Role of unsaturated permeability function in seepage analyses

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ABSTRACT: Unsaturated permeability function is the key parameter in seepage analyses. The unsaturated permeability function is often estimated from soil-water characteristic curve (SWCC) by means of estimation models. However, different estimation models result in different unsaturated permeability functions. In this paper, the effect of using different unsaturated permeability functions, as estimated by the different models, on seepage analyses is presented. The different unsaturated permeability functions can affect the results of seepage analyses which in turn may affect the design of earth structures. In this study, a column of residual soil was subjected to typical rainfall conditions in Singapore through numerical modelling. The results of the analyses show that infiltration characteristics of the residual soil column were noticeably affected when different unsaturated permeability functions were used for the same soil.

1 INTRODUCTION

Unsaturated permeability function is the key parameter in seepage analyses. The hydraulic properties of soil (soil-water characteristic curve, saturated permeability and unsaturated permeability) have been shown to significantly affect the results of transient seepage analyses and stability of slopes subjected to rainfall (Rahardjo et al., 2007; Rahimi et al., 2010a, Rahimi et al., 2010b). The unsaturated permeability function is often estimated from SWCC by means of estimation models. The SWCC of a soil is usually measured in the laboratory and best-fitted using a best-fit SWCC equation. A relative permeability equation is then used to integrate the best-fitted SWCC. The result of this process is the unsaturated permeability function of the soil. A generalized unsaturated permeability function can be expressed as follows (Rahimi, 2015):

$$k_w(\psi) = k_s\left[\int_{\psi(\theta_{wL})}^{\psi(\theta_w)} f(\psi)f'(\psi)d\psi \Big/ \int_{\psi(\theta_{wL})}^{\psi(\theta_s)} f(\psi)f'(\psi)d\psi\right]^d \quad (1)$$

where, $k_w(\psi)$ is the unsaturated permeability function as a function of suction; k_s is the saturated permeability; $f(\psi)$ is SWCC function; $\psi(\theta_{wL})$ is the lower limit of integration for suction and d is a parameter which varies for different models. However, different estimation models result in different unsaturated permeability functions (Mualem, 1986). The study by Rahimi et al. (2015, 2014) investigated the factors which cause the variation between all the estimated unsaturated permeability functions by various models. It was found that the measured SWCC data range, followed by best-fit SWCC equation and relative permeability equations were the most important factors causing this variation. Even though studies showed that hydraulic properties of soil significantly affect the results of seepage analyses, there is a lack of knowledge on the effect of the different unsaturated permeability functions on seepage analyses. Therefore, the objective of this paper is to quantify the effect of the identified parameters (i.e., measured SWCC data range, best-fit SWCC equation and relative permeability equation) of the unsaturated permeability functions on seepage analyses.

2 METHODOLOGY

In order to investigate the effect of the identified parameters (i.e., measured SWCC data range, best-fit SWCC equation and relative permeability equation) of the unsaturated permeability functions on seepage analyses, three estimation models namely, FCM, FMM and VMM, m=1-1/n (Rahimi, 2015) are selected as the unsaturated permeability estimation models. FCM estimation model is the combination of Fredlund and Xing (1994) SWCC equation with Childs and Collis-George (1950) relative permeability equation, FMM estimation model is the combination of Fredlund and Xing (1994) SWCC equation with Mualem (1976) relative permeability equation and VMM, m=1-1/n estimation model is the combination of van Genuchten (1980) SWCC equation with Mualem (1976) relative permeability equation. The measured SWCC data of a residual soil, collected from a construction site at a depth of 3 to 4 m from the ground surface, at Bukit Timah Road, Singapore (Rahardjo et al., 2012) was used to perform the seepage analyses. In practice, SWCC is generally measured in the laboratory over a limited suction range due to the cost and time constraints. For this residual soil (RS), SWCC was measured over a wide suction range of up to 1500 kPa suction. However, two different cases were assumed. First, it was assumed that the measured SWCC data were only available up to 100 kPa (i.e., the measured data beyond 100 kPa was omitted) and the measured SWCC data up to 100 kPa (i.e., DR1) were best-fitted using the two best-fit SWCC equations of Fredlund and Xing (1994) and van Genuchten (1980). The best-fitted SWCCs were then used for estimation of unsaturated permeability functions by FCM and VMM, m=1-1/n model. Second, it was assumed that the measured SWCC data were available up to 1500 kPa (i.e., all the measured SWCC data) and all the measured SWCC data up to 1500 kPa (i.e., DR2) were best-fitted using the two best-fit SWCC equations of Fredlund and Xing (1994) and van Genuchten (1980). The best-fitted SWCCs were then used for estimation of unsaturated permeability functions by FCM, VCM, m=1-1/n and FMM model by DR2. Therefore, five different cases, namely A, B, C, D and E were obtained to investigate the role of unsaturated permeability functions in seepage analyses as shown in Table 1.

Table 1. Five different cases

Case	SWCC	Relative permeability	k_w
A	[1]F&X (1994)-DR1	[3]CCG (1950)	FCM-DR1
B	F&X (1994)-DR2	CCG (1950)	FCM-DR2
C	[2]VG (1980)-DR1	CCG (1950)	VCM, m=1-1/n-DR1
D	VG (1980)-DR2	CCG (1950)	VCM, m=1-1/n-DR2
E	F&X (1994)-DR2	[4]M (1976)	FMM-DR2

1: Fredlund and Xing (1994); 2: van Genuchten (1980), m=1-1/n; 3: Childs and Collis-George (1950); 4: Mualem (1976)

All the five cases were compared according to Table 2. The effect of measured SWCC data range was investigated by comparing cases A and B and cases C and D and it was named as scenario 1 as shown in Table 2. The best-fitted SWCCs and unsaturated permeability functions of scenario 1 are shown in Fig. 1.

Table 1. Three different scenarios for study the effecting parameters, measured SWCC data ranges, best-fit SWCC equation and relative permeability equation.

Effecting parameter	Compared cases		Scenario
Measured SWCC data range	A & B	C & D	1
Best-fit SWCC equation	B & D		2
Relative permeability equation	B & E		3

The effect of best-fit SWCC equation was investigated by comparing cases B and D and it was named as scenario 2 as shown in Table 2. The best-fitted SWCCs and unsaturated permeability functions of scenario 2 are shown in Fig. 2. The effect of relative permeability equation was investigated by comparing cases B and E and it was named as scenario 3 as shown in Table 2. The best-fitted SWCCs and unsaturated permeability functions of scenario 3 are shown in Fig. 3.

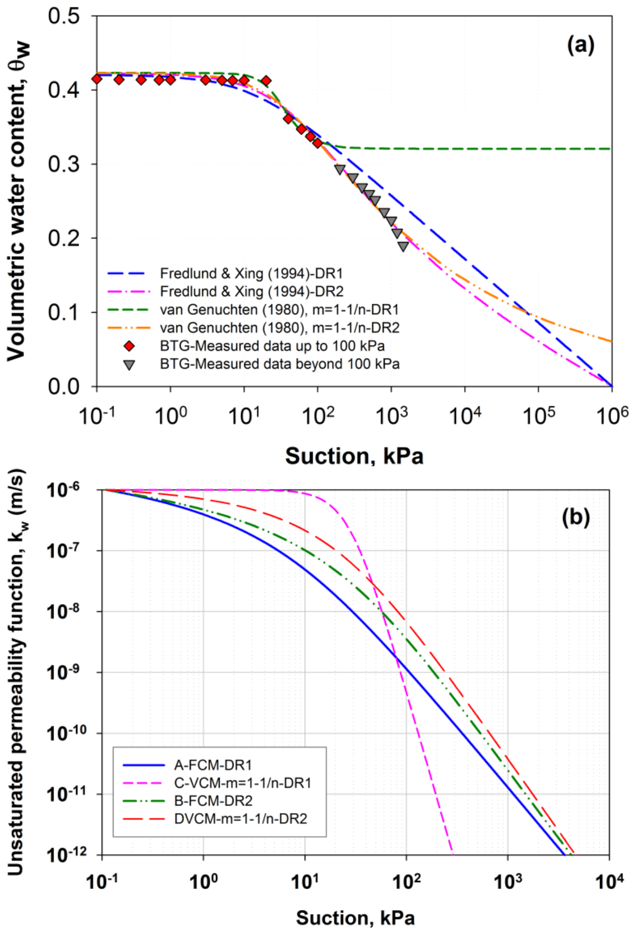

Figure 1. Soil hydraulic properties used in numerical modelling for scenario 1: (a) soil-water characteristic curve, (b) unsaturated permeability function

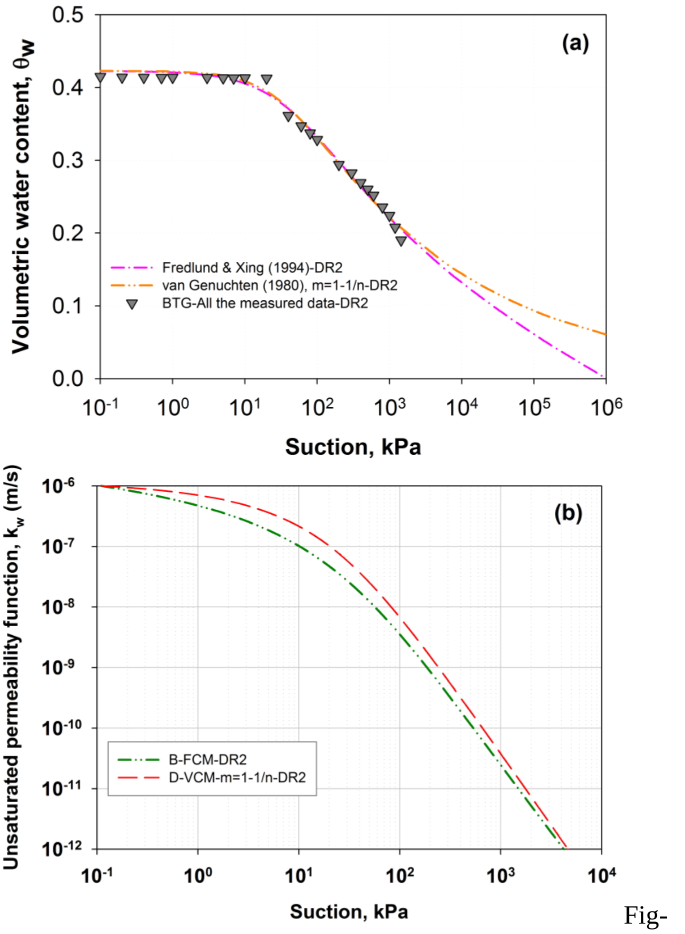

Figure 2- Soil hydraulic properties used in numerical modelling for scenario 2: (a) soil-water characteristic curve, (b) unsaturated permeability function

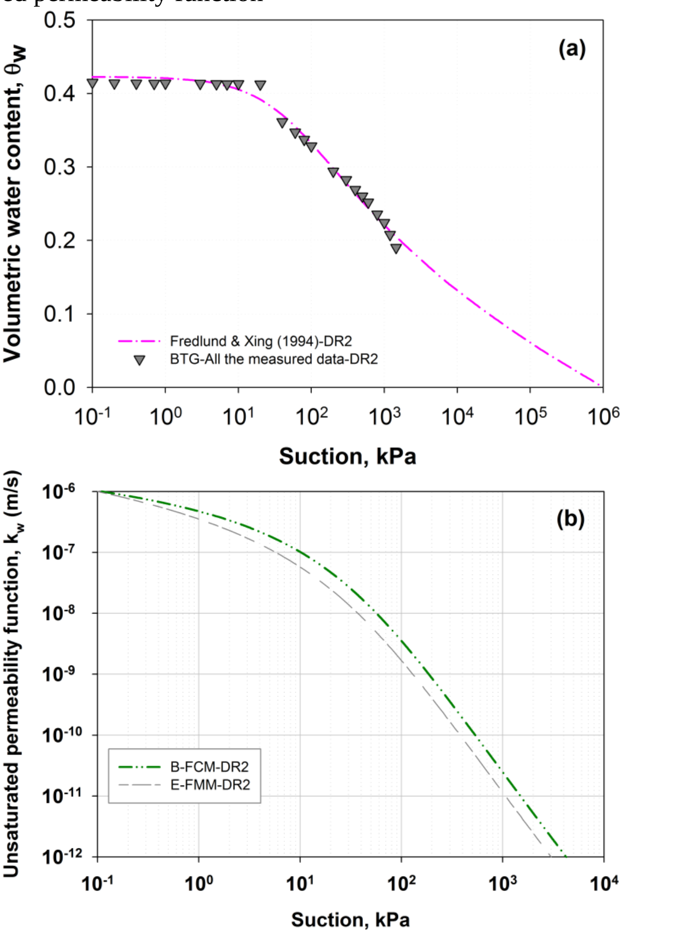

Figure 3. Soil hydraulic properties used in numerical modelling for scenario 3: (a) soil-water characteristic curve, (b) unsaturated permeability function

3 NUMERICAL MODELING

Seepage analyses were conducted using a numerical model of a homogenous soil column using SEEP/W (Geo-slope International, 2012) software. The soil column of 5 m height and 0.5 m width was subjected to rainfall intensity equal to the saturated permeability of the soil for a duration of 24 hours (see Fig.4).

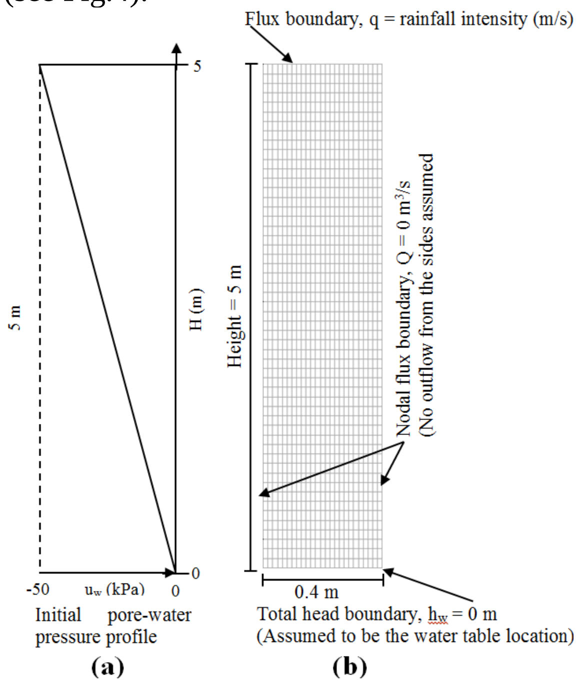

Figure 4. Finite element model and initial condition for the homogeneous soil column: (a) initial condition, (b) mesh and boundary condition (Figure not to scale)

The infiltration process of the rainwater into the soil column was conducted using a saturated-unsaturated, finite element seepage analysis. A value of 1×10^{-6} (m/s) was assumed for the saturated permeability of soil which is a typical value for residual soils of Singapore (Rahardjo et al., 2007). The soil column finite element mesh consisted of 3250 elements as shown in Fig.4b. The boundary and initial conditions applied to the soil column are also shown in Fig.4. A boundary flux, q, equal to rainfall intensity, I_r, was applied to the surface of the soil column. The nodal flux, Q, equal to zero was applied along the sides of the soil column to simulate no flow zone. The water table was assumed to be at the bottom of the soil column. The SWCC and unsaturated permeability function of the soil column were assigned for each of the five cases of A, B, C, D and E according to Table 1.

The water-flow governing equation used in the software for solving a one-dimensional transient seepage analyses is as follows:

$$m_w^2\gamma_w\frac{\partial h_w}{\partial t}=\frac{\partial}{\partial y}\left(-k_{wy}\frac{\partial h_w}{\partial y}\right)+q \qquad (2)$$

where m_w^2=slope of soil-water characteristic curve; γ_w= unit weight of water; h_w= hydraulic head or total head; t = time; k_{wy}= coefficient of permeability with respect to water as a function of matric suction in y-direction; and q = applied flux at the boundary. The numerical analyses were performed for all the five cases of A, B, C, D and E and the results were compared according to the three different scenarios shown in Table 2.

4 RESULTS AND DISSCUSSION

Two different parameters namely, normalized depth of wetting front, D'_{WF}, and matric suction reduction, R_M, were defined according to Equations 3 and 4 to compare and characterize the responses of the soil column for scenarios 1, 2 and 3.

$$D'_{WF} = \frac{D_{WF}}{H} \tag{3}$$

where D'_{WF} is normalized depth of wetting front; D_{WF} is depth of wetting front and H is height of the column.

$$R_M(\%) = ABS\left[\frac{PWP - PWP_i}{PWP_i}\right]100 \tag{4}$$

where R_M is the matric suction reduction; PWP is the pore-water pressure at any time step and PWP_i is the initial pore-water pressure. Depth of wetting front, D_{WF}, was defined as the depth from the soil surface over which the degree of saturation has changed due to water infiltration (Rahardjo et al., 2010).

4.1 *Effect of measured SWCC data range*

As explained earlier, the effect of measured SWCC data range on seepage analyses was investigated by comparing the results of cases A-FCM-DR1 and B-FCM-DR2 with the same best-fit SWCC and relative permeability equations and two different measured SWCC data ranges. In addition, the results of cases C-VCM, m=1-1/n-DR1 and D-VCM, m=1-1/n-DR2 with the same best-fit SWCC and relative permeability equations and two different measured SWCC data ranges were compared. The matric suction reduction, R_M, versus height, H, for cases A, B, C and D are shown in Fig.5 and the normalized depth of wetting front, D'_{FW}, versus time for all the four cases of A, B, C and D are shown in Fig.6. Fig.5 shows that for all the four cases, the matric suction decreased with time as the rainfall continued. After rainfall stopped (i.e., at t = 24 h), the matric suction increased in the upper part of the soil column but decreased in the lower part of the soil column. This behaviour shows that the rainwater infiltrated into the lower part of the soil column even after the rainfall has ceased.

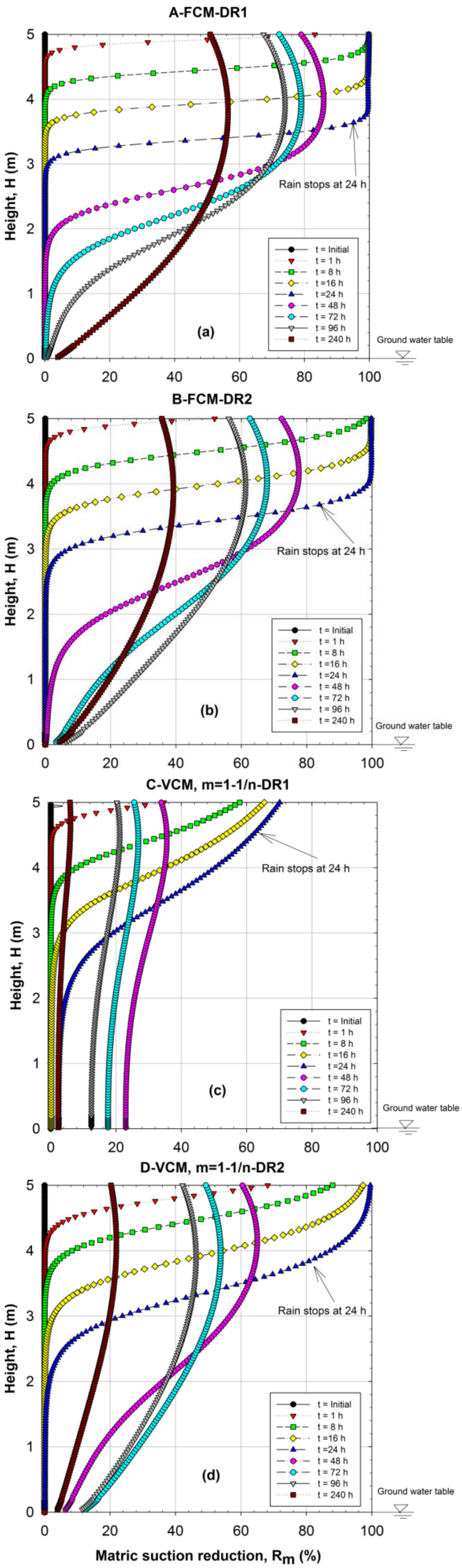

Figure 5. Matric suction reduction profiles-effect of measured data range: (a) A-FCM-DR1 (b) B-FCM-DR2 (c) C-VCM, m=1-1/n-DR1 (d) D-VCM, m=1-1/n-DR2

Comparison between A-FCM-DR1 (see Fig.5a) and B-FCM-DR2 (see Fig.5b) shows that the matric suction reduction and matric suction recovery was at a slightly higher rate for case B. This characteristic was reflected in the deeper normalized depth of wetting front, D'_{FW}, for B (i.e., 0.488) as compared to A (i.e., 0.584) as shown in Fig.6. This shows that using different measured SWCC data ranges resulted in different SWCC and k_w (see Fig.1) and in turn, in different matric suction profiles before and after rainfall. However, as shown in Figure 5, in both of the cases, A and B, the maximum matric suction reduction was about 100 % (at t = 8 h). Comparison between C-VCM, m=1-1/n-DR1 (see Fig.5c) and D-VCM, m=1-1/n-DR2 (see Fig.5d) shows that the matric suction reduction profile was at a significantly higher rate for C-VCM, m=1-1/n-DR1. However, the maximum matric suction reduction was about 70 % for case C (at t = 24 h) as compared to 100 % for case D.

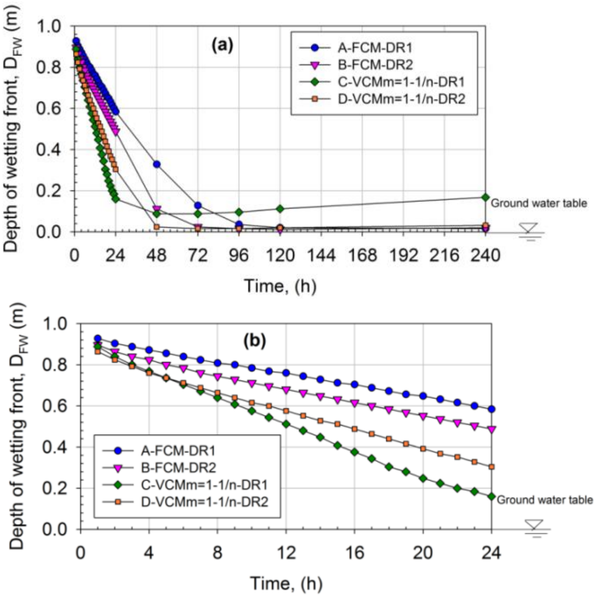

Figure 6. Normalized depth of wetting front-effect of measured SWCC data range

The higher k_w of case C as compared to that of case D caused the higher infiltration rate for the case C. Therefore, the rainwater infiltrated faster into the lower part of the soil column and caused reduction of matric suction in the lower part of the soil column. While the reduction of matric suction in the upper part of the column did not reach 100% as compared to that of case D. This characteristic was reflected in the deeper normalized depth of wetting front, D'_{FW}, for C (i.e., 0.16) as compared to D (i.e., 0.304) as shown in Fig.6. In general, it can be concluded that the effect of measured SWCC data range used to best-fit the SWCC and estimate the unsaturated permeability function had a significant effect on the seepage analyses results.

As shown in Fig.6, D'_{FW}, at t = 24 h was deeper for B (i.e., 0.488) as compared to A (i.e., 0.584). In addition, D'_{FW}, was deeper for C (i.e., 0.16) as compared to D (i.e., 0.304). Therefore, the difference between C and D was larger than the difference between A and B. This characteristic shows that VCM, m=1-1/n model has higher sensitivity then the FCM model to the measured SWCC data ranges. In the other words, Fredlund and Xing (1994) SWCC equation was less sensitive to the measured SWCC data ranges.

4.2 *Effect of best-fit SWCC equation*

The effect of best-fit SWCC equation on seepage analyses was investigated by comparing the results of cases B-FCM-DR2 and D-VCM, m=1-1/n-DR2 with the different best-fit SWCC, but the same relative permeability equations and the same measured SWCC data ranges (see Fig.2).

The matric suction reduction, R_M, versus height, H, for cases B and D are shown in Fig.5. The figure shows that for both of the cases, the matric suction reduced with time as the rainfall continued. After rainfall stopped (i.e., at t = 24 h), the matric suction increased in the upper part of the soil column but the matric suction decreased in the lower part of the soil column. Comparison between B-FCM-DR2 (see Fig.5b) and D-VCM, m=1-1/n-DR2 (see Fig.5d) shows that the matric suction reduction and matric suction recovery was at a higher rate for case D. Case B reached the maximum matric suction reduction of 100 % at t = 8 h, while Case D reached the maximum matric suction reduction of 100 % at t = 24 h. This was due to the higher unsaturated permeability of case D as compared to that of case B even when the same relative permeability equation and the same measured SWCC data range were used in the development of both models. Therefore, selection of the best-fit SWCC equation plays an important role when conducting seepage analyses.

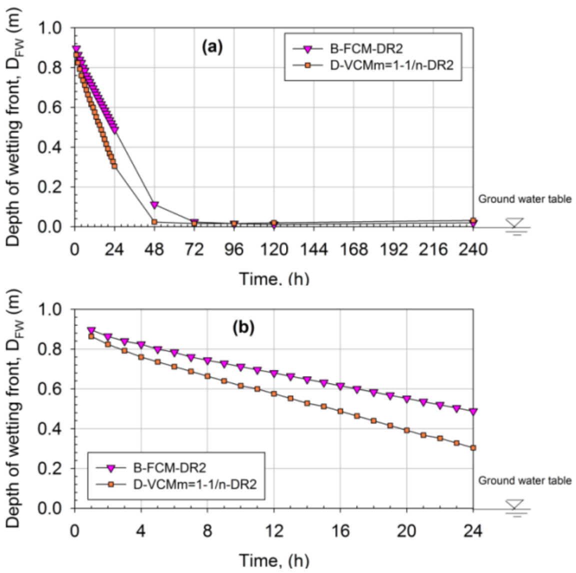

Figure 7. Normalized depth of wetting front-effect of SWCC equation.

The normalized depth of wetting front, D'_{FW}, versus time for cases B and D is shown in Fig.7. As shown in the figure, D'_{FW}, at t = 24 h was deeper for case D (i.e., 0.304) as compared to case B (i.e., 0.488). This shows that the best-fit SWCC equation can significantly affect the results of seepage analyses.

4.3 *Effect of relative permeability equation*

The effect of relative permeability equation on seepage analyses was investigated by comparing the results of cases B-FCM-DR2 and E-FMM-DR2 with the different relative permeability equations, but same best-fit SWCC and the same measured SWCC data ranges (see Fig.3). The matric suction reduction, R_M, versus height, H, for cases B and E are shown in Fig.8. The figure shows that for both cases, the matric suction decreased with time as the rainfall continued. After the rainfall stopped (i.e., at t = 24 h), the matric suction increased in the upper part of the soil column but the matric suction decreased in the lower part of the soil column.

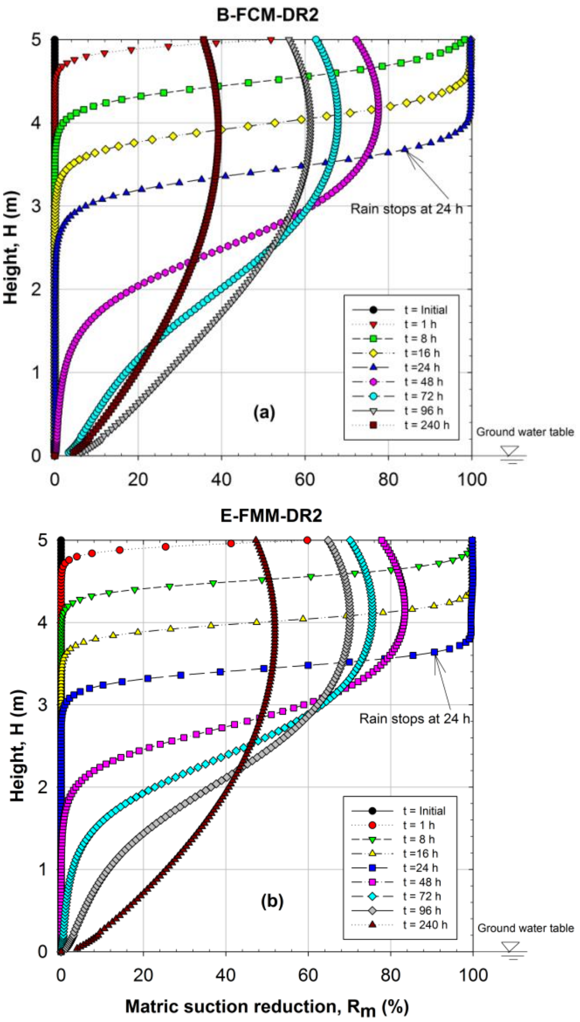

Figure 8. Matric suction reduction profiles-effect of relative permeability equation: (a) B-FCM-DR2 (b) E-FMM-DR2

Comparison between B-FCM-DR2 (see Fig.8a) and E-FMM-DR2 (see Fig.8b) shows that the matric suction reduction and matric suction recovery were more or less the same for both of the cases. Cases B and E reached the maximum matric suction reduction of 100 % at t = 8 h. The normalized depth of wetting front, D'_{FW}, versus time for the cases B and E is shown in Fig.9. As shown in Fig. 9, D'_{FW}, was slightly deeper for case B (i.e., 0.488) as compared to case E (i.e., 0.576). If the difference between D'_{FW} of cases B and E (i.e., effect of relative permeability equation) is compared to that of B and D (i.e., effect of best-fit SWCC equation), it can be seen that the effect of best-fit SWCC equation is more important than the effect of the relative permeability equation.

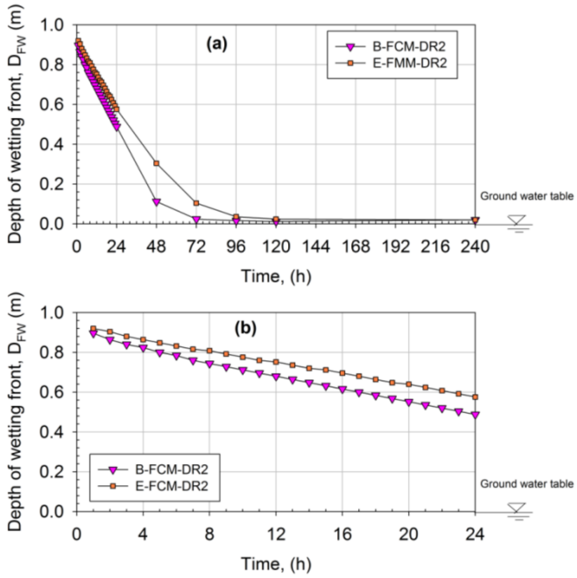

Figure 9. Normalized depth of wetting front-effect of relative permeability equation

5 CONCLUSIONS

Based on seepage analyses of a residual soil column subjected to 24 h rainfall of 1×10^{-6} m/s, it was found that the measured SWCC data range noticeably affected the results. The effect was more significant for van Genuchten (1980), m=1-1/n based models. The effect of best-fit SWCC equation was more important than the effect of relative permeability equation on the seepage analysis results.

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