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Numerate Life for Whom? A Non-book-review of John Allen Paulos's A Numerate Life: A Mathematician Explores the Vagaries of Life, His Own and Probably Yours

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A common, widespread view on the field of mathematics entails that "mathematics is dry as dust [and] as exciting as a telephone book" (Davis & Hersh, 1981, p. 169), and that the professional mathematician is merely "a kind of calculator" (Krull, 1987, p. 48). Even more provocatively, mathematics is conceived by many of the general public as a "deadend" discipline in which all questions have already been answered, where nothing worthwhile is left for further investigation (Movshovitz-Hadar, 2008). However, mathematics is an active, growing, open-ended field – a fact which can be illustrated, amongst others, by the exponential growth in the number of new articles that are published per year (Dunne, 2019). Furthermore, the common perception of the subject by the people who are working within the field – the mathematicians themselves – is that mathematical work is creative in nature and often driven by its intrinsic aesthetic dimension (e.g., Brinkmann & Sriraman, 2009; Gadanidis, 2012; Sinclair, 2004).

The dividing gap between how mathematics is perceived by the general public and how it is viewed by its practitioners, has been acknowledged repeatedly in the mathematics education literature (e.g., Gadanidis, 2012; Movshovitz-Hadar, 2008). Though, unfortunately, it seems this phenomenon still prevails nowadays. As noted by Gadanidis (2012), we may find one of the explanations in the interrelationship between how mathematics and mathematicians are viewed; that is, people's negative images of *mathematicians* may cause (as well as be caused by) their negative views of *mathematics*. When it comes to images of mathematicians, as illustrated in Picker and Berry's (2000) study, "mathematicians are essentially invisible, with the result that pupils appear to rely on stereotypical images from the media to provide images of mathematicians when asked"

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(p. 88). Students in this study believed mathematicians to be authoritarian and threatening; do arithmetic computations, accounting, and banking; as well as possess supernatural powers in making mathematics look like magic – the latter explicitly related to the invisibility of the mathematical processes. Accordingly, there may exist great value in mathematicians engaging in communications with the general audience while sharing aspects of their work, as well as their enthusiasm and passion for the field of mathematics.

John Allen Paulos, the author of "A Numerate Life: A Mathematician Explores the Vagaries of Life, His Own and Probably Yours", is a mathematics professor at Temple University, Philadelphia, PA, USA. While his academic background is in the fields of mathematical logic and probability theory, a large portion of his publications, as stated by Paulos (2016) in his CV, "lie at the border between the scholarly and the popular". One of these publications is the book discussed here, in which Paulos shares autobiographical memories from his personal and professional life, as well as explores connections he sees between the field of mathematics and our lives as human beings in general. While the book contains many stories from the author's life, Paulos explicitly declares the book is not an autobiography. Taking similar freedoms myself, while occasionally sharing my personal views on the book, this paper is not intended to be written or read as a book review. Rather, I attempt to address and examine several key ideas from the book while contextualising them within the field of mathematics education.

Memorable Events from One's Mathematical Studies

Memories are often inaccurate or fabricated, perspectives biased, 'laws' and assumptions unfounded, contingencies unpredictable; even the very notion of a self is suspect. (Introduction section, para. 3)

So how about our *mathematical* memories? Most people – whether mathematicians, mathematics teachers, students, or the general public – can recall memories from their own experiences of studying mathematics; and memories, as Paulos claims, are unreliable. Indeed, research has shown that people may recall inaccurate details of an event, remember an exaggerated version of what happened in reality, or even claim to remember events that in reality never occurred as a result of suggestion and imagination

(e.g., Loftus, 1997; Mazzoni & Memon, 2003). Nonetheless, as Paulos suggests in his book, people often create a narrative of their lives and tend to remember details in a way that fits their chosen narrative. Though as Paulos also admits, "I'm sure I'm somewhat guilty of that here" (Chapter 9, para. 8).

One of the life stories Paulos shares in his book is how his interest in mathematics was galvanised by an intense dislike of his elementary-school mathematics teacher, which Paulos describes as a "bullying martinet" (Chapter 1, para. 12). In one particularly memorable event during his elementary-school years, Paulos, who was interested in baseball statistics, calculated that a certain pitcher had an earned run average (ERA) of 135. Apparently (at least to someone like me, who is not familiar with Baseball), this is an "extraordinarily bad ERA" (para. 15) – a fact that Paulos found impressive. However, after Paulos had shared his discovery with his classmates, the teacher shouted at him that he was confused and wrong, and subsequently "cackled derisively" (para. 15). Nonetheless, as Paulos then describes:

Later that season the Milwaukee Journal published the averages of all the Braves players, and since this pitcher hadn't pitched again, his ERA was 135, as I had calculated. I remember thinking then of mathematics as a kind of omnipotent protector. I was small and quiet and he [the teacher] was large and loud, but I was right and I could show him. [...] Perhaps not surprisingly, the story still evokes the same emotions in me that it did decades ago. [...] this particular teacher did give me a potent reason to study mathematics that I think is underrated. Show kids that with it and logic, a few facts, and a bit of psychology you can vanquish blowhards no matter your age or size. (para. 16-17)

Paulos's story illustrates a wider phenomenon. That is, even though memory may not always be trustworthy, it seems that many people have strong memories of events from their mathematics studies, which they perceive as meaningful to the continuation of their (mathematical) life.

In relation to Paulos's story, findings in mathematics education research indicate that one of the themes that repeat in people's accounts of their mathematics studies is the impact of the mathematics teacher on forming their attitudes and beliefs towards the subject (e.g., Di Martino & Zan, 2010; Ellsworth & Buss, 2000; Towers, Hall, Rapke, Martin, & Andrews, 2017). This has also become evident in the data of a current research project I am involved in, where participants are asked to share strong, memorable events from their experience of studying mathematics. While it is perhaps not surprising that a supportive teacher may have a positive impact on students and an unsupportive teacher may have a negative one, Paulos's story illustrates a more complex situation – where a student's motivation to do well stems from a desire to prove the "bullying" teacher wrong (see also Adams, 2013, for a study that points towards a similar phenomenon).

Furthermore, a sentence that attracted my attention in Paulos's description above is his claim that "the story still evokes the same emotions in me that it did decades ago" (Chapter 1, para. 16). Accordingly, the question whether such strong memorable events from one's mathematics studies are remembered as originally experienced, intensified when remembered, or altered in the process of recalling in any way, is actually of lower importance – as ultimately what people recall and remember indicates the significance they attribute to the event, and what they carry with them into their future self.

In research on mathematical affect, there is a distinction between emotions, attitudes, and beliefs, where these constructs represent a decreasing order of affective intensity and an increasing order of stability (McLeod, 1989). Moreover, it has been claimed that in order to influence and shape the stable beliefs and attitudes towards mathematics for the better, students should go through a lengthy and slow process of experiencing repeated positive emotions during their mathematical engagements (e.g., Goldin, 2000; Zan, Brown, Evans, & Hannula, 2006). However, more recent findings have indicated that even one strongly positive experience may be enough to catalyse a swift change in a student's attitudes and beliefs (Liljedahl, 2005; Marmur, 2019; Marmur & Koichu, 2016; Weber, 2008). Particularly, in previous works (Marmur, 2019; Marmur & Koichu, 2018, 2021), I discuss how a teacher's instructional choices and efforts can be directed towards the creation of collectively and meaningfully shared affective experiences in class (referred to as *key memorable events*), that have the potential to shape students' long-term mathematical affect. Though not examined empirically, Paulos's story above might

be an evocative illustration of how a teacher's actions may have a strong impact on mathematical affect in both the short- and long-term.

Perhaps it is likely that a mathematician like Paulos, with a clear mathematical tendency from childhood (as indicated by other stories in the book), would have become a mathematician regardless, even had he not experienced the event discussed above. However, how about the mathematical biographies of the general public? How can we, as mathematics educators, try to shape these trajectories towards feelings of enjoyment from and interest in the subject? Clearly, this question is too big to address here and is simply raised as room for thought. I will simply end this section with Maciejewski's (2017) provocation that "mathematics education literature written from a cognitive psychology perspective has maintained too narrow a focus on the mathematical content of mathematics learning and ought to consider a student's broader remembered experience" (p. 209).

Mathematicians' Biographies in Mathematics Education

True to my doubts, what I've written is a meta-memoir, even an anti-memoir. (Introduction section, para. 5)

In his book, Paulos shares many biographical details from his personal and professional life as a mathematician. However, not only does he declare the book to not be an autobiography/memoir, but he also spends significant effort (and space) in arguing against biographies in general. Throughout the book, Paulos provides various arguments for his approach, among them the unreliability of our memory to recall details accurately, confirmation biases that can influence biographers when interviewing their subjects, and the constantly changing social-historical context that shapes how people's biographies are being evaluated. Using mathematics to illustrate his points – an approach Paulos applies throughout the book – he offers the following statistics analogy to explain his issue with biographies:

A common problem in statistics is finding the curve or surface that best fits or approximates a relatively small number of data points in space. Without intending to do harm to our notion of the richness of human biographies, I note that there is a suggestive analogy to biography here. It is the similarity between the linking of data points with the curve or surface that best fits them and the telling of the story of someone's life by invoking a relatively small number of remembered events. We infer from the events what must have linked them. If he did this, that, and the other thing, then he must have had such-and-such a period in his life or have been such-and-such a type of person. The events suggest a life's trajectory, but the narrative constructed from them is usually just an inference structured by social norms, conventional biases, passing fads, and personal attitudes. Joe Smith is a person who did X1, X2, X3,...and Xn, so based on these data points we construct his biography. (Chapter 4, para. 6)

However, when examining the mathematics-education research literature, we find arguments in favour of incorporating mathematicians' biographies as part of the discussions held in the mathematics classroom. For example, Radford (2008) suggested that exposure to mathematicians' life stories, such as Georg Cantor and Évariste Galois, can bring forth the human and social aspects involved in creative mathematical thinking. Additionally, the incorporation of mathematicians' biographies into mathematics lessons can also play a motivational role to counter the common perception of mathematics as a "dry subject" (e.g., Kleiner, 1988; Radford, 2008). For example, Kleiner (1988) proposed that the story of Gerolamo Cardano (1501-1576) and the birth of complex numbers, not only illustrates how the meaning of what a number is has changed over the years, but that the drama and intrigue in the story may also evoke enthusiasm towards complex numbers in teachers and students alike.

Furthermore, biographies may be used to counter the widespread belief that mathematical proofs cannot be disputed, as highlighted by Furinghetti and Radford (2008). Indeed, the research work of Kleiner on the history of mathematics (e.g., Kleiner, 2012) illustrates the constantly changing nature of mathematics, and how even "proofs" by famous mathematicians can later be found to be incorrect. For instance, Cauchy, one of the founders of modern Calculus, "proved" that an infinite sum (a convergent series) of continuous functions is a continuous function (assuming that the continuity of functions carries over from finite to infinite sums). However, a counterexample was provided by Abel in the 1820s, showing Cauchy's argument was wrong. Consequently, by highlighting

that also mathematicians can make mistakes, students may become more accepting of their own mathematical errors and failures during their process of learning.

Another example, which I personally find inspirational, comes from the recently published book edited by Kaufholz-Soldat and Oswald (2020), "Against All Odds: Women's Ways to Mathematical Research Since 1800". As the title suggests, the book presents the stories of female mathematicians from various countries and cultures, who managed to overcome the odds to be able to enter what was (and still is) a predominantly male-dominated domain. As noted by Moreau, Mendick, and Epstein (2010), the general perception of mathematicians by the public is that mathematicians are white, heterosexual, middle-class men. Accordingly, being exposed to life-stories of mathematicians from more diverse backgrounds can serve to motivate female students, as well as students of different underrepresented communities, to pursue mathematics studies.

Returning to Paulos's book, I would like to note that while I may see or understand the reasoning behind his concerns about the truthfulness of biographies, I am not convinced by the conclusion he reaches as a result – being anti-biographies. Rather than taking a dichotomous approach, where one is either for or against the idea of mathematicians' biographies, the use of biographies in mathematics education can (and should) include a careful discussion on both the affordances and limitations they present.

According to Paulos, our human tendency to seek confirmation of our biases typically influences biographers to the extreme, where they end up either loving or hating their subjects:

We're all subject to confirmation bias, the tendency to look largely for confirmation of our hunches and beliefs and rarely for disconfirmation, but perhaps few more so than biographers, who are often either in thrall to their subjects or else detest them. (Chapter 2, para. 10)

However, there is room for a more nuanced approach, where the biographer's perspective could be acknowledged and made transparent as much as possible in the text, in order to

help the reader gain a more objective perspective through an explicit discussion of the biographer's biases. For example, in one of the chapters in the book "Against All Odds" mentioned above, Fajstrup, Gjerløff, and Kjeldsen (2020) discussed the strengths and weaknesses in their interviewing methodology, as stemming from the relation between their professional lives as three female academics in Denmark, who are writing about female mathematicians in Denmark of the twentieth century. In the context of mathematics education, the use of biographies requires the responsibility from educators (and students alike) – to be critical readers in search of the biographer's voice, and to acknowledge the existence of different interpretations of a mathematician's life through an examination of various biographical/historical sources.

Regarding Paulos's claim on inferring the subject's personality based on a small number of remembered events – again, I suggest there is room for a discussion of biographical details while acknowledging the medium's limitations. Examining Paulos's book, on the one hand I found biographical stories I thought were rather charming – such as how he developed his interest in mathematics in elementary school, how he met his wife in an anti-Vietnam War rally, as well as additional stories that illustrated his love and care for his family. On the other hand, I also read biographical details that I personally found to be less appealing and sometimes also problematic – such as extremely long descriptions of his fame being a best-seller author while namedropping a list of TV shows he was interviewed in; stories of his risky sexual encounters while travelling in Kenya, Tanzania, and Uganda, including with the wife of a local chieftain; and a description of his friend, Jim Rakocy, and his relation with prostitution:

His [Jim's] reveries about his experiences with sex workers were oddly thoughtful, empathetic, and introspective. Hyperbole to be sure, but the neologism *proustitution*² occurred to me. (Chapter 12, para. 4)

Based solely on these accounts, I do not wish to suggest I know who Paulos is as a person. However, I would like to argue that there is a responsibility that is inherent to writing

² Italicised in the original text.

(auto)biographical stories that claim to represent and promote a larger community of mathematicians and mathematics.

In a thought-provoking paper, Eisenberg (2008) raised the ethical dilemma of whether aspects of a mathematician's life, personality, and political beliefs should be included in mathematics lessons. As an illustration, Eisenberg discussed, amongst others, Kurt Gödel's clinical paranoias, which seem to have led to his death; Alan's Turing homosexuality for which he was convicted through England's Indecency Act as homosexuality was illegal at the time; and Ludwig Bieberbach, Oswald Teichmüller, and George David Birkhoff's anti-Semitic acts during the Nazi period. Ultimately, Eisenberg claims that "although there are many things I do not feel comfortable in discussing, they cannot be left unsaid" (p. 10). While perhaps not always mathematically relevant, there is an ethical, social, and historical responsibility to not separate one's mathematics from other aspects of their life or the cultural atmosphere of their time. Therefore, we, as a mathematics education community at large, can discuss the values such stories represent and their potential impact on the public image of mathematicians.

A Numerate Life for Whom?

[The mathematician Paul Erdős when asked: why are numbers beautiful?] It's like asking why Beethoven's Ninth Symphony is beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is. (Hoffman, 1998, p. 44)

Like Erdős above and many other mathematicians, it is clear also Paulos shares a similar passion for the field of mathematics. In addition to meta-comments about biographies and life-stories, in his book Paulos attempts to illustrate the many connections he sees between the mathematical domain and our human lives. Though the one I personally liked most was when the connection was not made literal, but rather more poetic – using a metaphor of an empty set to discuss the death of his father (in the last paragraph of the book):

Although neither my father nor anyone nor anything is in the empty set, the empty set itself is procreative. A well-known fact from set theory (I'll mercifully

omit the details) is that from the empty set, one can construct or generate all the whole numbers, all the real numbers, in fact, all of mathematics. If we're made out of mathstuff in one sense or another, then maybe the procreativity of the empty set can be extended. Though it contains nothing at all, the empty set might be able to generate not only all of mathematics but my father's lop-sided Cheshire grin as well. (Chapter 12, para. 38)

While I enjoyed some of the connections suggested in the book, I must also admit I found many of them to be forceful in applying mathematical ideas to explain human nature and psychology. Moreover, considering the importance of communications between mathematicians and the general public – as suggested at the beginning of this paper – while reading this book I found myself reflecting not only on the *existence* of such communications, but also on their *tone, characteristics*, and *educational nature*. For example, in one of Paulos's stories, which I personally found worrisome from an education perspective, he discusses his short-lived experience of teaching mathematics to future nurses:

The people in my class – mostly women, a couple of men – seemed temperamentally suited to the profession, appearing to be compassionate, empathetic, and caring. Unfortunately, most of them were quite innumerate. Try as I did to get across the rudiments of percentages, numerical prefixes, proportions, simple calculations, and unit conversions, many never managed to distinguish 2 percent from .02 percent nor could they, despite my pleading, believe it was even very important to do so. The following semester I was scheduled to teach the course again and told the program supervisor that this level of mathematical naïveté was dangerous and that I wouldn't want to be attended to by many of the soon-to-be nurses were I to be hospitalized. (Chapter 8, para. 24)

Taking the freedom to insert here a short instance of a meta-review (inspired by Paulos's declaration of writing a meta-memoir), the following claim is taken from a previous review of Paulos's book:

Paulos can also come off as superior or self-congratulatory. The author frames himself as an ambassador for mathematics, but it's hard to believe he draws people in with his judgmental tone. (Kolderup, 2015, p. 101)

There seems to exist an elitist perception of mathematics that, unfortunately, some mathematicians endorse and promote. Many mathematicians see mathematics as creative, beautiful, and "all-encompassing", and accordingly, their lives may indeed be described, in some sense, as "a numerate life". Returning to Erdős's quote above, while he *knows* numbers are beautiful, he asserts that this appreciation cannot be shared with or explained to those who are not already in the know: "If you don't see why, someone can't tell you" (Hoffman, 1998, p. 44). Another well-known example comes from the mathematician Poincaré (1910), who claimed that aesthetic sensibility for mathematics is innate and does not belong to non-mathematicians. Alas, it is not uncommon to hear such declarations from mathematicians. However, what is the mathematicians' responsibility in sharing this perception with and educating the general population?

Fortunately, there are also mathematicians who hold a more inclusive view on mathematics education. For example, Papert (1980) presented ideas opposing Poincaré's perspective, and suggested that an aesthetic sensibility to and pleasure in mathematics should, and can be part of a general educational vision. I believe this goal should be shared by mathematicians and mathematics educators alike. As articulated by Jaworski (2008, p. 8; summarising the work of Krainer from the same book): "As long as mathematics remains unknown, elitist, separatist, feared by significant members of society and education, it cannot become accessible to all, nor bring its powerful potential for communication to fruition."

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