

Temporal Evolution of Mixing Regimes in Diurnally Heated Stratified Open Channel Flows

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1 Introduction

Diurnal solar radiation influences the stability of thermal stratification in shallow lakes, rivers, ponds and many other limnological water bodies. Because of the temporal variation in surface heat flux, the water bodies experience a wide range of flow states within their diurnal cycle (Yang et al. 2018). In this study, we carry out three-dimensional direct numerical simulations (DNS) of stably stratified open channel flow as a canonical representation of shallow lakes, rivers and ponds, all subject to a diurnal short-wave radiation. This study is based on the work of Williamson et al. (2015) and aims to examine the local flow states of the open channel and their variation across the diel cycle as the flow undergoes period of cyclic heating and adiabatic -destratification.

2 Approach

We model the open channel with a free-slip and adiabatic upper surface and a no-slip, adiabatic lower wall. The channel is periodic in the horizontal planes with a constant uniform pressure gradient applied to the stream-wise direction. A progressive absorption of the surface solar radiation is applied to the surface of the channel and is defined by a volumetric depth-varying heat source,

$$Q(Y, T) = I_s(T)\alpha e^{(Y-\delta)\alpha}, \quad (1)$$

where δ is the height of the channel, α is the absorption coefficient and I_s is the short-wave heat flux which varies with time. Wallace & Hamilton (1999) modelled the diurnality of I_s through a piecewise function described by:

$$I_s(T) = \begin{cases} I_m \sin^3\left(\frac{\pi(T-T_r)}{T_m}\right), & T_r < T < T_s \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where I_m is the maximum irradiance, T_r the rise time, T_s the set time, and T_m the length of the light period. The diurnal timescale \hat{t} is non-dimensionalised by the channel depth, diel cycle, and friction velocity at the bottom of the channel:

$$\hat{t} = \frac{U_\tau D}{\delta}. \quad (3)$$

Stratification is characterised by the bulk stability parameter λ_B , defined through the ratio of a bulk Obukhov length scale \mathcal{L}_B and the confinement scale δ :

$$\lambda_B = \frac{\delta}{\mathcal{L}_B}, \quad \mathcal{L}_B = \frac{(\tau_b)^{\frac{3}{2}} C_p}{g\beta\delta\rho_o^{\frac{1}{2}} Q_b}, \quad (4a, b)$$



where g is the gravitational acceleration and β is the coefficient of thermal expansion which relates the fluid density ρ to the temperature by $d\rho/\rho_o = -\beta d\phi$. τ_b and Q_b are the average shear stress and average depth-varying heat source (equation 1) profile, over $y = 0.6\delta$ to δ .

3 Results and Conclusion

Simulations with parameters $Re_\tau = 400$, $Pr = 1$, and the non-dimensional turbidity parameter $\alpha\delta = 8$ were run along with different \hat{t} values. Results reveal the relationship between the irreversible flux Richardson number R_f^* and the turbulent Froude number Fr collapses well on to previous parameterization of stratified flows by Garanaik & Venayagamoorthy (2019). For strongly stratified flows! it demonstrated $Fr \ll 1$ with $R_f^* \propto Fr^0$, while moderately stratified flows $Fr \sim O(1)$ exhibit the relation $R_f^* \propto Fr^{-1}$ and for weakly stratified conditions $Fr \gg 1$; $R_f^* \propto Fr^{-2}$. At relatively high \hat{t} , the vertical positions along the water column from the free surface to the wall, move through a wider ranger of Fr values. This indicates that points along the water column experience different stratification regimes and thermal buoyancy influences extend further down the depth of the channel compared to those with lower \hat{t} values which tend to stay within one regime along its heat cycle.

We have also identified two critical layers to quantify the strength of stratification; a laminar layer and a stratified layer depth. We define the depth of the laminar region, referred to as the laminar layer depth (LLD), as the depth from the free surface when the buoyancy Reynolds number $Re_B = 7$. The turbulent Froude number $Fr = 1$ is used to define the stratified layer depth (SLD). The LLD and SLD under certain conditions demonstrate three distinct behaviours. It is observed that the LLD or the SLD can persist with their depths greater than zero throughout their diurnal cycle. This behaviour is referred to as persistently laminar if the LLD exhibits this characteristic (PL) or persistently stratified for the SLD (PS). The behaviour for when the LLD or SLD during its diurnal cycle breaks down to zero and reforms to depths greater than zero is termed diurnally laminar when it is observed in the LLD (DL) and diurnally stratified for the SLD (DS). The last behaviour observed is a constant zero depth of either the LLD (NL) or the SLD (NS). The transition from these three behaviours have been revealed to be: NL to DL $\hat{t} = 2.7 \times 10^{-10} Re_{\mathcal{L}}^{4.5} - 0.1$, DL to PL $\hat{t} = 1.56 \times 10^2 Re_{\mathcal{L}}^{-0.5} - 18$, NS to DS by $\lambda_B = 1.012$ and DS to PS by $\hat{t} = 3\lambda_B$. The results of the regime map and its transition equations cover for a wide range of Froude numbers and is tested for a range of day lengths.

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