23rd Australasian Fluid Mechanics Conference - 23AFMC
Sydney, Australia
4 – 8 December 2022

Paper No: AFMC2022-102

Scaling of a Turbulent Natural Convection Boundary Layer Immersed in a Stably Stratified Medium

K.R. Maryada[1*], S.W. Armfield[2], P. Dhopade[1] and S.E. Norris[1]

[1]Department of Mechanical Engineering, The University of Auckland, Auckland, Auckland, 1010, New Zealand
[2]School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, New South Wales, 2006, Australia
*mailto: kmar699@aucklanduni.ac.nz

Abstract

Appropriate scales are proposed for the mean flow of a turbulent vertical natural convection boundary layer immersed in a stably stratified ambient medium. In the inner layer (near-wall flow), where viscous diffusion is dominant, the mean streamwise velocity field scales with the friction velocity, the outer Richardson number, viscous length scale and the friction Reynolds number. In the outer layer, the mean streamwise velocity field scales with the friction velocity, outer Richardson number and the boundary layer thickness. The mean potential temperature field in the inner layer scales with the viscous length scale, friction temperature and the Prandtl number of the fluid, while in the outer layer, friction temperature, boundary layer thickness and a non-dimensional stratification parameter are found to be the appropriate scales for the mean potential temperature field. Numerical simulations at several Reynolds numbers ($800 \leq Re \leq 1400$) for a Prandtl number of 0.71 demonstrate an excellent collapse of the mean streamwise velocity and potential temperature fields with the proposed scaling.

1 Introduction

Natural convection boundary layers immersed in a stably stratified medium are ubiquitous in several natural and industrial flows, and through the years, several studies have been dedicated to investigating such boundary layers.

Some of the early studies on natural convection boundary layers immersed in stably stratified media focused on the laminar flow (Gill & Davey 1969, Armfield et al. 2007, Lin et al. 2008). The steady laminar flow was shown to feature regions of temperature deficit and flow reversal due to ambient stable stratification (Gill & Davey 1969). For an evenly heated plate, it was shown that the flow is independent of the streamwise coordinate after the initial flow development during the start-up stage. The effect of the Prandtl number on the flow was quantified, and appropriate scaling laws were developed for different regions of the natural convection boundary layer (Armfield et al. 2007, Lin et al. 2008).

For a sufficiently high Reynolds/Grashof number, the initially steady laminar flow bifurcates and transitions into a turbulent state. Considerably fewer studies have been dedicated to investigating the flow dynamics of turbulent natural convection boundary layers immersed in stably stratified media, with most of the studies dealing with the flow's mean flow behaviour and one-point statistics at different Reynolds/Grashof numbers. It was shown that the turbulent mean flow exhibits a high level of qualitative similarities with the laminar flow (Fedorovich & Shapiro 2009). Like its laminar counterpart, the turbulent mean streamwise velocity field exhibits a velocity peak close to the wall and a region of flow reversal away from it. A localised region of potential temperature deficit was also

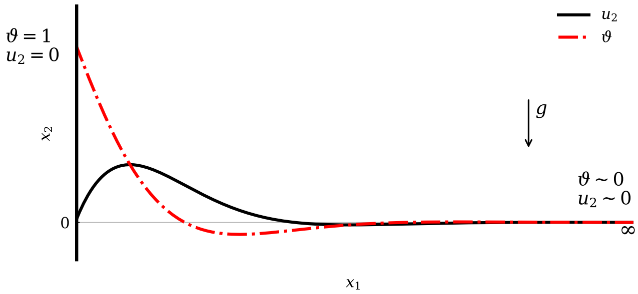


Figure 1. Geometric representation of the vertical buoyancy layer.

observed. A layered structure based on the turbulent kinetic energy for a natural convection boundary layer immersed in a stably stratified environment was proposed (Giometto et al. 2017). However, to date, there has not been any research concerning the scaling of turbulent natural convection boundary layers immersed in stably stratified media, with most of the scaling analyses being focused on unstratified turbulent natural convection boundary layers (e.g. George & Capp 1979, Shiri and George 2008, Wei 2020, Wei et al. 2021).

This paper proposes appropriate scales for the mean streamwise velocity and the mean potential temperature fields of a turbulent natural convection boundary layer immersed in a stably stratified ambient environment.

2 Governing Equations and Numerical Methodology

We consider a natural convection boundary layer developed over a linearly-heated vertical surface immersed in a stably stratified environment. If the rate at which the vertical surface is heated is equal to the vertical temperature gradient, then an equilibrium boundary layer can develop on the heated surface whose boundary layer thickness is constant in the downstream direction. This natural convection boundary layer is referred to as the buoyancy layer and is used to model the turbulent natural convection boundary layer immersed in a stably stratified medium. A schematic representation of the vertical buoyancy layer is shown in figure 1.

The flow was modelled using the following non-dimensional incompressible Navier–Stokes equations,

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1a}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} + \frac{2}{Re}\vartheta, \tag{1b}$$

$$\frac{\partial \vartheta}{\partial t} + u_j \frac{\partial \vartheta}{\partial x_j} = \frac{1}{RePr}\frac{\partial^2 \vartheta}{\partial x_j^2} - \frac{2}{RePr} u_2, \tag{1c}$$

where p is the pressure field, u_i is the velocity field, and ϑ is the potential temperature field, which is the temperature difference between the heated surface and the ambient surrounding with respect to the vertical temperature gradient (Gill & Davey 1969). The length is non-dimensionalised by $\delta_l = (4\nu\alpha/g\beta\gamma_s)^{0.25}$ and the velocity is non-dimensionalised by $\Delta T\,(g\beta\alpha/\nu\gamma_s)^{0.5}$. The temperature difference between the vertical surface and the ambient surroundings is represented using ΔT. Here, $\nu, \alpha, \beta, \gamma_s, g$ are the kinematic viscosity, thermal diffusivity, coefficient of thermal expansion, stable vertical temperature gradient and acceleration due to gravity, respectively.

The Reynolds number Re is $U_{\Delta T}\delta_l/\nu = (g\beta\Delta T\delta_l^3)/2\nu^2$, which is half the Grahsof number. The Prandtl number of the fluid is represented using $Pr = \nu/\alpha$, which is equal to 0.71.

An in-house collocated finite volume code was used to perform direct numerical simulation (Norris 2000, Armfield et al. 2003). A second-order Adams–Bashfort scheme and a second-order Crank–Nicolson scheme were used to solve the advection and the diffusion terms, respectively. A second-order central difference scheme was used to discretise the spatial terms. The code has been previously used to simulate natural convection flows, and its verification and validation are well-documented (Armfield et al. 2003, Maryada and Norris 2021). Periodic boundary conditions were imposed in the streamwise and spanwise directions, a no-slip wall with $u_i = 0$ and $\vartheta = 1$ was imposed at the heated wall, and an open-type boundary condition where the fluid was allowed to enter and exit the domain was used at the wall-normal far-field boundary.

A uniform mesh was used in the streamwise and spanwise directions, while a semi-logarithmic mesh with a stretching factor of 1.01 was used in the wall-normal direction. The domain sizes, the number of cells (N) and the cell spacings (Δ) used for the numerical simulations are shown in table 1. The domain sizes are normalised by the boundary layer thickness (δ_{bl}), while the cell sizes are normalised by the viscous length scale (δ_ν). The boundary layer thickness δ_{bl} is defined as the wall-normal distance at which the mean streamwise velocity changes sign for the first time. The domain sizes used are comparable to the domain sizes of Giometto et al. (2017). In table 1, Δx^+_{1min} refers to the wall-normal cell spacing adjacent to the wall, while Δx^+_{1bl} refers to the cell spacing at the edge of the boundary layer. The friction Reynolds number Re_τ is defined as $u_\tau\delta_{bl}/\nu$, where u_τ is the friction velocity, δ_{bl} is the boundary layer thickness and ν is the kinematic viscosity.

Re	$\mathbf{Re_\tau}$	**Size** $(\mathbf{x_1 \times x_2 \times x_3})$	$\mathbf{N_{x1} \times N_{x2} \times N_{x3}}$	$\mathbf{\Delta x_2^+ \times \Delta x_3^+}$	$\mathbf{\Delta x^+_{1min}}$	$\mathbf{\Delta x^+_{bl}}$
800	279	$6.8\delta_{bl} \times 7.6\delta_{bl} \times 6.8\delta_{bl}$	$280 \times 450 \times 375$	4.74×4.98	0.43	4.67
1000	336	$7.0\delta_{bl} \times 8.6\delta_{bl} \times 8.6\delta_{bl}$	$320 \times 600 \times 600$	4.78×4.78	0.42	5.02
1200	394	$6.4\delta_{bl} \times 7.8\delta_{bl} \times 7.8\delta_{bl}$	$320 \times 600 \times 600$	5.19×5.19	0.46	5.92
1400	454	$7.0\delta_{bl} \times 9.7\delta_{bl} \times 9.0\delta_{bl}$	$355 \times 906 \times 816$	4.85×4.82	0.42	6.30

Table 1. Parameters of the numerical simulations.

It is well-known that large-scale streamwise-elongated structures are present in the outer layers of natural convection boundary layers (Ke et al. 2021). Therefore, an additional simulation was performed at $Re = 800$ by doubling the domain size in the streamwise direction to verify the domain used in the current study. It was found that an increase in the streamwise domain size had a negligible influence on the mean flow and one-point statistics, and therefore, the current domain was used for further analysis at different Re.

The results of the numerical simulations reported in table 1 are used to validate the scaling laws proposed in the following section.

3 Mean Flow Scaling of the turbulent vertical buoyancy layer

The turbulent vertical buoyancy layer is a one-dimensional zero pressure-gradient parallel flow. Accordingly, for a fully-developed turbulent flow, the Reynolds-averaged momentum and buoyancy transport equations for the vertical buoyancy layer can be written as (Fedorovich & Shapiro 2009),

$$\frac{\partial\langle u_1 u_2\rangle}{\partial x_1} = \nu\frac{\partial^2\overline{u_2}}{\partial x_1^2} + g\beta\overline{\vartheta}, \tag{2a}$$

$$\frac{\partial\langle u_1 \vartheta\rangle}{\partial x_1} = \alpha\frac{\partial^2\overline{\vartheta}}{\partial x_1^2} - \gamma_s\overline{u_2}, \tag{2b}$$

where $\langle\cdot\rangle$ refers to streamwise, spanwise and temporal averaging and $\bar{\cdot}$ refers to the mean flow. The mean streamwise velocity and mean potential temperature fields are represented using $\overline{u_2}$ and $\overline{\vartheta}$, $\langle u_1 u_2\rangle$ is the mean Reynolds shear stress, γ_s is the mean vertical temperature gradient (stratification parameter) and $\langle u_1 \vartheta\rangle$ is the mean wall-normal turbulent heat flux.

The non-dimensional form of the Navier–Stokes equations is no longer used for the scaling analysis presented in this section, as the analysis is independent of the nondimensionalisation used.

3.1 Scaling of the Mean Streamwise Velocity Field

First, appropriate scales are developed for the mean streamwise velocity field in the outer layer. To this end, let us scale equation (2a) with the following differential scaling (Wei et al. 2021),

$$\partial x_1 \equiv l_c \partial x_1^{\times}, \quad \partial \overline{u_2} \equiv u_c \partial \overline{u_2}^{\times}, \quad \partial \langle u_1 u_2\rangle \equiv R_c \partial \langle u_1 u_2\rangle^{\times}, \quad \overline{\vartheta} \equiv \vartheta_c \overline{\vartheta}^{\times}, \tag{3}$$

where variables with $^{\times}$ refer to the outer-scaled variables, l_c, u_c, R_c and ϑ_c are the appropriate length, velocity, Reynolds shear stress and potential temperature scales.

In the outer layer, the viscous effects are negligible, and the vertical buoyancy layer can be approximated as a balance between the gradient of the Reynolds shear stress and buoyancy. Therefore, for an admissible scaling in the outer layer, the prefactors of the gradient of the Reynolds shear stress and buoyancy must be $O(1)$ and greater than the prefactor of viscous diffusion (Wei et al. 2021).

In the outer layer, as the effect of the wall is mimimal, it is reasonable to assume that the eddies have length scales on the order of the boundary layer thickness and, therefore, δ_{bl} can be used for the length scale l_c. As a starting point, if the velocity u_c, potential temperature ϑ_c and the Reynolds shear stress R_c scales are assumed to be the friction velocity ($u_\tau = \sqrt{\nu \partial \overline{u_2}/\partial x_1}$), friction temperature ($\theta_\tau = (-\alpha \partial \overline{\vartheta}/\partial x_1)/u_\tau$) and the square of friction velocity (u_τ^2), then, equation (2a) with the differential scaling in equation (3), after some manipulation to normalise the prefactor of the Reynolds shear stress stress term, can be written as,

$$\frac{\partial \langle u_1 u_2\rangle^{\times}}{\partial x_1^{\times}} = \frac{1}{Re_\tau} \frac{\partial^2 \overline{u_2}^{\times}}{\partial x_1^{2\times}} + Ri_o \overline{\vartheta}, \tag{4}$$

where $1/Re_\tau = \nu/\delta_{bl} u_\tau$ and $Ri_o = g\beta\theta_\tau \delta_{bl}/u_\tau^2$. It should be noted that Re_τ is the friction Reynolds number and Ri_o is the outer Richardson number, and $1/Re_\tau \approx 0$ for $Re_\tau \gg 1$. With this scaling, the prefactor to the buoyancy force is the outer Richardson number and not $O(1)$, invalidating our initial assumption regarding u_c, R_c and ϑ_c. As Ri_o appears in equation (4), it can be used as one of the scaling variables to normalise the prefactor of buoyancy.

If we assume,

$$u_c = u_\tau Ri_o, \quad \vartheta_c = \theta_\tau, \quad R_c = u_\tau^2 Ri_o, \tag{5}$$

equation (2a) can be written as,

$$\frac{\partial \langle u_1 u_2\rangle^{\times}}{\partial x_1^{\times}} = \frac{1}{Re_\tau} \frac{\partial^2 \overline{u_2}^{\times}}{\partial x_1^{2\times}} + \overline{\vartheta}, \tag{6}$$

where the prefactors of the gradient of Reynolds shear stress and buoyancy are $O(1)$, and the prefactor of viscous diffusion is negligible for $Re_\tau \gg 1$. Therefore, the following scales are valid for the mean streamwise velocity in the outer layer,

$$x_1^{\times} \equiv x_1/\delta_{bl}, \quad \overline{u_2}^{\times} \equiv \overline{u_2}/u_\tau Ri_o. \tag{7}$$

It should be noted that including Ri_o in the proposed scaling can be thought of as introducing buoyancy force into the scaling variables. Using the Ri_o scaling implicitly incorporates the $Re-$ (or) $Gr-$dependence of natural convection boundary layers into the scales.

To develop appropriate scales for the mean streamwise velocity in the inner layer, let us now introduce the following differential scaling,

$$\partial x_1 \equiv l_c \partial x_1^*, \quad \partial \overline{u_2} \equiv u_c \partial \overline{u_2}^*, \quad \partial \langle u_1 u_2 \rangle \equiv R_c \partial \overline{u_2}^*, \quad \overline{\vartheta} \equiv \vartheta_c \overline{\vartheta}^*, \tag{8}$$

where variables with * refer to the inner-scaled variables, and l_c, u_c, R_c and ϑ_c now refer to the appropriate length, velocity, Reynolds shear stress and potential temperature scales.

Close to the wall, for an admissible scaling in the inner layer, the prefactor of the diffusion term is no longer negligible and should be $O(1)$. Based on the inner scaling of several canonical boundary layers flows, it would seem reasonable to use ν/u_τ as the length scale in the inner layer. However, Shiri and George (2008), while investigating the scaling of turbulent natural convection boundary layers in a differentially heated channel, argue that the buoyancy force in the outer layer influences the flow in the inner layer. This implies that the eddies that scale with δ_{bl} in the outer layer interact with the eddies that scale with ν/u_τ in the inner layer. This effect can be quantified using the friction Reynolds number $Re_\tau = u_\tau \delta_{bl}/\nu$.

Using the above assumptions, let,

$$u_c = u_\tau Ri_o, \quad l_c = \sqrt{Re_\tau}\nu/u_\tau, \quad \vartheta_c = \theta_\tau, \tag{9}$$

be the scaling variables for the inner layer. It should be noted that the velocity scale is identical to the outer layer velocity scale (see equation (7)). For convenience, $\sqrt{Re_\tau}$ is used instead of Re_τ in the length scale l_c.

Then, equation (2a) with the differential scaling in equation (9), after some manipulation to normalise the prefactor of the diffusion term, can be written as,

$$\frac{\sqrt{Re_\tau}R_c}{u_\tau^2 Ri_o}\frac{\partial \langle u_1 u_2 \rangle^*}{\partial x_1^*} = \frac{\partial^2 \overline{u_2}^*}{\partial x_1^{2*}} + \overline{\vartheta}. \tag{10}$$

Close to the wall, in the inner layer of the turbulent vertical buoyancy layer, there is a balance between viscous diffusion and buoyancy. Therefore, for an appropriately scaled equation, the prefactors of viscous diffusion and buoyancy must be $O(1)$, and this is true in equation (10). The gradient of the Reynolds shear stress is not a dominant term, and its prefactor must be $O(1)$ or lower (Wei 2020, Wei et al. 2021). R_c should be appropriately chosen for this to be true; however, it is not investigated as the scaling of Reynolds shear stress is outside the scope of this paper.

From equation (10), the valid scales for the streamwise velocity field in the inner layer are,

$$x_1^* \equiv x_1 u_\tau/\nu\sqrt{Re_\tau}, \quad \overline{u_2}^* \equiv \overline{u_2}/u_\tau Ri_o. \tag{11}$$

Figures 2(a) and (b) show the inner- and outer-scaled mean streamwise velocity at different Reynolds numbers in the inner and the outer layers, respectively. In figure 2(a), the mean streamwise velocity field at different Reynolds numbers collapses onto a single curve until the velocity maximum with the proposed scaling shown in equation (11). In figure 2(b), the proposed Ri_o scaling, shown in equation (7), also collapses the mean streamwise velocity onto a single curve in the outer layer. This validates the proposed scaling for the mean streamwise velocity field. Also, the figure clearly shows that the outer layer scaling is valid from the velocity maximum to the bulk flow, including the flow reversal.

It should be noted that the proposed Ri_o scaling for the mean streamwise velocity field is equivalent to the scaling based on the velocity maximum developed for unstratified natural convection boundary layers (Wei 2020, Wei et al. 2021). This is the case as the velocity maximum is directly proportional to the outer Richardson number.

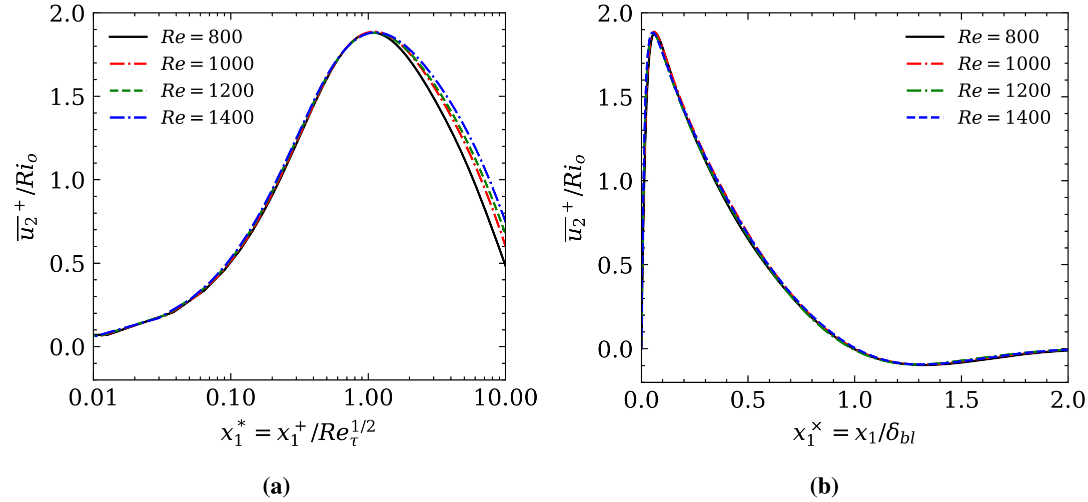


Figure 2. (a) Inner and (b) outer scaling of the mean streamwise velocity field.

3.2 Scaling of the Mean Potential Temperature Field

Let us introduce the following differential scaling to develop appropriate scales for the potential temperature field close to the wall (Wei et al. 2021),

$$\partial x_1 \equiv l_c \partial x_1^*, \quad \partial \overline{\vartheta} \equiv \vartheta_c \partial \overline{\vartheta}^*, \quad \partial \langle u_1 \vartheta \rangle \equiv H_c \partial \langle u_1 \vartheta \rangle^*, \quad \overline{u_2} \equiv u_c \overline{u_2}^*, \tag{12}$$

where the variables with * refer to the inner-scaled variables, and l_c, u_c, H_c and ϑ_c refer to the appropriate length, velocity, wall-normal turbulent heat flux and potential temperature scales.

Close to the wall, $\partial\langle u_1 \vartheta\rangle/\partial x_1 \to 0$ and $\overline{u_2} \to 0$, and only the diffusion term is dominant. This reduces equation (2b) to,

$$\frac{\alpha \vartheta_c}{l_c^2} \frac{\partial^2 \overline{\vartheta}^*}{\partial x_1^{2*}} \approx 0, \tag{13}$$

which is equivalent to appropriately scaled near-wall equation for an unstratified turbulent vertical natural convection boundary layer if $l_c = \alpha/u_\tau = \nu/u_\tau Pr$ and $\vartheta_c = \theta_\tau$ (George & Capp 1979, Wei 2020, Wei et al. 2021).

Therefore, the appropriate scales for the potential temperature field close to the wall are,

$$x_1^* \equiv x_1 u_\tau Pr/\nu, \quad \overline{\vartheta}^* \equiv \overline{\vartheta}/\theta_\tau. \tag{14}$$

Far away from the wall, in the outer layer, thermal diffusion has a negligible contribution to the flow. In this region, the gradient of the wall-normal turbulent heat flux and the stratification term are the dominant terms, and the boundary layer in this region can be approximated as a balance between these two terms. Therefore, for an admissible scaling, the prefactors of these terms can no longer be ignored and must be $O(1)$. To achieve this, let us introduce the following differential scaling,

$$\partial x_1 \equiv l_c \partial x_1^\times, \quad \partial \overline{\vartheta} \equiv \vartheta_c \partial \overline{\vartheta}^\times, \quad \partial \langle u_1 \vartheta \rangle \equiv H_c \partial \langle u_1 \vartheta \rangle^\times, \quad \overline{u_2} \equiv u_c \overline{u_2}^\times, \tag{15}$$

where the variables with $^\times$ refer to the outer-scaled variables, and l_c, u_c, H_c and ϑ_c refer to the appropriate length, velocity, wall-normal turbulent heat flux and potential temperature scales.

The length scale l_c in the outer layer is assumed to be δ_{bl}/Pr to incorporate the effect of the Prandtl number into the scaling variables. Further, let us assume that $\vartheta_c = \theta_\tau$, $u_c = u_\tau Ri_o$ and $H_c = u_\tau \theta_\tau$. With this scaling, in the outer layer, u_c is the same for the mean momentum and mean buoyancy equations.

Then, substituting these scales into equation (2b), after some manipulation to normalise the prefactor of the gradient of the wall-normal turbulent heat flux term, results in the following equation,

$$\frac{\partial \langle u_1 \vartheta \rangle^{\times}}{\partial x_1^{\times}} = \frac{1}{Re_\tau} \frac{\partial^2 \overline{\vartheta}^{\times}}{\partial x_1^{2\times}} - \sigma \overline{u_2}^{\times}, \tag{16}$$

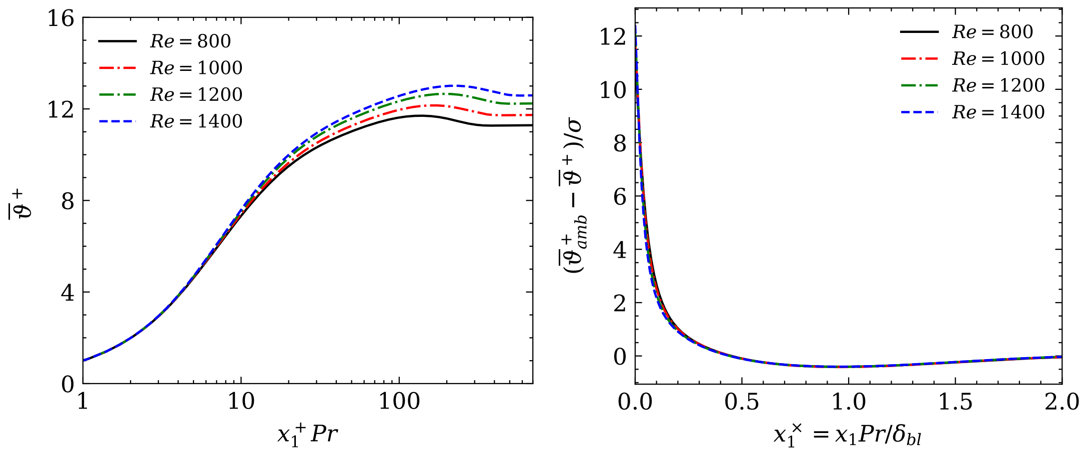
where $Re_\tau = \delta_b l u_\tau / \alpha Pr$ and $1/Re_\tau \approx 0$ for $Re_\tau \gg 1$. The nondimensional stratification parameter is represented using $\sigma = \gamma_s \delta_{bl} Ri_o / \theta_\tau Pr$. Equation (16) only reinforces the fact that thermal diffusion is negligible in the outer layers of turbulent natural convection boundary layers (Ke et al. 2021, Wei et al. 2021).

The prefactors of the stratification and the gradient of the wall-normal turbulent heat flux are not $O(1)$ in equation (16), and therefore the assumed scales are not accurate. As equation (16) includes the non-dimensional parameter σ, it can be used as a valid scaling variable. One way of normalising the prefactor of the stratification term to $O(1)$ is by assuming $\vartheta_c = \vartheta_c/\sigma$ and $H_c = u_\tau \theta_\tau / \sigma$. With this correction, the following scales are proposed for the potential temperature field in the outer layer,

$$x_1^{\times} \equiv x_1 Pr / \delta_{bl}, \quad \overline{\vartheta}^{\times} \equiv \left(\overline{\vartheta}_{amb} - \overline{\vartheta} \right) / \theta_\tau \sigma, \tag{17}$$

where $\overline{\vartheta}_{amb}$ is the potential temperature of the ambient medium. It should be noted that a deficit law is proposed for the mean potential temperature field in the outer layer, similar to the deficit law often used for unstratified turbulent natural convection boundary layers (Wei 2020, Wei et al. 2021).

Figures 3(a) and (b) show the mean potential temperature field using the proposed inner and the outer scaling for the inner (near-wall) and outer layers, respectively. In figure 3(a), the mean potential temperature field collapses onto a single curve at several Re until $x_1^{\times} \leq 10$, demonstrating the validity of the proposed scaling close to the wall. The mean potential temperature field also collapses onto a single curve in the outer layer ($0.2 \leq x_1^{\times} \leq 2.0$) with the proposed deficit-law scaling. This validates the proposed scaling of the mean potential temperature field for the inner and the outer layers.



(a) **(b)**

Figure 3. (a) Inner and (b) outer scaling of the mean potential temperature field.

4 Conclusions

This paper proposes appropriate scales for the mean streamwise velocity and the mean potential temperature of a turbulent vertical natural convection boundary layer immersed in a stably stratified environment. At $Pr = 0.71$, for the Reynolds numbers ($800 \leq Re \leq 1400$) investigated, data from numerical simulations demonstrate an excellent collapse of the mean streamwise velocity and the mean potential temperature field when scaled with the proposed scaling parameters.

The mean streamwise velocity field can be scaled using the friction velocity and the outer Richardson number in the inner (near-wall) and the outer layers. In the outer layer, the length scale for the mean streamwise velocity is the boundary layer thickness, defined as the wall-normal location where the mean streamwise velocity changes sign for the first time. In the inner layer, an appropriate length scale is the viscous length scale and the friction Reynolds number.

Close to the wall, the mean potential temperature field can be scaled with the friction temperature, the viscous length scale and the Prandtl number of the fluid. Away from the wall, in the outer layer, the mean potential temperature field scales with the boundary layer thickness, friction temperature and a nondimensional parameter σ, which is a function of the mean vertical temperature gradient (stable stratification) and the Prandtl number.

Acknowledgements

We thank New Zealand eScience Infrastructure (NeSI) for providing high-performance computing resources.

References

Armfield, S., Morgan, P., Norris, S. and Street, R., A parallel non-staggered Navier–Stokes solver implemented on a workstation cluster. In *Computational Fluid Dynamics 2002*, 2003, 30–45.

Armfield, S. W., Patterson, J. C. and Lin, W., Scaling investigation of the natural convection boundary layer on an evenly heated plate. *International Journal of Heat and Mass Transfer*, **50**, 2007, 1592–1602.

Fedorovich, E. and Shapiro, A., Turbulent natural convection along a vertical plate immersed in a stably stratified fluid. *Journal of Fluid Mechanics*, **636**, 2009, 41–57.

George Jr, W. K. and Capp, S. P., A theory for natural convection turbulent boundary layers next to heated vertical surfaces. *International Journal of Heat and Mass Transfer*, **22**, 1979, 813–826.

Gill, A.E. and Davey, A., Instabilities of a buoyancy-driven system, *Journal of Fluid Mechanics*, **35**, 1969, 775–798.

Giometto, M. G., Katul, G. G., Fang, J., and Parlange, M. B., Direct numerical simulation of turbulent slope flows up to Grashof number $Gr = 2.1 \times 10^{11}$. *Journal of Fluid Mechanics*, **829**, 2017, 589–620.

Ke, J., Williamson, N., Armfield, S. W., Komiya, A. and Norris, S. E., High Grashof number turbulent natural convection on an infinite vertical wall. *Journal of Fluid Mechanics*, **929**, 2021, A15.

Lin, W., Armfield, S. W. and Patterson, J. C., Unsteady natural convection boundary-layer flow of a linearly-stratified fluid with $Pr < 1$ on an evenly heated semi-infinite vertical plate. *International Journal of Heat and Mass Transfer*, **51**, 2008, 327–343.

Maryada, K. R. and Norris, S. E., Onset of low-frequency shear-driven instability in differentially heated cavities, in *Proceedings of CHT-21 ICHMT International Symposium on Advances in Computational Heat Transfer*, 2021, 107–118.

Norris, S. E., *A Parallel Navier–Stokes Solver for Natural Convection and Free Surface Flow*, PhD Thesis, The University of Sydney, 2000.

Shiri, A. and George, W. K., Turbulent natural convection in a differentially heated vertical channel. In *Proceedings of ASME Heat Transfer Summer Conference*, 2008, 285–291.

Wei, T., Analyses of buoyancy-driven convection. In *Advances in Heat Transfer*, **52**, 2020 ,1–93.

Wei, T., Wang, Y. and Abraham, J., Layered structure of turbulent natural convection over a vertical flat plate. *International Journal of Heat and Mass Transfer*, **181**, 2021, 121866.