Haptic Enumeration: Effects of Density and Distraction

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Abstract

Currently, studies on haptic enumeration have predominantly utilized vibratory stimuli applied to the fingertips or stimuli of varying shapes and sizes, with relatively few studies utilizing braille as a stimulus. Due to braille's unique characteristics, multiple stimuli can be perceived simultaneously on a single fingerpad. Therefore, experimental results utilizing braille as a stimulus may provide valuable insight into the study of haptic enumeration. In our study, we employed braille as stimuli to investigate the haptic enumeration of visually normal adults. Participants actively touched the raised dots on the braille in a distal to proximal direction while wearing sound-attenuating headphones and a blindfold. We examined the effects of four variables on haptic enumeration: the number of perceived dots (1-12), density (compression and dispersion), distractor (minimum (0) and maximum (6)), and hand and finger combination (homologous and non-homologous). Based on the experimental results, we drew the following conclusions and inferences: the perceived number showed a positive linear relationship with the actual number, and there was no evidence to support the bilinear fit model. However, based on the accuracy and confidence results, we speculated that haptic perception might have the subitizing ability for numbers one to three. Furthermore, based on this speculation, we deduced that participants used three enumeration modes during the enumeration process: groupitizing, counting, and estimation. Haptic enumeration appeared to be independent of non-numerical continuous magnitudes (density) in the current experimental paradigm. However, the interaction of density and the number of dots significantly affected the accuracy and confidence of haptic enumeration, probably due to the increased complexity of the stimuli in our experiment, which placed higher demands on cognitive abilities such as

attention and working memory. Regarding the use of two fingers for number perception, it was found that homologous fingers were more beneficial for haptic enumeration than adjacent fingers. Additionally, we observed that directed attention played a significant role in haptic enumeration.

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Table of Contents

List of Figures

Introduction

The British economist William Stanley Jevons observed through experiments that adults could easily handle smaller quantities (Jevons, 1871)). In his iconic "bean experiment," participants were instructed to examine a box containing beans briefly and then indicate the number of beans. Participants provided accurate responses when the number of beans in the box ranged from one to four. However, participants could not provide accurate answers when the number of beans exceeded five. According to the results of Jevons's experiment, the maximum number of beans participants can accurately provide is four when counting one by one is not possible. Jevons's bean experiment appears to reveal a phenomenon. Tracing back to ancient times, both the ancient Romans and the Mayans of Mesoamerica developed unique symbols to represent numerals equal to or larger than four. In ancient Rome, the numerals one, two, and three were written as I, II, and III, respectively. Interestingly, instead of simply repeating I for the number four, a new element, V, was introduced, resulting in the representation IV. Similarly, the Maya used horizontally arranged dots to symbolize the numerals one to four, while a horizontal line denoted the number five, rather than using five dots.

Both the bean experiment and the numeral representations of the Romans and the Mayans serve as illustrations of the challenges that arise in visual enumeration when humans are presented with quantities beyond the range of three or four. Enumeration is the ability to accurately report the number of items within a set, and the range of item numbers heavily influences its efficacy. To elaborate further, when the number falls within a smaller range, typically between one and four, humans employ subitizing, which is a rapid and accurate mode of enumeration. However, when the number exceeds this range, humans rely on counting or estimation modes to accomplish enumeration. In visual enumeration, numerous experiments have been conducted,yielding consistent findings. However, in contrast to the abundance of research in the visual domain, the haptic domain has been markedly deficient in studies on haptic enumeration, which refers to the ability to detect and discriminate between different numbers of objects through touch. This lack of research has resulted in a scarcity of consistent findings and methodological diversity, underscoring the need for increased attention and investment in this field.

Haptic, one of the five human senses, is crucial in conjunction with information from other senses, such as sight and hearing, to help us comprehend our surroundings and construct a comprehensive worldview. Humans rely heavily on haptic perception to investigate and differentiate the features of objects, particularly when visual cues are not available for verification. For instance, a visually impaired individual rely on haptic perception to ascertain the surrounding objects. Moreover, the progression of technology intertwines with the comprehensive investigation of haptic perception. For instance, haptic feedback devices, such as VR gloves, are capable of replicating the sensation of touching and manipulating objects in a virtual environment, which can boost the user's sense of presence and immersion. Multiple factors have the potential to influence haptic perception, encompassing the texture of objects, the number of objects, the presence of distractors, and the modality of touch. Given the insufficiency of haptic enumeration research, it is worthwhile to explore the extent to which these factors impact haptic enumeration and the specific mechanisms involved.

To understand haptic enumeration and its influencing factors, this study aims to investigate finger-based haptic enumeration through active touch. Specifically, we examine the impact of factors, numerosity, density, distractors, and hand and finger usage on haptic enumeration performance. Previous studies on haptic enumeration have primarily relied on vibrations or stimuli with varying shapes and sizes. However, there has been a relative dearth of research using braille as a stimulus for haptic enumeration. Hence, in this study, we employ braille as a stimulus to diversify the methodology in investigating haptic enumeration and to achieve density manipulation. Moreover, the distinctive properties of braille allow participants to perceive multiple stimuli simultaneously with a single fingerpad, thereby facilitating a more comprehensive comprehension of the mechanisms that underlie haptic enumeration.

In this section, we have synthesized relevant literature and experiments to discuss the physiological mechanisms of touch, braille and haptic perception and cognition. We also explored the effects of active and passive touch, as well as how attention and distractors influence haptic perception. In addition, we delved into the non-symbolic numerical processing system of numerical cognition, enumeration modes, and number sense, while examining the primary controversies within the field across various sensory modalities. Moreover, we review representative experimental studies on haptic enumeration and introduced comparable and insightful haptic enumeration experiments. Drawing upon the literature review and discussion, we then introduced our research topic, outlined the primary research objectives, and provided predictions regarding the potential experimental results that may arise from our study.

Haptic Physiology and Perception

How do our fingers perceive and process the sense of touch? Understanding the physiology of touch is crucial to comprehending haptic perception. Haptic physiology involves studying the various types of mechanoreceptors responsible for transmitting haptic information, as well as how this information is transmitted through neural pathways to the brain and subsequently processed. Through the study of haptic physiology, we can gain a better understanding of how our fingers recognize and interact with objects.

Four Low-Threshold Mechanoreceptors (LTMRs) in The Skin

The transmission and reception of haptic information are enabled by cutaneous mechanoreceptors and thermoreceptors beneath the skin, as well as kinesthetic mechanoreceptors in muscles, tendons, and joints (Lederman & Klatzky, 2009). The cutaneous and kinesthetic mechanoreceptors perform distinct haptic perceptual tasks in different combinations and proportions, which affect a person's ability to differentiate and comprehend objects (Lederman & Klatzky, 2009).

In areas of the body devoid of hair, such as the palms, fingertips, and soles, four distinct kinds of low-threshold cutaneous mechanoreceptors (LTMRs) are present (Abraira & Ginty, 2013). Haptic sensory discrimination, including pressure, vibration, sliding, and texture, is linked to the widespread distribution of these LTMRs on Aβ nerve fibers, which are large fibers with rapid conduction (McGlone et al., 2014). LTMRs consist of large and myelinated fibers with information conduction speeds between 35 and 70 m/s (Johnson et al., 2000). The rate of stimulus adaptation distinguishes two groups of LTMRs. The first group consists of the slowly adapting type I (SA 1) and type II (SA 2) neurons, which are responsible for continuous firing throughout sustained skin deformation (Abraira & Ginty, 2013; Johnson et al., 2000). The second group consists of rapidly adapting type I (RA) and type II (PC) neurons that only fire in response to dynamic tissue deformation (Johnson et al., 2000). SA1 and RA afferents end at dermal-epidermal margin receptors (Merkel and Meissner, respectively). SA1 and RA afferent nerves have substantially smaller receptor fields (RFs) than SA2 and PC afferent nerves (Johnson et al., 2000; McGlone et al., 2014). Consequently, the SA1 and RA fiber networks convey rather high spatial resolution neural pictures of skin surface events (Abraira & Ginty, 2013; Johnson et al., 2000). SA2 and PC afferents, on the other hand, terminate at deeper tissue receptors (Ruffini and Pacinian, respectively) and have bigger RFs (Johnson et al., 2000; McGlone et al., 2014). Therefore, the information they provide to the central nervous system (CNS) is more global (Johnson et al., 2000).

These four distinct LTMRs process different types of haptic inputs. SA1 receptors are sensitive to the spatial features of haptic inputs and are responsible for perception of structure and texture (Johnson et al., 2000). RA receptors are important for detecting tiny, low-frequency skin surface motions and providing signals necessary for grip control (Abraira & Ginty, 2013). PC receptors are primarily responsible for detecting and perceiving haptic stimuli at a particular distance via vibrations conveyed by hand-held items, probes, and tools (Johnson et al., 2000). SA2 receptors provide information about mechanical deformation of the skin and recognize forces operating on the hand, especially deep tissue strain from skin stretching and morphological changes in the hand and fingers (Abraira & Ginty, 2013; Johnson et al., 2000).

Haptic Information Transmission and Somatosensory Cortex

Once LTMRs receive haptic information, it needs to be transmitted to the central nervous system (CNS) for processing. How is this information delivered to the CNS? Abraira and Ginty (2013) demonstrated one direct ascending pathway and two indirect ascending pathways for information transmission in LTMRs.

The direct pathway is crucial for processing fine touch information. A substantial number of Aβ-LTMRs project haptic information from the dorsal column to the brainstem's dorsal column nucleus (DCN), then to the thalamus via the medial lemniscus route, and lastly to the somatosensory cortex via third-order thalamocortical neurons. In one indirect pathway, the postsynaptic dorsal column pathway (PSDC) receives indirect or direct input from nearly all LTMRs on the quality of touch (Abraira & Ginty, 2013). In another indirect pathway, the spinocervical tract pathway (SCT) does not receive input from the SAI-LTMRs (Abraira & Ginty, 2013). Since SAI-LTMRs play a crucial role in the discrimination of fine textural structures on the surface of objects, the PSDC is the primary and most likely only upstream pathway that transmits information from SAI-LTMRs, and is responsible for processing haptic information related to surface structures, playing a crucial role in haptic discrimination (Abraira & Ginty, 2013). Moreover, investigations on dorsal column injuries have revealed that dorsal column injury impairs an individual's ability to distinguish the texture, size, and shape of items (Abraira & Ginty, 2013). Thus, the DC and PSDC pathways may differentiate haptic inputs requiring sequential or spatiotemporal processing (Abraira & Ginty, 2013). However, there is scant evidence that lesions of the dorsal lateral funiculus alter touch discrimination, indicating that SCT has minimal influence on haptic discrimination (Abraira

& Ginty, 2013). Thus, when the LTMRs receive haptic information, the dorsal column can first represent, integrate, and process the LTMRs' information.

The information from the LTMRs eventually arrives at the somatosensory cortex. The somatosensory cortex, located in the brain's parietal lobe, receives input from the thalamus (Ten Donkelaar, 2011). Its primary function is to process information transmitted by low-threshold mechanoreceptors (LTMRs) to create a sensory perception of touch (ten Donkelaar, 2011). The somatosensory cortex comprises two major subdivisions: the primary somatosensory cortex (S1) and the secondary somatosensory cortex (S2) (Kaas, 2004). Haptic information is initially processed by the primary somatosensory cortex, followed by the secondary and posterior parietal cortex (Saito et al., 2021). It is widely believed that haptic information processing is more complex in S2 than in S1.

In summary, the LTMRs, ascending and descending pathways, and the somatosensory cortex all play critical roles in the processing and perception of haptic information.

Haptic Perception in Braille Reading

People who are unable to read print material benefit significantly from braille literacy (i.e., its writing and reading). Initially, blind individuals utilized braille by inscribing the Roman alphabet used by sighted readers in relief. However, this technique proved challenging for people who were born blind without any prior knowledge of the alphabet (Jimenez et al., 2009). Haptic symbols made of raised dots were subsequently found to lead to more accurate perception than characters made of straight and curved lines (Jimenez et al., 2009). This raised-dot approach not only exploits SA-driven haptic acuity but also allows for constructing an alphabet using various permutations inside a standardized rectangular configuration (Jimenez et al., 2009). Each dot's size and spacing can be accommodated by the fingerpads precisely, and the raised dots are suitable for haptic demands and readily identifiable by the fingerpads (Jimenez et al., 2009). Furthermore, the unique structural arrangement of braille is highly suitable for individuals who are blind. For instance, Fischer-Baum and Englebretson (2016) demonstrated that individuals with visual impairments who read braille benefit from the structure of the braille system. The benefits of the braille system arise from its specific components. First is sublexical units. The sublexical units of the braille system pertain to the representation or form of individual braille cells. It is noteworthy that adult readers of English braille demonstrate sensitivity towards these sublexical units (Fischer-Baum & Englebretson, 2016). Building upon the research conducted by Fischer-Baum and Englebretson (2016), braille readers actively utilize the sublexical structure, which holds significance in word recognition. Hence, each form of a raised dot possesses the capacity to constitute a distinct sublexical unit and can be organized to create a comprehensive sublexical structure that braille readers can process. Moreover, the advantages of the braille system are further influenced by the patterns of letter frequency, which reflect the frequency of occurrence of each letter in written text (Fischer-Baum & Englebretson, 2016). It has been observed that higher-frequency stem morphemes contribute to enhanced word recognition, as observed in print reading (Fischer-Baum & Englebretson, 2016). To illustrate, consider the word "careless," where both "care" and "less" serve as morphemes, with "care" functioning as the stem morpheme. Fischer-Baum and Englebretson (2016) asserted that braille readers possess sensitivity towards the morphological structure of words. Analyzing the morphological structure of a word involves breaking it down into its constituent morphemes. Each individual raised dot within the braille system can combine harmoniously to form diverse morphological structures. Each stem morpheme is represented by a specific arrangement of raised dots, which facilitates the proficiency of braille readers in recognizing stem morphemes and ultimately improves word recognition. Overall, the representation of raised dots and the structure arrangement of raised dots within the braille system enable individuals with visual impairments to enhance their reading experience by increasing both accuracy and efficiency.

Hughes et al. (2014) have found that the finger does not move at a steady speed during braille scanning. This variability in speed may be attributed to the discrete nature of braille characters and readers' need for accurate character discrimination (Hughes et al., 2014). Such discontinuity in velocities may impair perception, as the speed of fingertip scanning can affect texture perception (Weber et al., 2013). Both type I (SA1) and type II (RA) afferents may be affected by the speed of finger scanning during braille reading. However, their sensitivity and response properties are crucial for humans to extract braille information. SA1 and RA afferents are highly sensitive to spatial features, such as edge information, which is essential for discriminating different raised dots in braille (Cascio & Sathian, 2001). Moreover, Saal and Bensmaia (2014) have shown the critical role of SA1 and RA afferents in perceiving spatial detail and texture, which is likely involved in braille discrimination. Cascio and Sathian (2001) have also demonstrated that the sensitivity of these afferents remains unaffected by scanning speed, supporting their involvement in braille reading. Thus, the edge information of the braille dots and the SA1 and RA afferents play a crucial role in enabling humans to scan and read braille.

Haptic Exploration and Active Touch

Haptic perception should not only be seen as a means of sensory reception, but also for the effects of exploratory hand movements on the sense of touch (Wagner, 2016). The notion of differentiation between active and passive touch emerged in the 19th century (Wagner, 2016). Gibson (1962) stated in his work "Observation on Active Touch" that active touch should be distinguished from passive touch. Gibson argued that active touch and passive touch are different. Active touch is characterized by the purposeful and active exploration of the stimulus field, while passive touch is caused by an external agent, an unexpected or unconcerned haptic occurrence, and is based only on the sense of a stimulus to which the skin is exposed when at rest. Passive touch may become active touch if the organism responds to it for further selection or sensory information refinement (Lepora, 2016). Gibson asserted in 1962 and 1966 that active touch is superior to passive touch in object recognition tasks, which means that dynamic haptic stimuli are perceived more accurately than when they are passively applied. However, Gibson's viewpoint has been contested. Numerous studies have compared active and passive touch, with most concluding that the haptic perception abilities of active and passive touch are similar in most cases and involve the detection of minute surface irregularities, textures, scaling of textures, or raised patterns (Chapman, 1994). For example, Heller (1986) compared active and passive braille touch over a controlled touchng time and found that active touch outperformed all passive settings, with the static stimuli performing the poorest. Participants with no braille experience performed poorly while passively reading simple braille words, whereas active exploration improved word recognition (Heller, 1986). However, other investigations found no difference (Gmnwald, 1966; Vega-Bermudez et al., 1991) wh en comparing two touch modalities in relation to comparable pattern recognition tasks, including braille. In a recent master's thesis, researchers observed no significant difference between active and passive touch conditions when non-braille readers investigated the same or different tasks in braille using active and passive touch while controlling for the time participants were exposed to braille (Baciero de Lama, 2019). According to Chapman (1994), the manipulation of exploration duration is a critical component in understanding these discrepancies between experiments.

While tests of the differences between active and passive touch in the haptic domain yield mixed results, it is widely acknowledged that active touch surpasses passive touch in certain aspects (Chapman, 1994). Lederman and Klatzky (1987) conducted two experiments to examine the capacity of the hand to explore objects freely. They found that collecting information about an object's features through active hand exploration was adequate, ideal, and even indispensable (Lederman & Klatzky, 1987). They proposed that active hand movements act as "windows" into the procedural knowledge people possess when manually manipulating objects, and that the sensory capacities of the hand are vastly improved by its motor skills. (Lederman & Klatzky, 1987). Gibson (1962) noted that we only sense objects and surfaces through active touch when we actively investigate the external environment.

Numerous studies on human neuropsychology support that active touch improves object perception by engaging the sense of touch. For instance, Seminara et al. (2019) suggested that haptic recognition involves highly structured and purposeful movements, and that sensory neuron signals are received through active and intentional movements. Thus, touch is the junction of perception and action (Seminara et al., 2019). Binkofski et al. (2001)

demonstrated that specific parietal lobe lesions impair haptic sensory perception related to active exploratory movement. Even non-human animals show the advantages of active touch. For instance, Sinclair and Burton (1993) found that monkey somatosensory area II neurons did not respond to passively applied stimuli. However, they observed that graded changes in the surface texture of the contacting finger elicited graded firing changes in the task of active texture discrimination.

In active touch, the individual controls the exploration process, with their actively investigating finger controlling the speed and direction of movement to bring the most sensitive skin region into contact with the stimulus (Chapman, 1994). This exploratory approach may compensate for any drawbacks of active contact (Chapman, 1994). Attention plays a critical role in processing haptic information during active touch; specifically, attention can enhance the transmission of somatosensory signals to the brain, resulting in increased neural activity in the primary and secondary somatosensory cortex (Chapman, 1994). For example, Babadi et al. (2022) noted that active touch tasks requiring haptic discrimination involve attentional control and the participation of the somatosensory cortex, specifically SII. Therefore, individuals are better equipped to discriminate and detect haptic stimuli through active touch with attentional control.

Overall, compared to passive touch, active touch has its absolute advantage, as it allows individuals to control their hand's exploratory movements and strategies. Additionally, when attention is engaged, active touch can optimize touch performance.

Haptic Perception and Attention

Attention plays an important role in everyday haptic perception. For example, while people can estimate the pressure of a mobile phone in their jeans pocket or various contact points of clothing against their body, most of these stimuli remain unnoticed unless attention is consciously drawn to them. With the skin continuously receiving stimulation, directed attention helps sustain focus on a specific area or activity amidst distractions, allowing the brain selects one of multiple competing stimuli for proper processing (Sathian & Burton, 1991).

The concepts related to attention are of significant importance and merit highlighting. The attentional system comprises two subsystems: the endogenous attentional system and the exogenous attentional system, which depend on the presentation of stimuli. Endogenous attention refers to the voluntary, targeted allocation of attentional resources to specific events or spatial locations, based on internal goals, expectations, or prior knowledge (Spence, 2002). For example, we expect to feel haptic pressure when pressing a button. Exogenous attention, in contrast, denotes the automatic and reflexive allocation of attentional resources to salient or unexpected environmental stimuli, regardless of our goals or intentions (Spence, 2002; de Haan et al., 2008). For instance, when someone suddenly puts a piece of ice on our arm. Additionally, attention can be classified into two types based on the body's action response to a stimulus: overt and covert attention. Overt attention involves directing attention through apparent body movements, such as moving the eyes or head, to focus on particular objects or locations within the receptive field (Spence, 2002). Covert attention, on the other hand, is the capacity to direct attention to a specific object or location without making any movements and ignoring irrelevant information (Spence, 2002).

Directing attention to a particular spatial location can enhance the efficiency of perceptual processing for objects within that location (Sathian & Burton, 1991; Posner, 1978). For example, Posner (1978) developed a now familiar experimental paradigm to assess the effects of covert changes in spatial attention on vision. In this paradigm, participants are instructed to detect the target location (either left or right), and the location information related to the target is given to participants before the target appears. In 80% of the trials, the target location is correctly cued (valid), while in 20% of the trials, it is cued incorrectly (invalid). By comparing the reaction time and accuracy of valid and invalid trials, Posner (1978) found that participants performed best when the valid cue location corresponded to the target. Forster and Eimer (2005) suggested that accurate performance in this paradigm may result from prior knowledge about the target location, which can be used as a covert attention shift towards the intended target place. In other words, the valid cue directs endogenous attention towards the target location before the target is shown. As a result, participants expect the target to appear at that specific location, and when the target does appear, they have sufficient endogenous covert attention to respond accurately.

The paradigm of visually directed attention investigations has been adapted for haptic attention research. Butter et al. (1989) implemented Posner's paradigm with modifications to examine the impact of cue validity on haptic response speed. They categorized cues as valid, invalid, or neutral (no cues), and during the experiment, participants' hands were covered by a black curtain while a metal rod stimulated their index fingers. Participants' reaction time to the stimulus was recorded and analyzed, with results indicating that valid cues significantly affected haptic response speed, with participants responding the fastest in the valid cue trials. Whang, Button, and Shulman (1991) extended this line of research by exploring the relationship between directing attention and haptic perception. Their experiment featured a paradigm consisting of 80% valid and 20% invalid cues. Participants were presented with vibrotactile stimuli on four fingertips simultaneously and were asked to report which finger received a lower or higher amplitude of vibrotactile stimuli than the other fingers. The results revealed that directing attention prior to the appearance of the haptic target contributed to haptic perception, as participants had the highest accuracy in trials with valid cues.

Sathian and Burton (1991) investigated the impact of directed attention on haptic texture perception. During their experiment, participants' index and middle fingers of both hands were simultaneously presented with stimuli, and they were instructed to identify which finger received a different texture pattern. The cues used were 80% valid, indicating that the target appeared in the exact location as the cued finger, and the remaining 20% were invalid cues that stimulated different locations, such as the non-homologous fingers of the contralateral hands, the homologous fingers of the contralateral hands, and the non-homologous fingers of the ipsilateral hands. The study results indicated that valid cues enhance the ability to recognize textural differences haptically. However, when the stimuli had significant differences in texture features, the effect of directing attention was minimal. Similarly, Sinclair et al. (2000) used 80% valid cues to investigate the contribution of directed attention to feature detection. The study found that directing attention to haptic features improved discrimination, particularly in discriminating the probe's frequency (Sinclair et al., 2000). Sinclair et al. (2000) suggested that this may be due to an increased ability to process the feature being attended to, resulting in more accurate discrimination.

Experiments investigating attention in haptics have mainly focused on passive touch.

Nonetheless, Metzger et al. (2019) conducted a study to explore the effects of covert attention on discrimination performance during active touch for object shape and roughness. In the experiment, participants were asked to compare the shape or roughness of two objects without prior knowledge of the task. Metzger et al. (2019) induced directed attention by manipulating the degree to which participants anticipated the task. For instance, in a given trial, participants might expect a higher probability of judging the shape than the roughness task, thus directing their attention toward objects' shapes. The experiment's results suggested that if participants anticipate one task less than another, their discrimination threshold for the less anticipated task is significantly higher, indicating more difficulty in discriminating differences (Metzger et al., 2019). Therefore, the results of Metzger et al. (2019) study suggest that covert attention can influence the active touch of 3D objects.

These experiments demonstrate that intentionally directed attention to specific spatial locations or object features significantly affects haptic perception. The studies conducted by Sathian and Burton (1991) and Metzger et al. (2019) reveal that object features also play a crucial role in haptic perception. Moreover, Sathian and Burton (1991) found no significant performance differences between valid and invalid cueing trials that occurred on homologous fingers of both hands, and the effect of directed attention was not significant. Sathian and Burton (1991) suggested that haptic perception may be formed bilaterally, particularly with respect to the integration of homologous finger information. Overall, these experiments demonstrate that directed attention significantly contributes to haptic perception and that both object features and homologous/non-homologous finger location influence haptic perception.

Haptic Perception and Interference

When exploring an object with the hand, multiple fingers come into contact with the object's surface. The brain integrates haptic information from these different fingers to determine, for example, whether the two stimulation patterns are attributable to one or more objects. Similarly, a trained deaf-blind individual can use various fingers to explore a speaker's chin, lips, and neck, integrating the haptic information to understand communication. Geldard and Sherrick (1965) used a multi-contactor device simultaneously transmitting vibrational signals to ten distinct anatomical body regions. To enhance the discriminability of individual vibrators, they placed ten vibrators on widely separated locations of the participant's body. Geldard and Sherrick (1965) asked participants whether they perceived the vibratory patterns as identical or distinct. They found that the participants could simultaneously recognize the patterns presented on different skin locations. Experimental results from Craig (1985) also showed that the human body could combine haptic information from several touch points on the skin. The participants' left index and middle fingers were simultaneously presented with a half-spatial haptic pattern in the experiment. The participants were instructed to assemble the patterns perceived by their index and middle fingers and answer with the whole pattern. The results demonstrated that individuals could combine the two patterns and perform accurately, recognizing the patterns significantly better than chance (Craig, 1985).

Although humans are capable of processing and integrating touch information from various skin locations, haptic information is prone to different degrees of influence by various factors, such as the characteristics of stimuli and the combination of hand and fingers. In

Geldard and Sherrick's (1965) experiment participants were able to recognize patterns presented simultaneously on different skin locations, but their ability to discriminate between patterns was influenced by the characteristics of the stimuli. The stimuli in the experiments varied in complexity, which was defined by two factors: number and communality. Number refers to the number of vibrators used, while communality refers to the similarity of the patterns (Geldard & Sherrick, 1965). The results of the experiment showed that both number and communality affected the participants' error rates (Geldard & Sherrick, 1965). When the number of stimuli was constant, the commonality of the patterns was strongly correlated with the error rate (Geldard & Sherrick, 1965). Craig (1985) found that presenting a whole pattern to the middle or index finger alone resulted in more precise and faster assessments than combining half patterns provided to two fingers. He suggested that simultaneously providing haptic stimuli to two fingers of a hand had issues with attention deficits (Craig, 1985). Additionally, Craig (1985) examined the presentation of haptic stimuli to the homologous fingers of two hands, such as the index finger of the left and right hands. Craig (1985) found that performance on pattern recognition tasks, discrimination tasks, and tasks that required participants to combine pattern information from both fingers of opposing hands improved significantly. He concluded that attentional deficits were reduced when processing patterns simultaneously appeared on both hands' homologous fingers (Craig, 1985).

Tamè et al. (2011) confirmed the effect of hand and finger on haptic perception. They examined the haptic perception of healthy volunteers using double-simultaneous stimulation (DSS) on their fingertips. DSS refers to the competition between two simultaneous haptic stimuli. It has been discovered that this competition causes individuals with unilateral brain damage to lose their sense of touch (Tamè et al., 2011). Thus, the contralateral haptic experience cannot be registered when ipsilateral haptic stimuli occur concurrently (Tamè et al., 2011). However, even neurologically normal individuals can be impacted when two haptic stimuli occur in close proximity in time and space (Tamè et al., 2011). In their experiment. Tamè et al. (2011) presented participants with an image of two palms and a target finger before the start of the trial, and the participants' task was to indicate whether the target finger was stimulated. The study found that when two haptic stimuli were presented simultaneously, whether in the same hand or different hands, double simultaneous stimulation (DSS) interference occurred, compared to trials with only one stimulus. The DSS interference significantly affected response time and error rate for non-homologous fingers of two hands. Notably, the DSS interference in the non-homologous fingers was comparable when haptic stimuli were delivered with one or both hands. When homologous fingers on both hands were stimulated, the interference vanished, and the reaction time decreased. Overall, the interference of the non-homologous fingers in one hand was more significant than that of the homologous fingers in two hands.

Furthermore, Evans and Craig (1991) conducted three experiments to investigate the human haptic ability to detect the direction of pattern movement on different fingerpads. According to Evans and Craig (1991), certain studies have identified direction-sensitive neurons, suggesting that the direction of movement can be detected without attention. From this, Evans and Craig (1991) assumed that recognition difficulties experienced by participants when the direction of the target pattern motion contradicted that of the non-target pattern motion indicated the inability of participants to process information at a single location on the skin when two stimulations were presented simultaneously. Experiments 1 and 2 conducted by Evans and Craig (1991) required participants to identify the motion direction of a target pattern presented on one fingerpad, while a non-target pattern was presented on the adjacent fingerpad. Participants were instructed to focus solely on the target location. The results indicated that accuracy was high and response time was fast when both patterns moved in the same direction compared to when they moved in different directions (Evans & Craig, 1991). The lowest accuracy and slowest reaction time were observed when two movement direction-contradicting patterns were displayed simultaneously. In Experiment 3, the non-target location was the contralateral hand's fingerpad. The results showed that the non-target movement direction did not impact participants' performance (Evans & Craig, 1991). Evans and Craig (1991) found that haptic information was processed on adjacent fingers and caused interference even when participants were directed to focus on the target finger. Moreover, the participants could ignore the haptic information from the non-target finger and concentrate on the target finger when the haptic information was from the homologous fingers of both hands.

In conclusion, haptic perception is influenced by various stimulus features, including numerosity and feature differentiation, as well as by the hand and fingers used involved. Even when directed attention is applied, haptic perception of homologous fingers appears to perform better than non-homologous fingers. Additionally, the haptic information from adjacent fingers seems to have a more significant interference effect than homologous fingers.

Numerical Perception and Cognition

Estimating the number of apples on a tree or deciding on how many people are in a room are but two examples of the many situations in which we use cognition to determine quantities. However, humans are not the only species that demonstrate numerical cognition. In the animal kingdom, several species show the capacity of numerical cognition. For examples, bees can enumerate the number of petals on a flower(Leppik, 1953), and fish form schools to maximize their chances of survival and avoid being eaten (Agrillo & Bisazza, 2018). Both humans and animals use and have significantly benefited from their ability to negotiate numerical magnitudes. Therefore, investigating numerical cognition and enumeration is a field of study that connects humans universally with members of other species who have the ability to process numerical information

Two Non-Symbolic Numerical Processing Systems

Non-symbolic numerical processing systems are cognitive mechanisms that allow us to judge the numerosity of a set of objects without using symbols or language, such as Arabic numerals. These systems rely on features such as size, density, or texture to estimate the number of objects in a set. Experiments on visual perception significantly contribute to understanding human non-symbolic numerical cognition. Results from studies on non-symbolic numerical cognition in the visual field imply that the brain has at least two systems to process non-symbolic representations. An illustration from Feigenson et al. (2004) proposed two core systems for non-symbolic numerical cognition: core system one and core system two. These systems are also known as the approximate number system (ANS) and the parallel individuation system.

Core system one, or the ANS, is used to process larger sets of numbers (>4) and represent estimates of large numerical quantities or an approximation size number. Under core system one and the ANS, when there are more than four elements in a set, each object is represented by a single mental symbol (Hyde, 2011). These two systems become less accurate with larger numbers but have no upper limit on their capacity (Hyde, 2011). Additionally, discrimination between numbers by the system depends on the ratio between quantities, and the ratio of two adjacent numbers complies with Weber's law. Weber's law is a principle in psychophysics that states that the smallest detectable difference between two stimuli is proportional to the magnitude of the stimuli (Carriot et al., 2021). Core system one, or the ANS, is shared by newborns, children, and adults, and its sensitivity grows as individuals mature (Leibovich et al., 2017). For instance, infants as young as six months old can distinguish the difference between two numbers in a ratio of 1: 2 (e.g., 20 out of 40 items) but not 2 : 3, whereas adults can detect the ratio difference of 7:8 (Barth et al., 2003).

Core system two, or the parallel individuation system, can only represent a relatively low number of items, typically around three or four (Feigenson et al., 2004). Each item is represented by a distinct mental symbol in this system (Hyde, 2011). This system supports fine numerical discrimination, which is more precise and quicker, but the number of items must fall within the limited range (Leibovich et al., 2017). Performance in this system is independent of the ratio between numbers. For instance, infants can distinguish between two and three objects but not between four and six, even though the ratios are identical (Feigenson & Carey, 2003).

One way in which core systems one and two (or ANS and parallel individuation system) determine the number of objects in a set is through the use of numerosity enumeration modes. Enumeration is a generic capacity and refers to the cognitive strategies or methods we use to count or judge numerical information(Gliksman & Heni, 2019). These modes of enumeration not only involve non-symbolic numerical processing systems but also involve the use of symbolic representations, such as numbers or words. Therefore, understanding these modes of enumeration can help us gain insight into the mechanisms of number cognition.

Modes of Enumeration

Enumeration can be broadly categorized into three main modes: subitizing, counting, and estimation. Subitizing relies on core system two or the parallel individuation system; counting utilizes a symbolic numerical processing system; and estimation relies on core system one or the ANS.

Subitizing

Processing numbers within a small numerical range is called "subitizing." Subitizing, known to be accurate and rapid, utilizes the core system two or the parallel individuation system (Kaufman & Lord, 1949, Hyde, 2011). Subitizing can be performed quickly and precisely, even with insufficient time for directed attention (Trick & Pylyshyn, 1993). As a result, subitizing is considered pre-attentive, that is to say humans can automatically process sensory information before consciously attending to a specific stimulus; this mechanism is thought to operate in parallel across the visual field and can detect basic features such as colour, orientation and spatial frequency(Trick & Pylyshyn, 1993).

Subitizing has primarily been studied in the visual domain. Infants who lack symbolic computation skills have been shown to provide the clearest and most accurate representation of the difference between small and large numbers (Feigenson & Carey, 2003), which indicates subitizing. For example, in one study, after an experimenter consecutively placed one food item in one bucket and two in another, human newborns successfully searched for two food items in the second bucket (Feigenson & Carey, 2003). When comparing 1:3 and 2:3, infants consistently showed the ability to look for a higher number of items in the bucket (Feigenson & Carey, 2003). However, infants could not reliably choose the bucket holding more items when there were more than three items in the bucket (e.g., two vs. four; one vs. four) (Feigenson & Carey, 2003). Although infants showed a subitizing ability for numbers one to three, the maximum limit of subitizing remains debatable. However, visual research has established that the upper limit for subitizing is six (Katzin et al., 2019).

Subitizing has been subdivided into two categories based on the presentation of numerical stimuli. Simultaneous subitizing involves recognizing the number of objects presented simultaneously in the visual field, while sequential subitizing involves perceiving a small number of items presented one after the other over a short time interval which is too brief for counting. Anobile et al. (2019) observed both types of subitizing in children. In the experiment, sequential subitizing was tested by presenting a series of flashes or sounds randomly within a 2-second interval, with each item lasting 40 ms; simultaneous subitizing was tested by displaying sets of dots on the screen simultaneously. Children were asked to report the number of items perceived. The findings suggest that children possess both simultaneous and sequential subitizing abilities, with sequential subitizing also observed in the auditory modality. Sequential subitizing in the auditory modality has also been observed in the experiment by Repp (2007). This study presented participants with quick tone sequences ranging from 2 to 10 tones. The rate of correct enumeration declined as the number of tones increased. Response time was increased significantly when the number of tones was 5 or 6. However, the response time increase between 2 and 3 tones was less than the increase between 3 and 4 tones (Repp, 2007). Consequently, the auditory studies of Repp (2007) suggest that a single group of two or three tones might be subitized.

Furthermore, recent studies have shown that subitizing can involve grouping, known as groupitizing. This refers to the finding that visually grouped arrays can be counted faster and with greater accuracy than unstructured arrays (Ciccione & Dehaene, 2020; Starkey & McCandliss, 2014; Wege et al., 2022). For instance, Ciccione and Dehaene's (2020) study investigated whether visually grouping items could increase the subitizing limit. The experiment presented participants with visual stimuli consisting of unstructured (random) and structured dots patterns. The dots number either contained 1 to 5 or 6 to 10 dots. Participants were asked to report the number of dots as quickly as possible. Ciccione and Dehaene's (2020) results showed that participants could subitize up to 5 dots with both structured and unstructured patterns, but the limit increased to 10 dots when the dots were grouped. In addition, response times for grouped stimuli were faster than those for unstructured stimuli. Thus, groupitizing facilitates the efficiency and capacity of the enumeration.

Counting

Dehaene (2011) argues that counting relies on the approximate number system or core system one. He suggests that this non-symbolic system serves as a foundation for later, more complex mathematical abilities and continues to play a role in advanced skills such as estimation and mental arithmetic.

Counting is, by definition, a slow and precise procedure that takes place on long-duration displays (Kaufman& Lord, 1949). The numerical range of counting is between 5 and 7, according to Trick and Pylyshyn (1993). Counting can occur in three situations: when stimuli are present in one area at a sufficiently low rate, when stimuli are present in separate places simultaneously and remain there, or when stimuli appear in successive places at a low rate (Kaufman& Lord, 1949). Counting assigns a number from the number series to indicate each item of a group, and provides each item's numerical and verbal report (Kaufman& Lord, 1949).

While counting requires attentive processing shifted between items, subitizing is based on a preattentive visual process with a limited number of items (Trick & Pylyshyn, 1993). Thus, subitizing and counting are considered two entirely different systems. The latencies of enumeration as a function of numerosity are the first evidence showing that subitizing and counting are two distinct processes (Piazza et al., 2002). The sign of the change from subitizing to counting is the increase in slope in the latency function after three or four items (Trick & Pylyshyn, 1993). Consequently, counting and subitizing are viewed as two distinct systems. However, Piazza et al. (2002) conducted a PET investigation to evaluate if subitizing and counting were utilized as distinct or functionally overlapping processes at the neurological level. Participants completed enumeration on visually organized arrays of dots (1-4 and 6-9 dots). The Piazza et al's (2002) results demonstrated that subitizing and counting share a common network, which includes extrastriate middle occipital and intraparietal regions; the number and spatial arrangement of dots modified the network's spatial extent and intensity. The results of Piazza et al's (2002) experiment reveal that subitizing and counting are not considered two distinct systems at the neurological level; rather, they are inseparable in the nervous system.

Estimation

If counting is not possible, estimation becomes necessary when the numbers exceed the range for subitizing. Estimation is an imprecise form of enumeration used for large sets of items with limited exposure time. Although estimation is fast, it is associated with lower accuracy and confidence (Katzin et al., 2019). Like subitizing, estimation only allows for one response to a set of multiple items or quantities (Kaufman & Lord, 1949).

One hypothesis proposes that subitizing and estimation share one mechanism; however, according to Weber's law, the mechanism functions with high accuracy for subitizing small numbers, but less accurately for estimating larger numbers (Revkin et al., 2008). In order to investigate if subtizing and estimation share one mechanism, Revkin et al. (2008) used a masked forced-choice paradigm where participants were required to discriminate sets of numbers with different levels of difficulty, ranging from one to eight items and from 10 to 80 items. The results violated Weber's law, with discriminating one to four number being much more accurate than discriminating 10 to 40 number (Revkin et al., 2008). Thus, the results disconfirming the hypothesis that subitizing and estimation share a single estimation system. Furthermore, Cutini et al. (2014) used neuroimaging to investigate whether the neural activity of subitizing and estimation can be separated. In Cutini et al.'s (2014) study, multichannel near-infrared spectroscopy (fNIRS) was used to measure hemodynamic activity in the bilateral parieto-occipital cortex during a visual enumeration task where participants judged the number of dots in the arrays. For numbers within and above the subitizing range, different hemodynamic patterns were observed in the parietal cortex, including amplitude modulation and temporal framing (Cutini et al., 2014). Therefore, both the results from Revkin et al. (2008) and Cutini et al. (2014) studies suggest that subitizing and estimation are dissociable. Subitizing may rely on unique processing dedicated to small numbers (Revkin et al., 2008).

The Debate Over Two Non-Symbolic Numerical Cognition Systems

Several pieces of experimental evidence indicate that the ANS can process both small and large numbers. Cordes et al. (2001) conducted an experiment in which participants were instructed to press a key a certain number of times (referred to as the target number) while saying 'the' with each press as an articulatory inhibition. The results showed that the mean number of presses increased as the target number increased, while the variation remained constant within and beyond the subitizing range. Cordes et al. (2001) concluded that these findings suggest a continuum between the representation of small and large numbers. Brannon and Terrace (1998) trained two rhesus monkeys to respond to the numbers one through four in ascending order while controlling for non-numerical cues such as size, shape, and color. The monkeys' ability to sort the numbers 5 through 9 was then evaluated, and the results showed that both monkeys could respond to the new numbers in ascending order (Brannon & Terrace, 1998). These results suggest that the ability of rhesus monkeys to sort small and large numbers depends on the ratio between the sorted numbers and that the Approximate Number System (ANS) operates across the entire spectrum of number systems (Brannon & Terrace, 1998). In addition, the presumption of an approximation number system underlies most of the formal mathematical models of the brain's nonsymbolic number representation (Hyde, 2011). Therefore, some advocated a "one system view" of numerical cognition. It is important to note that those who hold the "one system view" do not dispute the existence of parallel differentiated systems but instead argue that approximative number systems may function over the full numeric spectrum (Hyde, 2011).

Nonetheless, according to the findings of event-related potential (ERP) research by Hyde and Spelke (2009), small and large numbers are represented differently. In their experiment, participants viewed dot arrays passively, and the ERP results showed that small numbers caused an early posterior parietal response (N1) that varied according to the number of items in the set, while large numbers produced a later, mid-latency component over the posterior parietal scalp (P2) that varied with the ratio of numerical change between sets (Hyde & Spelke, 2009). These results suggest that the approximation number system and the parallel individuation system are utilized differently in processing small and large numbers. In addition, research in psychophysics has provided further evidence that estimations of big and small numbers differ in terms of reaction time, accuracy, and answer distribution, further confirming the qualitative contrast between small and large number processing (Revkin et al., 2008). Furthermore, differences in individuals' range of small numbers were not linked to differences in the range of large numbers, and differences in the capacity of subitizing were not associated with differences in accuracy of large non-symbolic numbers (Revkin et al., 2008; Piazza et al., 2011). These findings suggest that subitizing relies on a distinct cognitive mechanism from that which supports estimation, and that small numbers are not represented as approximations of numerical magnitudes.

Burr et al. (2010) studied attention and working memory to enumerate small and big numbers by manipulating attentional load. Burr and colleagues examined respondents' ability to judge target numbers in the subitizing and estimating ranges quickly. Both spatial dual-task and "attentional blink" dual-task paradigms were employed as tasks to manipulate attentional load. During a partial dual-task experiment, subjects were required to complete a central task which involved identifying the presence of either a red square (low attentional load) or a combination of color and orientation (high attentional load), while simultaneously estimating the numerosity of a dot cloud. The dots and color square were presented together on the same screen.; During the attentional blink dual task, each trial involved presenting letter stimuli using rapid serial visual presentation (RSVP), which entails presenting a sequence of visual stimuli in quick succession at a constant rate. The trial commenced with a fixation point displayed for 1 second, followed by a stream of 12 letters presented one by one, with a blank interval in between. After that, the dot array was shown for 130 ms, appearing anywhere between 110 ms and 880 ms after the target letter, and then followed by a binary pixel noise mask displayed. The results of Burr et al.'s (2010) study indicated that subitizing and estimation performance were comparable under high attentional load conditions. However, subitizing performance improved while estimation performance was unaffected under low attentional load. Therefore, subitizing and estimation likely involve distinct operational processes(namely, two systems).However, Burr et al. (2010) propose that the processing of the two number ranges does not occur through completely independent mechanisms. Instead, they suggest the existence of pre-attentive estimation mechanisms that operate across all number ranges, encompassing both large and small numbers. Additionally, Burr et al. (2010) identify another attentive mechanism characterized by a limited capacity, capable of selectively attending to approximately four items. Notably, this attentive mechanism exhibits an extraordinary ability to achieve near-perfect performance specifically within the lower range of numbers (Burr et al., 2010). Another study found a correlation between individual variations in subitizing and working memory but not estimation abilities (Piazza et al., 2011). Subitizing ability is impaired when numerous items must be kept in working memory simultaneously, but dual-tasking with the same working memory demands does not degrade estimation ability (Piazza et al., 2011).

When stimuli were presented under attentional load, the experimental results revealed that participants exhibit superior performance in the range of small numbers (Burr et al., 2010; Piazza et al., 2011). Interestingly, this observation suggests that small numbers may not be represented solely by the approximate number system, thereby challenging the prevailing "one-system view" of numerical cognition. However, these results also contradict the "two systems view" by indicating that the two systems are not solely specialized for small or big numbers(numerosity). Instead, these two systems engagement is variable and depends on the nature of the stimuli provided and the restrictions of attention or working memory load (Hyde, 2011). Therefore, Hyde (2011) proposed that when items are presented under settings that allow for the selection of individual items, the items will be represented as distinct mental items through parallel separation, rather than as a numerical quantity. However, the ANS system processes items as a single mental numerical magnitude rather than using a parallel individuation system when the items are presented outside of the attentional load, such as if items are too numerous, too close, or when the attentional load is excessive.

Overall, these findings suggest that numerical cognition is a complex process that depends on various factors, including the features of stimuli, attentional demands, and working memory constraints. Further research is needed to investigate the interaction between the ANS and parallel individuation system, as well as how they contribute to the representation of small and large numbers under different conditions.

Number Sense and Non-Numerical Continuous Magnitudes

The two core systems, which are an expansion of the concept of number sense proposed by Dehaene in 1997 (Leibovich & Henik, 2013). Number sense enables humans and animals to quickly and accurately perceive the number of objects in an array. The sense of magnitude is closely related to number sense, as it allows individuals to differentiate between continuous magnitudes, such as the density of cherries or the surface area of tables. The sense of magnitude is considered more fundamental and automatic than the sense of number (Leibovich et al., 2017). Dehaene (1997) proposed that nonsymbolic numbers are processed independently of continuous magnitudes, like size, area, and density. Dehaene and Changeux (1993) suggest that each item is initially represented in geographic coordinates and subsequently mapped onto a brain topographic map. However, item characteristics such as continuous magnitude are ignored. Finally, specific neurons sum up the numbers on the topographical map, allowing people to estimate the number of items in a group (Dehaene & Changeux, 1993).

The independence of the continuum magnitude from the nonsymbolic number perception system is still a subject of debate. When comparing sets of stimuli with different numbers, there are often differences in total contour length, surface area, and density, suggesting that these continuum magnitudes may offer an alternative explanation for the meaning of numbers (Leibovich & Henik, 2013). This raises the question of whether number perception is derived from a more primary dimension. Despite efforts to determine the association between number perception and continuous magnitude, the correlation remains elusive (Leibovich & Henik, 2013). However, a reexamination of the literature shows that the effect of continuous magnitude on the outcome of paired number comparison tasks is not entirely ruled out (Leibovich & Henik, 2013). In most nonsymbolic number research, the nonsymbolic stimuli always contain non-numerical continuous magnitudes, making it challenging to produce sets of items that differ solely in number due to the effect of these possible non-numerical continuous magnitudes (Leibovich & Henik, 2013). Thus, a change in the number of elements always affects the continuous magnitude (Leibovich et al., 2017). This suggests that number sense and continuous magnitude may not be two distinct systems, as certain experimental findings have suggested.

For example, Gebuis and Reynaert's (2014) study investigated the impact of non-numerical continuous magnitude on ordinal number processing. The study showed participants five arrays of dots (groups of three, four, five, six, and nine dots, respectively). In the first four arrays of dots, both the numerical value and the five continuous magnitudes (convex hull, the smallest shape that can be drawn around a set of points in a way that the shape is convex; aggregated surface, a larger surface area created by combining smaller surface areas; density; diameter, and contour length) increased with the number of dots. In half of the trials for the final array, the continuous magnitude was consistent with the number, whereas it was inconsistent in the other half. Simultaneously taken EEG measurements of participants' brain activity revealed an interaction between the number and its sensory characteristics without a primary effect. Trials that shifted in the same direction elicited different neural responses compared to trials in which the numerical and sensory cues varied oppositely. These findings support the idea that visual cues (continuous magnitude) variations should be considered when interpreting numbers (Gebuis & Reynaert, 2014). The results indicate that sensory cues, specifically continuous magnitude, play a crucial role in numerosity processing and directly impact visual nonsymbolic number perception (Gebuis & Reynaert, 2014).

Leibovich et al. (2015) conducted an fMRI study targeting performance in the subitizing range. The study manipulated the consistency of the number of dots in arrays and total surface area, which were either congruent or incongruent. Half of the trials in each task were consistent (e.g., a large number and large surface area), while the other half were inconsistent (e.g., a large number and small surface area). Additionally, the task sequence was manipulated such that half of the participants started the task with number discrimination and the other half with area discrimination. The results indicated that the area discrimination task was completed faster and more precisely. It is important to note that, although the area factor influenced performance on the number test, the number of dots only affected performance on the area task for participants who first performed the number discrimination task. Interestingly, the sequence in which activities were completed altered brain activity levels. In congruency trials, the group that began with the number task showed activity in the right frontoparietal region, while the group that began with the area task demonstrated activation in the left frontoparietal region. This research lends credence to the idea that continuous magnitude influences numerical cognition performance, even within the subitizing range.

All of the studies mentioned above contradict the hypothesis that nonsymbolic numerical cognition and continuous magnitudes are two independent systems. However, according to theorists of the ANS, all magnitudes—whether discrete (e.g., number of items) or continuous (e.g., size, density, etc.)—are processed by a common system that follows Weber's law (Leibovich & Henik, 2013). To test the ANS hypothesis, Leibovich and Henik's (2014) study compared discrete (comparing the number of dots) and continuous (comparing the area of a square) tasks. Participants in the experiment compared each array of dots containing five to 25 dots. All continuous magnitudes were modified simultaneously with minimal correlation to numbers, so they could not be used as valid signals for numbers (Leibovich & Henik, 2014). The results showed that the discriminating threshold for the continuous task was greater than that for the discrete one. The performance of the discrete task complied with Weber's law, but the performance of the continuous task was inconsistent. Additionally, the continuous magnitudes of the dot arrays affected the performance of the discrete task (e.g., dot density, cumulative area of dots). Thus, continuous magnitudes affect task performance even though they are neither relevant nor predictive of the quantity of the task (Leibovich & Henik, 2014). This experimental finding contradicts the notion that a single ANS system applies to discrete and continuous magnitudes.

Here, we introduce another effect related to the sense of number, known as numerosity adaptation. Numerosity adaptation is a perceptual phenomenon that influences our perception of numerical quantities based on previous exposure to stimuli numerosity (Burr & Ross, 2008; Burr et al., 2011; Togoli et al., 2021). Specifically, exposure to stimuli with high numerosity leads to underestimation of subsequent stimuli, while exposure to stimuli with low numerosity leads to overestimation (Burr & Ross, 2008; Burr et al., 2011; Togoli et al., 2021).

Importantly, this effect has been demonstrated to occur independently of the format of the stimuli, such as size and density. (Togoli et al., 2021).Numerosity adaptation has been demonstrated in visual modalities. For instance, Burr and Ross (2008) conducted a visual experiment to establish the sensitivity of the numerosity system to adaptation by the human brain. In the experiment, the adaptation stimuli were positioned away from the fixation point, with half of the sessions above on the left and the other half below on the right.Each adaptation stimulus was displayed for a duration of 30 seconds. Following the adaptation phase, a test stimulus with varying numerosity appeared in the same position as the adaptor for 600 milliseconds. Subsequently, a probe stimulus with a fixed numerosity was displayed for 600 milliseconds directly above or below the test stimulus. A 400-millisecond pause occurred between each stimulus. Furthermore, to investigate whether adaptation is solely dependent on numerosity or influenced by factors such as texture density, Burr and Ross (2008) conducted a series of control experiments. The results, including the control experiments, showed that adapting to large numbers decreased apparent numerosity, while adapting to small numbers increased it. This effect was exclusively dependent on the numerosity of the adaptor and remained unaffected by contrast, size, orientation, pixel density, and low adaptor contrasts (Burr & Ross, 2008). However, in Burr and Ross's (2008) experiment, numerosity adaptation had little effect in the low subitizing range due to the single-task condition employed in their paradigm (Burr et al., 2011). To further investigate whether numerosity in the subitizing range can be adapted, Burr et al. (2011) conducted a dual-task experimental condition. The results revealed that even small numbers within the subitizing range can be adapted under high attentional load (Burr et al., 2011). These findings suggest that numbers are identified through a perceptual process that operates across the entire number range, with attention-based systems added for small numbers (Burr et al., 2011). Numerosity adaptation effects have been observed in various sensory modalities, including not only the visual modality but also the tactile modality. For example, in one of the experiments by Togoli et al. (2021), participants were assigned the task of performing with their left and right hands positioned in the left and right panels, respectively. The task consisted of two estimation conditions: adapted and non-adapted. In the adapted condition, the same hand that received the adaptation stimulus was used for the subsequent test. In the non-adapted condition, the adaptation and test stimuli were delivered to different hands. Different rates of adaptation were employed, specifically 1 or 2 Hz for low adaptation and 11 or 12 Hz for high adaptation. After the estimation phase, participants were instructed to verbally report the number of stimuli they had perceived. The results demonstrated a clear manifestation of adaptation effects when the adaptor and test stimuli were presented to the "adapted" hand (Togoli et al., 2021). Specifically, high adaptation induced underestimation, while low adaptation resulted in overestimation (Togoli et al., 2021). These findings suggest a distinct pattern of tactile numerosity adaptation aftereffects, where the extent of over- or underestimation depends on the adaptation rate (Togoli et al., 2021).

In conclusion, the relationship between number sense and non-numerical continuous magnitudes is a complex and debated topic. While some researchers argue for the distinctiveness of these systems, other studies suggest their interconnection and mutual influence. Experimental findings have consistently demonstrated the impact of continuous magnitudes, such as size, density, and surface area, on numerical cognition and task performance. These results highlight the crucial role of continuous magnitudes in numerosity processing and their direct influence on visual nonsymbolic number perception. Additionally, numerosity adaptation, a perceptual phenomenon, provides further evidence that our perception of numerical quantities, both within and beyond the subitizing range, can be recalibrated based on previous exposure to stimuli numerosity.Importantly, this effect remains independent of other stimulus characteristics, such as density, color, and size. Collectively, these findings challenge the notion of a single, independent system for numerical cognition, emphasizing the significance of considering continuous magnitudes in the understanding of number perception.

Haptic Enumeration

Most enumeration experiments have primarily focused on the visual domain. However, there have been instances where enumeration experiments have explored different sensory modalities. For instance, Repp (2007) and Anobile et al. (2019) reported auditory enumeration of subitizing, as mentioned in the "subitizing" section. In the haptic domain, parallels between haptic and visual enumeration are beginning to be discovered. Riggs et al. (2006) argued that the haptic domain also exhibits a subitizing phenomenon. In their experiment, participants' fingertips were stimulated using electromagnets, and they were asked to keep track of how many times each finger was stimulated. They discovered a striking divide in precision, with subjects executing precisely up to three fingers but suffering a precipitous decline in performance after three. However, around the same time, Gallace et al. (2006) asserted that subitizing does not occur in the haptic modality. Their study placed seven vibrotactile stimulators on the participant's body instead of their fingers. Parts of the body were stimulated concurrently in random order and for two durations (200 msec & 5 seconds), and the number of stimuli each participant passively experienced had to be reported. However, there was no evidence of subitizing. One likely reason for the disparities between Gallace et al. (2006) and Riggs et al. (2006) is the location of the skin being stimulated. The hand may be better at discerning tactile stimuli than the body surface because it occupies a greater proportion of the somatosensory cortex than the body surface (Gallace et al., 2008). Furthermore, it is important to note that Riggs' study split the linear function in half, one portion for 1–3 stimuli and another for 4–6 stimuli, to compare it to earlier research on visual subitizing (Gallace et al., 2008). This division of segments based on visual data rather than statistical evidence of experimental discontinuity does not effectively answer whether assessments of tactile stimuli include two distinct processing modes (subitizing and counting) (Gallace et al., 2008). Plaisier, Bergmann, Tiest, and Kappers (2009a, 2010a, 2010b) investigated the performance of active tactile enumeration using a variety of numerosities, geometric forms, and hand combinations (one and two hands). Participants were required to report the number of items in their hands accurately. The concluding findings of the experiments showed that, similar to how numbers are processed in vision, active touch combines subitizing and counting. However, it is important to note that, unlike the studies conducted by Gallace et al. (2006) and Riggs et al. (2006), participants in the studies by Plaisier and colleagues were allowed to freely explore their hands (active touch) in order to achieve the best possible performance in terms of enumeration.

A more recent study by Sharma et al. (2019) may have found evidence of tactile enumeration in subitizing. In the experiment, participants scanned arrays of raised dots on a braille surface using only their sense of touch. Two sub-experiments were conducted. In experiment one, the density of the raised dots was kept consistent, and participants scanned the raised dots with one or two fingers (index and middle finger) on either the right or both hands. The aim of experiment one was to investigate whether and how the number of fingers affected participants' haptic enumeration (Sharma et al., 2019). In experiment two, the density of the raised dots was manipulated. Two conditions of density, compressed and stretched, were used. In both experiments, reaction times were not recorded, but the accuracy and confidence of participants answers were recorded and analyzed. The experiments results found that participants' estimates of numbers increased as the actual numbers increased, while confidence decreased as actual numbers increased. Under the compressed condition, accuracy and confidence were reduced, and underestimation was exaggerated (Sharma et al., 2019). Furthermore, in experiment one, Sharma et al. (2019) found that accuracy was particularly high with up to six raised dots. However, after more than six, both accuracy and confidence decreased, and variability increased. This discontinuity may indicate evidence of subitizing (Sharma et al., 2019). However, since the two experiments did not consistently demonstrate this discontinuity before and after the number six, Sharma et al. (2019) concluded that the experimental results may provide evidence of haptic enumeration, but further research is necessary to establish the presence of subitizing.

In brief, haptic enumeration seems to demonstrate a subitizing ability similar to visual enumeration, whether through active or passive touch. However, unlike visual enumeration, the consistency of experimental evidence for tactile enumeration has yet to be firmly established. Therefore, there is a need for further research to explore and expand our understanding of tactile enumeration.

Aims and Hypotheses

A literature review reveals a dearth of studies on haptic enumeration under active touch, as well as limited research on haptic enumeration using braille as a stimulus. Braille provides a distinctive advantage by enabling the simultaneous perception of multiple stimuli on a single fingerpad due to its unique properties. As a result, the utilization of braille as a stimulus for haptic enumeration investigations possesses the potential to broaden and enrich the field of haptic numerical cognition.

The objective of our study is to build upon the work of Sharma et al. (2019), Browne (2019), and Rendell (2020) by utilizing braille as a stimulus to investigate haptic enumeration. These prior experiments involved visually normal braille novices using fingerpads to scan and perceive the number of raised dots on a braille sheet, with researchers recording and analyzing the participants' perceived number accuracy and their confidence in estimation. In Sharma et al.'s (2019) study, participants moved their fingerpad laterally from left to right, similar to how a braille reader would read, and perceived the number of raised dots. By comparing the velocity of fingers' movement, the accuracy of numerical perception, and confidence, Sharma et al. (2019) discovered that participants with no prior braille knowledge could recognize and extract braille dots based solely on touch. Furthermore, Sharma et al. (2019) observed a decrease in accuracy when the number of dots exceeded six, which they suggested could signal a change from subitizing to counting or from counting to estimating. While the exact nature of this change is uncertain, the results indicate that participants altered their enumeration mode and that this shift occurred close to six.

We modified the reference paradigm used in the experiments of Sharma et al. (2019), Browne (2019), and Rendell (2020) by introducing a new component: the distractors. During the experiment, participants were instructed to scan and perceive the number of dots in the target column while simultaneously scanning another non-target column with either a large number of distractors or no distractors. Additionally, we aimed to investigate two main objectives: first, to assess the impact of four factors (number of raised dots, presence of distractors, density, and hand and finger combination) on individuals' accuracy and confidence in haptic numerical perception; and second, to determine whether participants demonstrated a preference for using subitizing under varied stimulus circumstances or whether they employed one or more enumeration modes in this paradigm.

The studies conducted by Browne (2019) and Rendell (2020) modified the paradigm of Sharma et al.(2019). Browne and Rendell requested their participants to scan braille in the sagittal (midline) axis, in a distal to proximal direction, with each scanning finger encountering a different array of dots. Participants were then asked to perceive the total number of dots. While the results of their experiments were comparable to those of Sharma et al.(2019) in that the accuracy of numerical perception decreased when the number of dots exceeded six, there were differences in some of the findings. Specifically, Browne (2019) observed that participants performed better with a two-handed homologous finger (two index fingers) than with a single-handed non-homologous finger (index and middle fingers). In contrast, neither Sharma et al. (2019) nor Rendell (2020) found an effect of hand and finger use on the accuracy of perceived numbers. However, Rendell (2020) found higher confidence in the two-handed homologous finger condition and lower confidence in the one-handed non-homologous finger condition.

Aagten-Murphy and Burr (2016) against on the question about the temporal dynamics on numerosity adapation, they have developed a novel adaptation paradigm that effectively induces numerosity adaptation at multiple distinct locations simultaneously. Aagten-Murphy and Burr (2016) study's findings demonstrate the spatial specificity of this adaptation, revealing that different locations in the visual field can adapt (and simultaneously adapt) to different numerosity stimuli . The adaptation effects can be generated at specific locations based on the history of stimuli presented at those locations(Aagten-Murphy & Burr, 2016). Furthermore, investigation has shown that the number of unique adapting events has a primary influence on the adaptation effect, rather than the duration of each event or the total exposure duration to adapting stimuli(Aagten-Murphy & Burr, 2016).

Based on our literature review, we hypothesized that the presence of distractor dots in the non-target column could interfere with the accurate perception of the targeted column. This interference may result from attention's inability to filter out simultaneous stimuli. Alternatively, it may be due to perceiving a high number of distractors, which causes distortion of the actual number perception in the target column. Conversely, perceiving a low number of distractors may lead to a reduction in perceived numbers; this reduction or distortion may occur due to simultaneous adaptation. Furthermore, based on previous research outcomes by Sharma et al. (2019), Browne (2019), and Rendell (2020), we anticipated the following findings in our current study:

- H1) a systematic relationship between actual and perceived numbers, with perceived numbers increasing as actual numbers increase.

- H2) Density influences participants' performance, with better performance in the low-density condition than in the high-density condition.
- H3) Accuracy and confidence being impacted by the presence of distractors, with the non-target column with a large number of distractors resulting in a lower perceived number and worse performance compared to the non-target column without a distractor.
- H4) The performance of homologous fingers on different hands is better than that of non-homologous fingers on the same hand.)
- H5) If subitizing was present, there would be relatively high accuracy and confidence in task performance when the number of dots was less than six, and a discontinuity in the slope of a linear regression around the number six would be discovered.

Methods

Participants

The sample for this study comprised 20 sighted and braille-naive participants from the University of Auckland (13 females and 7 males, age ranging from 18 to 35 years old). All participants were recruited through Facebook pages for the University of Auckland School of Psychology and university student accommodation, as well as flyers. The study's experimental protocols were approved by University of Auckland Human Participants Ethics Committee.

All participants committed to complete two sessions within two weeks before their participation was confirmed. To incentivize participation, individuals who completed both sessions were awarded a $25 grocery voucher each hour. In addition, the participant with the greatest accuracy rate throughout both sessions was awarded a $50 supermarket voucher as an incentive to perform at a high level. For data analysis, all participant data were included.

Setup and Stimuli

This experiment was conducted in a dedicated lab at the city campus of the University of Auckland. Before and after the experiment, all laboratory equipment was sterilized with isopropyl alcohol to maintain a sanitary environment. Participants were encouraged to wear face masks for the sessions. During the experiment, participants sat at tables in a sound-isolated room. The braille sheets that served as the haptic stimuli were placed in the middle of the table. A board with a rectangular window measuring 4 cm × 13 cm was laid on top of the braille sheet to secure the display and to isolate what the participants could perceive. All arrays were placed on a flat table surface, centred on the participants' midline, and participants were instructed to scan all arrays in a distal-proximal direction, from the top of the rectangular window to bottom with their fingers. Each participant was blindfolded and instructed to wear noise-canceling headphones to eliminate visual and audial signals. Their index and middle fingers were joined with a delicate Velcro knot to guarantee that they moved at the same rate and unison.

The braille sheets, as shown in figure 1 and figure 2, used in this experiment were embossed by Blind + Low Vision New Zealand according to industry standards. There were four braille sheets in all. Each sheet was 28cm x 29cm and was divided into 12 rectangular sections measuring 4 cm x 13 cm, the same size as the rectangular window previously described. Each section included two vertical braille unit columns, one for each of two designated scanning fingers (see below), separated by distance between 1.0 cm and 1.5 cm. The intention was to scan one column and report its numerosity using one of two fingers, while the other column contained dots intended to serve as attentional distractors and be scanned by another finger. Each braille column contained one to 12 raised dots in the numerosity column, configured so as to maximize or minimize dot density (given actual numerosity). The other distractors column had either minimum distractors (no raised dot in any cell) or maximum distractors (12 braille cells, each containing six raised dots). The four sheets alternated between high density/low density number perception, and minimum/maximum distractors. The smallest distance between two dots in each column was 0.5 cm, whereas the greatest distance was 3.3 cm with low density. The smallest distance between two dots in each column at high density was 0.3 cm, whereas the maximum distance was 0.5 cm.

By combing distractors (maximum or minimum) and density (high or low) we created four unique conditions, which were presented to participants in a randomized order.

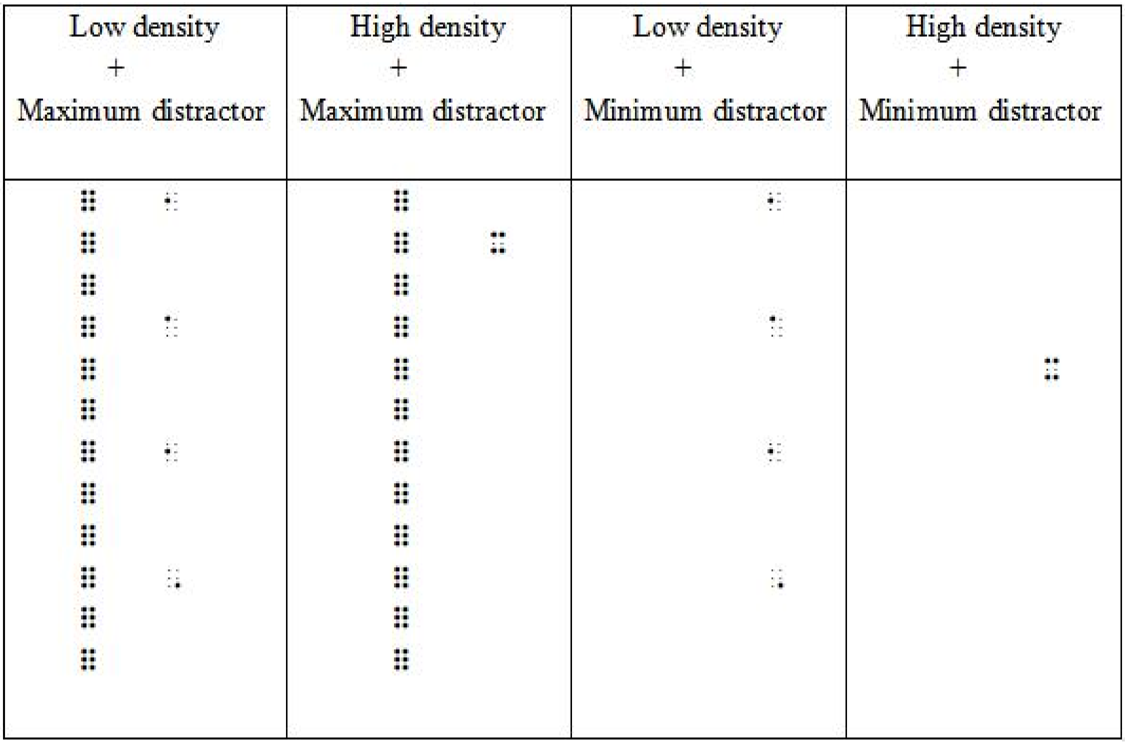

Figure 1. An example of the four raised dots (aN4) on the braille sheet under four different conditions: low density with maximum distractors, high density with maximum distractors, low density with minimum distractors, and high density with minimum distractors.

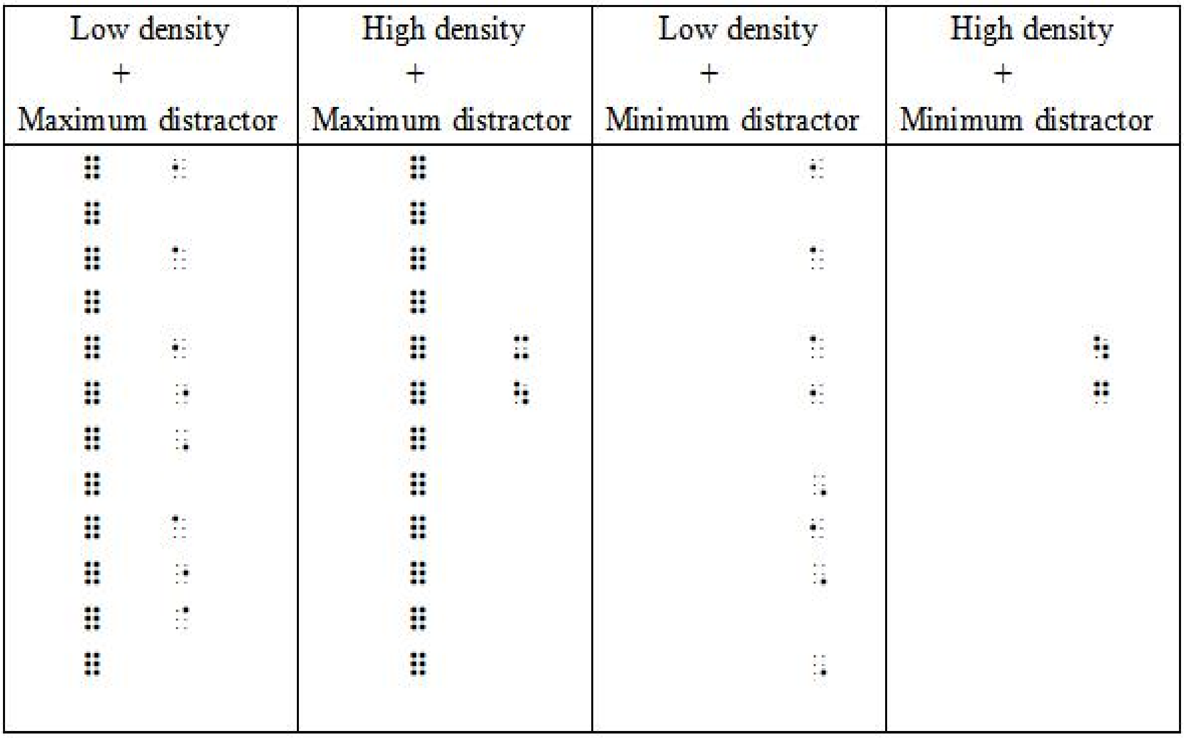

Figure 2. An example of the eight raised dots (aN8) on the braille sheet under four different conditions: low density with maximum distractors, high density with maximum distractors, low density with minimum distractors, and high density with minimum distractors.

Experimental Design and Procedure

In the experiment, participants were instructed to scan two columns using two fingerpads: one for the number perception column and one for the distractors column. Before the experiment began, participants were informed which two fingers were required to accomplish the task. When the participant's fingerpads made contact with the window's top edge, the trial began, and it ended when their fingers reached the window's bottom edge. They were instructed to move both fingers simultaneously down along the columns of braille sheets. Participants were allowed to scan each raised dot at any pace but were not allowed to stop, reverse, or rescan. Before each trial, participants were verbally instructed as to which finger would encounter the perception column. After scanning, participants were asked to verbally indicate the total number of raised dots perceived by their fingers and rate their degree of confidence on a five-point scale, with zero indicating no confidence and five indicating maximum confidence. Participants' responses and confidence ratings were immediately recorded manually in an experimental data file. Throughout the experiment, participants were not informed whether their responses were accurate or incorrect. The participants had visual access to the braille sheets only after both sessions.

This experiment was conducted in two sessions, with the sessions alternating between two sets of instructions. In one session, participants were instructed to utilize their left and right index fingers (two hands; Li+Ri; non-homologous fingers). In the other session, participants were instructed to utilize their right hand's index and middle fingers (one hand; Ri+Rm; homologous fingers). To avoid order effects, the sequence of sessions for each participant was counterbalanced. For instance, if a participant was instructed to begin with

Li+Ri, the next participant would be instructed to begin with Ri+Rm. Participants completed both sessions within two weeks, and halfway through the sessions, they were able to take a five- to ten-minute break to prevent weariness. Each session lasted no more than 90 minutes.

Before each experiment began, participants were given three practice trails to ensure that they understood the instructions, could scan the braille, and could report the results correctly. For each session, participants completed 96 trials corresponding to scanning all trials twice, with trial order generated through random number generator.

Data and Analysis

In this study, the accuracy of number perception was considered a crucial variable. Participants were instructed to touch and perceive the raised dots as precisely as possible to provide accurate responses. It should be noted that in many haptic measured. However, in our case, we judged this measure misleading, particularly since speed was never encouraged, and accuracy was prioritized to participants. Moreover, speed would be confounded in some cases, especially when comparing the enumeration of high-density arrays with low-density arrays. In this experiment, the recorded dependent variables were perceived numbers, from which we derived algebraic and absolute errors and confidence. This experiment was a within-subject design and had no control group in traditional sense. The participants serve as their own control group. This means that each participant is exposed to different conditions and their performance are compared within the same individual across these conditions.

In this experiment, participants used their index and middle fingers to scan and report perceived numbers. However, the effect of which finger scanned and reported the perceived number alone was not considered in this study. We only analyzed the hand and finger combination effect, considering homologous fingers from both hands and adjacent fingers from the right hand. Therefore, we only examined the number, density, distractors, and hand and finger combination effects, and any data variations resulting from the fingers alone, such as whether the middle finger is better than the index finger, were not examined. Separate analyses of variance (using SPSS v25) with perceived number (pN), algebraic error (calculated by subtracting pN from actual number), absolute error (algebraic error translated straight to absolute values) and confidence (RM-ANOVA) as dependent variables.

On braille sheet No. 4 with the condition of high density + distractors, the column for numerosity reported for number nine was incorrectly embossed as eight. Thus, in the current study, the data for the actual number nine in the condition of high density + distractors was missing. To address this problem, we referred to the data from numbers close to nine, such as the actual numbers eight and ten, and compared the performance of the actual number nine in the other three conditions to draw a corresponding conclusion.

Results

We focused on four dependent variables derived from each trial: (1) perceived number (pN), (2) algebraic error (i.e., signed), (3) absolute error (unsigned) of this value, and (4) reported confidence level. We analyzed these using a series of repeated measures analyses of variance (ANOVAs) that considered four factors: actual number (aN) (12), density (2), distractors (2), and finger-hand combination (2) using version 25 of SPSS.

Perceived numbers were those reported by participants after each scanning trial. The algebraic error is the difference between the perceived and actual numbers on each trial. For example, participants perceived that the array contained eight dots, but the actual number of dots in the array was six, resulting in an error of +2, indicating an overestimate; a negative error indicates an underestimate. The algebraic error provides a way to measure the magnitude of a participant's bias in one direction or the other. The absolute value of the deviation between the true number and reported number removes the sign and instead gives a measure of the variation or noise in individual response. The confidence level is the degree of certainty participants have regarding the number of dots in the array. The confidence level also predicts participants' enumeration modes when perceiving the number of dots.

We used Mauchly's test for sphericity to measure the magnitude of perceived number, algebraic error, absolute error, and confidence level. If the sphericity of Mauchly's test was significant ($< .05$), then the sphericity assumption was violated. At this point, we employed the Greenhouse-Geisser correction to adjust the degrees of freedom to produce a valid F value. If Mauchly's test for sphericity is non-significant, it implies that the sphericity assumption has not been violated, and hence no correction is necessary.

Perceived Numerical Magnitude

A four-way actual number (12) x density (2) x finger-hand combination (2) x distractors (2) ANOVA was conducted on perceived numerosity (pN). ANOVA revealed only one main effect, actual number (aN), $F(1.968, 37.397) = 1249.918$, $p < .001$, $\eta_p^2 = .985$. We found no main effects of distractors, $F(1, 19) = .00$, $p = .983$; density, $F(1, 19) = 1.605$, $p = .221$; and finger-hand combination $F(1, 19) = 3.107$, $p = .094$. Three interactions were significant: actual number by density, $F(2.868, 54.511) =7.502$, $p < .001$, $\eta_p^2 = .283$; distractors by density, $F(1, 19) = 11.475$, $p = .003$, $\eta_p^2= .377$; and density by finger-hand combination , $F(1,19)=4.864$, $p=.040$, $\eta_p^2=.204$.

The perceived number increased as the actual number increased, as shown in Figure 3. In the low-density condition, one-hand (non-homologous) fingers (RiRm) and two hands homologous fingers (RiLi) were equally accurate for aN1 to aN12. The linear pattern of RiRm and RiLi was generally consistent and highly accurate, with the perceived number increasing as the actual number increased. In low densities with maximum distractors conditions, the gap between RiRm and RiLi and the fitting line was narrow, and accuracy was high. In low densities with minimum distractors conditions, the RiRm and RiLi lines overlapped with the fitting line the most (essentially overlapped).

In the high-density with minimum distractors conditions, RiLi from aN1 to aN4 was deemed similarly accurate, with substantial overlap with the fitting line. There were slightly fluctuation from aN5 to aN12. In the condition of high density and maximum distractors, aN1 to aN8 had greater overlap with the fitting line. The underestimation increased when the actual number increased starting from aN8.

In high-density with minimum distractors conditions, RiRm from aN1 to aN4 showed good overlap with the fitting line. aN5 to aN7 were slightly overestimated, while aN12 was underestimated. In conditions of high density and maximum distractors, RiRm from aN1 to aN8 had greater overlap with the fitting line. There was an underestimation between aN11 and aN12.

(The linear regression pattern can be seen in Figure 3 below)

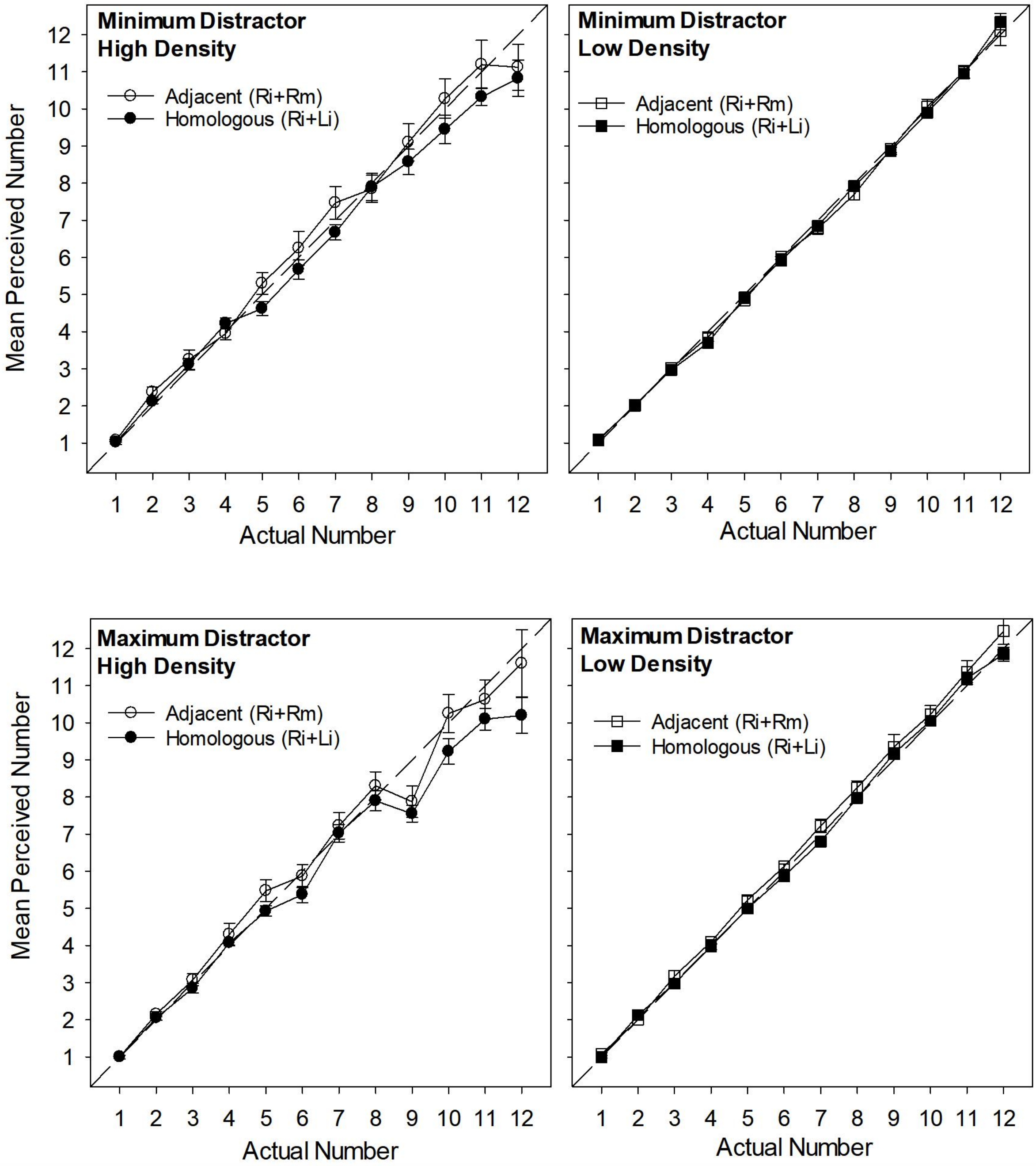

Figure 3. The mean perceived numerosity as a function of actual numerosity, finger-hand combination (Ri+Rm and Ri+Li), the density (high and low) and distractors (maximum and minimum). The dashed line corresponds to the ideal response pattern.

Algebraic Error

The algebraic error values were analyzed to better evaluate the extent to which the perceived number was systematically biased from the actual number, and to infer which enumeration mode was employed by the participants. For example, if participants used subitizing, then the algebraic linear would highly match the 0 regression linear.

A four way actual number (12) x density (2) x finger-hand combination (2) x distractors(2) ANOVA was conducted on perceived numerosity (pN). ANOVA revealed no main effects were found, actual number (aN) $F(1.968, 37.397) = 2.754$, $p = .077$; distractors, $F(1, 19) = .00$, $p = .983$; density, $F(1, 19) = 1.605$, $p = .221$; and finger-hand combination, $F(1, 19) = 3.107$, $p = .094$. Three interactions were significant: actual number by density, $F(2.868, 54.511) = 7.502$, $p < .001$, $\eta_p^2 = .283$; distractors by density, $F(1, 19) = 11.475$, $p = .003$, $\eta_p^2 = .377$; and density by finger-hand combination, $F(1,19) = 4.864$, $p = .040$, $\eta_p^2 = .204$.

In high density arrays with minimum distractors conditions, the algebraic error of RiLi increased in a positive direction (overestimation) with increasing numbers from aN1 to aN 4; however, all of them were close to the actual value. When the values range from aN5 to aN12, RiLi appears to be underestimated to varying degrees. As the number between aN8 to aN12 increased, algebraic error increased negatively. In high density with maximum distractors conditions, RiRm appeared to be overestimated and underestimated between aN1 and aN 12; no systematic pattern was found.

In high densities with maximum distractors conditions, RiRm exhibited varying degrees of overestimation and underestimation from aN1 to aN12, no obvious systematic pattern was shown. from aN1 to aN5, and aN7 to aN8, RiLi was near the actual number. Continouse underestimation occurred at aN8.

In low densities and with minimum distractors conditions, RiRm, and RiLi were near the actual number, but fluctuated slightly.

In low densities with the maximum distractors conditions, RiRm and RiLi were generally close to the actual number. However, RiRm was gradually overestimated as the actual number increased. RiLi was close to the actual number, with slightly fluction after aN5.

(The linear regression pattern can be seen in Figure 4 below)

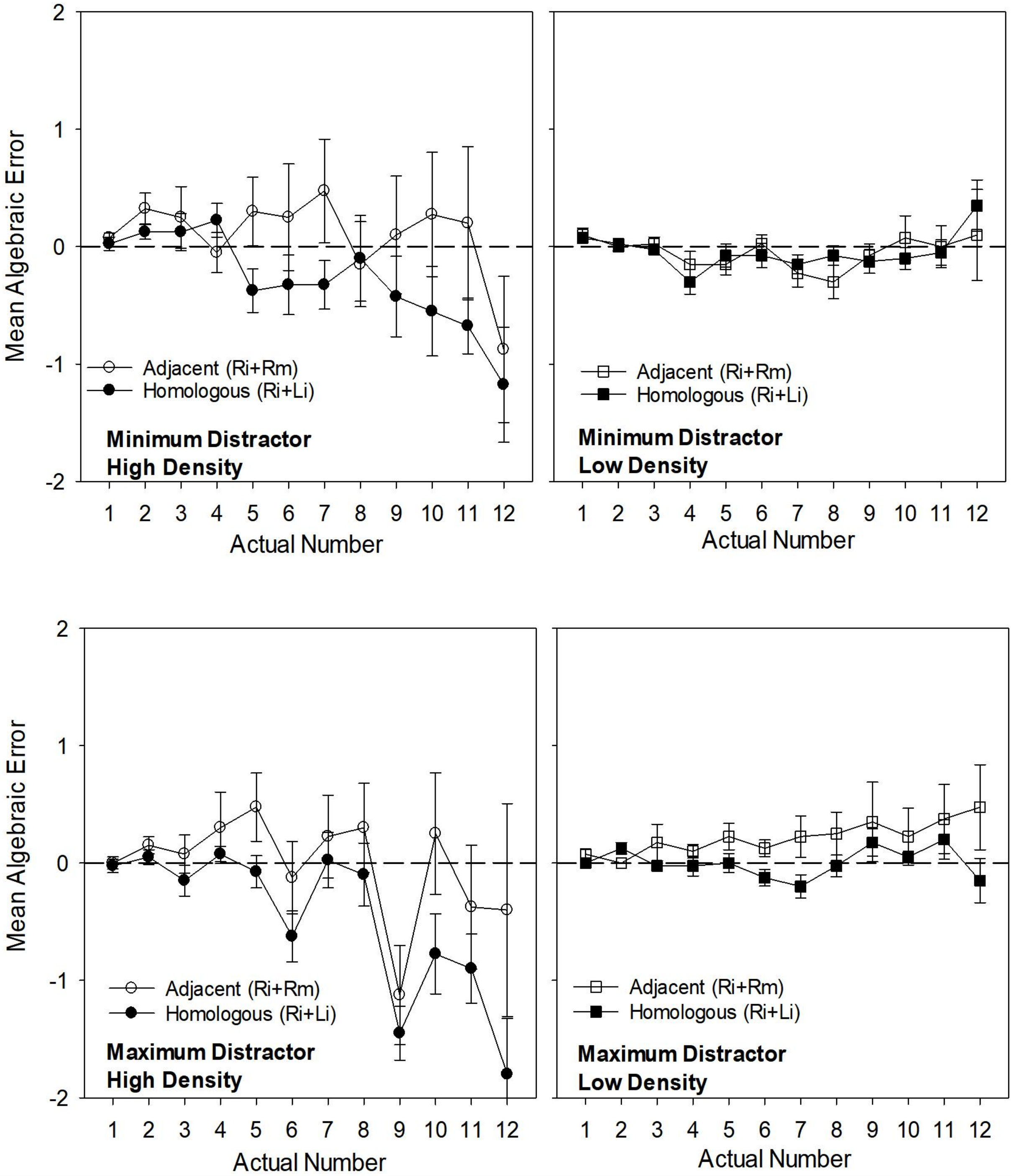

Figure 4. Mean algebraic error of the perceived as a function of actual numerosity, the finger-hand combination (Ri+Rm and Ri+Li), the density (high and low) and distractors (maximum and minimum). The dashed line corresponds to the ideal response pattern.

Absolute Error

Due to the influence of the average value and estimation direction (overestimation/underestimation), the magnitude of the perceived number and algebraic error may lead the perceived number to differ from the actual number to a smaller extent. The study of absolute error was investigated to measure the variability (or the noise) of the perceived numbers more precisely.

A four-way actual number (12) x density (2) x finger-hand combination (2) x distractors (2) ANOVA was conducted on perceived numerosity (pN). ANOVA revealed three main effects were found, actual number (aN): $F(2.460, 46.740) = 31.713$, $p<.000$, $\eta_p^2=.625$; density $F(1,19)=115.140$, $p<.000$; and finger-hand combination, $F(1, 19) = 4.511$, $p = .047$, $\eta_p^2 = .192$. We found no main effects of distractors, $F(1,19)=1.022$, $P=0.325$. Two interactions were significant: actual number by density, $F(4.026, 76.500) = 22.730$, $p < .001$, $\eta_p^2 = .545$; and density by finger-hand combination , $F(1,19)=4.922$, $p=.039$, $\eta_p^2 = .206$.

In high density with minimum distractors conditions, the overall patterns for RiRm and RiLi were similar: the absolute error increased as the actual number increased. However, RiRm and RiLi were similar when the absolute error became smaller as the actual number changed from aN3 to aN4. Then, the absolute error increased from aN4 to aN5, and the slope of the absolute error steepened markedly.

In high density with maximum distractors conditions, the overall pattern of absolute error of both RiRm and RiLi increased with increasing actual numbers. RiRm and RiLi both showed that, at aN4, both exhibited a minor downturn, and the absolute error fell below aN3's absolute error. At aN5, however, the absolute error increased dramatically (from aN4 to aN5).

In high-density condition, it is notable that the absolute error of RiRm was consistently greater than that of RiLi. From aN10 to aN11 and aN11 to aN12, RiLi tended to fall suddenly and then significantly increase. In addition, there was a decline in the absolute error at aN4 and a dramatic increase in slope between aN4 and aN5 for both RiRm and RiLi.

In low density with minimum distractors conditions, RiRm and RiLi were closer to the actual number at aN1 to aN3. RiRm and RiLi largely overlapped from aN1 to aN8. RiRm absolute error increased after aN9; but RiLi almost kept constant from aN4.

In low density with maximum distractors conditions, the overall trend of RiRm rose as the actual number increased. The absolute error of RiLi was nearly identical from aN1 to aN10, with minor variations; from aN11 to aN12 showed a greater absolute error than aN1 to aN10. RiRm and RiLi exhibited greater overlap between aN1 and aN7.

(The linear regression pattern can be seen in Figure 5 below)

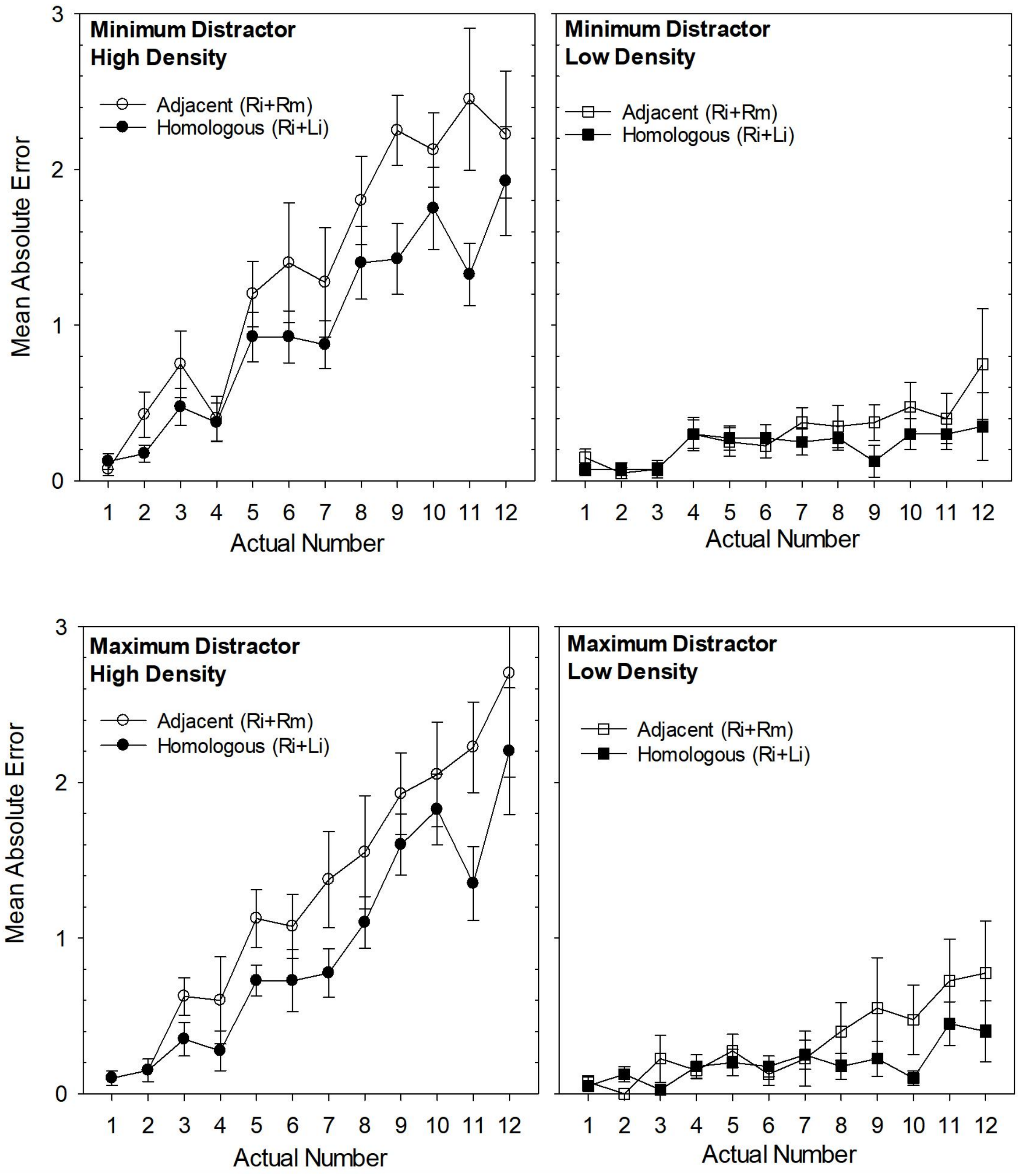

Figure 5. Mean absolute error of the perceived as a function of actual numerosity, the finger-hand combination (Ri+Rm and Ri+Li), the density (high and low) and distractors (maximum and minimum).

Confidence

Confidence levels provide two pieces of primary information. Firstly, they reflect the level of certainty that participants have about their perceived numerical answers. Secondly, if participants use estimation mode, their confidence value may be lower compared to when they use subitizing and counting mode.

A four-way actual number (12) x density (2) x finger-hand combination (2) x distractors (2) ANOVA was conducted on perceived numerosity (pN). ANOVA revealed four main effects, actual number (aN): $F(4.181, 79.437) = 47.490$, $p<.000$, $\eta_p^2 = .714$; distractors, $F(1,19)=14.791$, $P=0.001$, $\eta_p^2 = .438$; density, $F(1, 19)=97.571$, $p<.000$, $\eta_p^2 = .837$; and finger-hand combination, $F(1, 19) = 4.891$, $p = .039$, $\eta_p^2 = .205$. One interaction were significant: actual number by density, $F(4.026, 81.139) = 19.517$, $p < .001$, $\eta_p^2 = .507$.

In high density with minimum distractors conditions, the confidence levels of RiRm and RiLi decreased as the actual numbers increased. At aN1 to aN3, RiRm, and RiLi confidence levels substantially overlapped. Interestingly, at aN4, RiRm and RiLi confidence levels appeared to improve.

In high density with maximum distractors conditions, RiRm and RiLi's confidence levels decreased as the actual number increased. Both RiRm and RiLi were less certain at aN10, with aN10 being the least confident. In this condition, RiRm was less confident than RiLi.

In high-density condition, it is important to note that RiLi was typically more confident than RiRm.

In low density with minimum distractors conditions, the confidence levels of RiRm and

RiLi did not decline significantly as the number increased but stayed virtually constant. The patterns of RiRm and RiLi's confidence levels were essentially identical, with both remaining at a high level of confidence.

In low density with maximum distractors conditions, RiRm and RiLi did not demonstrate a significant decrease in confidence with increasing numbers, although they exhibited a slight negative trend. The linear trends of RiRm and RiLi were essentially the same, with both remaining at high levels of confidence.

(The linear regression pattern can be seen in Figure 6 below)

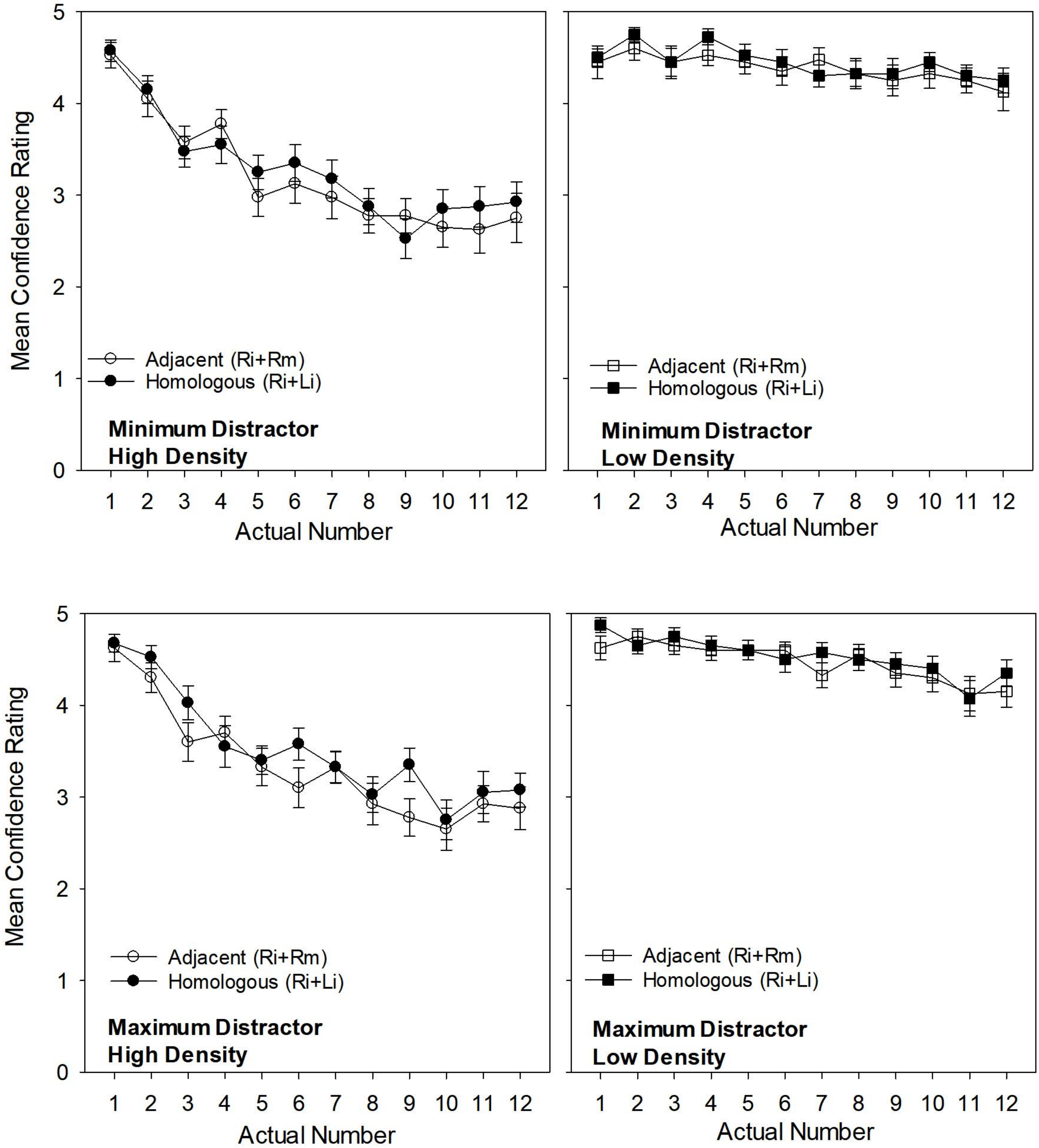

Figure 6. Mean confidence ratings of the perceived as a function of actual numerosity, the finger-hand combination (Ri+Rm and Ri+Li), the density (high and low) and distractors (maximum and minimum).

Discussion

The present study extended the paradigms of Sharma et al. (2018), Browne (2019), and Rendell (2020) by adapting the experimental procedure and factors to meet the objectives of this study. Our aim was to investigate whether participants could accurately report the number of raised dots they touched with their fingerpads after a single scan from far to near. We varied the number of raised dots from 1 to 12 while manipulating three other factors: density of dots, distractors, and hand-finger combinations. The experiment aimed to examine the relationship between actual numerosity (aN) and perceived numerosity (pN) and determine how the four mentioned factors affect this relationship. One unique feature of this experiment was the inclusion of distractors as a factor; this aspect set it apart from previous studies conducted by Sharma et al. (2018), Browne (2019), and Rendell (2020).

The behavioral results data in this experiment supported the following hypotheses: H1 - There is a systematic relationship between the actual number and perceived number during haptic enumeration. H2 - Density affects enumeration accuracy. H4 - Homologous fingers perform better than non-homologous fingers in the enumeration task. However, we did not find any evidence to support H5, and no discontinuity was observed at or around the actual number six. Furthermore, while the presence of distractors was found to be related to accuracy and confidence, the most surprising finding was that we did not find significant evidence to support H3, which assumed that a large number of distractors would lead to lower perceived numbers and worse performance.

To obtain a comprehensive understanding of the results, we will discuss and explain the behavioural results against the four factors of numerosity (number of dots), density, distractors, and hand-finger combinations in turn.

Haptic Enumeration and Numerosity

Our experiment found that, as expected, actual numbers had a strong relationship with the perceived number, absolute error, and confidence. As the actual number increased, the magnitude of the perceived number and absolute error increased, while confidence decreased. This suggests that participants' performance becomes less accurate at larger numerosities. Although, overall, the perceived number increased as the actual number increased, we did not find that the actual number had a main significant effect on algebraic error. Furthermore, and all linear plots of algebraic error did not reveal systematic under- and overestimation. In addition, our experimental data failed to identify a discontinuity in the linear slope discovered by Sharma and colleagues (2019), Browne (2019), and Rendell (2020) at or about aN=6. In what follows, we discuss the inconsistency with the findings of previous studies and the possible enumeration modes used by participants.

Subitizing and Bi-Linear Fit Model

Sharma and colleagues (2019), Browne (2019), and Rendell (2020) found that when aN was close to six, the line graphs exhibited a different slope, which is crucial in confirming the existence of subitizing and may diagnostic a change in enumeration modes. In the visual domain, when measuring enumeration latency as a function of the number of dots on the display, an apparent discontinuity in slope is observed (Trick & Pylyshyn, 1993). When plotting reaction time versus the number to be estimated, the data can typically be described by two lines with different slopes: a slope of zero or close to zero (within the subitizing range) and a positive slope (above the subitizing range) (Leibovich-Raveh et al., 2018). The intersection of these two lines is then considered the upper limit of the subitizing range (Leibovich-Raveh et al., 2018). This is also known as bilinear fit, a prevalent method for calculating subitizing range (Leibovich-Raveh et al., 2018).

Therefore, if subitizing exists, observing a bilinear fit model based on accuracy measures, rather than reaction time is possible. The results from Sharma et al. (2019), Browne (2019), and Rendell (2020) showed that when aN ranged from one to six, the slope of the algebraic error function was close to zero; when aN was greater than six, the slope remained highly linear but increased, resembling the pattern observed in the bilinear fit model. However, the algebraic error linear plots from our experiment exhibited only irregular positive and negative fluctuations, and the overall trend of absolute error linearity increased as the actual number increased. Neither the algebraic nor the absolute error linear plots provided any evidence of a discontinuity favoring a bilinear fit.

Subitizing and Pre-Attentive Mechanisms

Subitizing enables rapid and accurate perceptual processing, even when directed attention is not fully engaged. Therefore, subitizing must rely on mechanisms that can process multiple items simultaneously. Trick and Pylyshyn (1993) proposed pre-attentive mechanisms to explain subitizing. The mechanism underlying subitizng operates in parallel but has a limited capacity stage after feature detection and grouping and prior to the allocation of spatial attention. For aNs lower than the bilinear fits inflection point, enumeration can rely on pre-attentive mechanisms. According to these mechanisms, humans have a limited capacity to distinguish between groups of features with only a small number of spatial reference tokens. When the number of items exceeds this capacity, a different process, such as counting, is required (Trick & Pylyshyn, 1993). Thus, the theory of pre-attentive mechanisms is said to explain the bilinear fit model's discontinuity.

It is important to note that the pre-attentive mechanism requires processing received information simultaneously and treating it as a whole. In the visual domain, for instance, the pre-attentive mechanism parallels the processing of target objects across the entire visual input (Zhaoping & Dayan, 2006). This is evident in the visual experimental paradigm, where participants are simultaneously presented with varying targets on a display screen. In haptic experiments examining subitizing, multiple stimuli are presented simultaneously on the participants' skin, as demonstrated in studies conducted by Gallace et al (2006) and Riggs et al (2006).

Evidence of Subitizing

After analyzing the data from our experiment, we did not observe a bilinear model. Instead, the data exhibited a linear pattern across the entire range of aNs. As a result, there was no direct or indirect evidence to support subitizing. However, we could still gain insights into subitizing by considering pre-attentive mechanisms and factors such as accuracy, confidence, and experimental paradigm.

Subitizing is a cognitive phenomenon where individuals can quickly and accurately perceive the number of items in a visual display without counting. This ability is thought to be due to pre-attentive mechanisms that enable simultaneous and parallel processing of multiple stimuli. However, our experiment's paradigm posed a challenge in achieving the simultaneous and parallel perception of the stimuli through participants' fingerpads. This was because the total length of the dot array increased with the actual number of dots, requiring participants to sequentially move their fingerpads from far to near to touch and perceive all the dots.

Zhang and colleagues (2017) conducted a study to compare the effects of finger movements in a distal and proximal direction on the fingertip's fingerprint area and contact area when in contact with a flat surface. They found that proximal direction movement reduced the contact area of the fingertip on a flat surface, causing fingerprints to stack on one side of the fingertip while those on the other side near the finger pulp remained relatively smooth. Zhou and colleagues (2020) also investigated how finger movement direction affected the fingertips' contact area, and their results were consistent with those of Zhang and colleagues (2017). In our experiments, we did not have an instrument available to measure the area of contact between participants' fingertips and the stimulus. However, a distal to proximal direction touch may have reduced the contact area between participants' fingertips and the stimulus, which could have limited their ability to perceive multiple dots simultaneously. For instance, the total area for three dots was greater than that for two dots. Nonetheless, due to the far to near sequential touch pattern, participants' fingertips could perceive the raised dots in the same row simultaneously, at least in principle. In our experiment, the maximum number of dots appearing in the same row was two, so participants could perceive one to two dots simultaneously.

After analyzing the linear plots of absolute error and confidence, we found that the minimum absolute error for RmRi and RiLi under high-density conditions occurred at aN=4, between aN=3 and aN=5. The highest confidence for RiRm and RiLi occurred at aN=4, between aN=3 and aN5=, under high-density and minimum distractors conditions. Although in high-density and maximum distractors conditions, RiLi's confidence decreased as the actual number increased, RiRm's confidence was highest at aN=4, between aN=3 and aN=5. These results indicate that participants performed better under high-density conditions when the actual number was four between aN=3 and aN=5. In our experiments, when the actual number of dots was four, the arrangement of the dots was identical to that of a four-sided die, with a distance of 0.5 cm between the two rows of dots. The smallest distance humans can distinguish is one millimeter (Lamb, 1983), and 0.5 cm may represent two discrete stimuli for tactile perception. Thus, participants could perceive two consecutive and simultaneous dots as two distinct stimuli. Using our experimental data and the characteristics of the arrangement of the raised dots when the number of dots was four, we speculated that tactile perception might be capable of subitizing the number two.

Furthermore, in Sharma et al.'s (2019) experiment, participants touched the Braille dots horizontally, from left to right. This suggests that the fingerpads were capable of detecting the presence of raised dots simultaneously when they appeared in the same column. In the experiment, raised dots representing the numbers two (aN=2) or three (aN=3) were arranged and appeared in the same column under high-density conditions. Sharma et al. (2019) found that participants' performance was accurate for both aN=2 and aN=3, and their confidence levels were similarly high.

Overall, we did not find a bilinear fit model to support the existence of subitizing in our experiment. However, if subitizing requires the support of pre-attentive mechanisms, then it is feasible to subitize only those stimuli that are perceived simultaneously by the fingers. Considering the characteristics of our experimental paradigm and results from Sharma and colleagues (2019), it is observed that two and three dots could be scanned by one fingerpad simultaneously, with high accuracy and confidence.) This finding suggests that haptics may have the ability to subitize numerosity only in the range of one to three. It is important to note that in both Sharma et al.'s (2019) study and our own, the low-density condition allowed the dots to be perceived as distinct individual dots. Therefore, our discussion of subitizing focuses on its occurrence in high-density conditions.

Haptic Enumeration Modes

We found that the absolute error, representing the degree of deviation or distance between perceived and actual number, increased with actual numbers and was higher in the high-density condition than in the low-density condition. However, we did not observe systematic over- or underestimation and found a bilinear fit model. Considering these results, we raise the question of which enumeration strategy was employed by the participants in this study.

Recent studies in the field of vision suggest that individuals perceive numerosity more quickly and accurately when items are grouped into small clusters within the subitizing range (Maldonado Moscoso et al., 2022). This process, known as groupitizing, may automatically engage precise computational and fact retrieval strategies such as multiplication tables or simple addition (Maldonado Moscoso et al., 2022). Thus, groupitizing relies on accurate subitizing and calculation abilities (Maldonado Moscoso et al., 2022).

Starkey and McCandliss (2014) conducted a study on groupitizing in children. In this study, children were asked to rapidly enumerate one to eight dots that were either randomly distributed or grouped together (with a maximum of three items per group). The results showed that grouping items led to faster and more accurate enumeration. Interestingly, the study found that the groupitizing effect was not evident in kindergarten children but was enhanced in children from grades 1-3 (Starkey & McCandliss, 2014). In a more recent study, Maldonado Moscoso and colleagues (2022) used fMRI to investigate the neural mechanisms underlying groupitizing. In this study, participants were presented with 8, 12, and 16 dots, which were either randomly distributed or grouped in clusters; each cluster included 2-4 or 2-6 dots. Participants were asked to report the number of dots seen. Maldonado Moscoso and colleagues (2022) found that the brain activity patterns of participants were altered when participants employed the groupitizing strategy instead of simply estimating numbers without grouping; these altered brain regions involved basic calculation. According to Starkey and McCandliss (2014) and Maldonado Moscoso and colleagues (2022), groupitizing effects were present in children from first grade to adult, and groupitizing was associated with subitizing and calculation abilities.

Based on our previous speculation (that participants could perceive a minimum of two dots simultaneously in our experiment and might be able to subitize the number two), we inferred that participants in our study might have employed a groupitizing mode to perceive the number of dots. The participants likely processed the number of dots perceived by their fingertpads simultaneously by moving their fingers downwards and then calculated the final total number of dots. When the number of dots in each cluster was one, grouping could also be interpreted as counting, i.e., encoding the items one by one. Furthermore, as mentioned previously, humans can distinguish the smallest distance of one millimeter (Lamb, 1983). In our study, we observed that in the high-density condition, the smallest distance between two dots was approximately 0.3 cm, whereas in the low-density condition, the smallest distance between two dots exceeded 0.3 cm. Hence, when the dot spacing is wide enough for participants to perceive them as separate entities, it is highly probable that participants encoded the dots individually (ie., counting)

Henik (2021) noted that attention was necessary to perceive numbers, to focus on their numerical features while excluding irrelevant distractions. Hyde (2011) hypothesized that when items were presented under conditions that allowed for individual selection, they were represented as distinct mental items through parallel separation, rather than as a numerical magnitude. Conversely, when items were presented outside of attentional constraints (e.g., too many, too close, or under a high attentional load), they were processed by the approximate number system (ANS) and represented as a single mental numerical magnitude. Therefore, we inferred that participants in our experiments used groupitizing and counting as numerosity enumeration modes. However, the attentional load and distribution varied depending on the number, density, distractions, and hand-finger combination. Therefore, depending on their attentional resources, participants might have switched between the three numerosity enumeration modes of groupitizing, counting, and estimation.

Haptic Enumeration and Density

According to the results of our experiment, the data showed that density had a significant effect on absolute error and confidence. Density interacted with number and had a significant effect on the perceived number, algebraic error, absolute error, and confidence. Thus, the interaction between density and number strongly influenced the enumeration performance in our experiment. Furthermore, the interaction between density and distractors, and the interaction between density and hand-finger combination also had a significant influence on the enumeration accuracy. Therefore, density played an important role in haptic enumeration in our study.

Density Discrimination

Reading braille is a skill for blind individuals. The braille block is a six-cell, three-by-two rectangle, with each character corresponding to a distinct dot pattern. Accordingly, it has been suggested that the recognition of braille patterns is based on recognizing the shape outlined by the dots of each character (Loomis, 1981). However, extensive research with sighted and blind individuals has revealed that braille recognition is related to texture, and variations in the spacing or density of the dots (Millar, 1986). The linear plots and p-values from our experiment demonstrated that, even when density significantly affected accuracy, the non-braille readers still demonstrated high accuracy in both density conditions. This indicates that the participants recognize braille relatively effectively. Some studies, such as the one by Grant et al (2000), have shown that even inexperienced braille readers can distinguish between different textural densities. In Grant et al (2000) experiment, embossed dots similar to those used in braille were used, and blind and sighted participants were required to make perceptual judgments based on the offset of three central dots in a longitudinal arrangement. The standard stimulus was a column of three relief dots measuring approximately 0.3 mm in diameter and 2 mm from center to center. The comparison stimulus was a similar pattern in which the central dots were horizontally offset by a range of 0.1 to 1 mm. Participants were instructed to place the fingerpad of their index finger vertically on the pattern, and any scanning movements were prohibited. Participants were instructed on each trial to compare side-by-side standard and comparison stimuli. The goal was to determine whether the stimulus with the offset was on the right or left (Grant et al., 2000). participants were free to move back and forth between the two modalities as often as they wanted, with no time limit. The experiment was tested on participants four times. The initial thresholds were determined by the first test's results, while the final thresholds were determined by combining the performance values of the last two tests. Due to the similarity of the stimuli to braille dots, the experiment predicted that the blind individual's experience with braille reading would contribute to their performance on this task, resulting in an excess threshold value (Grant et al., 2000). However, the experiment's results revealed that blind participants performed better than sighted participants, but only for the initial thresholds. Within three or four sessions, normally-sighted individuals attained thresholds comparable to those of blind individuals. This confirms, according to Grant et al. (2000), that perceptual learning on this task is rapid and that blind people are not truly super tactile but rather acquire greater proficiency with non-visual modalities due to complex processes such as learning and attention. Moreover, Grant et al. (2000) argued that this result is consistent with Gibson's (1969) assertion that selective attention and fine-tuning active exploration are crucial for perceptual learning. As participants were not permitted to move during the Grant et al. (2000) experiment, the optimal performance of haptics was not taken advantage of (Grant et al., 2000). Scanning enhances tactile performance on the texture task, possibly as a result of an increased firing rate of SAl afferents nerves and recruitment of purely dynamic mechanical receptor types (Grant et al., 2000). Our participants were still able to achieve the same range of values as braille readers even under conditions that prohibited hand movement. Notably, Grant et al. (2000) utilized a much narrower range of densities than the current study.

The sensitivity of non-braille readers to density was also demonstrated by the experiments of Lamb (1983), in which plastic strips were made with either a square arrangement of raised dots (standard surface) or a number of rectangular arrangements of raised dots (modified surface), with the spacing of the raised dots differing slightly between the standard and modified surfaces. The dots were centered at a distance of 1 or 2 mm (period), and their diameter was one-third of the period. Participants looked pairs of surfaces and were asked to determine whether each pair contained two identical standard surfaces or a standard surface and a modified surface. The direction of touch was categorized as either vertical or lateral. The results showed when participants actively moved their fingers over the surfaces, their performance was nearly unbiased(Lamb, 1983). There was a linear relationship between discrimination and the distance between the dots on the two surfaces (Lamb, 1983). At a 75% accuracy rate, participants were able to distinguish surfaces in which the period of the dots varied by only 2% (Lamb, 1983). Although there was substantial variation in the velocity profiles of the various movements, the participants' performance was nearly independent of the method of movement employed (Lamb, 1983). When the performance of lateral touch surfaces was compared to that of vertical touch, it was found to be twice as poor, but they still demonstrated extremely high discrimination (Lamb, 1983). In addition, their performance decreased significantly when participants' fingers touched the surface for only 0.3 seconds as opposed to 1 to 2 seconds (Lamb, 1983). Moreover, participants described surfaces with increasing period sizes as rougher (Lamb, 1983). Frequency coding was suggested by Lamb(1983) to explain this phenomenon. By comparing the difference in the period to the frequency of a continuous line of dots on a standard surface as it passes over a point on the skin (Lamb, 1983). These distinguishable period increments can also be considered in terms of frequency (Lamb, 1983). It is investigated from a frequency encoding standpoint that a 3% increase in the period of the longitudinal dot alignment dimension results in a 3% decrease in the frequency of the dotted line for any given velocity(Lamb, 1983). Thus, it was observed that the ability of participants to discriminate differences in the frequency of dots striking the skin generated density discrimination (Lamb, 1983). This is comparable to Hughes, Van Gemmert, and Stelmach's (2011) findings. Hughes, Van Gemmert, and Stelmach (2011) suggested that motor control plays a significant role in actions involving texture perception. Although finger movements appear to be smooth when reading braille, acceleration and deceleration alternate continuously. Contact between the skin and the textured surface may contribute to fluctuations in finger velocity. The slower the average speed of the hand/finger movement, the higher the frequency of the number of fluctuations between acceleration and deceleration. In contrast, as finger movement speed increases, acceleration, and deceleration fluctuations decrease (Hughes et al., 2011). Sharma and colleagues (2019) experiment also examined the fingers of the most and least accurate participants and found no difference in the type of movement they executed. Both participants performed continuous acceleration and deceleration; however, the participant with the highest accuracy who scanned at a slower speed had more acceleration and deceleration (Sharma et al., 2019)

In our study, the experiment did not impose any time constraints on the participants, participants could move their fingers at any speed they chose, and the active longitudinal touch paradigm served as the guiding principle throughout the experiment. Based on the preceding experiments' findings and conclusions, it is likely that the participants had a high frequency of acceleration and deceleration in perceiving the density of braille by using slow-moving velocity. So that participants perceived the space between each point as complete as possible. Moreover, a braille novice reader also has the ability to extract braille information. Therefore, it is reasonable to believe that even at high densities, the non-braille readers in this experiment could demonstrate excellent perception. Current experiment data revealed at least one and no more than three interaction effects of density with number, distractors, and hand and finger combination under different dependent variables. These interactions significantly affected the participants' response accuracy. In the previous experiment (Grant et al., 2000 and Lamb,1983), the participants' tasks were to judge the density; in the present study, the participants' tasks were to report the number of perceived dot sets. Clearly, the task presented to the participants in this study was more challenging than in previous studies. Observing the linear plots and p-values revealed a decreasing trend in correctness as the number increased in the high-density condition. Consistent with Sharma's (2019) and Rendell's (2020) experiments with comparable outcomes.

Haptic Enumeration and Non-numerical Continuum Magnitude

The question of whether non-numerical continuum magnitudes, such as density, can affect enumeration remains a topic of debate. In the field of vision, some researchers argue that numerical perception and non-numerical continuum magnitudes cannot be distinguished. For example, Lourenco and Aulet (2023) contend that numerical continuum magnitude information is continuous throughout visual perception and that number perception is intertwined with non-numerical continuum magnitudes. A theory proposed by Dakin and colleagues (2011) suggests that numerosity is not directly perceived, but rather calculated using the product of density and area. This theory is supported by Picon and colleagues' (2019) demonstration of a direct effect of density on numerical perception. In their experiments, two convex hulls with different densities are displayed simultaneously, and dot arrays are presented in both high- and low-density configurations. The four conditions tested are: both expanded (low density), both compressed (high density), blue expanded (blue dots low density, yellow dots high density), and blue compressed (blue dots high density, yellow dots low density). The Picon and colleagues' (2019) results of the study show that participants are biased towards underestimation when densities are compressed.

In the field of vision, however, a growing body of experimental data demonstrates that numerosity and non-numerical continuum magnitudes can be distinguished under certain conditions. Visual numerosity can be perceived without recalculating area and density (Kingdom, 2016). Anobile and colleagues (2017) segmented visual stimuli using thin lines and investigated the impact of segmentation on visual number perception at varying densities (high, medium, and low densities). The results of the experiment demonstrated that numbers are not derived indirectly from density, texture, or other low-level features and confirmed that at low numerosity, density judgments are unreliable; density seems to be derived indirectly from numerosity (Anobile et al., 2017). Anobile and colleagues' (2017) experimental findings also support the notion that visual number are directly perceived rather than recalculated from area and density. Density and visual number perception are investigated in the experiments of Anobile and colleagues (2016). In the experiments, the number of items exceed the subitizing range, and the number of items are divided into two densities for comparison. In low densities, items are sufficiently spaced apart to be viewed as distinct, individual objects, whereas in high densities, items are crowded together to form a texture. As long as items are not overcrowded, experimental results suggest that estimates of numbers do not depend on a response mechanism to the texture that reflects the ability to discriminate numbers rather than the ability to discriminate texture (Anobile et al., 2016).

According to our experimental data, density significantly affected absolute error, but not algebraic error. Observing the linear plots for algebraic errors revealed no systematic under- or overestimation. This indicates that our experimental data do not support Picon et al.'s (2019) conclusion that participants are biased towards underestimating values when density is compressed. In other words, our experimental findings demonstrate that numerical perception is independent of non-numerical continuum magnitudes (density). However, Rendell (2020) discovered that density significantly impacted both algebraic and absolute error; additionally, linear plots demonstrated systematic underestimation. Our experiments are not consistent with Rendell's findings (2020). Based on the experimental findings of Anobile and colleagues (2016), there is no evidence that the estimation of numbers depends on the response mechanism to texture if the items are not overcrowded enough. We speculate that our inconsistency with Rendell (2020) may be due to the threshold of density. First, in our experiments, the dot array was sufficiently sparse at low densities to be perceived as distinct objects. In high density condition, our experiments did result in a more concentrated distribution of dots; however, in Rendell's (2020) experiments, the densities of dots (both high and low densities) are significantly higher than in our experiments. Inferring from this, our experiments' range of density threshold has not yet reached a specific value that would allow density as a non-numerical continuum magnitude to influence tactile enumeration. Our inferences are subject to additional testing in the future.

Effects of Density Interact With Other Factors

Why does density have many significant effects on the experimental results when interacting with other variables? Perhaps this can be answered by the results of an experiment conducted by Foulke and Warm (1967). They investigated the tactile recognition of metric numerals by sighted secondary school students and blind individuals. Additionally, they attempted to evaluate the effects of two information parameters: complexity and redundancy. In Foulke and Warm's (1967) experiment, four random and redundant forms of complexity were utilized. The stimuli consisted of raised dots with standard braille for the height and spacing of dots. Their complexity is determined by the maximum number of figures that can be generated by the matrix upon which they are based, which in turn is determined by the number of cells in the matrix (Foulke & Warm, 1967). Experiments were performed to generate graphical 3 x 3, 4 x 4, 5 x 5, and 6 x 6 matrices at four levels of complexity, with the average amount of information required to specify a possible random number at each level of complexity being 4.8, 8.0, 11.6 and 15.5 bits, respectively (Foulke & Warm, 1967). Each level of complexity required an average of 2.6, 4.6, 6.9, and 9.5 bits to specify one of all possible redundant digits (Foulke & Warm, 1967).. To generate redundant digits at each level of complexity, a random sample of column heights was taken for each digit, limiting the occurrence of each column height to a single instance in each digit (Foulke & Warm, 1967). Participants were required to report the number of touched points. The participants were told that speed and accuracy were essential to judgments. At the beginning of each experiment, participants were given a chance to practice. The data revealed that as stimulus complexity increased, blind and sighted individuals' performance efficiency decreased (Foulke & Warm, 1967). Braille readers performed better than blind controls in the tactile recognition of dot patterns in the 3 x 3 and 4 x 4 matrices but not in the 5 x 5 and 6 x 6 matrices (Foulke & Warm, 1967).. The difficulty of figures from the two lower complexity levels was roughly comparable to figures from the two higher complexity levels (Foulke & Warm, 1967). However, figures from the two higher complexity levels were manifestly more difficult than figures from the lower complexity levels (Foulke & Warm, 1967).. This sudden increase in difficulty suggests a change in one or more determinants, such as the nature of the task participants must complete (Foulke & Warm, 1967).. It is possible that perceiving the numbers generated by two larger matrices would require participants to modify the method due to their larger area (Foulke & Warm, 1967).. The figures produced from the two smaller matrices are, in most cases small enough that a single fingertip can cover the whole figure (Foulke & Warm, 1967).. On the other hand, the remaining matric is too large to be examined at once, necessitating a continuous perception of the pertinent details(Foulke & Warm, 1967).

In our experimental study, linear plots of algebraic errors, absolute errors, and confidence in the high density + distractors condition revealed that participants in this condition made more mistakes and had less confidence than those in other conditions. Since the complexity of the stimuli was most complex in this condition, as the number of stimuli increased, their complexity also increased. For instance, as the number exceeds six, the length of the dot matrix increases (from one braille block to two braille blocks). According to Foulke and Warm's (1967) conclusion, this is when the participants' fingertips must sequentially perceive the dot matrix's details. High densities are intrinsically more complex than low densities. In current experiments, adding other independent variables (number, distractors, and hand and finger combination) made the task more challenging. Consequently, it may be possible to explain why density significantly affects the absolute error rate. In addition, the interaction of density was significant across all four dependent variables; for example, there were three significant interactions of density for the magnitude of the number perceived. Comparing the experiments to those of Rendell (2020), which also involved longitudinal touch, Rendell (2020) discovered a main effect of density and a significant interaction effect. By examining the linear plots of our experiments, we discovered that our participants were more accurate than Rendell's (2020). As the density used by Rendell (2020) was more condensed than the density of the present experiment, it may be that the complexity of the stimuli in the present experiment was lower than in Rendell's (2020). Therefore, speculating based solely on density, could the high accuracy of participants' responses in the current experiment result from the stimuli's reduced complexity.

It is noteworthy that the results of the experiment conducted by Foulke & Warm (1967) revealed a general tendency toward more efficient performance with random figures than redundant figures. The authors concluded that when humans perceive a form, they do not necessarily process all the information in a figure. Instead, distinctions are typically based on the parts of the figure where the observer has the most uncertainty - the unique details. Therefore, random numbers perform better than redundant numbers. This is comparable to a few of the current experimental participants' experiences. At the end of the current experiment, some participants reported that when identifying high numerosity dots, they preferred to rely on the absence of bumps or vacancies in the dots. This phenomenon may be comparable to the results observed for random and redundant numbers (Foulke & Warm, 1967). Furthermore, another interesting idea arises: could participants have employed other strategies (e.g., pattern recognition) to perceive numerosity by becoming familiar with the pattern through multiple scans? Future studies can explore this potential possibility.

Haptic Enumeration and Distractors

In the same distal to proximal direction touch experimental paradigm, Browne (2019) and Rendell (2020) did not manipulate or analyze distractors. In our study, we manipulated distractors arrays to test whether they affected the accuracy and confidence of haptic enumeration.

Our data analysis revealed that distractors arrays did not strongly influence enumeration accuracy, in terms of perceived number, algebraic error, or absolute error. Additionally, distractors arrays did not bias participants in any specific direction; only confidence was affected by the distractors arrays. This finding is interesting because it suggests that participants were able to largely ignore distractors arrays during simultaneous haptic perception. Additionally, our results do not support our original assumption of simultaneous numerosity adaptation, as no systematic underestimation or overestimation was observed. To further understand these results, we discuss haptic perception and cognition in relation to attention in the following.

Haptic Perception and Directed Attention

In the experiments conducted by Browne (2019) and Rendell (2020), participants were instructed only to scan raised dots with one or two fingers and verbally report the total number of raised dots. In contrast, in our experiment, participants received clear and specific instructions on which finger to use to scan the stimulus before each trial, and only one finger was needed to report the number of raised dots. As a result, participants could fully trust and predict the location of the stimulus. As mentioned before, many haptic studies have demonstrated that haptic attention can be directed, and that directed attention can influence haptic task performance.

For instance, Posner's (1978) paradigm, which included 20% invalid cues and 80% valid cues, demonstrates that a priori knowledge of potential target locations results in a shift in covert attention to the intended target location. In other words, prior attentional localization increases attention to the stimulus within that location. The attentional network model proposed by Posner and Petersen (1990) adequately explains the attentional component of haptic directed attention experiments. This model describes the workings of the human attentional system and identifies three sub-components of human attention: alerting, orienting, and executing. Alerting involves the use of a warning signal to produce a phase change in alertness before a target event, with the warning signal altering the rate of directed attention in response to the signal (Petersen & Posner, 2012). Orientation emphasizes the capacity to prioritize sensory input by selecting a pattern or location. However, moments of target detection can disrupt the entire system, delaying the detection of another target. These processes are related to the attentional system's limited capacity and consciousness itself and are frequently referred to as focal attention (Petersen & Posner, 2012).

In our experiment, participants were instructed before each trial on which finger to focus and report the number of perceived dots. Therefore, it is highly probable that participants activated the alert signal before the start of the trial and then directed their attention to the instructed finger. This suggests that participants' attention was successfully directed due to the prior instruction. Since our results suggest that participants were able to ignore raised dot stimulation on the other finger, the question arises whether they were able to allocate attention exclusively to the target finger.

Non-Single Channel Model of Haptic Perception

First, it needs to be refuted that participants can automatically block or ignore tactile information from the distractors. Due to the fact that tactile perception is not a driven by single-channel model, participants can perceive the presence of distractors even if the distractors had no effect on number perception. In a model with a single channel, only one piece of information can enter the system at any given time. In addition, the single-channel model assumes that attention rapidly shifts between the various sensory channels (Shiffrin et al., 1973). If the single-channel model holds, then it is possible that, in the current experiments, participants selectively block information from distractors due to their ability to predict the location of the target stimulus and allocate their attention to the target's finger. Shiffrin and colleagues (1973) used a paradigm to determine whether the ability of humans to detect near-threshold vibrotactile stimuli at a given spatial location depends on the total number of signals that must be monitored simultaneously in order to determine whether the tactile sensation is a single-channel model. Two conditions comprised the experiment: sequential and simultaneous. In the simultaneous condition, one of three locations would present a vibrotactile stimulus. The participants were asked to report the location of the stimulus verbally. As the participants did not know which location would receive the signal, all three locations must be monitored concurrently. In the sequential condition, the signal appeared sequentially at the designated location. The experiment predicted that performance would be inferior in the simultaneous condition, which required participants to divide their attention among three potential sources of tactile stimuli. In addition, the single-channel model predicted a significant advantage for the sequential mode, as attention was always focused on the pertinent signal point. However, experimental data revealed that performance was identical under both conditions, and experimental model calculations demonstrated that the single-channel model could be excluded.

In numerous subsequent experiments utilizing various paradigms of tactile perception, it has been demonstrated that participants can perceive the total number of stimulus points when the skin is simultaneously stimulated at multiple sites. Participants can even recognize stimuli patterns present simultaneously on different bodylocations (e.g., Gallace et al., 2006; Geldard & Sherrick, 1965; Craig, 1985). As tactile sensation is not a single-channel model; it is difficult or impossible for participants to block tactile information from a distractor automatically. The question that needs to be asked, then, is whether the brain can, through conscious allocation of attention, leave the received distractors signal unprocessed to have sufficient resources to allow for adequate processing of the target, assuming that perception cannot be blocked or ignored.

Haptic Attentional Blink

The phenomenon of the "attentional blink" (AB) has received extensive research in the field of vision. AB is an inability to detect a second target (T2) when it appears within

180-500 ms of the first target (T1) in the field of vision (Beanland & Pammer, 2012). The attentional blink occurs when other task-processing overwhelms the attentional process (Beanland & Pammer, 2012). Chun and Potter (1995) proposed a two-stage model to explain the attentional blink. In the first stage of processing, stimuli are quickly detected, and various characteristics are analyzed to provide a basis for target selection. In stage 1, nearly all stimuli (including distractors items) are processed, but stimulus presentation is brief and highly susceptible to overwriting, particularly in the RSVP situation(which entails presenting a sequence of visual stimuli in quick succession at a constant rate) . Task-related items are selected on the basis of initial Stage 1 processing, but to reach consciousness, stimuli must move to a second processing stage of limited capacity. Stage 1 processing is fast, whereas stage 2 processing is more intensive. The only exception to this rule is when T2 comes after T1 and uses the same target recognition criteria (for instance, both targets are letters), in which case both targets enter stage 2 simultaneously, and a lag will occur with T1 retained (Chun & Potter, 1995). Once T1 processing is complete (up to 500 milliseconds), T2 can proceed to stage 2, which explains the recovery from the late lag observed in the attentional blink (Chun & Potter, 1995).

Few studies have examined the role of attention in the conscious perception of tactile events. However, an interesting experiment involving the sequential presentation of individual stimuli has been interpreted within the context of the 'attentional blink' phenomenon. In experiments conducted by Dell'Acqua et al. (2006), it was demonstrated that attentional capture in the tactile context is not fully automatic. In the Dell'Acqua et al. (2006) experiment, the participants' fingers, on one hand, were presented with a fixed time interval of 600 ms before the first T1. Two possible SOAs (360 or 800 ms), followed T1, followed by a second (T2), were presented to the fingers of the other hand. Participants had to identify which finger was stimulated first. There were two experiments blocks: either T1 and T2 needed to be reported, or T1 was ignored, and only T2 was reported. The experiment comprised four sub-experiments. In the first experiment, the T1-T2 pair was presented with the same probability and pattern. In the second experiment, on the basis of experiment one, participants were informed of the order of the position of the stimulated hand. In the third experiment, T1 was presented in a different pattern from T2; the rest of the experimental conditions were the same as in experiment one. In the fourth experiment, the presentation pattern of T1 was different from that of T2, and participants were informed beforehand which hand would receive the stimulus first. The results of the fourth experiment revealed that the accuracy of reporting T2 decreased significantly with decreasing SOA when T1 was required to be reported, but SOA had no effect when T1 was required to be ignored. This result suggests that, under appropriate conditions, participants can ignore tactile distractions that precede the tactile target. The lack of an SOA effect in the ignore T1 trials contradicts the results of the first three experiments. Combining a predictable sequence of hand stimuli with differences in tactile patterns between T1 and T2 clearly allowed participants to ignore T1. This finding suggests that, from a haptics perspective, attentional capture cannot be entirely automatic. Furthermore, the missed T2 observed in Experiments 1–4 was not because the hand passing through T2 was momentarily unnoticed but rather because the processing of information in T2 was temporarily delayed in the shorter SOA due to the AB effect. In terms of haptics, although certain stimuli sometimes seem to attract attention involuntarily (especially emergent stimuli), there is good evidence that the extent to which a stimulus attracts attention (even if it is emergent) is controlled by the interaction between aspects of the stimulus and the observer's current intentions and goals (Dell'Acqua et al., 2006). Attentional capture can be modulated by top-down attention (Dell'Acqua et al., 2006).

While there are numerous experiments showing that directed attention contributes to haptic perception, these experiments are based on haptic detection tasks that do not involve receptor movement, such as head movement. In our experiments, participants were required to move their fingers to continuously scan the stimuli, which involves overt attention; and attention orientation should be multiple and prolonged compared to haptic detection tasks. Therefore, based on the paradigm of our experiment and previous haptic AB effect experiments, it is possible to speculate that participants may have relied on a mechanism similar to attentional blink when perceiving the distractors arrays and target.

In the present experiment, the distractors arrays appeared before the target in the high density + distractors condition. Only when the fingers moved to a certain location could the target and distractors be perceived simultaneously. Therefore, when participants started to scan the two columns of dots, even though they knew which finger would scan the target, their attention (exogenous attention) might have been drawn to the distractors arrays due to the sudden haptic stimulation that occurred after the start of each trial. Thus, participants might have treated the distractors arrays as T1. However, as participants continued to scan the braille, their attention was once again drawn to the target finger when the finger reached the target, due to the unpredictable timing of its appearance. Since participants were instructed to focus on the target finger prior to the trial, they might have consciously paid more attention to it. According to the previously mentioned attention model, attentional orientations emphasize the capacity to prioritize sensory input by selecting patterns or locations. However, the moment of target detection can disrupt the entire system, thereby delaying the detection of another target (Petersen & Posner, 2012). Consequently, when T1 and T2 were simultaneously perceived by fingers, the movement of fingers might have shown an intermission, and conscious attention would have caused the target to be perceived as T1, with the distractors arrays shifted to T2. Even though two fingerpads simultaneously received haptic input from the target and distractors arrays, the attentional system and consciousness prioritized T1 (the target), and the information from T2 (the distractors arrays) was processed only after T1 had been fully processed, resulting in a temporary delay in processing T2, similar to the AB effect.

Similar to the high-density+distractors condition, the distractors arrays appeared before the target in the low-density+distractors condition. However, when the participants' fingers were moved to a certain height, their fingerpads would simultaneously scan the target and distractors arrays. The difference between the target and distractors patterns' features was greater in this condition, with the distractors arrays being more condensed than the target. As a result, participants in the low-density+distractors condition may also employ an AB-like mechanism. The predictable location of stimuli with different pattern features between T1 and T2 enabled participants to successfully ignore distractors arrays.

Therefore, we inferred that directed attention contributes to reducing noise from distractors arrays (considerations in terms of the level of distractors factor) during simultaneous haptic perception, thereby resulting in an insignificant main effect of distractors arrays. Furthermore, during sequential haptic perception, participants may use a mechanism comparable to the attentional blink to assist in the attentional shift between target and distractors arrays. However, due to the lack of relevant literature, this speculation needs to be examined in future experiments

Haptic Perception and Homologous and Non-Homologous Fingers

In our experiment, hand and finger combinations were separated into homologous fingers from both hands (RiLi) and non-homologous fingers from one hand (RiRm). Absolute error and confidence were significantly influenced by the hand and finger combinations. Interactions between hand and finger combinations and density emerged in analyses of magnitude of perceived number, algebraic error, and absolute error. Sharma and colleagues (2019) did not find a main effect or any interaction involving hand and/or fingers on magnitude of number perceived, algebraic error, and confidence; Rendell (2020) only found a significant hand and fingers interaction with response confidence. In order to observe the effect of hand and finger combinations on the degree of perceived deviation from actual numbers, the linear plot of absolute errors was chosen as the primary reference for this study to avoid the effect of averages from overestimation and underestimation. Consistent with algebraic errors, absolute errors revealed that RiLi and RmRi performed better at lower than higher densities. Notably, the absolute errors for RiLi were consistently smaller than those for RmRi, regardless of density and distractors. This pattern was also observed in linear confidence measures, regardless of density and distractor, RiLi typically exhibiting more confidence than RmRi. Thus, our experiments suggested that homologous fingers from both hands performed better than non-homologous fingers from one hand, consistent with

Browne's findings (2019). In order to explore the reasons why RiLi outperformed RiRm in terms of absolute error and confidence, this thesis investigated competitive inhibition in the somatosensory cortex and the effect of attention on the competition for tactile information.

Competitive Inhibition in Somatosensory Cortex

Studies on tactile perception have revealed that inhibition of tactile information occurs in the somatosensory cortex with simultaneous stimulation of multiple fingers. For instance, Biermann et al (1998) utilized whole-head magnetoencephalography (MEG) to measure the cortical interaction of tactile inputs from adjacent and non-adjacent fingers. A pneumatic stimulator was used to apply brief tactile stimuli to the participants' right fingers I, II, or V (I, II, and V denotes the cortical representation of the fingers). Experiment 1 stimulated digits I and II separately or simultaneously; experiment 2 stimulated digits I and V separately or simultaneously. The stimuli were presented at random intervals of 500 milliseconds. Biermann and colleagues (1998) discovered that simultaneous stimulation of adjacent fingers II and I resulted in stronger inhibition than simultaneous stimulation of non-adjacent digits II and V. Base on neural theory, Biermann and colleagues (1998) suggested that when the periphery of two adjacent representations is stimulated, excitatory and inhibitory synaptic connections may overlap and interfere, resulting in a weakening of the tactile information. As the cortical representations of digits I and II are closer together than those of digits II and V, this interference may weaken as the spacing between stimulated cortical representations increases (Biermann et al., 1998). Ishibashi et al (2000) used MEG to investigate inhibition by applying electrical stimulation to both index (II) and middle (III) fingers simultaneously or both index (II) and little (V) fingers simultaneously. The stimulation lasted for 0.2

milliseconds. Ishibashi et al (2000) findings align with those reported by Biermann and colleagues (1998): the inhibition of somatosensory tactile information was most affected when the receptive fields of the fingers were close together. Similar studies can also be found in Ching-Liang Hsieh et al (1995) and Hoechstetter and colleagues (2001). All of these experiments demonstrate that the simultaneous presentation of multiple tactile stimuli within the same hand inhibits information processing in the somatosensory cortex and this inhibition effect decreases as the distance between stimuli increases.

Hoechstetter and colleagues (2001) investigated the presentation of stimuli on the same hand in an experiment in which multiple tactile stimuli were presented in a two-handed condition. They observed no inhibition in SI when both hands were stimulated simultaneously. In SII, the responses to simultaneous two-hand, two-finger stimulation had the same amplitudes, latencies, and waveforms as the responses to separate stimulation of the one hand and one finger (Hoechstetter et al., 2001). No significant activation of the SI was observed following the simultaneous stimulation of both hands, nor was any interaction induced by the stimulation of both hands (Hoechstetter et al., 2001). They concluded that the two hemispheres' and regions' activities were nearly separate (Hoechstetter et al., 2001).

All of these experiments demonstrated the inhibition of tactile information in the somatosensory cortex when simultaneous stimuli are received within the hand; this inhibition effect reduces as the distance to the stimulus source increases. Tactile information between the hands is received and processed by both hemispheres, and since the activity of these two hemispheres can be almost completely separated, there is no significant inhibition of somatosensory information when both hands are presented with simultaneous tactile stimuli.

Considering that our experiments had directed attention (focused on one finger), does directed/oriented attention contribute to tactile perception within the hand?

Effect of Attention on Homologous and Non-homologous Fingers

C. Breitwieser and colleagues (2011) investigated how directed attention affects tactile stimulation using steady-state somatosensory evoked potentials (SSSEPs). The subject's right hand was simultaneously stimulated with vibrotactile stimulation, and the subject's EEG signal was recorded. In the reference stimulus condition, participants were not informed of which finger would be stimulated; in the thumb or middle finger stimulus conditions, participants were instructed to shift their attention to the thumb or middle finger. The experiment consisted of three tasks: attention to the thumb stimulus versus the reference stimulus, attention to the thumb stimulus versus the reference stimulus, and attention to the middle finger stimulus versus the reference stimulus. Participants needed to report which finger received stimulation. In the reference stimulus vs. thumb stimulus and reference stimulus vs. middle finger stimulus tasks, all but one subject significantly outperformed and was accurate, whereas only two subjects performed accurately in the thumb stimulus vs. middle finger stimulus condition. The findings of C. Breitwieser et al (2011) revealed that directed attention contributes to tactile perception, but difficulties occurred when attention was continuously shifted between two fingers on the same hand.

C. Breitwieser and colleagues' (2011) experiments were comparable to Shiffrin and colleagues (1973) in that they compared the tactile perception differences between simultaneous stimulation of different locations on the skin when attention was focused on a single location versus undirected attention. Shiffrin and colleagues (1973) concluded that subjects performed equally well with or without directed attention. This result contradicts the findings of C. Breitwieser and colleagues (2011). Therefore, it is essential to note that the stimuli in Shiffrin and colleagues' (1973) experiment were located at the thenar eminence on the right hand, 1 cm from the fingertip of the left index finger, and on the forearm; the physical distance between the three stimuli was greater than the distance between the stimuli (two fingertips of the same hand) in C. Breitwieser and colleagues' (2011) experiment.

Consider the finding of tactile competition in the somatosensory cortex that the effect of inhibition is smaller when the stimuli are more widely spaced or when cortical representations are more widely spaced; the differences in the results of the experiments of Shiffrin et al. (1973) and C. Breitwieser et al. (2011), it appears possible that as the physical distance between tactile stimuli reduces , the more important becomes the orienting of attention to accurate perception of tactile stimuli.

An experiment by Pang and Mueller (2015) compared the effects of directed attention on the accuracy of perceiving tactile stimuli for two fingers under conditions between and within hands, as well as EEG signals. In the within-hand condition, participants were instructed to focus on the index finger, ring finger, or both fingers; in the between-hands condition, participants were instructed to focus on the left middle finger, the right middle finger, or both middle fingers. There were two types of tasks: focusing on one stimulus location while ignoring the other or focusing on both stimulus locations. Participants were asked to identify the stimulated finger. Before each trial, participants were informed of the task to be performed and the finger to be focused. Their analysis revealed that the performance between hands was superior to that of the within-hand condition (Pang &

Mueller, 2015). This suggests that when processing one or two stimulation locations, the within-hand condition is more difficult than the between-hand condition (Pang & Mueller, 2015). Furthermore, within-hand vibrotactile stimuli result in greater competing interactions than between-hand stimuli (Pang & Mueller, 2015). When Pang and Mueller (2015) correlated the attentional effects in SI and SII, they found correlations only within the hemispheres but not between the hemispheres. These findings support that tactile perception is superior in the between-hand condition for reasons having to do with attentional allocation.

The within-hands condition was associated with smaller SSSEPs and lower accuracy than the between-hands condition, not only when performing a two-position task but also when performing a single-position task. Thus, Pang and Mueller (2015) argued that attention acted as an “independent gain control” mechanism, independent of spatial competition, because if the intra-hemispheric resources need to be distributed to process information is the only reason for these effects, then SSSEPs and accuracy should be smaller/lower only when both locations are attended to within the same hemisphere (i.e., attention must be divided); but not when only one location is attended to (Pang & Mueller, 2015). Therefore, Pang and Mueller (2015) argued that attention and competitive inhibition within the hand are unrelated. Nevertheless, the SSEPs data suggest sustained spatial attention significantly enhanced the SSSEPs for the attended task compared to ignored task overall. Thus, Pang and Mueller's (2015) experiment found that directed and undirected attention did not cause a difference in the task difficulty in the two experimental conditions. However, with simultaneous stimuli, sustained spatial attention could be directed to a specific finger and cause a significant increase in SSSEP amplitude for the engaged stimuli. Attention appears as an independent gain factor, independent of spatial competition.

Collectively, the experiments conducted by Hoechstetter et al (2001), Pang and Mueller (2015), and C. Breitwieser et al (2011) demonstrate that between-hand performance is superior to within-hand performance when two simultaneous stimuli are presented to the fingers. Consequently, the behavioral outcomes of our experiments are interpretable. As tactile inhibition within the hand was greater than that between hands; in our experiment, the absolute error of RiLi was consistently less than RiRm, and the subjects' confidence between hands was greater than within hands. Furthermore, the Pang & Mueller (2015) and C. Breitwieser et al (2011) experiments demonstrated that directed attention contributes to tactile perceptual accuracy. This finding may explain why participants in our experiment found the perceived number to be so close to the actual numbers, regardless of magnitude. This is due to the fact that in our experimental paradigm, subjects were informed of which finger to focus on. Compared to Sharma and colleagues (2018), Browne (2019), and Rendell (2020) (without directed attention), our experiment data showed significantly greater accuracy from the subjects.

Hand and Finger Combinations and Experimental Paradigms

It is important to note that the results of Browne's (2019) experiment demonstrated that subjects were more accurate and more confident in between-hand condition than within-hand condition. However, in the within-hand (RiRm) condition, a significant underestimation and greater variance were observed, particularly when the actual number was greater than eight (Browne, 2019). This result contradicts our findings. First, our SPSS output showed that the hand and finger combination is not the main significant factor for the mean perceived number and algebraic error. Second, the linear graphs revealed that the hand and finger combination did not show any particular pattern regarding its effects on numerosity perception across all experimental conditions. For example, the analysis of algebraic error in our experiment revealed that in the high-density condition, RiLi and RiRm fluctuated between underestimation and overestimation when the actual number was between 1 and 8. However, when the actual number was greater than 8 (aN > 8), RiLi exhibited continuous underestimation, while RiRm continued to fluctuate between overestimation and underestimation. In the low-density condition, the algebraic error for RiLi and RiRm was accurate and deviated from 0 to a similar degree. Additionally, in the low-density condition with distractors, RiRm consistently overestimated as the number increased, but RiLi did not show a similar pattern as RiRm. Therefore, it is difficult for our experimental results to reach a similar conclusion as Browne (2019).

Experiments conducted by Pang and Mueller (2015) demonstrated that simultaneous stimulation within one hand led to integrated processing, whereas two-handed stimulation did not. Moreover, within hand, integration tended to be greater when subjects attended to both locations simultaneously than when they attended to only one location, which appeared to be a distraction (Pang & Mueller, 2015). As the density of stimuli in Browne's (2019) experiment was comparable, this may have resulted in comparable tactile information between the two fingers intensifying the competing inhibition. In addition, no directed attention was required in Browne (2019). In our experiment, the density of target and non-target stimuli was inconsistent, and directed attention was given; consequently, based on the findings of Pang and Mueller (2015), it is possible that tactile integration in the present experiment may differ from that of Browne's (2019) subjects, resulting in inconsistent behavioral outcomes.

Interestingly, Sharma and colleagues (2019) and Rendell (2020) did not find a significant effect of hand and finger on the magnitude of perceived number and errors. These differences could be due to different experimental paradigms and experimental tasks. For example, Sharma et al (2019) required subjects to move laterally. The theories of tactile perception covered in this section are derived from passive touch experiments, whereas the paradigm for this experiment is active touch. There are some cognitive mechanisms differences between active and passive touch, and there are fewer reports on the effects of between/within hands and hand moving orientation on the competitive inhibition effect of the somatosensory cortex under the active touch, pending future research.

Conclusion

To better understand haptic enumeration, the current experiment used braille as the stimulus and required participants to report the number of dots they perceived. The study also examined the effects of four variables, namely density, distractors, hand and finger combinations, and the number of dots on haptic enumeration. Compared to the previous series of experiments conducted by Sharma et al. (2018), Browne (2019), and Rendell (2020), the inclusion of the distractors variable is a unique feature in our study.)

In our experiment, the distractors factor did not show a significant main effect on magnitude of perceived numbers, algebraic error, or absolute error, but it did have a main effect on confidence. Furthermore, we observed an interaction effect between distractors and density on perceived number and algebraic error. However, when it comes to linear graphs, we did not find any consistent or systematic influence of the interaction effect of the distractors on haptic enumeration. It is important to note that neither the main effect nor any interaction effect of the distractors on absolute errors was observed in our study. Therefore, based on our experimental data, it appears that the distractor has no significant effect on haptic enumeration.) Based on these results, we inferred that directed attention played a crucial role in the experiment. Participants were able to accurately predict the target location and consciously modulate and allocate their attention to the target column rather than the non-target column (distractors), effectively avoiding the influence of the distractors. Furthermore, we speculated that the participants' attention shift during the experiment might resemble the partial AB effect in haptic, although this conjecture necessitates confirmation through future research.

The main effect of density was found to have an impact on absolute error and confidence, while the interaction effect between density and the number of dots was significant in terms of the magnitude of perceived numbers, algebraic error, absolute error, and confidence. However, the linear plot of algebraic error revealed no systematic overestimation or underestimation patterns. Therefore, we concluded that haptic enumeration is independent of nonnumerical continuous magnitudes(density) in the paradigm of this experiment, although density makes impact task difficulty.

The hand and finger combinations produced a main effect on the absolute error and confidence. The interaction between hand and finger combinations with density, influenced the magnitude of perceived numbers, algebraic error, and absolute error. As observed by the linear plots of absolute error, the absolute error was smaller for homologous fingers of both hands than for non-homologous (adjacent) fingers of one hand. Moreover, no regular patterns were found in the linear plots of the algebraic errors. Therefore, we concluded that in our experiment, haptic enumeration was more accurate for homologous fingers of both hands compared to non-homologous fingers of one hand)

The number of dots (numerosity) had a significant main effect on the magnitude of perceived number, algebraic error, absolute error, and confidence. The interaction between the number of dots and density also significantly impacted these variables. Although no consistent pattern of overestimation or underestimation was found, the absolute error increased with the number of dots, suggesting that the number of dots influences the degree of deviation from the actual number in the haptic enumeration. Moreover, although no discontinuities in slope were observed before and after the number six in our study, we combined our results with the experimental paradigm and data of Sharma et al. (2019) to postulate that haptic enumeration exhibits subitizing ability for numbers one to three. Based on this postulation, we further inferred that participants employed three enumeration modes: groupitizing, counting, and estimation; and they switched between these modes depending on the task's attention and working memory demands.

All four variables - number of dots, distractors, density, and hand and finger combinations - significantly impacted the participants' confidence. The interaction between density and the number of dots had the most significant effect on haptic enumeration in this experiment, as evidenced by its significant influence on all four dependent variables. In addition,we inferred that the number of dots, distractors, density, and hand and finger combinations increased the complexity of the enumeration task and affected the participants' perception of touch. Specifically, the complexity of the stimuli increased when density and number interacted, leading to higher demands on cognitive abilities such as attention and working memory. Therefore, we concluded that haptic enumeration is highly sensitive to the physical characteristics of stimuli, and the complexity of stimuli directly impacts tactile enumeration accuracy.

In conclusion, our results suggested that haptic enumeration may exhibit subitizing ability for numbers one to three. Using homologous fingers of both hands is more conducive to haptic enumeration. Density can increase the task's difficulty, especially when combined with an increased number of dots. Directed attention contributes to haptic enumeration and helps reduce the effects of the distractors.

Directions for future studies

Through the present experiments, we obtained relevant conclusions and inferences, but there are still some questions that need to be further investigated.

First, there was no main effect of the distractors on haptic enumeration in the present study. Our results showed that haptic enumeration was independent of the distractors, which was surprising. We inferred that this might be related to directed attention, and there may also be a mechanism similar to the attentional blink effect that allows participants to allocate and switch their attention between the target and the distractors. This remains to be further investigated and confirmed.

Second, Our experiment found no significant min effect of density on haptic enumeration accuracy, which contradicts previous findings. One possible inference for this discrepancy is that it may be related to the density threshold.) In our study, the density might not have been sufficient to induce systematic changes in haptic enumeration, such as overestimation or underestimation. To explore this further, future studies could incorporate a quantitative design of stimulus density thresholds, including high, medium, and low densities, similar to research methods used in the visual enumeration. This approach would enable us to investigate whether non-numerical continuous magnitudes, such as density, have systematic effects on haptic enumeration.

Third, Pang and Mueller (2015) examined how spatial attention affects the SSSEP amplitude when participants perceive stimuli on different fingers within and between hands. They discovered that attention acted as an independent gain factor, increasing the SSSEP amplitude for attended fingers regardless of finger spatial competition. Additionally, they observed that fingers between hands outperformed within-hand fingers. Our study replicated this finding and also indicated that directed attention plays a role in haptic perception. Since within-hand haptic sensitivity to stimuli is lower. Is it possible to specifically train directed attention for those fingers, which may yield beneficial results?. Attention can act as an independent gain factor, unaffected by spatial competition between fingers, and this training could reduce the difference between the SSSEP amplitude of fingers between hands and those within the same hand. Such training could be particularly advantageous for visually impaired individuals, as it may increase their directed attention and improve within-hand haptic sensitivity, leading to an overall improvement in haptic perception.

Fourth, in this experiment, we did not investigate how the arrangement of dots affects haptic enumeration. The size of the contact area on the fingerpads determines the number of dots that can be perceived simultaneously, but we did not measure the size of the area in contact with the stimulus. Therefore, it is difficult to determine the number of raised dots that form an arrangement that participants' fingerpads could perceive simultaneously. Haptic enumeration may have been affected by the arrangement of dots (for example, the Gestalt law), participants recognize regular arrangements of dots faster than random ones, and the arrangement of the items also influences subitizing and grouping. Future experiments could measure the contact surface between the participants' fingerpads and the stimuli to determine the number of dots that can be perceived simultaneously and the arrangements that the dots can form, given the contact area. Additionally, combining the contact area with the bilinear fit model could better analyze the subitizing range.

Appendix A

Numerical Cognition, Active Touch and Visual Impairment

Persons with a visual impairment rely on other senses for perceptual access to the world.

- **Does this mean that these other senses become more acute?**
- **Does this mean that areas of the brain normally dedicated to visual processing get used for tactile or auditory processing?**

But almost everyone has an acute natural ability to use the sense of touch, especially through the hands and fingers. It so acute that we often value touch more than sight:

- **Is this surface totally smooth?**
- **Is this material silk or cotton?**
- **Is this sandpaper smoother or rougher than that one?**

Yiming Li doing her Masters degree under the supervision of Dr **Barry Hughes**, and we are looking for sighted participants to contribute to our investigation of such questions. In our study, we will be working with braille symbols and we will investigating how blindfolded **sighted people** perceive these symbols and what role attentional focus plays in being able to do so.

Participants will need to take part in two sessions, scheduled at their convenience. Each session will last approximately one hour.

Specific details will be provided to you upon request.
You must be over 18 y of age and have no problems with your sense of touch (no diabetes, no scares or injury).

Participants will be reimbursed for their efforts with a $25 gift voucher per hour of participation. If the study requires more than session, payment is for both sessions, not just one.
If you are interested in our scientific pursuit of answers to our questions, contact Yiming and she will arrange for more detailed information to be sent to you. Yiming <yli589@aucklanduni.ac.nz>

Approved by the University of Auckland Human Participants Ethics Committee on 26 February 2021 until 18 May 2023. Ref 019220

Appendix B

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SCIENCE
SCHOOL OF PSYCHOLOGY

Numerical cognition, visual impairment, and active touch
PARTICIPANT INFORMATION SHEET
Please retain for your records

My name is **Yiming Li** and I am a Masters degree student in the School of Psychology, University of Auckland. I am carrying out research to investigate the brain's ability to keep track of numbers of raised dots during various touch-related (or tactile) tasks. I am working on this project with my supervisor, **Dr Barry Hughes**.

What is the project about?
Some braille readers read as fast and as accurately as the sighted do with print. Braille readers exhibit remarkable skill when reading: through one or two fingerpads they can read up to a hundred words per minute. This means they *encode* multiple individual cells per second. *Encode* here means they accuratey decide: How many raised dots? What pattern do they form? What letter or symbol does that number and pattern mean? What word does that group of cells mean?

The aims of this project are to investigate how the brain encodes raised dots and how it does so during brief scanning movements of the fingerpads. We are interested in your ability to do this. You are like someone encountering braille for the first time. Of course, you are not blind. But you are likely to have a sophisticated natural sense of touch: you can judge texture, temperature, slipperiness, shape, contours, even weight through touching objects. This is a natural skill most people possess.

Here, you will be asked to scan with two fingerpads two columns of braille dots: one finger, one column. You will be asked to make judgments: **How many raised dots did I touch**? **How confident am I in this judgment**?

Who is eligible?

We are looking for sighted individuals to participate in our research. All participants should meet the following conditions:

1. No underlying neurological and/or other sensory deficit
2. No tactile impairment
3. Must not have severe diabetes
4. Must have the capacity to consent
5. Older than 18 years of age

If you have any other disability or condition that you think might be relevant, we ask that you advise us and we will make a decision as to whether the research is still suitable for you (it may not be for everyone). If you are currently affected by any acute illness or wound/cuts on hands, we might not be able to include you now, but you will be welcome to take part when you recover. If you agree to participate, you may be asked to be blindfolded and to wear earplugs during the study. This is ensure that your judgments are based on what you feel through the fingerpads only.

What will the study involve?

We anticipate that each participant will take part in two testing sessions, each lasting no more than 90 minutes, and each scheduled no more than one week apart at a time that is mutually agreeable.

The study is completely safe, with no known associated risks. All the patterns and objects used are non-invasive and should cause no discomfort. We will have breaks, whenever you wish.

Where will the study be done?

The research will take place in our laboratory on the City campus: Science Centre Building (i.e., 302.), 23 Symonds Street in the Haptics/Braille laboratory, Room 389.

Participation and withdrawing from study

Your participation is voluntary, and you may decline this invitation to participate without penalty. The decision to be involved in the research is up to you entirely. Even if you agree to take part, you can always stop and withdraw without having to give a reason. Even if you fully participate, you have the right to withdraw your results and any other information that you have provided within 3 months of completion of your participation.

In return for participation (whether complete or not), we will reimburse you with a voucher (Countdown grocery). This will be calculated at $25 per hour. If you participate in a multisession study, we will pay at the completion of the entire series, for the entire series. We cannot pay for incompletion. We will also offer a $50 bonus for the participant who is calculated to have been the most accurate across the study. Note that the while the grocery cards do not expire, they cannot be used for online grocery shopping.

If you are a student in any course taught by me or my students, you cannot receive course credit for participating. And your decision whether to participate will have no effect on your standing it that course. You may contact the Psychology HOS if you feel that this assurance has not been met (details are below).

What will happen to the information collected during the study?

All personal information and any information that we obtain from our studies will be completely confidential and known only to the research team. All the personal information and other data will be password protected and securely held on the University IT system or locked in a filing cabinet. All of the results from the study will be stored in a database, and access will be restricted to the research team. Personal data will be stored separately from other data and will not be disclosed to anyone. All the data collected will be kept for six years following the end of the study. After that, the paper data will be shredded and the electronic data will be erased.

We anticipate being able to report the findings at scientific and/or educational conferences and in peer-reviewed journals. Under no circumstances will individual participants be named or identifiable in these reports. Your participation will take place under strict conditions of confidentiality. A copy of the research findings will be made available to you if you wish.

We will ask you if you would like to be contacted for future studies. If you agree, we will update your details on our database so that we can contact you in future for more studies.

Thank you for taking the time to read this information sheet. Please retain for your records.

We would be happy to answer any questions that you might have concerning the research. Our contact details are as follows:

Yiming Li, School of Psychology, University of Auckland
Email: yli589@aucklanduni.ac.nz

Dr Barry Hughes, School of Psychology, University of Auckland, Private Bag 92019, Auckland.
Telephone: (09) 373-7599 ext. 85265; Email: b.hughes@auckland.ac.nz

The Head of School is Professor Suzanne Purdy who may be contacted at Department of Psychology, University of Auckland, Private Bag 92019, Auckland.
Telephone: (09) 3737-599; sc.purdy@auckland.ac.nz

If you have any queries regarding ethical concerns, you may contact the Chair, University of Auckland Human Participants Ethics Committee, The University of Auckland, Research Office, Private Bag 92019, Auckland 1142.
Telephone: (09) 373-7599 ext. 83711; Email: humanethics@auckland.ac.nz

Appendix C

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SCIENCE
SCHOOL OF PSYCHOLOGY

Researchers: Yiming Li, Dr Barry Hughes

Numerical cognition, visual impairment, and active touch
CONSENT FORM
THIS FORM WILL BE HELD FOR A PERIOD OF SIX YEARS

- I understand that I am being asked to participate in a project on active touch and pattern/object exploration being conducted by the above research team.
- I have read the Participation Information Sheet, have had the research described to me, and I have had the opportunity to ask questions to the researchers and have had them answered to my satisfaction.
- I have understood the general nature of research and understand that more specific information will be provided prior to each study
- I am clear that participation is voluntary and that I am free to withdraw my consent to participate at any time and without the need to provide reasons. I know that I can withdraw any data that I have provided within three months of my participation without any consequence.
- I understand that I will be reimbursed with a gift voucher valued at $25 per hour in compensation for participating for both sessions.
- I understand that the researchers undertake to treat my participation confidentially, to store my data in a secure University of Auckland facility, and to destroy all raw data (including this form) that I may provide after six years.

- I consent to participate in this research.

Name ________________________________ Date ____/____/____

Signature ____________________________

☐ Check this box if you are happy to be **contacted** for related on-going/future studies.
☐ Check this box if you wish to **receive a summary report** of the research at email/postal address:

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Approved by the University of Auckland Human Participants Ethics Committee on 26 February 2021 until 18 May 2023. Ref 019220

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