Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand). This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.
http://researchspace.auckland.ac.nz/feedback

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form
Qualitative Topological Coverage of Unknown Environments by Mobile Robots

Sylvia Wong
February 2006

Supervisors: Dr Bruce A. MacDonald
            Dr George Coghill

A thesis submitted in partial fulfillment of the requirements of
Doctor of Philosophy in Engineering
Abstract

This thesis considers the problem of complete coverage of unknown environments by a mobile robot. The goal of such navigation is for the robot to visit all reachable surfaces in an environment. The task of achieving complete coverage in unknown environments can be broken down into two smaller sub-tasks. The first is the construction of a spatial representation of the environment with information gathered by the robot’s sensors. The second is the use of the constructed model to plan complete coverage paths.

A topological map is used for planning coverage paths in this thesis. The landmarks in the map are large scale features that occur naturally in the environment. Due to the qualitative nature of topological maps, it is rather difficult to store information about what area the robot has covered. This difficulty in storing coverage information is overcome by embedding a cell decomposition, called slice decomposition, within the map. This is achieved using landmarks in the topological map as cell boundaries in slice decomposition. Slice decomposition is a new cell decomposition method which uses the landmarks in the topological map as its cell boundaries. It decomposes a given environment into non-overlapping cells, where each cell can be covered by a robot following a zigzag pattern. A new coverage path planning algorithm, called topological coverage algorithm, is developed to generate paths from the incomplete topological map/slice decomposition, thus allowing simultaneous exploration and coverage of the environment.

The need for different cell decompositions for coverage navigation was first recognised by Choset. Trapezoidal decomposition, commonly used in point-to-point path planning, creates cells that are unnecessarily small and inefficient for coverage. This is because trapezoidal decomposition aims to create only convex cells. Thus, Choset proposed boustrophedon decomposition. It introduced ideas on how to create larger cells that can be covered by a zigzag, which may not necessarily be convex. However, this work is conceptual and lacking in implementation details, especially for online creation in unknown environments. It was later followed by Morse decomposition, which addressed issues on implementation such as planning with partial representation and cell boundary detection with range sensors. The work in this thesis was developed concurrently with Morse decomposition.
Similar to Morse decomposition, slice decomposition also uses the concepts introduced by boustrophedon decomposition. The main difference between Morse decomposition and slice decomposition is in the choice of cell boundaries. Morse decomposition uses surface gradients. As obstacles parallel to the sweep line are non-differentiable, rectilinear environments cannot be handled by Morse decomposition. Also, wall following on all side boundaries of a cell is needed to discover connected adjacent cells. Therefore, a rectangular coverage pattern is used instead of a zigzag. In comparison, slice decomposition uses topology changes and range sensor thresholding as cell boundaries. Due to the use of simpler landmarks, slice decomposition can handle a larger variety of environments, including ones with polygonal, elliptical and rectilinear obstacles. Also, cell boundaries can be detected from all sides of a robot, allowing a zigzag pattern to be used. As a result, the coverage path generated is shorter. This is because a zigzag does not have any retracing, unlike the rectangular pattern.

The topological coverage algorithm was implemented and tested in both simulation and with a real robot. Simulation tests proved the correctness of the algorithm; while real robot tests demonstrated its feasibility under inexact conditions with noisy sensors and actuators.

To evaluate experimental results quantitatively, two performance metrics were developed. While there are metrics that measure the performance of coverage experiments in simulation, there are no satisfactory ones for real robot tests. This thesis introduced techniques to measure effectiveness and efficiency of real robot coverage experiments using computer vision techniques. The two metrics were then applied to results from both simulated and real robot experiments. In simulation tests, 100% coverage was achieved for all experiments, with an average path length of 1.08. In real robot tests, the average coverage and path length attained were 91.2% and 1.22 respectively.
Acknowledgements

It has been a long time since I first arrived at the E&E department in the University of Auckland as a wide-eyed undergraduate. I am happy to have this opportunity to thank some of the people who have helped make it an enjoyable experience.

First of all, I would like to thank my supervisor, Dr Bruce MacDonald for his support and guidance throughout my PhD years. Without his encouragement and occasional stern looks, I would not have been able to make it to the end.

I would also like to thank my parents. They provided much financial support, which is very appreciated. I hope they feel proud when they see this 200 page masterpiece.

The technical staff in the department have also been of tremendous help. Lance Allen and the workshop designed and built the wooden enclosure for the Khepera robot. Jamie Walker and Evans Leung helped me with all sorts of computer and networking problems. Bev Painter, Nichola Kavacevich and Grant Sargent have also been very helpful.

I greatly enjoyed the interactions with other PhD students in the department, from inspiring technical discussions to playing age of empires. They are (in alphabetical order): Geoff Biggs, Toby Collett, Barry Hsieh, Lee Middleton, Adrian Pais, Russell Smith, Brad Sowden, Chris Waters, Joseph Wong and David Yuen.

Lastly, I would like to thank Jorge Cham, the creator of piled higher and deeper. His comic strips provided me with laughter when I sorely needed it.
## Contents

1 Introduction .............................................. 1  
  1.1 Overview of problem domain .................................. 1  
  1.2 Applications of coverage path planning ......................... 3  
  1.3 Description of the thesis ..................................... 3  
    1.3.1 Scope of the research ................................... 3  
    1.3.2 Overview of the thesis ................................... 4  
  1.4 Contributions of the thesis ................................... 5  

2 Coverage navigation and path planning ......................... 7  
  2.1 Voronoi diagram ........................................... 8  
  2.2 Cell decomposition .......................................... 10  
  2.3 Grid map ................................................... 28  
  2.4 Topological map ............................................. 34  
  2.5 Reactive robots ............................................. 35  
  2.6 Coverage with multiple robots ................................ 36  
  2.7 Coverage of 3-dimensional surfaces ............................ 36  
  2.8 Performance metrics .......................................... 37  
    2.8.1 Simulation ............................................... 37  
    2.8.2 Real robots ............................................... 37  
    2.8.3 Complexity of coverage navigation ....................... 38  
  2.9 Discussion .................................................. 39  
  2.10 Summary .................................................... 40  

3 Slice decomposition ......................................... 41  
  3.1 Slice Decomposition I ........................................ 42  
    3.1.1 Events .................................................. 43  
    3.1.2 Algorithm ............................................... 46  
  3.2 Slice Decomposition II ........................................ 47  
    3.2.1 Events .................................................. 47
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.2</td>
<td>Algorithm</td>
<td>49</td>
</tr>
<tr>
<td>3.3</td>
<td>Effects of step size and sweep direction</td>
<td>53</td>
</tr>
<tr>
<td>3.4</td>
<td>Tethered robots</td>
<td>56</td>
</tr>
<tr>
<td>3.5</td>
<td>Discussions</td>
<td>56</td>
</tr>
<tr>
<td>3.6</td>
<td>Summary</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>Topological Coverage Algorithm</td>
<td>59</td>
</tr>
<tr>
<td>4.1</td>
<td>Finite State Machine</td>
<td>60</td>
</tr>
<tr>
<td>4.1.1</td>
<td>State – Normal</td>
<td>60</td>
</tr>
<tr>
<td>4.1.2</td>
<td>State – Boundary</td>
<td>62</td>
</tr>
<tr>
<td>4.1.3</td>
<td>State – Travel</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>Cell boundaries</td>
<td>63</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Event – Split</td>
<td>64</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Event – Merge</td>
<td>67</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Event – End</td>
<td>70</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Event – Lengthen</td>
<td>72</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Event – Shorten</td>
<td>73</td>
</tr>
<tr>
<td>4.2.6</td>
<td>Combination of split and merge events</td>
<td>76</td>
</tr>
<tr>
<td>4.3</td>
<td>Topological Map</td>
<td>77</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Nodes</td>
<td>80</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Edges</td>
<td>80</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Map updates</td>
<td>81</td>
</tr>
<tr>
<td>4.4</td>
<td>Travel between cells</td>
<td>90</td>
</tr>
<tr>
<td>4.5</td>
<td>Completeness</td>
<td>93</td>
</tr>
<tr>
<td>4.6</td>
<td>Complexity</td>
<td>95</td>
</tr>
<tr>
<td>4.7</td>
<td>Summary</td>
<td>97</td>
</tr>
<tr>
<td>5</td>
<td>Performance metrics</td>
<td>99</td>
</tr>
<tr>
<td>5.1</td>
<td>Metrics</td>
<td>100</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Effectiveness: percentage coverage</td>
<td>100</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Efficiency: path length</td>
<td>100</td>
</tr>
<tr>
<td>5.2</td>
<td>In simulation</td>
<td>101</td>
</tr>
<tr>
<td>5.3</td>
<td>In real robot experiments</td>
<td>104</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Creating composite images</td>
<td>104</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Correcting perspective warp</td>
<td>108</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Computing percentage coverage</td>
<td>110</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Calculating normalised path length</td>
<td>110</td>
</tr>
<tr>
<td>5.4</td>
<td>Summary</td>
<td>111</td>
</tr>
</tbody>
</table>
## Contents

6 Implementation 113
6.1 Khepera robot ........................................ 113
6.2 Simulation ............................................. 115
6.3 Topological map ........................................ 119
6.4 Robot controller ....................................... 120
6.5 Summary ................................................ 122

7 Results and discussion 123
7.1 Landmark Detection ................................... 123
7.1.1 Discontinuity on side of robot ................... 124
7.1.2 Topology changes in front of robot ............. 135
7.2 Coverage Experiments ................................. 138
7.2.1 Simulation ......................................... 138
7.2.2 Real Robot ......................................... 143
7.3 Zigzag as coverage pattern ......................... 143
7.4 Evaluating composite images ....................... 149
7.5 Performance Metrics ................................. 155
7.5.1 Simulation ......................................... 155
7.5.2 Real robot experiments ......................... 156
7.6 Composite image for non-circular robots .......... 159
7.7 Path length L and complexity of environment ... 159
7.8 Summary ................................................. 163

8 Future Work and Conclusions 165
8.1 Future work ............................................. 165
8.1.1 Tethered robot ..................................... 165
8.1.2 Simultaneous localisation and coverage (SLAC) 166
8.1.3 Multi-robot coverage .............................. 167
8.2 Conclusions .............................................. 167

A Landmark Recognition using Neural Networks 171
A.1 Pattern classification with Neural Networks ........ 171
A.2 Multilayer Perceptron (MLP) ......................... 172
   A.2.1 Forward propagation ............................ 173
   A.2.2 Error back-propagation ....................... 175
A.3 Learning Vector Quantisation (LVQ) ................. 176
   A.3.1 Vector Quantisation ............................ 176
   A.3.2 Learning the reference vectors ............... 176
A.4 Landmark recognition .............................. 177
A.4.1 Preprocessing ........................................ 178
A.4.2 Results .................................................. 179
A.5 Summary .................................................. 180

B  Computer Vision ........................................ 183
B.1 Canny Edge Detection ................................. 183
   B.1.1 Gaussian smoothing ............................... 185
   B.1.2 Sobel edge detection ............................. 187
   B.1.3 Non-maximal suppression ....................... 188
   B.1.4 Hysteresis thresholding ......................... 189
B.2 Hough Transform ........................................ 190