraint is irrelevant. Thus, one does not need to consider explicitly the MoE constraint under autarky.
The MoE constraint may become binding under financial integration, as shown in the proof of proposition 3. For simplicity, we focus on the case of $\underline{\varepsilon}_t > 1$ in section 3, while the proof of proposition 3 covers the case where the MoE constraint is binding under financial integration.

denotes the entrepreneurial net wealth share[17] and captures the extensive-margin effect. Use equations (10)-(12) to solve for the *partial elasticities* of M_t with respect to the two variables,

$$\frac{\partial \ln M_t}{\partial \ln w_t} = \underbrace{\frac{\partial \ln w_t L}{\partial \ln w_t}}_{\text{Int. margin effect }=1} + \underbrace{\frac{\partial \ln \delta_t}{\partial \ln \tau_t}\frac{\partial \ln \tau_t}{\partial \ln w_t}}_{\text{Ext. margin effect }=\frac{1}{\theta}-1} = \frac{1}{\theta}, \tag{13}$$

$$\frac{\partial \ln M_t}{\partial \ln u_t} = \underbrace{\frac{\partial \ln \frac{1}{u_t}}{\partial \ln u_t}}_{\text{Int. margin effect }=-1} + \underbrace{\frac{\partial \ln \delta_t}{\partial \ln \tau_t}\frac{\partial \ln \tau_t}{\partial \ln u_t}}_{\text{Ext. margin effect }=-(\frac{1}{\theta}-1)} = -\frac{1}{\theta}. \tag{14}$$

For $\theta \in (0,1)$, the two partial elasticities (in absolute value) exceed unity, due to the extensive margin effect. By definition, θ is the inverse of the shape parameter for the Pareto distribution. The smaller the θ, the smaller the dispersion of labor endowment and net wealth among young agents, the more elastic the mass of entrepreneurs $\frac{\partial \ln \tau_t}{\partial \ln w_t} = -\frac{\partial \ln \tau_t}{\partial \ln u_t} = \frac{1}{\theta}$, the stronger the extensive margin effect $\frac{\partial \ln \delta_t}{\partial \ln w_t} = -\frac{\partial \ln \delta_t}{\partial \ln u_t} = \frac{1}{\theta} - 1$, the more elastic the domestic investment demand.[18]

Remark 3: if the borrowing constraints are binding, the partial elasticities of the domestic investment demand (in absolute value) depend negatively on the net wealth heterogeneity, due to the endogenous extensive margin.

Market Clearing Conditions under Autarky versus under Financial Integration

Under autarky, the markets for credit and capital goods clear domestically,[19]

$$\int_{\varepsilon_t}^{\infty} (m_{j,t} - n_{j,t}) dG(\varepsilon_j) = \int_{1}^{\varepsilon_t} n_{j,t} dG(\varepsilon_j), \;\Rightarrow\; M_t = w_t L, \tag{15}$$

$$K_{t+1} \equiv \int_{\varepsilon_t}^{\infty} \Phi_{j,t} m_{j,t} dG(\varepsilon_j) = \Phi_t M_t. \tag{16}$$

So far, we have focused on the case of $q_{t+1}\Phi_t > r_t$ where the borrowing constraints are binding. In the case of $q_{t+1}\Phi_t = r_t$, the borrowing constraints are slack and entrepreneurs choose $a_{j,t}$ to maximize the project productivity $\Phi_{j,t}$. As a result, the intangible fraction of investment is equal to the factor share of intangibles in capital formation, $a_t = \eta$.[20] Those who can meet the MIR do not have strong incentive to put their labor income in the project or borrow to the limit. Despite the indeterminacy at the individual level, domestic investment is still fully financed by domestic saving, $M_t = w_t L$, and the productivity is constant at $\Phi_t = 1$.

[17] As young agents save their entire labor income, domestic savings in period t is $w_t L$.

[18] One needs to distinguish between two related but different concepts. M_t specified in equation (12) refers to the domestic investment demand. According to equations (13) and (14), it is a log-linear, increasing function of w_t and a log-linear, decreasing function of u_t, independent of whether country N is under autarky or under financial integration. In contrast, the actual size of domestic investment refers to the equilibrium value determined jointly by the domestic investment demand and the total funds available for country N, which differs under autarky versus under financial integration. In the following analysis, domestic investment refers to its actual size.

[19] As mentioned above, all entrepreneurial projects have the same productivity, $\Phi_{j,t} = \Phi_t$.
In the no-default equilibrium, no liquidation costs are incurred. According to the Walras' law, the market for final goods clears in equilibrium, $M_t + \int_{\varepsilon_t}^{\infty} c_{j,t}^e dG(\varepsilon_j) + \int_1^{\varepsilon_t} c_{j,t}^h dG(\varepsilon_j) = Y_t$.

[20] Alternatively, one can consider the case where the borrowing constraints are weakly binding and $q_{t+1}\Phi_t = r_t$. According to equation (9), $\psi_t = 1$ implies $a_t = \eta$.

Definition 1. *Under autarky, a market equilibrium in country N is a set of choices of agents,* $\{n_{j,t}, m_{j,t}, a_t, \Phi_t, c^e_{j,t}, c^h_{j,t}, u_t, u_{T,t}\}$*, aggregate quantities* $\{Y_t, K_t, M_t\}$*, the prices* $\{q_t, \psi_t, w_t, r_t\}$*, and the threshold value* $\{\varepsilon_t\}$*, satisfying equations* (1)-(2)*,* (5)-(11)*, and* (16)-(15)*.*

Country N and the rest of the world are inherently identical, except that country N is more financially developed and its population share in the world economy is negligible. Under financial integration, agents can borrow and lend aboard[21]. Due to its negligible country size in the world, the interest rate in country N is aligned to the world level $r_t = r^*_t$, while the domestic investment-savings gap is covered by financial capital flows, $FCF_t = M_t - w_t L$.

Definition 2. *Under financial integration, a market equilibrium in country N is a set of choices of agents,* $\{n_{j,t}, m_{j,t}, a_t, \Phi_t, c^e_{j,t}, c^h_{j,t}, u_t, u_{T,t}\}$*, aggregate quantities* $\{Y_t, K_t, M_t\}$*, the prices* $\{q_t, \psi_t, w_t\}$*, and the threshold value* $\{\varepsilon_t\}$*, satisfying equations* (1)-(2)*,* (5)-(11)*, and* (16)*, given* $r_t = r^*_t$*.*

At the aggregate level, M_t units of final goods are invested in period t, which yields $K_{t+1} = \Phi_t M_t$ units of capital goods in period $t+1$. Define the social rate of return as $\frac{q_{t+1}K_{t+1}}{M_t} = q_{t+1}\Phi_t$.

2 The Autarkic Equilibrium

In this section, we use the law of motion for wage to analyze the dynamics of aggregate income and allocative efficiency under autarky. Combine equations (15)-(16) with (1)-(2) to get

$$w_{t+1} = \left(\frac{K_{t+1}}{L\rho}\right)^{\alpha} = \left(\frac{M_t\Phi_t}{L\rho}\right)^{\alpha} = \left(\frac{w_t\Phi_t}{\rho}\right)^{\alpha}, \text{ where } \rho \equiv \frac{\alpha}{1-\alpha}. \tag{17}$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \Big(\underbrace{\frac{\partial \ln M_t}{\partial \ln w_t}}_{\text{Inv. size effect}} + \underbrace{\frac{\partial \ln \Phi_t}{\partial \ln w_t}}_{\text{Prod. effect}} \Big)[1 - \underbrace{(1-\alpha)}_{\text{Decr. MPK effect}}]. \tag{18}$$

A rise in current income w_t affects capital formation K_{t+1} through two channels. First, it raises the size of domestic investment M_t; second, it may affect *within-firm* allocative efficiency and the productivity of capital formation Φ_t. Meanwhile, due to the decreasing MPK, a change in capital formation K_{t+1} affects future income w_{t+1} *less than proportionally*.

In the following, we characterize the investment size effect and the productivity effect in two cases before addressing the steady-state property of the model economy.

The Unconstrained Case

Let $\bar{w}_A \equiv [1-\lambda(1-\eta)]^{\frac{1}{1-\theta}} \frac{m}{1-\theta}$. For $w_t \geq \bar{w}_A$, aggregate income is sufficiently large and so are the mass of entrepreneurs and the domestic credit demand, which keeps the interest rate equal to the social rate of return,[22] $r_t = q_{t+1}\Phi_t$. Thus, the borrowing constraints are slack, $a_t = \eta$, and the productivity is constant at $\Phi_t = 1$. Besides, domestic investment is fully financed by domestic saving, $M_t = w_t L$. As the productivity effect is mute $\frac{\partial \ln \Phi_t}{\partial \ln w_t} = 0$ and the investment

[21] Following Matsuyama (2004), we exclude FDI flows by assumption. von Hagen and Zhang (2014) analyze the joint determination of financial capital flows and FDI flows.

[22] See the proof of Proposition 1 for the technical derivation of the autarkic equilibrium.

size effect is constant at unity $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$, the decreasing MPK effect is a convergence force and drives country N towards a steady state, $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - (1-\alpha) < 1$. The smaller the α, the stronger the decreasing MPK effect, the faster the convergence. As long as the borrowing constraints are slack, wealth inequality *does not matter* for aggregate dynamics.

The Constrained Case

For $w_t < \bar{w}_A$, aggregate income is inefficiently low and so are the mass of entrepreneurs and the domestic credit demand, which keeps the interest rate below the social rate of return, $r_t < q_{t+1}\Phi_t$. Thus, entrepreneurs borrow against the tangibles to the limit and, due to the unit-cost differential $u_{T,t} = 1 - \frac{1}{1-a_t}\frac{\lambda(1-\eta)}{\psi_t} < u_{I,t} = 1$, they invest inefficiently less (more) intangibles (tangibles) in the project, i.e., $a_t < \eta$, and hence the productivity is inefficiently low, $\Phi_t < 1$.

According to equations (5) and (9), the productivity is an increasing function of a_t, while the latter is proportional to u_t. According to equations (12) and (14), the domestic investment demand M_t is a decreasing, log-linear function of u_t. Thus, u_t is key to the productivity effect and the investment size effect specified in equation (18). Next, we first use the domestic investment-saving diagram to determine u_t and then characterize the two effects analytically.

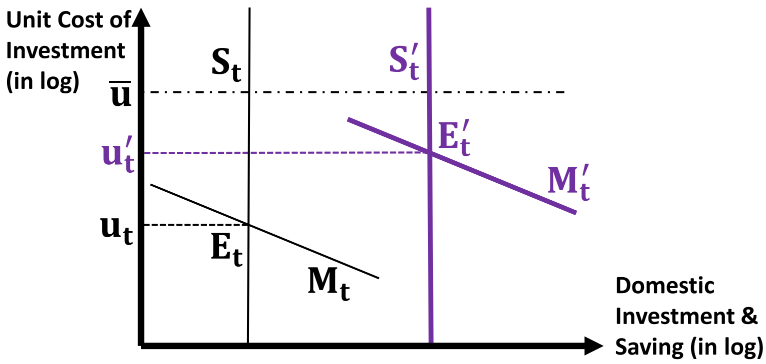

Figure 1: The Response of Domestic Saving-Investment Balance to Income Change

Given w_t, point E_t in figure 1 denotes the domestic investment-saving balance where domestic investment demand M_t and domestic saving $S_t = w_t L$ intersect, and both axes are in logarithms. For a rise in current income $w_t' > w_t$, domestic saving shifts rightwards proportionately $\frac{\partial \ln S_t}{\partial \ln w_t} = 1$ to S_t', while domestic investment demand shifts rightwards more than proportionately $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta} > 1$ to M_t', according to equation (13). As domestic saving is perfectly inelastic, the equilibrium size of domestic investment is $M_t' = S_t' = w_t' L$ and hence, the investment size effect is constant at unity $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$.[23] Meanwhile, the unit cost investment rises to

[23]In equation (13), $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta}$ is the partial elasticity, reflecting the direct impact of an income change on domestic investment demand, given u_t. Under autarky, the income change also affects domestic investment demand indirectly via u_t, as shown in figure 1; the domestic investment-savings balances $M_t = \frac{\delta_t w_t L}{u_t} = S_t = w_t L$ gives $\delta_t = u_t$, implying that the extensive margin effect is fully offset by the unit-cost-of-investment effect, $\frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{\partial \ln u_t}{\partial \ln w_t}$, and hence, the investment size effect in equation (18) is constant at unity $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$. Under financial integration, capital flows break the domestic investment-saving balance, $M_t \neq S_t$ and $\delta_t \neq u_t$, so that the investment size effect is not constant at unity, $\frac{\partial \ln M_t}{\partial \ln w_t} \neq 1$, according to equation (25).

restore the domestic investment-saving balance $u_t' > u_t$. Intuitively, a higher aggregate income allows more agents to become entrepreneurs, which raises (reduces) the domestic credit demand (supply) simultaneously. Thus, ψ_t rises to clear the domestic credit market. By reducing the collateral value of tangibles, the rise in ψ_t raises the unit cost of tangibles as well as u_t. Thus, entrepreneurs shift the project investment from tangibles towards intangibles and the rise in a_t leads to a higher productivity. Here, the productivity moves positively with aggregate income, due to the extensive margin effect.[24] Combine equations (5), (9), (10), and (12)-(15) to solve for some key variables as the functions of w_t,

$$\tau_t = \frac{w_t(1-\theta)}{\mathfrak{m}},\ u_t = \delta_t = \tau_t^{1-\theta},\ \psi_t = \frac{\lambda(1-\eta)}{1-\delta_t},\ \text{and}\ a_t = \frac{u_t\eta}{1-\lambda(1-\eta)}, \tag{19}$$

$$\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \frac{\partial \ln \Phi_t}{\partial \ln a_t}\frac{\partial \ln a_t}{\partial \ln u_t}\frac{\partial \ln u_t}{\partial \ln \psi_t}\frac{\partial \ln \psi_t}{\partial \ln \delta_t}\frac{\partial \ln \delta_t}{\partial \ln \tau_t}\frac{\partial \ln \tau_t}{\partial \ln w_t} = \left(1-\frac{1-\eta}{1-a_t}\right)(1-\theta). \tag{20}$$

If the borrowing constraints are binding, $a_t < \eta$ and the productivity effect depends negatively on the degrees of wealth inequality and financial frictions. First, according to equation (13), the smaller the θ, the stronger the extensive margin effect $\frac{\partial \ln \delta_t}{\partial \ln w_t} = \frac{1}{\theta} - 1$, the large the rightward shift of the domestic investment demand $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta}$, the large the rises in the unit cost and the intangible fraction of investment $\frac{\partial \ln a_t}{\partial \ln w_t} = \frac{\partial \ln u_t}{\partial \ln w_t} = 1-\theta$, the stronger the productivity effect. Second, given the MIR, the lower the λ, the larger the investment distortion, the lower the a_t, the stronger the productivity effect.

The Steady-State Properties

Put $\frac{\partial \ln M_t}{\partial \ln w_t} = 1$ and equation (20) into (18). There is a unique, autarkic steady state in country N,[25] iff the decreasing MPK effect always dominates the productivity effect at any steady state. The decreasing MPK effect is negatively related to α (the capital share), while the productivity effect is negatively related to θ (wealth inequality) and λ (financial frictions). As shown in proposition 1, the steady-state property of the model economy depends on the relative size of these parameters. Let X_A denote the steady-state value of variable X_t under autarky.

Proposition 1. *Let* $\underline{\theta} \equiv \max\{1-\frac{1-\alpha}{\alpha\eta}, 0\}$, $Z \equiv \frac{1-\theta}{\rho^\rho \mathfrak{m}}$, *and* $\tilde{\lambda}_A \equiv \frac{1-Z^{1-\theta}}{1-\eta}$.

$\theta \in [\underline{\theta}, 1)$ *is a sufficient condition for the existence of a unique, autarkic steady state.*[26]

Given $\theta \in [\underline{\theta}, 1)$ *and* $\lambda < \min\{\tilde{\lambda}_A, 1\}$, *the borrowing constraints are binding, the interest rate is below the social rate of return, the intangible fraction of investment and the productivity are inefficiently low in the autarkic steady state, i.e.,* $\psi_A \equiv \frac{r_A}{q_A \Phi_A} < 1$, $a_A < \eta$ *and* $\Phi_A < 1$.

Besides, $\frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_A}{1-a_A}\frac{\partial \ln a_A}{\partial \ln \lambda} > 0$, $\frac{\partial \ln w_A}{\partial \ln \lambda} = \rho \frac{\partial \ln \Phi_A}{\partial \ln \lambda} > 0$, *and* $\frac{\partial \ln r_A}{\partial \ln \lambda} = \frac{\partial \ln \psi_A}{\partial \ln \lambda} > 0$.

Assumption 1. $\theta \in [\underline{\theta}, 1)$, $0 < \lambda^* < \lambda < \min\{\tilde{\lambda}_A, 1\}$, *and* $\frac{L}{L+L^*} \to 0$.[27]

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[24] In appendix C, I develop an alternative model with a fixed mass of entrepreneurs. In the absence of the extensive margin effect, $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{\partial \ln S_t}{\partial \ln w_t} = 1$ so that u_t is independent of income change and so are ψ_t and a_t.

[25] The sufficient and necessary condition for the existence of a unique, stable steady state in country N is that the law of motion for wage has a slope less than unity at any steady state, $\frac{\partial w_{t+1}}{\partial w_t}|_{w_{t+1}=w_t} < 1$.

[26] See figure 11 in the proof of proposition 1 for a graphical illustration of the threshold values.

[27] By definition, the composite parameter Z is independent of the level of financial development and the population size. Thus, Z takes the same value for both regions.

Under assumption 1, there exists a unique steady state in country N and in the rest of the world, respectively; the borrowing constraints are binding in both regions; as country N is more financially developed, it has a higher interest rate, a higher productivity, and a higher income per capita than the rest of the world, $r_A^* < r_A < \rho$, $a_A^* < a_A < \eta$, $\Phi_A^* < \Phi_A < 1$, and $w_A^* < w_A$.

3 Productivity Dynamics under Financial Integration

Suppose that the world economy is initially in the autarkic steady state. From period 0 on, agents can borrow and lend abroad. The initial interest rate spread $r_A^* < r_A$ leads to "uphill" financial flows towards country N, a well-known result in the literature on global imbalances (Caballero et al., 2008; Ju and Wei, 2011; Mendoza et al., 2009; von Hagen and Zhang, 2014).

Uphill financial flows align the interest rate in country N to the world level, $r_t = r^* = r_A^* < r_A$ and cover the domestic investment-saving gap, $FCF_t = M_t - w_t L$. The other equilibrium conditions are identical as under autarky. The law of motion for wage is featured by equations (17)-(18), except that domestic investment is not constrained by domestic saving, $M_t \neq w_t L$.

Let $\bar{w}_F \equiv [1-\lambda(1-\eta)]\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^{\theta}(r^*)^{-\frac{\theta}{1-\alpha}}$. For $w_t \geq \bar{w}_F$, $q_{t+1}\Phi_t = r^*$ so that the borrowing constraints are slack, $a_t = \eta$, the productivity is constant at $\Phi_t = 1$, and $w_{t+1} = (r^*)^{-\rho}$.

In this section, we focus on the case of $w_t < \bar{w}_F$ where the borrowing constraints are binding in country N. Financial inflows affect domestic capital formation and aggregate income in two ways. First, by augmenting domestic investment, they stimulate domestic capital formation and raise aggregate income over time.[28] Second, they have non-trivial impacts on allocative efficiency and the productivity of capital formation. In particular, the productivity may initially fall and then rise over time; whether the productivity eventually exceeds its autarkic level depends critically on the the partial elasticities of domestic investment demand.

3.1 Immediate Responses upon Financial Inflows

In this subsection, we first use the domestic investment-saving diagram to show the response of u_0 to financial inflows and then characterize the productivity effect analytically.

In figure 2, point E_A denotes the domestic investment-saving balance in the autarkic steady state where domestic saving S_A intersects with domestic investment demand M_A, given $\theta \in (0,1)$. In period 0, financial inflows do not affect aggregate income $w_0 = w_A$, and hence, the two lines stay put. Given the downward-sloping M_0, the unit cost of investment must fall so as to accommodate financial capital inflows, $FCF_0 > 0$. According to equation (9) and (5), $u_0 < u_A$ implies the fall in the normalized interest rate $\psi_0 < \psi_A$, which then leads to the falls in the intangible fraction of investment $a_0 < a_A$ and the productivity $\Phi_0 < \Phi_A$.

Let us now explore the economic logic behind the change in the normalized interest rate, $\psi_0 \equiv \frac{r_0}{q_1\Phi_0}$. By lowering the interest rate in country N $r_0 = r^* < r_A$, financial inflows tend to reduce ψ_0. Meanwhile, by augmenting domestic investment, financial inflows reduce the MPK and the social rate of return, $q_1\Phi_0 < q_A\Phi_A$, which tends to raise ψ_0. The net impact depends

[28]See appendix A and the proof of proposition 3 for a detailed analysis of financial integration.

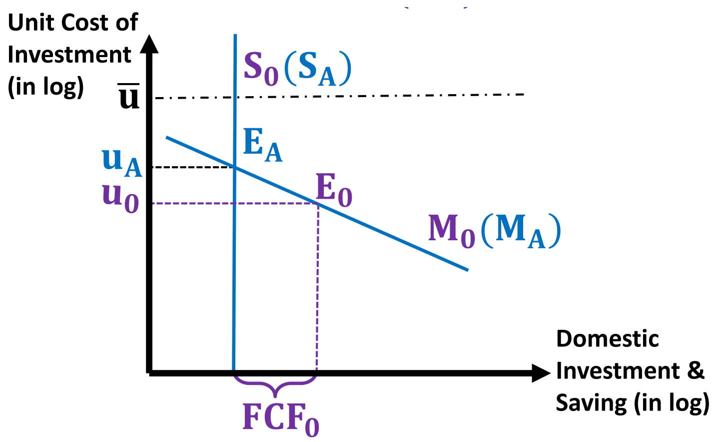

Figure 2: Financial Integration and Domestic Investment-Saving Imbalance in Period 0

on the relative size of these two effects, as shown in equation (21).

$$\frac{\partial \ln \psi_0}{\partial \ln r_0} = \underbrace{1}_{\text{interest rate effect}} - \underbrace{\frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0}}_{\text{MPK effect}} = \frac{1}{1 + \frac{1-u_0}{u_0}\left(\frac{1-\alpha}{\theta} + \alpha\frac{\eta - a_0}{1-a_0}\right)} > 0. \tag{21}$$

Given the interest rate decline $r_0 = r^* < r_A$, $\frac{\partial \ln x_0}{\partial \ln r_0} > 0$ implies a fall in variable x_t, while $\frac{\partial \ln x_0}{\partial \ln r_0} < 0$ implies a rise in variable x_t in period 0. Intuitively, for a given interest rate decline, the smaller the financial inflows, the weaker the MPK effect, the larger the fall in ψ_0, the larger the fall in productivity. What determines the size of financial inflows?

For a given interest rate decline, the larger the θ, the smaller the extensive margin effect $|\frac{\partial \ln \delta_t}{\partial \ln u_t}| = \frac{1}{\theta} - 1$, the less elastic the domestic investment demand $|\frac{\partial \ln M_t}{\partial \ln u_t}| = \frac{1}{\theta}$, the steeper the M_0 line in figure 2, the smaller the financial inflows and domestic investment expansion, the smaller the MPK effect, the larger the declines in ψ_0 and the productivity Φ_0. Here, *the endogenous extensive margin* is the key channel through which wealth inequality dampens the domestic investment expansion and amplifies the productivity decline in period 0.

$$\frac{\partial \ln M_0}{\partial \ln r_0} = -\frac{1}{(1-\alpha) + \theta\left(\frac{u_0}{1-u_0} + \alpha\frac{\eta - a_0}{1-a_0}\right)}, \tag{22}$$

$$\frac{\partial \ln \Phi_0}{\partial \ln r_0} = \frac{\partial \ln \Phi_0}{\partial \ln a_0}\frac{\partial \ln a_0}{\partial \ln u_0}\frac{\partial \ln u_0}{\partial \ln \psi_0}\frac{\partial \ln \psi_0}{\partial \ln r_0} = \frac{\frac{\eta - a_0}{1-a_0}}{\frac{1-\alpha}{\theta} + \left(\frac{u_0}{1-u_0} + \alpha\frac{\eta - a_0}{1-a_0}\right)}. \tag{23}$$

According to equations (22)-(23), financial inflows directly raise the size of domestic investment and indirectly reduce the productivity in period 0. As the former dominates the latter, financial inflows stimulate capital formation and raise aggregate income in period 1.

$$\frac{\partial \ln w_1}{\partial \ln r_0} = (\underbrace{\frac{\partial \ln M_0}{\partial \ln r_0}}_{\text{Inv. size effect}} + \underbrace{\frac{\partial \ln \Phi_0}{\partial \ln r_0}}_{\text{Prod. effect}})[1 - \underbrace{(1-\alpha)}_{\text{Decr. MPK effect}}] = -\frac{\frac{\alpha}{1-\alpha}\left(1 - \theta\frac{\eta - a_0}{1-a_0}\right)}{\frac{\theta}{(1-\alpha)}\left(\frac{u_0}{1-u_0} + \alpha\frac{\eta - a_0}{1-a_0}\right) + 1}. \tag{24}$$

In order to highlight the role of the extensive margin effect in determining the productivity effect, we analyze in appendix B a special case of $\theta \to 0$ where the labor endowment distribution degenerates into a unit mass at $l_j = 1$ and agents have the same labor income, $n_t = w_t$.

3.2 Dynamic Responses along the Convergence Path

Financial inflows stimulate capital formation in period 0 and aggregate income rises in period 1. As shown in figure 1, the higher aggregate income shifts domestic saving rightwards proportionately $\frac{\partial \ln S_t}{\partial \ln w_t} = 1$, while it shifts domestic investment demand rightwards more than proportionately $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta} > 1$, due to the extensive margin effect. In period 1, the widening domestic investment-saving gap amplifies financial inflows, which stimulates capital formation and lowers the MPK. As the interest rate is constant at the world level $r_t = r^*$, the fall in the MPK raises ψ_t. Thus, entrepreneurs shift investment towards the intangibles, which raises the productivity. By lowering the partial elasticity $\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\theta}$, wealth inequality dampens the investment size effect and the productivity effect over time.

$$\frac{\partial \ln M_t}{\partial \ln w_t} = \frac{1}{\frac{1-\alpha}{\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)}+\theta} > 0, \tag{25}$$

$$\frac{\partial \ln \psi_t}{\partial \ln w_t} = \underbrace{\frac{\partial \ln r_t}{\partial \ln w_t}}_{\text{interest rate effect (0)}} - \underbrace{\frac{\partial \ln q_{t+1}\Phi_t}{\partial \ln w_t}}_{\text{MPK effect (-)}} = \frac{\frac{u_t}{1-u_t}}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)+1} > 0, \tag{26}$$

$$\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \frac{\partial \ln \Phi_t}{\partial \ln a_t}\frac{\partial \ln a_t}{\partial \ln u_t}\frac{\partial \ln u_t}{\partial \ln \psi_t}\frac{\partial \ln \psi_t}{\partial \ln w_t} = \frac{\frac{\eta-a_t}{1-a_t}}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)+1} > 0. \tag{27}$$

The investment size effect and the productivity effect compete with the decreasing MPK effect, which drives the dynamics of aggregate income over time.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = (\underbrace{\frac{\partial \ln M_t}{\partial \ln w_t}}_{\text{Inv. size effect}} + \underbrace{\frac{\partial \ln \Phi_t}{\partial \ln w_t}}_{\text{Prod. effect}})[1 - \underbrace{(1-\alpha)}_{\text{Decr. MPK effect}}] = \frac{\frac{\alpha}{1-\alpha}\left(\frac{u_t}{1-u_t}+\frac{\eta-a_t}{1-a_t}\right)}{\frac{\theta}{1-\alpha}\left(\frac{u_t}{1-u_t}+\alpha\frac{\eta-a_t}{1-a_t}\right)+1}. \tag{28}$$

So far, we have shown that, in the presence of financial inflows, the productivity falls in period 0 and then rises over time. Can the subsequent rises dominate the initial fall so that the productivity eventually exceeds its autarkic level?

According to equations (13)-(14), the larger the wealth dispersion, the weaker the extensive margin effect, the smaller the elasticities of domestic investment demand (in absolute value), the smaller the financial inflows and domestic investment expansions, the weaker the MPK effect, the larger the initial decline and the smaller the subsequent rises in productivity over time. Thus, in the presence of financial inflows, *wealth inequality has a negative effect on the long-run level of productivity.*

According to equations (24) and (28), the larger the capital share, the weaker the decreasing MPK effect, the larger the contribution of capital formation to aggregate income $\frac{\partial \ln Y_t}{\partial \ln K_t} = \alpha$, the higher the long-run income level w_F, the larger the mass of entrepreneurs and δ_F, the larger the financial inflows, the lower the MPK, the higher the ψ_F. Let X_F denote the steady-state value of variable X_t under financial integration. Thus, in the presence of financial inflows, *the capital share has a positive effect on the long-run productivity level.*

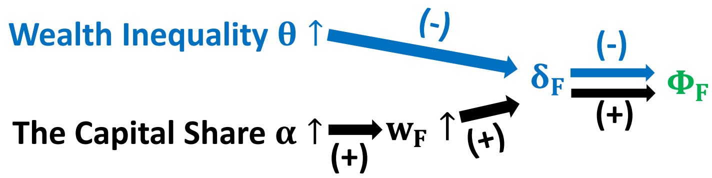

Figure 3: Two Competing Factors Relevant for the Long-Run Productivity Implications

Figure 3 features the entrepreneurial wealth share δ_F as the key channel through which wealth inequality and the capital share have the opposite effects on the long-run productivity implications of financial inflows.

Proposition 2. *Given $\theta \in (0,1)$ and $r^* < r_A < \rho$, the productivity falls in period 0, $\Phi_0 < \Phi_A$, and then rises over time, $\Phi_{t+1} > \Phi_t$. For $\theta \in (0,\alpha)$, the productivity eventually exceeds its autarkic level, i.e., $\Phi_0 < \Phi_A < \Phi_F$; for $\theta \in (\alpha,1)$, the opposite applies, i.e., $\Phi_0 < \Phi_F < \Phi_A$.*

Proof. Let subscript $s \in \{A,F\}$ denote the steady state under autarky versus under financial integration,[29] respectively. Given the binding borrowing constraints, the law of motion for wage takes the same functional form under the two scenarios, as specified by equations (17) and (12). Thus, the steady-state wage rate also takes the same functional form under the two scenarios, $w_s = \left(\frac{\delta_s}{u_s}\frac{\Phi_s}{\rho}\right)^{\frac{\alpha}{1-\alpha}}$. Combine (10) and (12) to get $\delta_s = \left(\frac{w_s}{u_s}\frac{1-\theta}{m}\right)^{\frac{1}{\theta}-1}$. Combine these two conditions with equations (5) and (9) to get

$$\frac{u_s}{\Phi_s^\theta}\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^\theta = w_s^{\frac{\alpha-\theta}{\alpha}} \tag{29}$$

$$\Rightarrow \frac{\partial \ln LHS}{\partial \ln a_s} = 1-\theta\frac{\eta - a_t}{1-a_t} > 0 \quad \text{and} \quad \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha-\theta}{\alpha}. \tag{30}$$

The left-hand side (LHS) of equation (29) is an increasing function of a_s, while the relative size of α versus θ determines the monotonicity of the right-hand side (RHS) as a function of w_s.

As financial inflows stimulate the domestic capital formation, the long-run income level is strictly higher than in the autarkic steady state, $w_F > w_A$.

- For $\theta = \alpha$, the wealth-inequality effect and the capital-share effect cancel out so that the RHS of equation (29) is constant at unity, $RHS = 1$. Thus, the LHS is also constant at unity in the two steady state, implying that the intangible fraction of investment is identical in the two steady state $a_F = a_A$ and so is the productivity $\Phi_F = \Phi_A$. In this case, the initial fall in the productivity is exactly offset by the subsequent rises.

- For $\alpha > \theta$, the capital-share effect dominates the wealth-inequality effect so that $\frac{\partial \ln RHS}{\partial \ln w_s} > 0$. Thus, $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$ also holds. Combine them with $w_F > w_A$ to get $a_F > a_A$ and $\Phi_F > \Phi_A$. For $\theta > \alpha$, the opposite applies so that $a_F < a_A$ and $\Phi_F < \Phi_A$.

□

[29] Appendix A specifies the conditions for the existence of a stable steady state under financial integration.

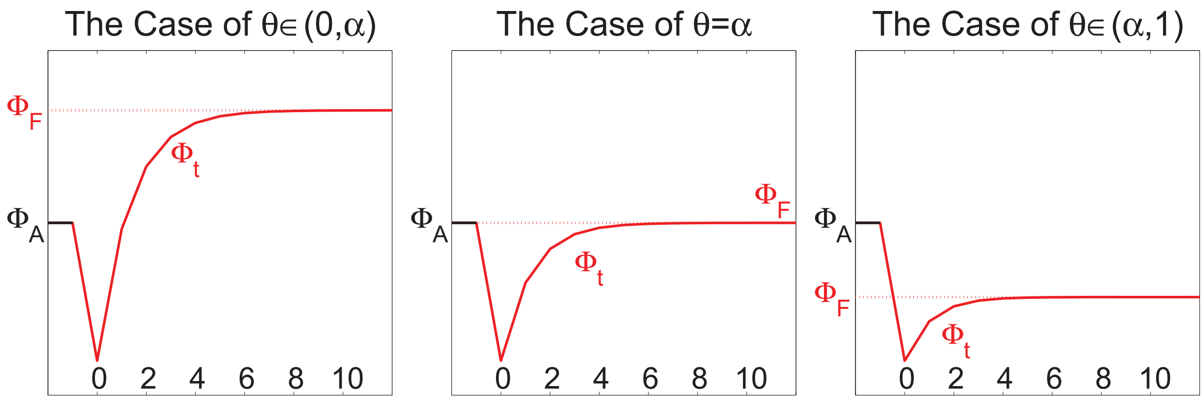

Figure 4: Impulse Responses of Productivity under Financial Integration

Figure 4 shows the impulse responses of productivity in three cases,[30] with the horizontal axis denoting the time period $t = 0, 1, 2, \ldots$

As shown in figure 1 and 2, what really matter for the dynamics and the long-run level of productivity are the two partial elasticities of domestic investment demand,[31] which happen to be negatively related to wealth inequality in this model. Thus, one should not over-interpret the role of wealth inequality in our findings. The mechanism behind our findings may also hold if other market frictions or regulatory requirements hamper entrepreneurial entry.

Our findings imply that a country may obtain productivity gains from debt inflows, if the elasticity of domestic investment exceed some "threshold" value. It resonates with the spirit of Kose et al. (2011, 2009a) and calls for the empirical research identifying the institutional factors relevant for the elasticity of domestic investment and exploring the "threshold" conditions.

According to equations (13)-(14), the *endogenous* entrepreneurial wealth share δ_t is a key channel through which domestic investment demand responds by a larger proportion than the change in w_t and in u_t. The stronger the extensive margin effect, the more elastic the domestic investment demand, the more likely the productivity exceed its autarkic level in the long run. For comparison purpose, we set up an alternative model in appendix C where the mass of entrepreneurs is *exogenous* and hence, δ_t is constant. Given $r^* < r_A$, the productivity falls upon financial integration and then rises over time, similar as in the current model. However, in the absence of the extensive margin effect, the unit cost of investment in the long run is strictly lower than its initial level, $u_F < u_A$, and so is the productivity, $\Phi_F < \Phi_A$.[32]

So far, we have focused on a special case, i.e., country N is initially in the autarkic steady state where the borrowing constraints are binding, and the world interest rate is marginally lower than that in country N; under financial integration, country N converges to a new steady

[30] As the steady-state value of productivity is a function of wealth inequality, we scale the vertical axes of figure 4 purely for illustration purpose. One should not compare the productivity levels literally in the three cases.

[31] The partial elasticity $\frac{\partial \ln M_0}{\partial \ln u_0}$ determines the magnitude of the initial fall in productivity, while the partial elasticity $\frac{\partial \ln M_t}{\partial \ln w_t}$ determines the magnitude of the subsequent rises in productivity.

[32] In the current model, for $\theta \to 1$, the extensive margin effect in equations (13)-(14) vanishes $\frac{1}{\theta} - 1 \to 0$ and hence, δ_t is independent of changes in w_t or u_t. Thus, ψ_t and Φ_t have the same dynamic patterns as those in the alternative model in appendix C. In other words, the responses of ψ_t and Φ_t in the alternative model constitute the lower bound for those in the current model. See subsection 3.3.5 for endogenizing δ_t in an alternative setting.

state in the neighbourhood of the initial one. In short, we conduct the **marginal analysis**. Our findings also hold in an alternative case where country N is initially in the steady state under financial integration and the world interest rate falls marginally from period 0 on.

In section 4, we conduct the **global analysis** and explore the model dynamics over the entire state spaces. Before moving on to the global analysis, let us first discuss the robustness of our findings under alternative settings.

3.3 Robustness and Extensions

3.3.1 Elasticity of Substitution between the Tangibles and the Intangibles

In the current model, tangible and intangible investments are combined for capital formation in a Cobb-Douglas fashion. Following Falato et al. (2018), one can assume alternatively that capital formation takes the CES (constant elasticity of substitution) form,

$$k_{j,t+1} = \left[\eta \left(\frac{m_{j,I,t}}{\eta} \right)^{\frac{\sigma-1}{\sigma}} + (1-\eta) \left(\frac{m_{j,T,t}}{1-\eta} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{31}$$

In so doing, aggregate productivity and the unit cost of investment become,

$$\Phi_t = \left[\eta \left(\frac{a_t}{\eta} \right)^{\frac{\sigma-1}{\sigma}} + (1-\eta) \left(\frac{1-a_t}{1-\eta} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \tag{32}$$

$$u_t = a_t + (1-\lambda) \left(\frac{1-\eta}{\eta} \right)^{\frac{1}{\sigma}} a_t^{\frac{1}{\sigma}} (1-a_t)^{\frac{\sigma-1}{\sigma}}. \tag{33}$$

$$\frac{\partial u_t}{\partial a_t} = 1-(1-\lambda) \left[\frac{1-\eta}{\eta} \frac{a_t}{1-a_t} \right]^{\frac{1}{\sigma}} \left(1 - \frac{1}{\sigma a_t} \right) > 0, \text{ for } a_t \leq \eta \tag{34}$$

Equation (9) is a special case of equation (33) with $\sigma = 1$. Since $\frac{\partial u_t}{\partial a_t} > 0$ holds in this general setting, the mechanism of our current model still holds. The higher the σ, the more substitutable the two types of investments in the project of capital formation, the larger the within-project investment reallocation triggered by financial inflows. Allowing $\sigma \neq 1$ only affects the magnitude of the productivity dynamics, while the opposite dynamic pattern still exists.

Let $\mathbb{A}_t \equiv \left[\frac{1-\eta}{\eta} \frac{a_t}{1-a_t} \right]^{\frac{1}{\sigma}}$. For $a_t < \eta$, $\mathbb{A}_t < 1$. In the general setting, condition (30) becomes

$$\frac{\partial \ln LHS}{\partial \ln a_s} = \frac{1-(1-\lambda)\left(1-\frac{1}{\sigma a_t}\right)\mathbb{A}_t}{1+(1-\lambda)\frac{1-a_t}{a_t}\mathbb{A}_t} - \theta \frac{1-\mathbb{A}_t}{1+\frac{1-a_t}{a_t}\mathbb{A}_t} > 0, \text{ and } \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha - \theta}{\theta}. \tag{35}$$

In this case, $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$ still holds and so does the proof of proposition 2. The relative size of θ and α still determines the long-run productivity effect, the same as in our current model.

3.3.2 The MIR on the Tangibles

In the current model, the MIR and financial frictions jointly endogenize the mass of entrepreneurs, while the resulting endogenous extensive margin of domestic investment demand is key to the productivity dynamics under financial integration. Instead of assuming the MIR for the total project investment, one can assume that the MIR applies to tangible investment, $m_{j,T,t} \geq \mathfrak{m}$. Accordingly, the cutoff value specified by equation (10) becomes

$$\frac{w_t(1-\theta)\underline{\varepsilon}_t}{u_t}(1-a_t) = \mathfrak{m}, \Rightarrow \underline{\varepsilon}_t = \frac{u_t}{(1-a_t)w_t}\frac{\mathfrak{m}}{1-\theta}, \tag{36}$$

while other conditions in section 1 are unaffected. In particular, the two partial elasticities of domestic investment demand are still specified by equations (13)-(14). Thus, the core mechanism of our current model still holds and so does the opposite productivity dynamics.

In the proof of proposition 2, δ_s and equation (29)-(30) become

$$\delta_s = \underline{\varepsilon}_s^{-\frac{1-\theta}{\theta}} = \left(\frac{w_s(1-a_s)}{u_s}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1}{\theta}-1} \tag{37}$$

$$\frac{u_s}{\Phi_s^{\theta}(1-a_s)^{1-\theta}}\left(\frac{\mathfrak{m}}{1-\theta}\right)^{1-\theta}\rho^{\theta} = w_s^{\frac{\alpha-\theta}{\alpha}} \tag{38}$$

$$\Rightarrow \frac{\partial \ln LHS}{\partial \ln a_s} = 1+(1-\theta)\frac{a_t}{1-a_t} - \theta\frac{\eta-a_t}{1-a_t} > 0 \text{ and } \frac{\partial \ln RHS}{\partial \ln w_s} = \frac{\alpha-\theta}{\alpha}. \tag{39}$$

In this case, $\frac{\partial \ln LHS}{\partial \ln a_s} > 0$ still holds and so does the proof of proposition 2. The relative size of θ and α still determines the long-run productivity effect, the same as in our current model.

Assuming the MIR for the tangibles does not change the findings of our current in terms of the dynamic pattern of productivity and the conditions for the long-run productivity effect. However, it does make the analytical solution more cumbersome. For example, the productivity effect specified in equation (20) becomes

$$\frac{\partial \ln \Phi_t}{\partial \ln w_t} = \left(1-\frac{1-\eta}{1-a_t}\right)\frac{1}{\frac{1}{1-\theta}+\frac{a_t}{1-a_t}}. \tag{40}$$

3.3.3 Collateral Constraints Revisited

In the current model, only the tangibles can serve as collateral for loans, while the intangibles have to be fully financed by the entrepreneur's own funds. Using a large sample of syndicated loans to US corporations, Falato et al. (2018) have verified that only 3% of secured syndicated loans have patents or brands used as collateral. Here, we check the robustness of our findings in a generalized setting (Caggese and Pérez-Orive, 2018) where the intangibles can also serve as collateral for loans but they have a lower degree of pledgeability than the tangibles.

To be specific, if agent j defaults, lenders can seize and liquidate not only the tangibles but also the intangibles; after deducting the liquidation costs, the lenders get $\lambda_I p_{I,t+1}$ as the liquidation value per unit of intangibles, where $p_{I,t+1}$ and λ_I denote respectively the market price and the pledgeability of intangibles. Let $\kappa \equiv \frac{\lambda_I}{\lambda} < 1$ denote the intangible-tangible pledgeability ratio. If the borrowing constraints are binding, the unit cost of intangibles is $u_{I,t} = 1 - \frac{\kappa\lambda p_{I,t+1}}{r_t}$, while the unit return is $q_{t+1}\frac{\partial k_{j,t+1}}{\partial m_{j,I,t}} - \kappa\lambda p_{I,t+1}$. The agent equalizes the internal rates of return,

$$\frac{q_{t+1}\frac{\partial k_{j,t+1}}{\partial m_{j,I,t}} - \kappa\lambda p_{I,t+1}}{u_{I,t}} = \frac{q_{t+1}\frac{\partial k_{j,t+1}}{\partial m_{j,T,t}} - \lambda p_{t+1}}{u_{T,t}} \Rightarrow \frac{1-\frac{\lambda p_{t+1}}{r_t}}{1-\frac{\kappa\lambda p_{I,t+1}}{r_t}} = \frac{q_{t+1}\Phi_{j,t}\frac{1-\eta}{1-a_{j,t}} - \lambda p_{t+1}}{q_{t+1}\Phi_{j,t}\frac{\eta}{a_{j,t}} - \kappa\lambda p_{I,t+1}}. \tag{41}$$

According to equation (41), the optimal choice of $a_{i,t}$ is a function of the parameters (λ, κ, η) and the market prices $(p_{t+1}, p_{I,t+1}, r_t, q_{t+1})$. Thus, the agents who meet the MIR optimally choose the same intangible fraction of investment. Hereafter, we drop off subscript j and use a_t to denote it. Agent j equalizes its marginal revenue of intangibles to the market price,

$$q_{t+1}\frac{\partial k_{j,t+1}}{\partial m_{j,I,t}} = q_{t+1}\Phi_t\frac{\eta}{a_t} = p_{I,t+1}, \tag{42}$$

Combine equations (5) and (41)-(42) to solve for the unit cost of investment,

$$u_t \equiv a_t u_{I,t} + (1-a_t)u_{T,t} = 1 - [\eta\kappa + (1-\eta)]\frac{\lambda}{\psi_t}, \tag{43}$$

$$u_{I,t} = 1 - \frac{\eta}{a_t}\kappa\frac{\lambda}{\psi_t} = \frac{\eta}{a_t}(1-\kappa\lambda)\frac{u_t}{1-[\kappa\eta+(1-\eta)]\lambda} \tag{44}$$

$$u_{T,t} = 1 - \frac{1-\eta}{1-a_t}\frac{\lambda}{\psi_t} = \frac{1-\eta}{1-a_t}(1-\lambda)\frac{u_t}{1-[\kappa\eta+(1-\eta)]\lambda}, \tag{45}$$

$$\frac{a_t}{\eta}\{1-[\kappa\eta+(1-\eta)]\lambda\} = 1-\kappa\lambda - \frac{(1-\eta)(1-\kappa)\lambda}{\psi_t} = \frac{(1-\kappa)}{1+\frac{\eta}{1-\eta}\kappa}u_t + \frac{(1-\lambda)+\lambda\eta(1-\kappa)}{\eta+\frac{1-\eta}{\kappa}} \tag{46}$$

Our current model is a special case of this generalized setting. One can put $\kappa = 0$ into equations (43)-(46) to get $u_{I,t} = 1$ and equations (6)-(9) in our current model. Except them, other equilibrium conditions are identical as in our current model.

In our current model, λ measures the level of financial development. In the generalized setting, κ is another measure of financial development, reflecting the heterogeneous pledgeability across different asset classes. If the intangibles and the tangibles have the same pledgeability $\kappa = 1$, use equation (44)-(46) to get $a_t = \eta$ and $u_{I,t} = u_{T,t} = u_t$. In this case, the unit costs of tangibles and intangibles equalize, the within-project investment composition is efficient, and the productivity is constant at $\Phi_t = 1$, despite the binding borrowing constraints $\lambda < \tilde{\lambda}_A$. If the intangibles has a lower pledgeability than the tangibles, $\kappa < 1$ creates the unit-cost differential $u_{I,t} > u_{T,t}$ and hence, entrepreneurs invest less (more) in the intangibles (tangibles) $a_t < \eta$, which distorts allocative efficiency and undermines the productivity $\Phi_t < 1$. Thus, it is κ rather than λ that distorts allocative efficiency in the general setting.

Besides, in the case of $\kappa \in [0,1)$, the unit-cost differential $u_{I,t} > u_{T,t}$ implies that the unit cost of tangibles is more elastic to the change in ψ_t than that of intangibles,

$$\frac{\partial \ln u_{T,t}}{\partial \ln \psi_t} = \frac{1}{u_{T,t}} - 1 > \frac{\partial \ln u_{I,t}}{\partial \ln \psi_t} = \frac{1}{u_{I,t}} - 1 > 0. \tag{47}$$

In period 0, financial inflows lower ψ_0, which causes $u_{T,0}$ to fall by a larger proportion than the change in $u_{I,0}$. It widens the unit-cost differential and induces entrepreneurs to shift investment further towards the tangibles. From period $t = 1$ on, the rise in ψ_t also causes $u_{T,t}$ to rise by a larger proportion than the change in $u_{I,t}$, and the narrowing unit-cost differential induces entrepreneurs to shift investment towards the intangibles. Besides, the relative size of θ and α still matters for the long-run productivity effect in the same way as specified in proposition 2. In comparison with our current model, allowing intangibles to serve as collateral with $\kappa \in [0,1)$

narrows the unit-cost differential and weakens the magnitude of the productivity dynamics, while our findings still hold.

Remark A: in Matsuyama (2004) and Zhang (2017), the productivity of capital formation is constant at R and entrepreneurs can borrow up to a fraction of the future project revenue, i.e. $b_{j,t} = \frac{\lambda q_{t+1} R m_{j,t}}{r_t}$. In the current model, we assume that entrepreneurs can borrow up to a fraction of the future market value of tangibles, as shown in equation (3). Although the borrowing constraints take different forms in the two papers, the two specifications are technically equivalent. According to equation (44), if the intangibles have the same degree of pledgeability $\kappa = 1$, the unit cost of investment is $u_t = 1 - \frac{\lambda q_{t+1}\Phi_t}{r_t}$, the same as in Matsuyama (2004). Introducing heterogeneous pledgeability essentially reduces the pledgeable value of the project revenue. For $\kappa = 0$, the unit cost of investment is higher, $u_t = 1 - \frac{(1-\eta)\lambda q_{t+1}\Phi_t}{r_t}$.

Remark B: In order to ensure the binding borrowing constraints in the steady state, we adopt the two-period, overlapping-generation model and exclude the possibility that agents overcome the entry barrier by accumulating net wealth over time. Besides finite lifetime (Bernanke et al., 1999), researchers may also assume impatient entrepreneurs (Kiyotaki and Moore, 1997) to ensure the binding borrowing constraints in the long run. Moll (2014) develops a model with idiosyncratic productivity shocks and shows that, if shocks are transitory, self-financing cannot fully mitigate the borrowing constraints at the aggregate level so that steady-state capital misallocation is large; if shocks are relatively persistent, self-financing undoes capital misallocation from financial frictions in the long run, but transitions to this steady state take a very long time.

3.3.4 Within-Project vs. Cross-Project Investment Reallocation

In the current model, agents are endowed with the same project and entrepreneurs choose the same intangible fraction of investment in equilibrium. Thus, the productivity at the aggregate level coincides with that at the individual level. By triggering *within-project* investment reallocation along the tangibles-intangibles margin, financial inflows affect allocative efficiency.

Financial inflows may affect allocative efficiency by triggering investment reallocation across projects with heterogeneous productivity. Consider a model with two projects indexed by $h \in \{1,2\}$. In order to feature explicitly the cross-project margin, we turn off the within-project margin by assuming that both projects are linear with the exogenous productivity, $k_{h,j,t+1} = \phi_h m_{h,j,t}$. The borrowing constraints are project-specific, $b_{h,j,t} \leq \lambda_h \frac{\phi_h q_{t+1}}{r_t} m_{h,j,t}$. Project 2 is more productive than project 1, $\phi_2 > \phi_1$, but it is subject to the tighter borrowing constraint, $\lambda_2 < \lambda_1 \frac{\phi_1}{\phi_2}$.[33] Besides, the two projects are subject to the same MIR.

[33] Matsuyama (2007) introduces such a model with exogenous heterogeneity in productivity and pledgeability. One can endogenize the cross-project differences in pledgeability as follows. Suppose that entrepreneurs can choose between a traditional project indexed by $h = 1$ and a modern project indexed by $h = 2$ for capital formation. The traditional project is linear $k_{1,j,t+1} = \phi_1 m_{1,j,t}$, with the input of tangibles $m_{1,j,t}$ only; the modern project takes the Leontief form, $k_{2,j,t+1} = \phi_2 \min\left\{\frac{m_{2,j,I,t}}{\eta}, \frac{m_{2,j,T,t}}{1-\eta}\right\}$, with the inputs of tangibles $m_{2,j,T,t}$ and intangibles $m_{2,j,I,t}$. Let $m_{2,j,t} \equiv m_{2,j,T,t} + m_{2,j,I,t}$ denote agent-j's total investment in the modern project. In equilibrium, agent j chooses $\frac{m_{2,j,I,t}}{\eta} = \frac{m_{2,j,T,t}}{1-\eta} = m_{2,j,t}$ and the modern project is linear, $k_{2,j,t+1} = \phi_2 m_{2,j,t}$. By assumption, intangible investment improves the project productivity $\phi_2 > \phi_1$, while only tangibles can be used as the collateral for loans. When running the traditional project, agents face the borrowing constraints $b_{1,j,t} \leq \lambda \frac{\phi_1 q_{t+1}}{r_t} m_{1,j,t}$. When running

Agents prefer to invest in the more productive project if they can meet the MIR. In the case where the borrowing constraints are binding for both projects, there are two cutoff values, $\underline{\varepsilon}_{2,t} > \underline{\varepsilon}_{1,t} > 1$, that split agents into three groups. Those with $\varepsilon_j \in (1, \underline{\varepsilon}_{1,t})$ lend out the labor income and are called households; those with $\varepsilon_j \in [\underline{\varepsilon}_{1,t}, \underline{\varepsilon}_{2,t})$ invest in project 1 and are called group-1 entrepreneurs; those with $\varepsilon_j \geq \underline{\varepsilon}_{2,t}$ invest in project 2 and are called group-2 entrepreneurs. The aggregate productivity is the weighted average of the project productivity, $\Phi_t = \chi_t \phi_2 + (1 - \chi_t)\phi_1$, where $\chi_t \equiv \frac{M_{2,t}}{M_{1,t}+M_{2,t}}$ denotes the fraction of domestic investment allocated in project 2.

Our preliminary analysis shows that, given $r^* < r_A$, domestic investment first shifts disproportionately towards project 1 upon financial inflows and then towards project 2 over time. Thus, aggregate productivity falls in period 0 and then rises over time. The higher the wealth inequality, the less elastic the mass of entrepreneurs in each group, the smaller the elasticities of domestic investment for each type of project, the larger the initial fall and the smaller the subsequent rises in aggregate productivity. This way, by reducing the elasticities of domestic investment demand, wealth inequality dampens the productivity dynamics along the within-project and the cross-project margins in the same way.

3.3.5 Alternative Way of Endogenizing the Entrepreneurial Wealth Share

As shown in subsections 3.1-3.2 and appendix C, and *endogenous* entrepreneurial wealth share δ_t and the extensive margin effect are the key channel through which wealth inequality affects the elasticity of domestic investment demand and the productivity dynamics. In this two-period, OLG framework, agents save their entire labor income when young and consume their entire investment return when old. Thus, the labor income is *the only source* of individual net wealth that matters for domestic investment, $n_{j,t} = l_j w_t$, and hence, the entrepreneurial wealth share is driven purely by the changes in the mass of entrepreneurs, $\delta_t = \tau_t^{1-\theta}$.

Besides, one can endogenize δ_t by allowing for individual wealth accumulation over a longer time horizon. Keeping the mass of entrepreneurs exogenous, one can embed the core elements of the model specified in appendix C into a continuous-time, perpetual youth framework (Blanchard, 1985). For $q_{t+1}\Phi_t > r_t$, the borrowing constraints are binding and, due to the leverage effect, entrepreneurs earn a higher rate of return on their net wealth than households.[34] The rate-of-return differential allows entrepreneurs to accumulate wealth at a faster rate than households, which endogenizes the entrepreneurial wealth share.

By reducing the interest rate, financial inflows widens the rate-of-return differential, which raises δ_t along the intensive margin in country N. The mechanism of the current model still applies. A complete analysis of the productivity implications in that model is beyond the scope of the current paper and we leave it for future research.

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the modern project, agents face the borrowing constraints $b_{2,j,t} \leq \lambda \frac{\phi_2 m_{2,j,T,t} q_{t+1}}{r_t} = \lambda(1-\eta)\frac{\phi_2 q_{t+1}}{r_t} m_{2,j,t}$. Thus, the cross-project difference in pledgeability reflects the cross-project difference in tangibility, $\frac{\lambda_2}{\lambda_1} = 1 - \eta$.

[34]For each unit of investment in period t, an entrepreneur has to put down $u_t = 1 - \frac{(1-\eta)\lambda q_{t+1}\Phi_t}{r_t}$ units of own fund. In period $t+1$, it uses the project revenue $q_{t+1}\Phi_t$ to pay off the debt $(1-\eta)\lambda q_{t+1}\Phi_t$ and consumes the rest, $[1-(1-\eta)\lambda]q_{t+1}\Phi_t$. The gross rate of return on entrepreneurial wealth is $\Gamma_t \equiv \frac{[1-(1-\eta)\lambda]q_{t+1}\Phi_t}{u_t} = r_t + \frac{q_{t+1}\Phi_t - r_t}{u_t}$, where $\frac{q_{t+1}\Phi_t - r_t}{u_t}$ features the leverage effect. In contrast, households lend out their own funds for the gross interest rate r_t. Given $q_{t+1}\Phi_t > r_t$, the leverage effect is positive, which ensures $\Gamma_t > r_t$.

3.3.6 Exogenous vs. Endogenous Wealth Inequality

In the current model, the distribution of labor endowment is exogenous and so is the wealth distribution of young agents. A larger wealth inequality weakens the responses of δ_t with respect to the changes in the interest rate and aggregate income, which then dampens capital inflows and productivity dynamics.

The wealth distribution in the model mentioned in subsection 3.3.5 is endogenous. By widening the rate-of-return differential, financial integration raises the entrepreneurial wealth share, which amplifies capital inflows as well as widens wealth inequality. In contrast to the findings of the current model, the *endogenous* wealth inequality moves ex post *positively* with the size of capital inflows in that model.[35] The dynamic, two-way interactions between wealth inequality and capital inflows deserves further research.

4 Equilibrium Shifts and Productivity Patterns

According to equations (22)-(28), the investment size and the productivity are the two channels through which financial inflows affect the domestic capital formation and the income dynamics. Iff the borrowing constraints are binding, $a_t < \eta$ and the productivity channel is active. The tighter the borrowing constraints and the lower the wealth inequality, the more strongly the δ_t responds to financial inflows, the stronger the domestic capital formation effect.

The capital formation effect is negatively related to λ and θ, while the decreasing MPK effect is negatively related to α. According to equation (28), if the former dominates the latter at any steady state, multiple steady states arise. Proposition 3 specifies the threshold conditions for multiple steady states in terms of $\{\lambda, \theta, \alpha\}$, while appendix A offers a detailed analysis.

Proposition 3. *For $\theta < \alpha$ and $\lambda < \tilde{\lambda}_F \equiv \min\{\frac{\alpha-\theta}{(1-\theta)(1-\eta)}, 1\}$, financial integration may lead to multiple steady states in country N.*

Corollary 1. *Given $\theta < \alpha$, $\lambda < \tilde{\lambda}_F$, and $Z \in (\underline{Z}_F, \bar{Z}_F)$, there exist two threshold values, $\hat{r}^*$ and $\tilde{r}^*$, where $\hat{r}^* < r_A < \tilde{r}^*$. Under financial integration, multiple steady states arise if the world interest rate is moderate $r^* \in [\hat{r}^*, \tilde{r}^*]$, while there is a unique steady state if $r^* < \hat{r}^*$ or $r^* > \tilde{r}^*$.*

According to corollary 1, if r^* either falls below $\hat{r}^*$ or rises above $\tilde{r}^*$, country N shifts from the equilibrium with multiple steady states to the one with a unique steady state, which has substantial impacts on income and productivity. We clarify this mechanism with two examples.

4.1 From International Autarky to Financial Integration

In the first example, we revisit the long-run implications of financial integration, taking into account the possibility of equilibrium shift. Given $\theta < \alpha$, $\lambda < \tilde{\lambda}_F$, and $Z \in (\underline{Z}_F, \bar{Z}_F)$, the dashed curve in the left panels of figure 5 shows the law of motion for wage under autarky, while point A denotes the autarkic steady state where the borrowing constraints are binding and $\Phi_A < 1$.

[35] The exogenous wealth inequality in the current model can be regarded as an ex ante measure of inequality, while the endogenous wealth inequality in that model can be regarded as an ex post measure of inequality.

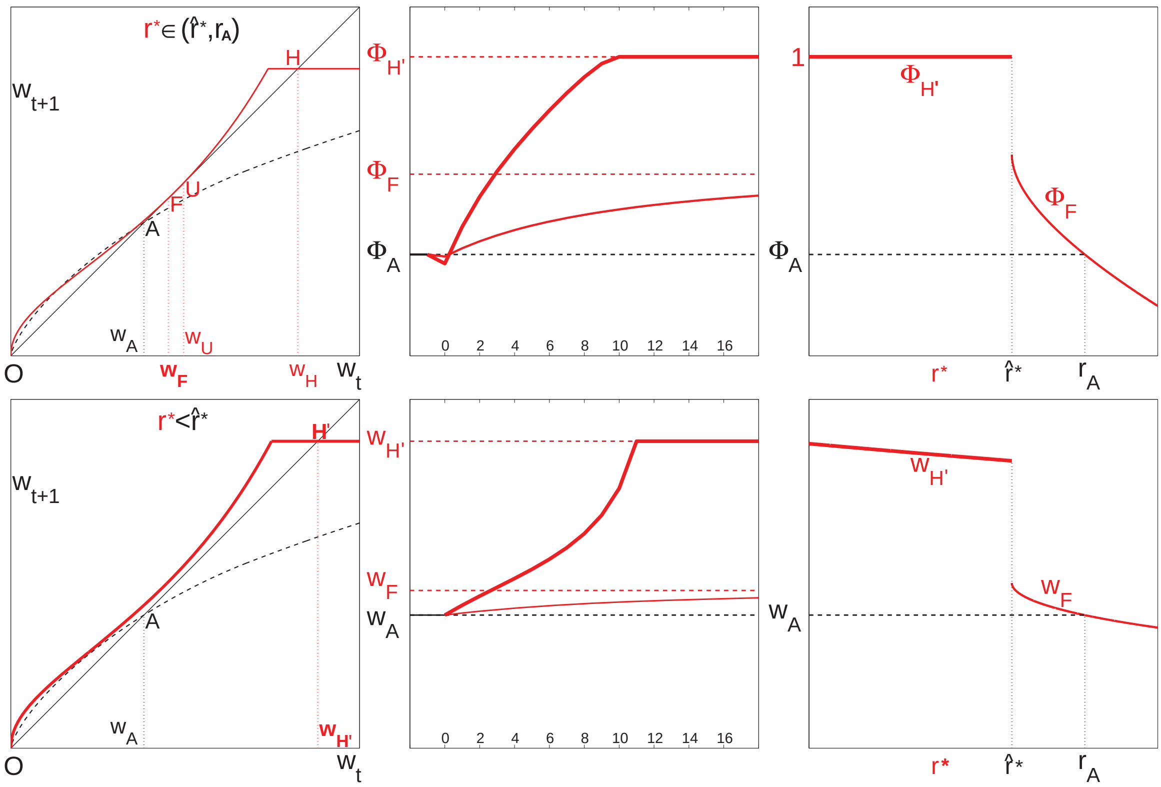

Figure 5: Impacts of Financial Integration: the Case of $w_0 = w_A$

Country N is initially at the autarkic steady state A. If the world interest rate is slightly below the autarkic interest rate in country N $r^* \in (\hat{r}^*, r_A)$, financial inflows are moderate and and so are their impacts on domestic capital formation. The thin, solid curve in the upper-left panel of figure 5 shows the law of motion for wage under financial integration[36]. From period 0 on, country N converges from the autarkic to a new steady state F where aggregate income is higher, $w_F > w_A$ and the borrowing constraints are still binding. The thin, solid curve in the upper-middle (lower-middle) panel shows the impulse responses of the productivity (income), with the time period $t = 0, 1, 2, \ldots$ on the horizontal axis. Given $\theta < \alpha$, the productivity in the long run is higher than its initial level, despite its initial fall in period 0, $\psi_0 < \psi_A < \Phi_F < 1$.

Besides the stable steady state F, there are an unstable steady state (U) and another stable steady state (H), as shown in the upper-left panel of figure 5. Starting from the autarkic steady state A, country N converges along a unique path to the stable steady state F. Thus, in the presence of multiple steady states, the initial condition matters for the long-run allocation.

For $r^* < \hat{r}^*$, the initial interest rate difference $r_A - r^*$ is so large that financial inflows shift the law of motion for wage upwards, as shown by the thick, solid curve in the lower-left panel.[37] From period 0 on, country N converges from the autarkic to the new, unique steady state H′ where the borrowing constraints are slack, the within-firm investment composition is

[36]Under financial integration, the law of motion for wage is piecewise: for $w_t < \bar{w}_F$, it is upward-sloping, the borrowing constraints are binding, and $\Phi_t < 1$; for $w_t \geq \bar{w}_F$, it is flat at $w_{t+1} = (r^*)^{-\rho}$, the borrowing constraints are slack, and $\Phi_t = 1$. See appendix A and the proof of proposition 3 for the technical analysis.

[37]For $r^* = \hat{r}^*$, the law of motion for wage is tangent with the 45° line in the convex part and crosses the 45° line in the flat part. In this case, there are two steady states.

efficient, $a_t = \eta$ and $\Phi_{H'} = 1 > \Phi_A$, and the income is substantially higher than its initial level, $w_{H'} = (r^*)^{-\rho} \gg w_A$. The thick, solid curve in the upper-middle (lower-middle) panel shows the impulse responses of productivity (income).

To sum up, given $w_0 = w_A$, if the world interest rate falls marginally from above to below $\hat{r}^*$, country N shifts from the equilibrium with multiple steady states to the one with a unique steady state. The equilibrium shift causes a discontinuous jump in the long-run pattern of productivity (income), as shown by the piecewise, solid curve in upper-right (lower-right) panel of figure 5. The horizontal axis denotes the world interest rate r^*, while $\hat{r}^*$ and r_A denote respectively the threshold value and the autarkic interest rate in country N.

4.2 The World Interest Rate Hike under Financial Integration

In the second example, we analyze the impacts of a permanent rise in the world interest rate. Given $r^* < \hat{r}^*$, the dashed curve in the left panels of figure 6 shows the law of motion for wage under financial integration. Country N is initially in the steady state denoted by point H$'$ where the borrowing constraints are slack and $\Phi_{H'} = 1$.

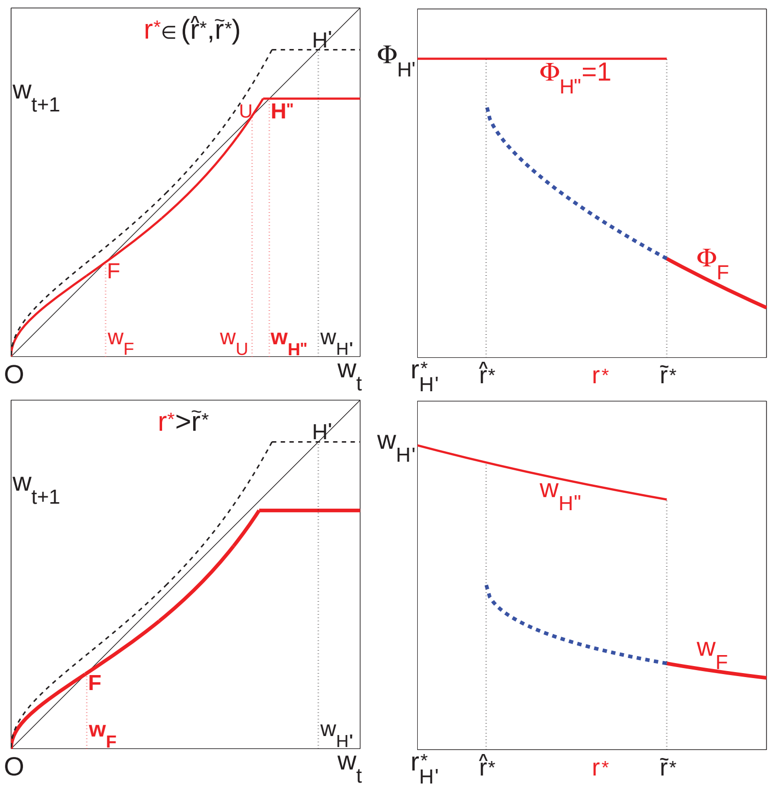

Figure 6: Impacts of the World Interest Rate Hike: the Case of $r^*_{H'} < \hat{r}^*$ and $w_0 = w_{H'}$

From period 0 on, a rise in the world interest rate reduces financial inflows and undermines the capital formation in country N, which shifts the law of motion for wage downwards. In figure 6, the solid curves in the upper-left and the lower-left panels show respectively the laws of motion for wage in the cases of a mild rate rise $r^* \in (\hat{r}^*, \tilde{r}_t)$ vs. a large rate rise $r^* > \tilde{r}^*$.

- For the mild interest rate rise, country N converges from point H$'$ to H$''$ where the income level is lower, $w_{H''} < w_{H'}$, but the productivity level is unaffected, $\Phi_{H''} = \Phi_{H'} = 1$.

- For the large interest rate rise, country N converges from point H′ to F where the levels of income and productivity are substantially lower, $w_F < w_{H'}$ and $\Phi_F < \Phi_{H'}$.

Given $w_0 = w_{H'}$, if the world interest rate rises marginally from below to above $\tilde{r}^*$, it shifts country N from the equilibrium with multiple steady states to the one with a unique steady state. The equilibrium shift causes a discontinuous jump in the long-run pattern of productivity (income), as shown by the piecewise, solid curve in the upper-right (lower-right) panel of figure 6. The horizontal axis denotes the world interest rate r^*, while $r^*_{H'}$, $\hat{r}^*$, and $\tilde{r}^*$ denote respectively the initial world interest rate and the two threshold values.

4.3 Path Dependence and Equilibrium Shift as the Cause of Discontinuity

For a moderate world interest rate $r^* \in [\hat{r}^*, \tilde{r}^*]$, multiple steady states arise under financial integration, which has two implications. First, the existence of multiple steady states implies *path dependence*, i.e., the initial condition matters for the long-run allocation. Second, if r^* rises above $\hat{r}^*$ or falls below $\tilde{r}^*$, country N witnesses the *equilibrium shift* from multiple steady states to a unique steady state. Both are crucial for the discontinuity in the long-run patterns.

In the first example, given $r^* < r_A$, the discontinuity occurs around $r^* = \hat{r}^*$, if the initial income is sufficiently low, e.g., $w_0 = w_A$. However, if the initial income is sufficiently high, e.g., $w_0 > w_U$, country N converges to steady state H in the case of $r^* \in [\hat{r}^*, r_A)$ and to steady state H′ in the case of $r^* < \hat{r}^*$. See the left panels of figure 5. In both cases, the long-run level of productivity is equal to unity and the long-run level of income takes the form of $(r^*)^{-\rho}$. In other words, the long-run patterns of income and productivity are continuous for $r^* < r_A$, despite the equilibrium shift around $r^* = \hat{r}^*$. Hence, *the initial condition is crucial for the discontinuity.*

Only the equilibrium shift from multiple steady states to a unique steady state may cause the discontinuity.[38] In the first example, financial integration lowers the domestic interest rate, $r_t = r^* < r_A$ and the equilibrium shift relevant for the discontinuity occurs when r^* falls marginally below the lower bound $\hat{r}^*$. In the second example, the world interest rate hike raises the domestic interest rate $r_t = r^* > r_{H'}$ and the equilibrium shift relevant for the discontinuity occurs when r^* rises marginally above the upper bound $\tilde{r}^*$. Hence, *the direction of the equilibrium shift is crucial for the discontinuity.*

The discontinuous patterns of endogenous variables has two implications. First, the long-run impact of a world interest rate change depends crucially on whether it crosses the relevant threshold value ($\hat{r}^*$ or $\tilde{r}^*$). In other words, *the initial level of the world interest rate matters.* Second, a rise and a decline in the world interest rate may have *asymmetric* effects on endogenous variables. In the first example, country N is initially at the autarkic steady state and a

[38]The equilibrium shift from a unique steady state to multiple steady states does not cause the discontinuity. As shown in the upper-left panel of figure 6, given $r^* < \hat{r}^*$, country N is initially in the equilibrium with a unique steady state H′ under financial integration where $w_{H'} = (r^*)^{-\rho}$ and $\Phi_{H'} = 1$. A mild world interest rate rise, i.e., $r^* \in (\hat{r}^*, \tilde{r}^*)$, shifts the law of motion for wage downwards and the model economy moves from the equilibrium with a unique steady state to the one with multiple steady states. Then, country N converges from the initial steady state H′ to the new steady state H″ where $w_{H''} = (r^*)^{-\rho}$ and $\Phi_{H''} = 1$. Thus, for $r^* < \tilde{r}^*$, the long-run patterns of income and productivity are *continuous*.

marginal decline in the world interest rate in the interval of $r^* \in (\hat{r}^*, r_A)$ *raises* the long-run level of productivity, as shown by the downward-sloping part of the solid curve in the upper-right panel of figure 5. In the second example, country N is initially at the steady state H' with $r^*_{H'} < \hat{r}^*$ and a marginal rise in the world interest rate in the interval of $r^* \in (\hat{r}^*, r_A)$ *does not affect* the long-run level of productivity, as shown by the flat part of the solid curve in the upper-right panel of figure 6. The long-run patterns of productivity in the two examples differ in the interval of $r^* \in (\hat{r}^*, r_A)$. For a direct comparison, we use the dashed curve in the upper-right panel of figure 6 to show the long-run pattern of productivity in the first example. Obviously, *depending on the initial condition, the productivity may respond asymmetrically to a rise and a fall in the world interest rate within the same range.*

The global analysis allows us to identify the equilibrium shift as an amplification mechanism through which a minor change in the world interest rate around the threshold values may bring country N far away from its initial allocation and causes the substantially large changes in endogenous variables in the long run. One can use this mechanism to evaluate the implications of the ongoing U.S. interest rate hikes. Since the 2009 global financial crisis, the world interest rate has stayed at the record low level for nearly a decade. If the U.S. interest rate hikes only lead to a moderate rise in the world interest rate, it may cause moderate changes in income and productivity for small open economies. However, if the U.S. interest rate exceeds the threshold value $\tilde{r}^*$, the impacts on the income and productivity in small open economies can be disproportionately large. This approach differs fundamentally from those in the classical business cycles literature (Bernanke et al., 1999; Carlstrom and Fuerst, 1997; Kiyotaki and Moore, 1997) which takes the local approximation around a deterministic steady state and features the credit multiplier as the amplification mechanism.

5 Final Remarks

This paper proposes a novel channel, i.e., *within-firm intangible-tangible investment composition*, through which financial capital inflows may have opposite effects on allocative efficiency as well as on the productivity of capital formation over time. Heterogeneous pledgeability is the key factor behind the dynamics of productivity, while the elasticities of domestic investment demand determine the long-run productivity effects of financial inflows.

In our model, the borrowing constraints and the MIR act as an entry barrier for potential entrepreneurs, while wealth inequality reduces the elasticities of domestic investment demand via the extensive margin effect. Other market frictions and regulatory distortions that hamper entrepreneurial entry may also reduce the elasticity of domestic investment and dampen the productivity dynamics. Identifying the institutional factors that shape the elasticity of domestic investment may improve the empirical estimates on the productivity effects of financial integration, in the spirit of Kose et al. (2011, 2009a). For individual countries, ameliorating these frictions and distortions may raise the elasticity of domestic investment and enhance the productivity gains via the intangible investment channel.

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Appendices for Online Publication

A Steady-State Property under Financial Integration

This section analyzes the steady-state property of the current model under financial integration.

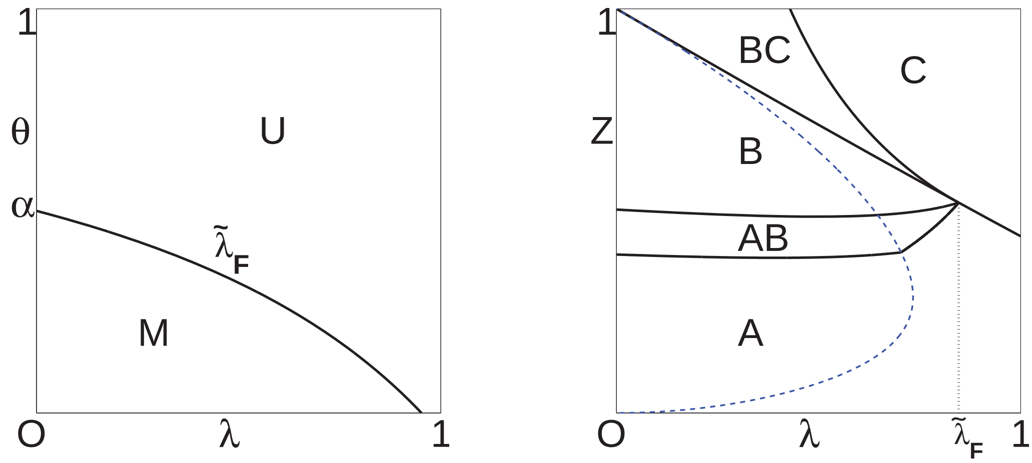

Figure 7: Steady-State Property under Financial Integration: the Case of $r^* = r_A$

Let $\boldsymbol{\tilde{\lambda}_F \equiv \min\{\frac{\alpha-\theta}{(1-\theta)(1-\eta)}, 1\}}$. In figure 7, the downward-sloping curve in the left panel shows $\tilde{\lambda}_F$ as a function of θ; for (λ, θ) in region U, the autarkic steady state is still the unique steady state under financial integration; for (λ, θ) in region M, the right panel shows the parameter constellations for five cases in the (λ, Z) spaces, while the solid (dashed) curves in figure 8 show the laws of motion for wage under financial integration (under autarky) in these five cases, respectively.[39] The proof of proposition

[39]Under financial integration, the law of motion for wage consists of two or three parts, depending on the bindingness of the borrowing constraints and the MoE constraint. See footnote 16 for the definition of the MoE constraint. The dashed curve in the right panel of figure 7 shows a threshold value $\check{\lambda}_F$ in the (λ, Z) space.

- For (λ, Z) to the right of the dashed curve, $\lambda > \check{\lambda}_F$ and the MoE constraint is always slack as long as the

3 characterizes the law of motion for wage and specifies the threshold values. Given $r^* = r_A$,[40] multiple steady states arise in three cases.

- In case B, financial integration destabilizes the autarkic steady state (point A), which leads to two stable steady states (point H and point L), with $w_L < w_A < w_H$.
- In case AB, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point H) arise, with $w_A < w_U < w_H$.
- In case BC, the autarkic steady state (point A) is still stable under financial integration, while an unstable steady state (point U) and another stable steady state (point L) arise, with $w_L < w_U < w_A$.

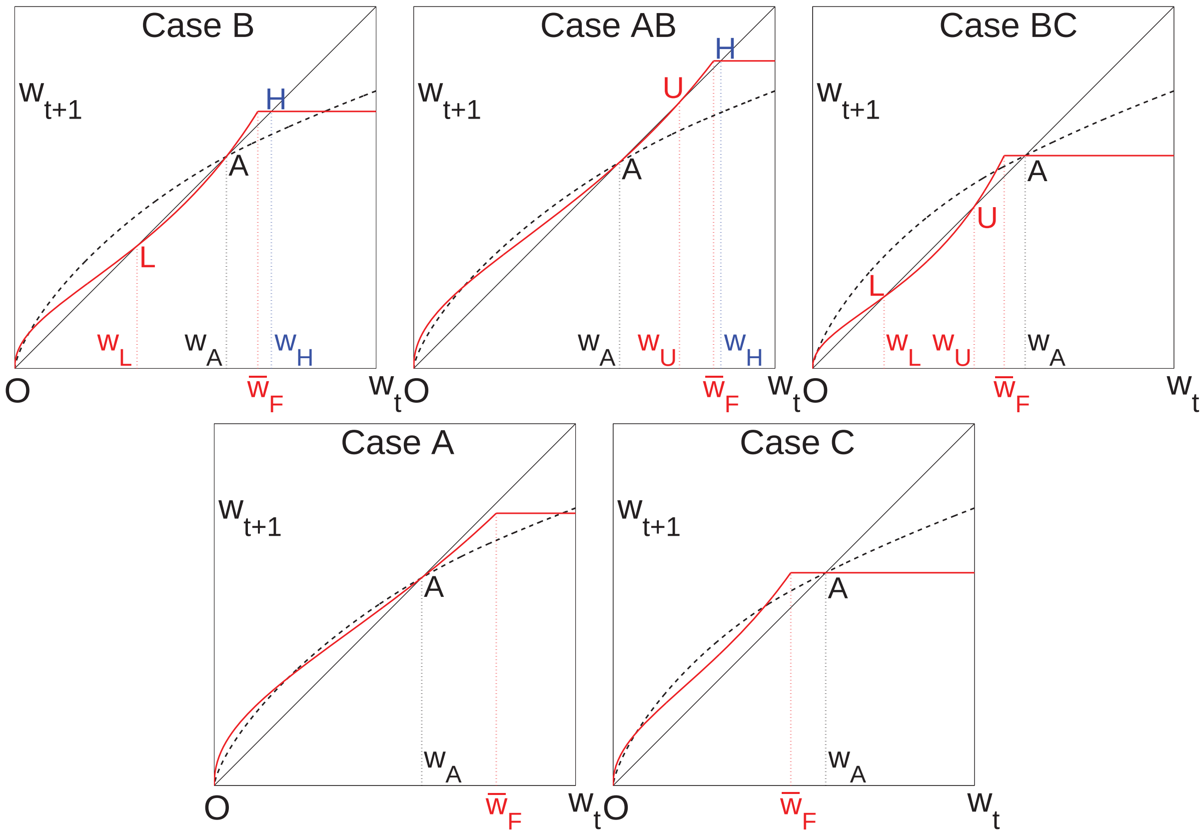

Figure 8: Laws of Motion for Wage under Financial Integration: $\theta < \alpha$ and $\lambda < \tilde{\lambda}_F$

borrowing constraints are binding; when the borrowing constraints are slack, agents who can overcome the MIR do not have strong incentive to run the project and hence, the MoE constraint is irrelevant. Let $\bar{w}_F \equiv [1-\lambda(1-\eta)]\left(\frac{m}{1-\theta}\right)^{1-\theta}\rho^{\theta}(r^*)^{-\frac{\theta}{1-\alpha}}$. For $w_t > \bar{w}_F$, the borrowing constraints are slack; for $w_t < \bar{w}_F$, the borrowing constraints are binding. Hence, the law of motion for wage consists of two parts and the solid (dashed) curves in figure 8 show the laws of motion for wage under financial integration (autarky).

- For (λ, Z) to the left of the dashed curve, $\lambda < \check{\lambda}_F$. There are two threshold values $\tilde{w}_F < \bar{w}_F$ such that, for $w_t < \tilde{w}_F$, the borrowing constraints are binding and the MoE constraint is slack; for $w_t \in [\tilde{w}_F, \bar{w}_F)$, both the borrowing constraints and the MoE constraint are binding; for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant. Hence, the law of motion for wage consists of three parts and the solid (dashed) curves in figure 13 show the laws of motion for wage under financial integration (autarky).

[40] One can use the solution approach described in the proof of proposition 3 to analyze the case of $r^* \neq r_A$.

B The Case of Homogeneous Wealth Distribution: $\theta \to 0$

In order to further highlight the critical role of the extensive margin effect in determining the productivity effect, we analyze a special case of $\theta \to 0$ where the distribution of labor endowment degenerates into a unit mass at $l_j = 1$ and agents have the same labor income, $n_t = w_t$. If $w_t < \mathfrak{m}$, an agent has to borrow at least $\mathfrak{m} - w_t$ to run the project. As argued in Matsuyama (2004), the equilibrium allocation involves credit rationing, i.e., the credit is allocated randomly to a fraction of agents who become entrepreneurs, while the rest are denied credit and become households. Due to competition on the credit market, each entrepreneur only demands for the credit of $\mathfrak{m} - w_t$ and invests at the level of the MIR, $m_t = \frac{n_t}{u_t} = \mathfrak{m}$.[41] In this case, domestic investment demand is endogenous along the extensive margin only $M_t = \mathfrak{m}\tau_t L$.

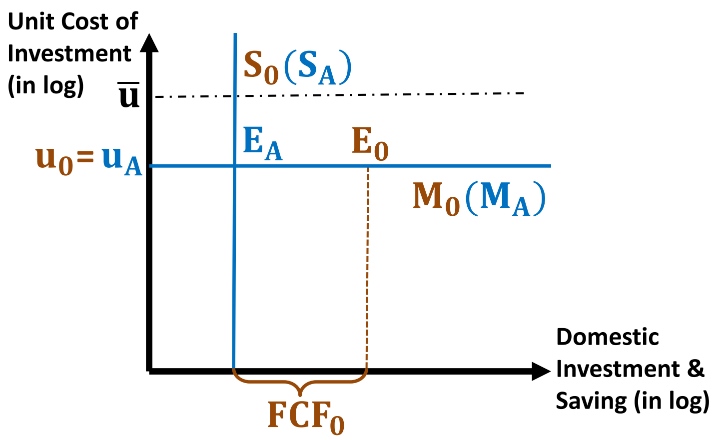

Figure 9: Financial Inflows and Domestic Investment-Saving Imbalance in Period 0: $\theta \to 0$

In figure 9, point E_A denotes the domestic investment-saving balance in the autarkic steady state. According to equation (14), $\theta \to 0$ makes domestic investment demand perfectly elastic with respect to u_t and the line of M_A is flat. In period 0, given $w_0 = w_A$ and $m_0 = m_A = \mathfrak{m}$, the credit demand of each individual entrepreneur is the same as before $m_0 - w_0 = m_A - w_A$ and so is the unit cost of investment $u_0 = \frac{w_0}{m_0} = \frac{w_A}{m_A} = u_A$. Thus, financial capital inflows stimulate domestic investment demand only along the extensive margin. As a result, the domestic investment expansion is so large that the social rate of return falls by the same proportion as the change in the interest rate $\frac{\partial \ln q_1 \Phi_0}{\partial \ln r_0} = 1$. Thus, $\psi_0 = \psi_A$ holds and so does $\Phi_0 = \Phi_A$. One can confirm these findings by putting $\theta \to 0$ into equations (21) and (23).

C A Model with the Exogenous Mass of Entrepreneurs

The endogenous entrepreneur wealth share is key to our findings in the current model. For comparison, we set up a model which differs from the current model in two aspects. First, there is no MIR. Second, only a constant fraction τ of agents in each generation are endowed with the investment project and they are called *entrepreneurs*, while the others do not have the project and are called *households*. Agents are equally endowed with one unit of labor when young[42] and their labor income is homogeneous at w_t. Due to the exogenous mass of entrepreneurs, domestic investment adjusts only along the **i**ntensive **m**argin. Thus, we call it model IM.

[41] According to equations (19), for $\theta \to 0$, $u_t = \tau_t = \frac{w_t}{\mathfrak{m}}$ holds under autarky and so does $m_t = \frac{n_t}{u_t} = \frac{w_t}{u_t} = \mathfrak{m}$. According to equation (63), this result also holds under financial integration.

[42] As wealth distribution does not matter for our findings in this model, we assume it away for simplicity.

Assumption 2. $\tau < \eta$ *and* $\lambda \in (0, 1]$.

Under assumption 2, the entrepreneurial wealth share is less than the efficient share of intangible investment $\delta_t = \frac{\tau w_t L}{w_t L} = \tau < \eta$. Thus, the borrowing constraints are binding under autarky and the individual optimization is the same as shown in section 1. A rise in current income raises the net wealth of all agents in equal proportions. Due to the fixed masses of entrepreneurs and households, the aggregate credit demand and the aggregate credit supply rise along the intensive margin in equal proportions so that the normalized interest rate stays put and so do the unit cost of investment, the intangible fraction of investment, and the productivity.

$$M_t = \frac{\tau w_t L}{u_t} = w_t L, \;\Rightarrow\; u_t = u_A = \tau, \;\; \psi_t = \psi_A = \frac{(1-\eta)\lambda}{1-u_t} = \frac{(1-\eta)\lambda}{1-\tau} < 1,$$
$$a_t = a_A = \eta\frac{u_t}{1-\lambda(1-\eta)} = \eta\frac{\tau}{1-\lambda(1-\eta)} < \eta, \;\; \Phi_t = \Phi_A < 1. \tag{48}$$

Combine them with equations (1)-(2) to get the law of motion for wage,

$$w_{t+1} = \left(\Phi_A \frac{w_t}{\rho}\right)^{\alpha}, \quad \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{Decr. MPK effect}} < 1. \tag{49}$$

The dynamics of aggregate income are purely driven by the decreasing MPK effect.

Proposition 4. *Under autarky, the borrowing constraints are binding, the normalized interest rate is constant at* $\psi_t = \psi_A < 1$*, the intangible fraction of investment is constant at* $a_t = a_A < \eta$*, and the productivity is constant at* $\Phi_t = \Phi_A < 1$*. Besides,* $\frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_t}{1-a_t}\frac{\partial \ln a_A}{\partial \ln \lambda} > 0$.

There is a unique, autarkic steady state where the social rate of return is $q_A \Phi_A = \rho$ *and the interest rate is* $r_A = \psi_A \rho$*, while* $\frac{\partial r_A}{\partial \lambda} > 0$ *and* $\frac{\partial w_A}{\partial \lambda} > 0$.

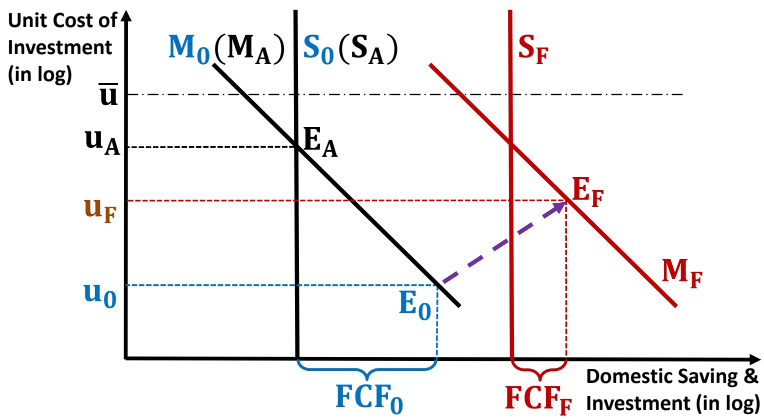

Figure 10: Financial Integration and Domestic Investment-Saving Imbalance in Model IM

Figure 10 shows the impacts of financial integration on domestic investment-saving imbalances. Point E_A denotes domestic investment-saving balance in the autarkic steady state where the domestic investment demand M_A and the domestic saving S_A intersect. Due to the exogenous mass of entrepreneurs and the homogeneous wealth distribution, the entrepreneurial wealth share is constant at $\delta_t \equiv \frac{\tau w_t L}{w_t L} = \tau$. Given $w_0 = w_A$ and $\delta_t = \tau$, the two lines stay put in period 0. Thus, the normalized interest rate must fall $\psi_0 < \psi_A$ so as to create the excess domestic credit demand and absorb financial inflows. The equilibrium moves downwards from point E_A to E_0. From period $t = 1$ on, the rise in aggregate income shifts the two lines rightwards in equal proportions, while domestic investment expansion reduces the social rate of return; given the interest rate constant at the world level $r_t = r^*$, ψ_t rises over time; the dashed arrow shows the path along which country N converges to the new steady state F. Given $\delta_t = \tau$, $u_F < u_A$ must hold so as to justify capital inflows with the excess domestic credit demand in the new steady state. Proposition 5 summarizes the productivity implications of financial integration, which follows closely the dynamics of ψ_t.

Proposition 5. *In model IM, given $r^* < r_A$, the productivity falls upon financial integration, $\Phi_0 < \Phi_A$ and then rises over time, $\Phi_t > \Phi_{t-1}$; in the long run, the productivity is strictly lower than its initial level, $\Phi_0 < \Phi_F < \Phi_A$.*

In model IM, the entrepreneurial wealth share is constant; upon financial integration, the domestic investment demand and the domestic saving stay put, while the subsequent rises in aggregate income raise them in equal proportions. In the current model, due to the endogenous entrepreneurial wealth share, the domestic investment demand responds to income rises by a larger proportion than the change in domestic saving and hence, financial inflows are larger than in model IM. This way, the endogeneity of the entrepreneurial wealth share is key to the different patterns of the normalized interest rate and the productivity between the two models.

D Proofs

Proof of Proposition 1

Proof. The proof consists of four steps.

Step 1: Solve the Individual Optimization Problem and Derive the Unit Costs

Agent j chooses tangible and intangible investments as well as loans to maximize its net investment revenue, subject to the budget constraint and the borrowing constraints.

$$\mathbb{L}_{j,t+1} \equiv \max_{m_{j,T,t}, m_{j,I,t}, b_{j,t}} q_{t+1}k_{j,t+1} - r_t b_{j,t} - \xi_{j,t}(m_{j,I,t} + m_{j,T,t} - n_{j,t} - b_{j,t}) - \zeta_{j,t}\left(b_{j,t} - \lambda \frac{p_{t+1}}{r_t} m_{j,T,t}\right).$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial b_{j,t}} = -r_t + \xi_{j,t} - \zeta_{j,t} = 0, \qquad \Rightarrow \zeta_{j,t} = \xi_{j,t} - r_t, \tag{50}$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial m_{j,I,t}} = MR_{j,I,t+1} - \xi_{j,t} = 0, \qquad \Rightarrow MR_{j,I,t+1} = \xi_{j,t}, \tag{51}$$

$$\frac{\partial \mathbb{L}_{j,t+1}}{\partial m_{j,T,t}} = MR_{j,T,t+1} - \xi_{j,t} + \zeta_{j,t}\lambda \frac{p_{t+1}}{r_t} = 0, \qquad \Rightarrow MR_{j,T,t+1} = \xi_{j,t} - (\xi_{j,t} - r_t)\lambda \frac{p_{t+1}}{r_t}. \tag{52}$$

Let $\mathbb{A}_{j,t} \equiv \frac{a_{j,t}}{1-a_{j,t}} \frac{1-\eta}{\eta}$. The marginal revenues of intangibles and tangibles are respectively,

$$MR_{j,I,t+1} \equiv q_{t+1} \frac{\partial k_{t+1}}{\partial m_{j,I,t}} = q_{t+1} \mathbb{A}_{j,t}^{\eta-1}, \text{ and } MR_{j,T,t+1} \equiv q_{t+1} \frac{\partial k_{t+1}}{\partial m_{j,T,t}} = q_{t+1} \mathbb{A}_{j,t}^{\eta}.$$

If the borrowing constraint is *slack* for agent j, $\zeta_{j,t} = 0$ and, according to equations (50)-(52),

$$MR_{j,I,t+1} = MR_{j,T,t+1} = \xi_{j,t} = r_t, \Rightarrow a_{j,t} = \eta,\ \Phi_{j,t} = 1, \text{and } q_{t+1} = r_t.$$

As the private and the social rates of return coincide, those who can meet the MIR do not have strong incentive to invest their entire net wealth in the project or borrow to the limit. Nevertheless, those who run the project choose $a_{j,t} = \eta$. Thus, $a_t = \eta$ and $\Phi_t = 1$ hold at the aggregate level.

If the borrowing constraint is *binding* for agent j, $\zeta_{j,t} > 0$ and equations (50)-(52) imply

$$MR_{j,I,t+1} = \xi_{j,t} = \frac{MR_{j,T,t+1} - \lambda p_{t+1}}{1 - \lambda \frac{p_{t+1}}{r_t}}, \tag{53}$$

which is equivalent to equation (4). As the right hand side of equation (4) increases in $\mathbb{A}_{j,t}$, there exists a unique $\mathbb{A}_{j,t}$ that solves this equation. Since all agents face the same market prices, they choose the same value for $\mathbb{A}_{j,t}$ in equilibrium and subscript j can be left out, i.e., $\mathbb{A}_{j,t} = \mathbb{A}_t$ and $a_{j,t} = a_t$. Besides, as the price of tangibles is equal to the marginal revenue of tangibles at the aggregate level, we simplify the unit cost of tangibles as equation (6). Use it to derive equation (9) specifying the unit cost of total investment.

The binding borrowing constraints imply that $\zeta_{j,t} > 0$. Combine it with equations (50)-(52) to get $MR_{j,I,t} = \xi_{j,t} > r_t$ and $MR_{j,T,t} = (\xi_{j,t} - r_t)u_{T,t} + r_t > r_t$. Thus, the social rate of return exceeds the interest rate in equilibrium,

$$q_{t+1}\Phi_t = a_t MR_{j,I,t} + (1 - a_t) MR_{j,T,t} > r_t.$$

Step 2: Derive the Condition for the Binding Borrowing Constraints under Autarky

In the case of the slack borrowing constraints, $r_t = \Phi_t q_{t+1}$, $a_t = \eta$, and $\Phi_t = 1$. Combine equations (16)-(15) with (1)-(2) to get equation (17) specifying the law of motion for wage.

In the case of the binding borrowing constraints, $r_t < \Phi_t q_{t+1}$ and equations (19) specify the major endogenous variables in the autarkic equilibrium.

When the borrowing constraints are weakly binding, $\Phi_t = 1$ and $q_{t+1} = r_t$. Use equation (9) to get $u_t = 1 - \lambda(1 - \eta)$. Combine it with equation (19) to get the threshold value $\bar{w}_A = [1 - \lambda(1 - \eta)]^{\frac{1}{1-\theta}} \frac{m}{1-\theta} > 0$.

When the borrowing constraints are strictly binding, use equations (19), (9), and (5) to get

$$\frac{\partial \ln a_t}{\partial \ln w_t} = \frac{\partial \ln u_t}{\partial \ln w_t} = 1 - \theta < 1, \text{ and } \frac{\partial \ln \Phi_t}{\partial \ln a_t} = \frac{\eta - a_t}{1 - a_t} > 0, \text{ if } a_t < \eta. \tag{54}$$

For $w_t < \bar{w}_A$, the borrowing constraints are binding, with $u_t < 1 - \lambda(1 - \eta)$, $a_t < \eta$, and $r_t < q_{t+1}\Phi_t$. Combine equations (16)-(15) with (1)-(2) to get (17) for the law of motion for wage.

Step 3: Derive the Condition for the Unique, Stable Steady State under Autarky

Under autarky, there is a unique, stable steady state, if the slope of the law of motion for wage at any steady state is less than unity, $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} < 1$.

- According to equation (17), this condition holds, $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A > \bar{w}_A} = \alpha < 1$, if there is an autarkic steady state with $w_A > \bar{w}_A$.

- If there is an autarkic steady state with $w_A < \bar{w}_A$, use equation (17) to get

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_A<\bar{w}_A} = 1-(1-\alpha)\left[1-\rho\eta(1-\theta)\frac{1-\frac{a_t}{\eta}}{1-a_t}\right]. \quad (55)$$

Given $w_A < \bar{w}_A$, $a_A < \eta$ and hence. $\frac{1-\frac{a_t}{\eta}}{1-a_t} < 1$. A sufficient condition for $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A<\bar{w}_A} < 1$ is $\rho\eta(1-\theta) \leq 1$ or equivalently, $\theta \geq \underline{\theta} \equiv \max\{0, 1-\frac{1}{\rho\eta}\}$.

Step 4: Derive the Autarkic Steady-State Patterns of Endogenous Variables

Given $\theta \geq \underline{\theta} \equiv \max\{0, 1-\frac{1}{\rho\eta}\}$, there exists a unique, stable steady state under autarky. Let $\tilde{\lambda}_A$ denote a threshold value such that, for $\lambda = \tilde{\lambda}_A$, the borrowing constraints are weakly binding at the autarkic steady state, with $w_A = \bar{w}_A$, $a_A = \eta$. In this case, $\Phi_A = 1$ and

$$w_A = \rho^{-\rho} = \bar{w}_A = (1-\tilde{\lambda}_A)^{\frac{1}{1-\theta}}\frac{\mathfrak{m}}{1-\theta}, \Rightarrow \tilde{\lambda}_A = 1-Z^{1-\theta}, \text{ where } Z \equiv \frac{1-\theta}{\rho^{\rho}\mathfrak{m}}. \quad (56)$$

For $\lambda > \tilde{\lambda}_A$, the borrowing constraints are slack at the autarkic steady state where the endogenous variables $\{a_A, u_A, r_A, \psi_A, w_A\}$ are constant and independent of λ. For $\lambda < \tilde{\lambda}_A$, the borrowing constraints are binding at the autarkic steady state. Combine equations (5), (17) and (19) to get

$$w_A = u_A^{\frac{1}{1-\theta}}\frac{\mathfrak{m}}{1-\theta}, \Rightarrow \rho\ln\Phi_A = \frac{1}{1-\theta}(\ln a_A + \ln[1-\lambda(1-\eta)] - \ln\eta) - \ln Z$$

$$\rho(1-\theta)\frac{\partial\ln\Phi_A}{\partial\ln a_A}\frac{\partial\ln a_A}{\partial\ln\lambda} = \frac{\partial\ln a_A}{\partial\ln\lambda} - \frac{\lambda(1-\eta)}{1-\lambda(1-\eta)}, \quad \frac{\partial\ln a_A}{\partial\ln\lambda} = \frac{\frac{\lambda(1-\eta)}{1-\lambda(1-\eta)}}{1-\rho(1-\theta)\eta\left(1-\frac{\frac{1}{\eta}-1}{\frac{1}{a_A}-1}\right)}.$$

Given $\theta > \underline{\theta}$ and $\lambda < \tilde{\lambda}_A$, $a_A < \eta$ and $\frac{\partial\ln a_A}{\partial\ln\lambda} > 0$. Use equations (5) and (9) to get

$$\frac{\partial\ln u_A}{\partial\ln\lambda} = \frac{\partial\ln a_A}{\partial\ln\lambda} > 0, \quad \frac{\partial\ln\Phi_A}{\partial\ln\lambda} = \frac{\eta-a_A}{1-a_A}\frac{\partial\ln a_A}{\partial\ln\lambda} > 0, \quad \frac{\partial\ln w_A}{\partial\ln\lambda} = \rho\frac{\partial\ln\Phi_A}{\partial\ln\lambda} > 0.$$

Under autarky, domestic investment in period t is $M_t = w_tL$, while the revenue of capital goods in period $t+1$ is $q_{t+1}K_{t+1} = \frac{\alpha}{1-\alpha}w_{t+1}L$. Then, the social rate of return is

$$\frac{q_{t+1}K_{t+1}}{M_t} = q_{t+1}\Phi_t = \rho\frac{w_{t+1}}{w_t}. \quad (57)$$

In the steady state, $w_{t+1} = w_t$ implies that the social rate of return is constant at ρ and the interest rate $r_A = \psi_A\rho$ rises strictly with λ.

Let $\psi_t \equiv \frac{r_t}{q_{t+1}\Phi_t}$ denote the normalized interest rate. According to equation (57), the social rate of return in the steady state is independent of λ, i.e., $q_A\Phi_A = \rho$. Use equation (9) to get

$$r_A = \rho\psi_A, \ \psi_A = \frac{\lambda}{1-u_A}, \Rightarrow \frac{\partial\ln r_A}{\partial\ln\lambda} = \frac{\partial\ln\psi_A}{\partial\ln\lambda} = 1+\frac{u_A}{1-u_A}\frac{\partial\ln a_A}{\partial\ln\lambda} > 0. \quad (58)$$

In figure 11, the left panel shows the threshold value $\underline{\theta}$ in the $\{\alpha, \theta\}$ space; given $\{\alpha, \theta\}$ in region U, the right panel shows the threshold value $\tilde{\lambda}_A$ in the (λ, Z) space. For (λ, Z) in region UB (US) of the right panel, there is a unique, autarkic steady state where the borrowing constraints are binding (slack) with $w_A < \bar{w}_A$ ($w_A > \bar{w}_A$).[43]

[43] See figure 12 in the proof of proposition 1 for the laws of motion for wage in these two cases.

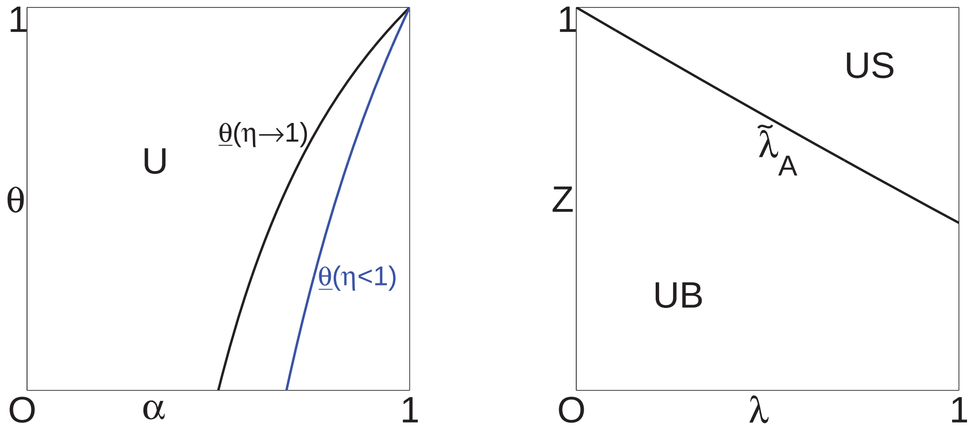

Figure 11: Threshold Values for the Autarkic Steady State

Given $\{\alpha, \theta\}$ in region U of the left panel and (λ, Z) in region UB (US) of the right panel in figure 11, the solid curve in the left (right) panel of figure 12 show the law of motion for wage. For comparison purpose, the dashed curves in figure 12 show the laws of motion for wage in the absence of financial friction.[44] For $w_t \in (0, \bar{w}_A)$, the solid curve lies below the dashed curve and the gap reflects the efficiency losses, $(1-\Phi_t^\alpha)\left(\frac{w_t}{\rho}\right)^\alpha$.

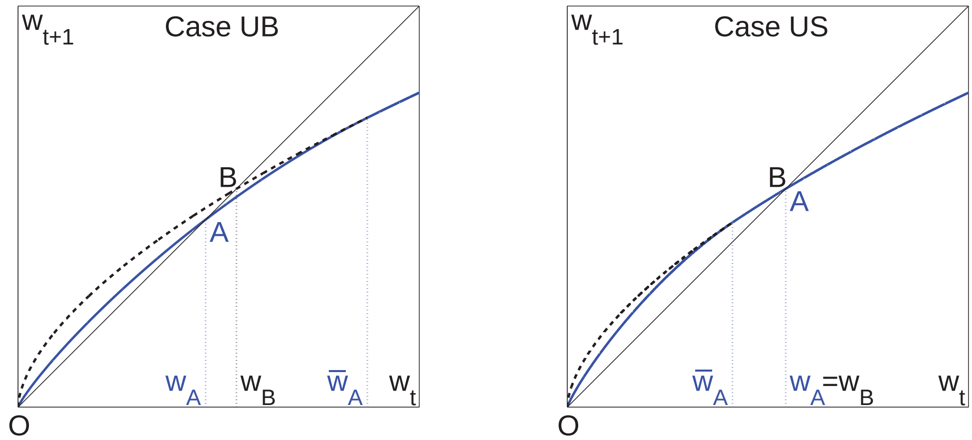

Figure 12: Laws of Motion for Wage under Autarky: $\theta \geq \underline{\theta}$

□

Proof of Proposition 3

Proof. we first derive the law of motion for wage under financial integration. Then, we use it to analyze the dynamic and the steady-state properties of the model economy.

Step 1: Derive the Law of Motion for Wage under Financial Integration

Combine equations (1)-(2) to get the factor price equation,

$$q_t^\alpha w_t^{1-\alpha} = 1. \tag{59}$$

[44]In the absence of financial frictions, the intangible fraction of investment is always equal to its factor share in capital formation $a_t = \eta$ and the productivity is efficient at $\Phi_t = 1$. Then, the entire law of motion for wage under autarky is specified by equation (17) and there is a unique, autarkic steady state with the wage rate $w_B = \rho^{-\rho}$.

- Iff $q_{t+1}\Phi_t = r^*$, the borrowing constraints are slack and $\Phi_t = 1$. Combine them with equation (59) to get

$$w_{t+1} = (r^*)^{-\rho}, \tag{60}$$

which is constant and independent of w_t. Thus, when the borrowing constraints are slack, the law of motion for wage is flat.

- Iff $q_{t+1}\Phi_t > r^*$, the borrowing constraints are binding and the model dynamics are determined jointly by five equations. First, use $r_t = r^*$ and equation (59) to rewrite the binding borrowing constraints (3) as

$$r_t = \frac{\lambda(1-\eta)q_{t+1}\Phi_t}{1-u_t} = r^*, \;\Rightarrow\; w_{t+1}^{\frac{1}{\rho}} = q_{t+1}^{-1} = \frac{\lambda(1-\eta)\rho}{r^*(1-u_t)}\frac{\Phi_t}{\rho}. \tag{61}$$

Second, combine equations (1)-(2) with (16) to get equation (17) specifying the law of motion for wage. Third, the mass of entrepreneurs cannot exceed the total population in each generation, $\tau_t \leq 1$. Combine this constraint with equations (10) and (12) to get

$$\delta_t = \tau_t^{1-\theta} = \begin{cases} \left(\frac{w_t}{u_t}\frac{1-\theta}{\mathfrak{m}}\right)^{\frac{1-\theta}{\theta}} < 1, & \text{if } w_t < u_t\frac{\mathfrak{m}}{1-\theta}; \\ 1, & \text{if } w_t \geq u_t\frac{\mathfrak{m}}{1-\theta}. \end{cases} \tag{62}$$

Under financial integration, when the borrowing constraints are binding, equations (5), (9), (12), (17), (61), (62) jointly determine $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t, which characterizes the law of motion for wage.

In the special case of $\theta = 0$ and $\delta_t < 1$, combine equations (12), (17), and (61)-(62) to get

$$u_t = \frac{w_t}{\mathfrak{m}}, \tag{63}$$

implying that each entrepreneur borrows $\mathfrak{m} - w_t$ and invests at the level of the MIR in equilibrium, $m_{j,t} = \mathfrak{m}$.

Step 2: Derive the Conditions under which the Two Constraints are Binding

For a sufficiently low level of aggregate income, the mass of entrepreneurs is inefficiently low so that the borrowing constraints are binding and the MoE constraint is slack. The rise in aggregate income allows more agents to become entrepreneurs, which may trigger two events: (1) the borrowing constraints become slack; (2) the MoE constraint becomes binding. The law of motion for wage is piecewise and its characterization depends on which event comes first. The MoE constraint matters only when the borrowing constraints are binding.[45]

In the following analysis, we first derive the condition for the MoE constraint to be binding, given that the borrowing constraints are binding. Then, we specify the law of motion for wage in two scenarios.

Define two threshold values,

$$\bar{w}_F \equiv \left[\frac{Z\rho^{\frac{1}{1-\alpha}}}{(r^*)^{\frac{1}{1-\alpha}}}\right]^{\theta} \frac{1-\lambda(1-\eta)}{Z\rho^{\rho}} \quad \text{and} \quad \tilde{w}_F \equiv \frac{\tilde{a}_F}{\eta}\frac{1-\lambda(1-\eta)}{Z\rho^{\rho}},$$

$$\text{where } \tilde{a}_F \text{ is a solution to } \frac{\lambda(1-\eta)\rho}{r^*\left\{1-\frac{\tilde{a}_F}{\eta}[1-\lambda(1-\eta)]\right\}}\left[\left(\frac{\tilde{a}_F}{\eta}\right)^{\eta}\left(\frac{1-\tilde{a}_F}{1-\eta}\right)^{1-\eta}\right]^{\rho} Z = 1. \tag{64}$$

[45] When the borrowing constraints are slack, the agents who can overcome the MIR do not have strong incentive to be entrepreneurs and hence, the MoE constraint is irrelevant.

Given r^*, solve for the threshold value $\check{\lambda}_F$ as the function of Z such that $\bar{w}_F = \tilde{w}_F$. The dashed curve in the right panel of figure 7 shows $\check{\lambda}_F$. Here are two scenarios under financial integration.

- **Scenario 1:** for (λ, Z) to the left of that dashed curve, $\lambda < \check{\lambda}_F$ and $\tilde{w}_F < \bar{w}_F$. In this scenario, the MoE constraint becomes binding before the borrowing constraints become slack. Combine $\delta_t = 1$ with equations (5), (9), (12), (17), (61) to solve for w_t, which gives the threshold value $\tilde{w}_F$ as specified in equation (64). Meanwhile, combine $a_t = \eta$ and $\delta_t = 1$ with equations (5), (9), (12), (17), (61) to solve for w_t, which gives another threshold value $\bar{w}_F \equiv \frac{[1-\lambda(1-\eta)]\rho}{(r^*)^{\frac{1}{1-\alpha}}} > \tilde{w}_F$. The law of motion for wage consists of three parts:
  - for $w_t < \tilde{w}_F$, the borrowing constraints are binding and the MoE constraint is slack, and hence, the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (5), (9), (12), (17), (61) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$;
  - for $w_t \in (\tilde{w}_F, \bar{w}_F)$, both the borrowing constraints and the MoE constraint are binding, and hence, the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (5), (9), (12), (17), (61) and $\delta_t = 1$;
  - for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant, and hence, the law of motion for wage is characterized by equation (60).
- **Scenario 2:** for (λ, Z) to the right of that dashed curve, $\lambda > \check{\lambda}_F$ and $\bar{w}_F < \tilde{w}_F$. In this scenario, the MoE constraint never binds as long as the borrowing constraints are binding. Combine $a_t = \eta$ and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$ with equations (5), (9), (12), (17), (61) to solve for w_t, which gives the threshold value $\bar{w}_F$ as specified in equation (64). The law of motion for wage consists of two parts:
  - for $w_t < \bar{w}_F$, the borrowing constraints are binding and the MoE constraint is slack, and hence, the law of motion for wage is characterized by $\{w_{t+1}, a_t, u_t, \Phi_t, \delta_t\}$ as the functions of w_t satisfying equations (5), (9), (12), (17), (61) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$;
  - for $w_t > \bar{w}_F$, the borrowing constraints are slack and the MoE constraint is irrelevant, and hence, the law of motion for wage is characterized by equation (60).

The solid (dashed) curves in figure 8 shows the laws of motion for wage under financial integration (under autarky), given $\lambda > \check{\lambda}_F$ and (λ, Z) in the respective region of the right panel in figure 7. Figure 13 shows the laws of motion for wage, given $\lambda < \check{\lambda}_F$ and (λ, Z) in the respective region of the right panel of figure 7.

Step 3: Derive the Threshold Conditions for Multiple Steady States

Under financial integration, multiple steady states arise if there exists one unstable steady state, i.e., the law of motion for wage has a slope more than unity at a steady state. Let X_U denote the value of variable X_t at the **u**nstable steady state. As shown in equation (60), for $w_t \geq \bar{w}_F$, the borrowing constraints are slack and the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$. Thus, if there exists an unstable steady state, $w_U < \bar{w}_F$ must hold and the borrowing constraints must be binding there. How about the bindingness of the MoE constraint at the unstable steady state?

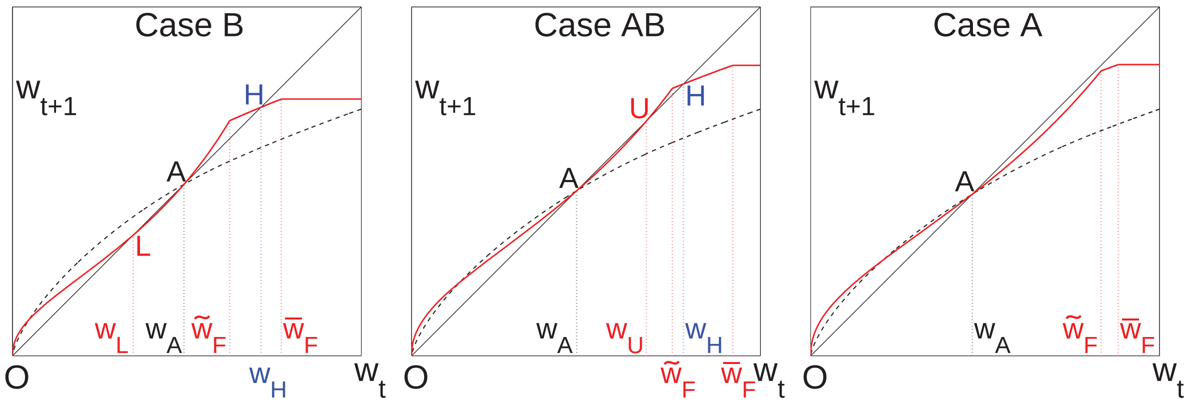

Figure 13: Laws of Motion for Wage under Financial Integration: $\theta < \alpha$ and $\lambda < \check{\lambda}_F$

In the case where both the MoE constraint and the borrowing constraints are binding, combine equations (5), (9), (12), (17), (61) with $\delta_t = 1$,

$$\frac{\partial w_{t+1}}{\partial w_t}\frac{w_t}{w_{t+1}} = \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \frac{1}{1+\rho\left(u_t\frac{1-\eta}{1-a_t}+\frac{\eta-a_t}{1-a_t}\right)} < 1. \tag{65}$$

Thus, if there exists a steady state where the MoE constraint is binding, the slope of the law of motion for wage is strictly less than unity there and hence, this steady state must be stable.

To sum up, if there exists an unstable steady state, the borrowing constraints must be binding and the MoE constraint must be slack there. Combine equations (5), (9), (12), (17), (61) with $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$ to get equation (28) describing the slope of the law of motion for wage in logarithm. In a threshold case where the law of motion for wage has a slope equal to unity at a steady state, combine $\frac{\partial \ln w_{t+1}}{\partial \ln w_t}|_{w_{t+1}=w_t=w_U} = 1$ with (28) to get a_U as a function of λ and other parameters,

$$\frac{1}{1-\frac{a_U}{\eta}[1-\boldsymbol{\lambda}(1-\eta)]}\frac{1-\frac{\theta}{\alpha}}{1-\theta} - \frac{1-\eta}{1-\boldsymbol{a_U}} = \frac{1-\alpha}{\alpha}. \tag{66}$$

Next, we specify the conditions for the existence of multiple steady states, given $r^* = r_A$.

3.1. Parameter Constellation for Multiple Steady States in the (λ, θ) Space

Combine equation (66) with $a_U \leq \eta$ and $\lambda \leq 1$ to get a threshold value $\boldsymbol{\tilde{\lambda}_F \equiv \min\{\frac{\alpha-\theta}{(1-\theta)(1-\eta)}, 1\}}$, as shown by the downward-sloping curve in the left panel of figure 7. For (λ, θ) in region U, equation (66) does not have a solution with $a_U \in (0, \eta]$ and hence, the autarkic steady state is still the unique, stable steady state under financial integration; for (λ, θ) in region M, multiple steady states may arise as proved below.

3.2. Parameter Constellation for Multiple Steady States in the (λ, Z) Space

Given (λ, θ) in region M of the left panel of figure 7, multiple steady states arise in three scenarios with (λ, Z) in region BC, B, and AB of the right panel in figure 7, respectively.

Scenario BC: for (λ, Z) in region BC, $\lambda > \bar{\lambda}_A$ and the borrowing constraints are slack in the autarkic steady state, with $r_A = \rho$, as shown in the right panel of figure 11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (L) arise, $w_L < w_U < \bar{w}_F$, as shown in the upper-right panel of figure 8. In the threshold case, the law of motion for wage is tangent with the 45° line at the unstable steady state and accordingly, two conditions hold. First, equation (66) specifies a_U as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, combine equations (61), (12), (17), (5), (9) with $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{\text{m}}\right)^{\frac{1}{1-\theta}}$, $w_{t+1} = w_t = w_U$, and $r^* = r_A = \rho$ to characterize the unstable steady state,

$$\left[\frac{\lambda(1-\eta)}{1-\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\right]^{\frac{\theta-\alpha}{1-\alpha}} \frac{a_U}{\eta}[1-\lambda(1-\eta)] = \left\{\left[\left(\frac{a_U}{\eta}\right)^{\eta}\left(\frac{1-a_U}{1-\eta}\right)^{1-\eta}\right]^{\rho} Z\right\}^{1-\theta}, \tag{67}$$

which specifies a_U as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (66) and (67) jointly specify Z as a function of λ, featuring the border between region BC and C in the right panel of figure 7.

Scenario B: for (λ, Z) in region B, $\lambda < \bar{\lambda}_A$ and the borrowing constraints are binding in the autarkic steady state, with $r_A < \rho$, as shown in the right panel of figure 11. Financial integration destabilizes the autarkic steady state, while two stable steady states (L and H) arise, $w_L < w_A < w_H$, as shown in the upper-left panel of figure 8 and in the left panel of figure 13. In the threshold case, the law of motion for wage is tangent with the 45°line at the autarkic steady state and accordingly, two conditions hold. First, $a_A = a_U$ is a function of $\{\lambda, \alpha, \theta, \eta\}$, according to equation (66). Second, combine $w_{t+1} = w_t = w_A$ with equations (5), (9), (19), (17) to characterize the autarkic steady state,

$$u_A^{\frac{1}{1-\theta}} = w_A\frac{1-\theta}{\text{m}} = Z\Phi_A^{\rho}, \Rightarrow \left\{\frac{a_A}{\eta}[1-\lambda(1-\eta)]\right\}^{\frac{1}{1-\theta}} = Z\left[\left(\frac{a_A}{\eta}\right)^{\eta}\left(\frac{1-a_A}{1-\eta}\right)^{1-\eta}\right]^{\rho}, \tag{68}$$

which specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (66) and (68) jointly specify Z as a function of $\lambda \in (0, \tilde{\lambda}_F)$, featuring the border between region B and AB in the right panel of figure 7.

Scenario AB: for (λ, Z) in region AB, $\lambda < \bar{\lambda}_A$ and the borrowing constraints are binding in the autarkic steady state, with $r_A < \rho$, as shown in the right panel of figure 11. Under financial integration, although the autarkic steady state is still stable, an unstable steady state (U) and another stable steady state (H) arise, as shown in the upper-middle panel of figure 8 and in the middle panel of figure 13; as explained in step 2, the law of motion for wage is piecewise and consists of two or three parts; multiple steady states arise if at least one kink point in the law of motion for wage is above the 45° line.

- **Scenario AB.R:** for (λ, Z) in region AB and to the **r**ight of the dashed curve, $\lambda > \check{\lambda}_F$ and the law of motion for wage has one kink point at $w_t = \bar{w}_F$, where $\bar{w}_F$ is specified in equation (64). For $w_t \geq \bar{w}_F$, the borrowing constraints are slack, $\Phi_t = 1$, and the law of motion for wage is flat at $w_{t+1} = (r^*)^{-\rho}$. In the threshold case, the kink point is on the 45°line and accordingly, two conditions hold. First, combine $r^* = r_A$ and $w_{t+1} = (r^*)^{-\rho} = w_t = \bar{w}_F$ with equations (64) and (68) to characterize the kink point

$$\frac{1}{\frac{1-\frac{a_A}{\eta}}{\lambda(1-\eta)} + \frac{a_A}{\eta}} = \left(\frac{a_A}{\eta}\frac{1}{\Phi_A^{(1-\theta)\rho}}\right)^{\frac{1-\alpha}{\alpha-\theta}}, \text{ where } \Phi_A = \left(\frac{a_A}{\eta}\right)^{\eta}\left(\frac{1-a_A}{1-\eta}\right)^{1-\eta}. \tag{69}$$

It specifies a_A as a function of $\{\lambda, \alpha, \theta, \eta\}$. Second, equation (68) specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (68) and (69) jointly specify Z as a function of λ. It is shown as the part of the border between region AB and A, which is to the right of the dashed curve in the right panel of figure 7.

- **Scenario AB.L:** for (λ, Z) in region AB and to the **l**eft of the dashed curve, $\lambda < \check{\lambda}_F$ and the law of motion for wage has two kinks at $w_t = \tilde{w}_F$ and $w_t = \bar{w}_F$, respectively. In the threshold case, the kink point at $w_t = \tilde{w}_F$ is on the 45°line, where $\tilde{w}_F$ is specified in equation (64). In this case, three conditions hold. Let $\tilde{X}_F$ denote the value of variable X_t at that kink point. First, combine equations (64) and (68) with $r^* = r_A$ to characterize the kink point

$$\frac{1-\frac{a_A}{\eta}[1-\lambda(1-\eta)]}{1-\frac{\tilde{a}_F}{\eta}[1-\lambda(1-\eta)]}\left[\left(\frac{\tilde{a}_F}{a_A}\right)^{\eta}\left(\frac{1-\tilde{a}_F}{1-a_A}\right)^{1-\eta}\right]^{\rho}\left\{\frac{a_A}{\eta}[1-\lambda(1-\eta)]\right\}^{\frac{1}{1-\theta}} = 1, \tag{70}$$

which specifies $\tilde{a}_F$ as a function of a_A and $\{\lambda, \alpha, \eta, \theta\}$. Second, combine equations (5), (9), (12), (17), (61) with $\delta_t = 1$ and $w_{t+1} = w_t = \tilde{w}_F$ to characterize that kink point as a steady state

$$\left(\frac{a_A}{\tilde{a}_F}\right)^{\eta}\left(\frac{1-a_A}{1-\tilde{a}_F}\right)^{1-\eta} = \left\{\frac{a_A}{\eta}[1-\lambda(1-\eta)]\right\}^{\frac{1-\alpha}{\alpha(1-\theta)}}\left\{2-\frac{a_A}{\eta}[1-\lambda(1-\eta)]\right\}^{\frac{1}{\alpha}}, \tag{71}$$

which specifies $\tilde{a}_F$ as a function of a_A and $\{\lambda, Z, \alpha, \eta, \theta\}$. Third, equation (68) specifies a_A as a function of $\{\lambda, Z, \alpha, \theta, \eta\}$. Then, equations (68), (70), and (71) jointly determine $\{Z, \tilde{a}_F, a_A\}$ as the functions of λ. The relationship between Z and λ is shown as the part of the border between region AB and A in the right panel of figure 7, which is to the left of the dashed curve.

□

Proof of Corollary 1

Proof. $\underline{Z}_F$ and $\bar{Z}_F$ are characterized in the proof of proposition 3 as the functions of λ. They are shown as the upper and lower borders of region AB in the right panel of figure 7.

- Define $\hat{r}^*$ as a threshold value such that, for $r^* = \hat{r}^*$, the law of motion for wage under financial integration is tangent with the 45° line where a_U denote the steady-state value of a_t. Use equation (66) to solve a_U as a function of λ and other parameters. Combine $w_{t+1} = w_t = w_U$ with equations (5), (9), (12), (17), (61) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$ to get

$$\hat{r}^* \equiv \frac{\lambda(1-\eta)\rho}{1-\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\left[\frac{(\Phi_U^{\rho}Z)^{1-\theta}}{\frac{a_U}{\eta}[1-\lambda(1-\eta)]}\right]^{\frac{1-\alpha}{\alpha-\theta}}, \text{ where } \Phi_U = \left(\frac{a_U}{\eta}\right)^{\eta}\left(\frac{1-a_U}{1-\eta}\right)^{1-\eta}.$$

- Define $\tilde{r}^*$ as a threshold value such that, for $r^* = \hat{r}^*$, the law of motion for wage under financial integration has the kink point on the 45○ line. At the kink point, $a_k = \eta$, $\Phi_k = 1$, $u_k = 1-\lambda(1-\eta)$, and $w_{t+1} = w_t = w_k = \bar{w}_F$. Combine them with equations (5), (9), (12), (17), (61) and $\delta_t = \left(\frac{w_t}{u_t}\frac{1-\theta}{m}\right)^{\frac{1-\theta}{\theta}}$ to get

$$\tilde{r}^* \equiv \rho\left[\frac{Z^{1-\theta}}{1-\lambda(1-\eta)}\right]^{\frac{1-\alpha}{\alpha-\theta}}$$

□

Proof of Proposition 4

Proof. Consider first the case of the binding borrowing constraints under autarky. As domestic investment is financed by domestic saving, the unit cost of investment is constant and so are other major endogenous variables, as specified by equations (48). Under assumption 2, $\psi_A < 1$ implies that the borrowing constraints are binding. According to equations (48),

$$\frac{\partial \psi_A}{\partial \lambda} = \frac{1-\eta}{1-\tau} > 0, \;\; \frac{\partial \ln a_A}{\partial \ln \lambda} = \frac{\lambda(1-\eta)}{1-\lambda(1-\eta)} > 0, \;\; \frac{\partial \ln \Phi_A}{\partial \ln \lambda} = \frac{\eta - a_A}{1-\eta} \frac{\partial \ln a_A}{\partial \ln \lambda} > 0. \tag{72}$$

As the project productivity is constant, the law of motion for wage is log-linear with the slope less than unity, according to equations (49). Thus, there exists a unique, autarkic steady state. The social rate of return is defined by equation (57) and, in the autarkic steady state

$$w_{t+1} = w_t = w_A, \;\; \Rightarrow \;\; q_A \Phi_A = \rho, \;\; \text{and} \;\; r_A = \psi_A q_A \Phi_A = \psi_A \rho < \rho. \tag{73}$$

□