REACHING DECISIONS VIA INTERNAL DIALOGUE: ITS ROLE IN A LECTURER PROFESSIONAL DEVELOPMENT MODEL

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In this paper we consider the professional development of university mathematics lecturers. We describe two exemplars from a two-year research process to engage mathematics educators and research mathematicians in a constructive dialogue about teaching. In this study lectures were video recorded and then discussed in a supportive community of practice. Using Schoenfeld's Resources, Orientations and Goals (ROG) theoretical framework we analyse two lecture segments and describe how the two lecturers' ROGs caused them to make decisions that moved them toward different outcomes. The value of explicit ROG-based discussion of small-scale lecture moments to a professional development model is considered.

INTRODUCTION

Delivering mathematics content to large numbers of students via the medium of lectures presents a number of pedagogical difficulties that are seldom explicitly addressed (Speer, Smith, & Horvath, 2010). This paper presents a small part of a two-year project examining a professional development model, with two key questions: whether an effective lecturing professional development strategy can be built around a community of practice; and whether Schoenfeld's Resources, Orientations and Goals (ROG) theoretical framework (see details below) can be adapted to analyse university mathematics lecturing. While the effectiveness of various approaches to teacher professional development has been extensively examined, comparatively little is known at collegiate level (Speer et al., 2010). Addressing this need, this project structured community interactions to prioritise three practices identified as effective. Firstly we focused on "small, but meaningful, aspects of practice" (Speer, 2008, p. 219), "at the very level of detail when development and change appear to occur—the moment-to-moment decisions and practices of teachers" (Speer, 2008, p. 263). Such fine-detailed examination has been successfully used in microteaching in teacher education. Secondly, all discussions within the group followed the protocol that these should develop from concerns identified by the lecturer themselves. This aligns well with Robinson (1989), who argues that professional development that recognises the teacher as a professional and works from the 'bottom-up' is empowering, providing teachers with opportunities to make meaningful choices, and with Paterson and Barton (2009), who showed that teachers evidenced positive changes when working on self-identified areas of concern in their practice. Finally, the development of a community of practice was actively fostered (Buckley & du Toit, 2010).

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THEORETICAL FRAMEWORK

Schoenfeld (2008, 2010) has developed a theory of teaching-in-context, with a goal of answering how and why teachers make the in-the-moment choices they do while they are engaged in the act of teaching. The current framework is based on Resources, Orientations and Goals (ROG) that teachers bring to their practice. Thus, what a teacher decides to do while engaged in teaching is a function of the teacher's goals, orientations, including dispositions, beliefs, values, tastes and preferences, (which serve to prioritize goals), and resources (particularly knowledge) (Schoenfeld, 2008). Hence, when a teacher enters a classroom they use their orientations to adjust to the situation. Goals are established based on the orientation, and relevant knowledge is activated. Decisions consistent with goals are made, consciously or unconsciously, about the directions to pursue and the resources to use (Schoenfeld, 2010). Classroom decisions, including those made in-the-moment, are crucial, since "The quality of people's decision making…affects how successfully people attain the goals they set for themselves." (Schoenfeld, 2010, p. 36), and analysing these decisions should be part of a professional development programme. However, an individual's ROG may contain competing goals inspired by differing orientations. This latter situation is the subject of this paper, as we seek to analyse how the conflict of competing goals arising from an internal dialogue between mathematician and teacher are resolved.

METHOD

In this study four research mathematicians and four mathematics educators, all from Auckland University, formed a community to re-examine lecturing practice in the light of educational theories. This paper discusses lectures of two research mathematicians that were video-recorded; each lecturer then chose a small section of less than five minutes that the whole group watched and discussed together in a supportive manner. This focussed discussion was audio-recorded and later transcribed. We found that the discussion often started with the video content but then moved on to examine other relevant, related issues of learning, practice and mathematics. The video and audio transcription data was supplemented by a lecturer ROG written before the lecture and interviews. The data was analysed by focussing on the relationship between decision points and the ROGs.

DESCRIPTION OF TWO SITUATIONS

The two situations we describe involved experienced male lecturers, who we call Sandy and Simon, presenting an applied mathematics lecture to first year students and a postgraduate lecture in number theory, respectively. The primary purpose of Sandy's lecture was to consider solutions, for various values of the parameter q, of a difference equation that reduced to the form $x_m = qx_{m-1}(1-x_{m-1})$. He revealed a number of orientations relevant to this lecture segment. Firstly, as a teacher he values demonstrating results, even without precise prior knowledge of what may occur:

O1: "It is good to demonstrate things to the students rather than just tell them."

O2: "I'm pretty happy to experiment in this course and things will occasionally go wrong when you do that."

He also has a pedagogical belief that, since he is part of a teaching team:

O3: It is important to stick to the course book and cover all the material.

With regard to the use of Matlab, his orientations were:

O4: It has value for exploration "I just wanted to explore, show them the graphs on the screen using Matlab and change the Matlab a little bit to get a closer look at the graphs"

These were some of the orientations that led to the establishment of a number of goals, some of which Sandy explicitly wrote down, and some we infer, including:

G1: To show students that interesting, unexpected things happen to the solution as the parameter changes.

G2: Students understanding, from the demonstration, that the solution of the difference equation is a periodic function.

G3: To show how easy it is to discover these things by using Matlab.

G4: To have students appreciate the value of Matlab as a mathematician's research tool. "…the use of computers to um.. explore mathematics that's something that I see as a mathematician and that I'd like to impart that...".

G5: To keep students interested. "…we wanted to keep them interested and so this was an extra lecture showing some more advanced features of the logistic equation that's usually taught at graduate level."

G6: To explain clearly the mathematical basis of the construction and solutions of the difference equation.

G7: To stick to the course book and cover all the assigned material.

To achieve these goals he called on resources, including his mathematical knowledge (R1) and the computer program Matlab (R2), which was used to plot solutions of the equation for various values of the parameter. The solutions were then displayed using an overhead projector (R3). At one point Sandy made the decision to move to the projected graph and show the students the periodicity of the function.

Why did he decide to do this? In line with his orientations O1, 2 and 4 it was part of his attainment of the goals G2, 3 and 6. O1 was crucial here, the belief that demonstrating and not just telling leads to understanding better, as he said "In fact the solutions are periodic and it was a bit hard to look and see that straight off that the solution's periodic, so that's why I wanted to do the counting." However, it was during this process of counting the function local minima that a crucial decision point arose. The lecture transcription follows.

1. What's happening here it looks even more complicated, 3.6…[*3 to 4 secs*] yeh so you can see that if you look at it closely…[*walks to screen*]

2 Suppose you start by looking at this value here [pointing at the graph on the projection] then there's going to be 1, 2, 3, 4, 5, 6, 7, you can count to 8 I think maybe.. do I ever get back to where I started, maybe not [*realises there's a problem*]

3 9, 10, 11, 12 ,13, 14 um.. how many values? So it looks like there's a period of um.. let's see 1, 2, 3, 4, 5, 6, 7, 8.. [*starts to count again*] it looks like there's a period of 14. Whether that's the case or not I'm not sure.

4 We might not have got to the limiting value yet. But it looks like we've settled down to a period of 14. By a period of 14 I mean that it takes 14 um.. we need n to change by 14 to get back to where you started from…So it seems to be settling down to some complicated periodic um.. solution.

We see here that the count of the period arrives at 14, but Sandy knows that the true value is 16, as he later explained "…because the period doubles each time, so it goes from 2 to 4 to 8 to 16, so.. and so on, so there's a theory that actually says the period has to double." This was unexpected, "I guess the thing that I was probably concerned about was um.. observing something that I didn't expect and not being about to explain it immediately". Hence, in-the-moment, he has to decide whether to address what is, for him, a mathematical discrepancy. How did he make the decision?

Analysing the decision—Sandy

Arriving at the decision involved an internal dialogue between the lecturer as a mathematician [M] and as a teacher [T]. This dialogue had as its aim the resolution of conflict between the competing pedagogical goals, G1, 2, 3 and 5, and the mathematician's goals of G4 and G6. We see this from Sandy's comments about this.

> Yeah, in fact my decision was based on the fact that I'd already spoken far longer than I'd planned to [G7 T] on the existing equation and it was time to actually go and do some problems [G7 T] which was supposed to be the rest of the lecture so I got onto that [G7 T].
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> Actually I would have liked to have pursued it a bit [G6 M] but we had already spent more than the allotted amount of time on this demonstration [G7 T] and I had shown them periodicity for shorter periods already [G2 T] so I think they had grasped the concept quite well, so the fact that I didn't actually get a period of 16 bugged me a bit [G6, mathematician] but not enough to ruin the rest of the lecture [G7 T].
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> I certainly made a decision not to continue with an unexpected outcome on a graph in the first part of the lecture. Part of me wanted to address this at the time [G4, 6 M] but I had already gone over time with this part of the lecture and had achieved the goals I desired [G1, 2, 3 T].

We see that, in *this* situation, with *these* students, the teacher wins out over the mathematician. The reason seems to be that the predominant goal was G2, to demonstrate that 'that the solution of the difference equation is a periodic function'. This had been accomplished, and meeting his pedagogical goals released him from the need to explain the mathematical anomaly, as he said "Actually I didn't get any reaction from the students…I never did tell them it was really 16." He confirmed that, as a teacher, he was happy with the outcome of the lecture, including this decision:

> I'm pretty happy with the way the lecture went. Students seemed interested [G5 T], in our exploration of the logistic equation [O1, 2, 4; G1, 3 T] and participated in the exploration. In response to questions, we changed the Matlab code to zoom in on graphs of solutions, which allowed us to clearly see the periodicity [G2 T].
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> The actual lecture itself went pretty well I was pleased with the demonstration [O1, 2, 4; G1, 2, 3 T] um.. I think it was clear enough. The students were able to see what was going on [G2 T].

Simon's lecture introduced the students to continued fractions. He too revealed a number of orientations relevant to this lecture but we only present here the ones related to the decision we examine in detail.

O1: To emphasise to students that the right theoretical tools and proof techniques can tame a mathematical problem.

O2: Some proofs are more interesting and important than others. "The real reason I think it's a cool proof is the fact that you prove a more general result. It's one of these things that happens a lot in mathematics, you don't see it much at the junior level."

O3: Mathematics needs to be correct. "Oh, this is not really right, I don't like it not to be right."

O4: Mathematical notation needs to be consistent and accurate. "Right so the symbols h_j over k_j will from now on will mean precisely one of these things for the specific numbers I am interested in."

O5: Students who are talented at mathematics, such as this class, can cope when a lecturer dwells on the finer points in mathematics. "There is also a confidence that the students can cope with – if I go off on my own little journey the students will have the tools to deal with that … Whereas in another class you would be worried that if I'd lost them after 15 minutes then that's it."

O6: Some (but not all) all students at this level are ready to be inducted into mathematics. "Last year's class I didn't feel like they were being inducted into mathematics so it wasn't necessary to dwell on this particular issue of the more general result."

In the ROG he writes before the lecture he states the following goals for the course:

G1: To increase the students' mathematical maturity by helping them understand the theory and do proofs.

G2: To provide good general preparation for post-graduate study in number theory.

G3: To give exposure to different proof techniques.

G4: The most important theoretical part, for this first lecture, is to state and prove correctness of the recurrence formulae for computing the convergents.

Some of Simon's goals emerge during discussion.

G5: To engage with the mathematics for its own sake, 'it's fun.'

G6: To ensure that the mathematics he does is 'right'; it's part of his role as a lecturer

G7: To use notation that is consistent.

G8: To induct (some) students into thinking and behaving like mathematicians.

In order to satisfy his goals he draws on a number of resources, including his knowledge of mathematics in general and number theory in particular, (R1) and his assessment of the students' mathematical ability and interest (R2). The decision we will discuss here is one he made when he suddenly realised that he was going to encounter a 'notational conundrum' while proving the correctness of the recurrence formulae for computing the convergents. The lecture transcription follows.

1. So, by the inductive hypothesis, [*starts to write*] I know what this is. [*gestures in swirl over previous line*] It is some $\frac{h_i}{k_i}$. [*looks at board, as if he is thinking*]
2. I'm going to call it…Did I give it a name? [*Looks at paper*] I didn't give it a name. It's just some $\frac{h_i}{k_i}$. But whatever that $\frac{h_i}{k_i}$ is it apparently satisfies the recurrence formula [*points to paper looks at class*] [*Stands back, looks at the board, pauses*]
3. Yeah I mean this is *an* $\frac{h_i}{k_i}$ but it's not *the* $\frac{h_i}{k_i}$ that I am really thinking of [*gestures back to previous expression*] This is a very subtle point.
4. Let's define h_i over…Let's define $\frac{h_j}{k_j}$ to be these things up to a_j where I have worked these out already all the way up to *i* [*writes down and puts in rectangular box above previous 2 expressions*] *Right* so the symbols $\frac{h_i}{k_i}$ will from now on mean precisely one of these things for the specific numbers I am interested in. This thing I have written down here [*gestures*] is *not* the $\frac{h_i}{k_i}$ in that notation because this end term is wrong.

Analysing the decision—Simon

When asked why he made a decision to 'labour the point' and disentangle a problem of which the students were not (yet) aware, Simon said it was the mathematician within going "Oh this is not really right" [G6 M] and added "at that point the whole world disappeared and it's just me and the mathematics." [G5 M]. "I suddenly realised that it is sort of not quite fitting how I was using those symbols previously. [G7 M] I was thinking ahead to where the proof was going and suddenly it becomes clear to me that there is a problem ahead but it's not clear to anyone else yet." [R1] He likens himself to the driver of a group of interested, but unworried, tourists (the students) who to his surprise notices a 'Danger Ahead" sign on the roadside while the others are all still happily chatting at the back of the bus. He could 'put his foot down and hope' and tell the students "it's in the notes … it would be a good exercise for you to do carefully." But he chose to sort it out, to labour the point, and take the bus down the bumpy road. [G5 M and G2, 10 T].

When describing why he chose this section to re-view he spoke about a mismatch between what he did and his written ROG. Discussion uncovered higher order goals that drove the decision—his unwritten orientation and goals as a mathematician [G5, 6, 7 M] and his desire to induct talented students into mathematics [G2, 8 T]. He is not alone in needing to resolve the "tensions experienced by the lecturer in satisfying student needs and mathematical values." (Joworski et al., 2009 p. 249). It was his estimation of students' ability [O5] that allowed him to take the 'detour'. His original assumption that because he saw them as good students [O5] he would say 'Just do it' proved incorrect in-the-moment. As he says 'last year there was not the same ability so I went through the proof much more lightly with them. It wasn't necessary to dwell on this particular issue of the more general result.' The need to 'get it right' [G6, 7 M] and to induct them [G8 T] won the day.

DISCUSSION

Can we say that this professional development model works? Is the analysis in depth of small parts of a lecture chosen by the lecturer, against an explicit framework of ROGs within a community of practice, of pedagogical value? The feedback from all involved suggests that the answer is yes. A number of important aspects contribute to this. One is that having a 'mixed' group of mathematics educators and mathematicians enables cross-fertilisation of ideas. Sandy commented, "I gained a mathematics education perspective … which clarified in my own mind what I do when I teach", and "…you come in with your theory from time to time explaining some of the things that we all do and that's very useful." One valuable aspect of the process was the opportunity to see others' teaching. As Sandy said "And also seeing other people teaching, that's wonderful." Towards the end of the year it was agreed that it would be a good thing if the practice of watching others became 'business as usual.' While we agree, we contend that the subsequent, focussed discussion is extremely important.

Secondly, the community was deliberately set up to be supportive. While Simon's comment, "It's pretty revealing watching yourself being videoed isn't it?" shows that lecturers were sensitive about exposing their practice, Sandy maintained "It was reassuring that nobody thought that [he looked silly]…that was good." and that the group was "very supportive, very supportive." He valued feedback and discussion from a practical, teaching perspective: "So it's good to get that feedback from other people and in some cases people identify things that I do that I wasn't even aware of…I have my usual techniques for teaching but it's good to get some opinions on these." The development of a community of practice is evidenced in the fact that when we observed that "you [Simon] looked how [Sandy] looked when he was worried about the thing", we all knew 'the thing' referred to his 14/16 dilemma and subsequent decision. As we work the repertoire of shared decision moments is growing and enables new ones to feed off them. The process of thinking about, and then writing, one's ROG was another feature commented on, with Sandy saying how it improved his lecture. In Simon's case the 'dissonance' he perceived between his stated ROG and his decision, led to a discussion of his higher order goals. This led him to say that he would like to think more about the audience and that "anything that encourages me personally, to put more thought into who the audience really are, what actually they know, that's extremely useful…and it's astonishing to think that that is not automatically done". The ROG structure also provides a framework for discussing the 'objects of practice' the lecturers chose. Engaging mathematicians (even more than teachers) in a conversation about pedagogy—particularly their own—is enabled by linguistic and theoretical support that is grounded in their practice. We suggest that a fine-grained analysis of small-scale lecturer-chosen lecture segments, against the ROG framework, activates an awareness of the basis on which we make teaching decisions, prompting examination of these decisions leading to development of practice.

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