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Modelling Levator Ani Mechanics During the Second Stage of Labour

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A thesis submitted in partial fulfilment of the requirements for the degree of
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Abstract

This thesis presents a modelling framework that quantitatively analyses the mechanical behaviour of the levator ani (LA) muscle during the second stage of labour. This modelling framework would enable the identification of risk factors (for muscle damage) associated with vaginal delivery.

Detailed three dimensional (3D) geometries of thirteen female pelvic floor structures were segmented from magnetic resonance (MR) images. The LA muscle model was augmented with surrounding structures to simulate the second stage of labour. Twelve individualised pelvic floor models were constructed. Preliminary feature analysis showed considerable inter-individual variations in LA muscle size and orientation.

The mechanism of labour was modelled by applying minimal movement constraints to the fetal skull. In order to improve simulation performance during the finite element solution procedure, an “energy norm” convergence criterion and a line search method were implemented. It was demonstrated that allowing the fetal skull to find its own minimum potential energy state substantially affected the mechanical response of the LA muscle during delivery compared to employing a fixed head path, which has been used in all previous studies.

The constitutive parameters for the LA muscle were fitted to experimental data from in vitro mechanical tests on fresh cadavers. The elastic nonlinear and anisotropic properties of the constitutive relations were investigated through a series of child-
birth simulations. Both factors showed marked effects on the mechanical response of the LA muscle. The dorsal-caudal aspect of the LA muscle and the insertions to the pubis were identified as high risk areas of muscle damage, due to large stretches (>2.5) predicted by the model. LA muscle trauma at these locations has also been observed clinically.

This modelling framework was then applied to simulate the use of an intra-vaginal device test in order to investigate the passive muscle properties in control and high impact, frequent intense training (HIFIT) women. Results of this study suggest no difference in the passive stiffness between these two groups, which is in contrast to previous findings based on a simplified conceptual model. However, the relative difference in the average force was comparable with previous results. The study demonstrated the ability of the modelling framework to quantitatively interpret clinical data in silico.
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Lastly, I wish to dedicate the following paragraph to my friends and families who have kept me going for all of this time. To my corner buddies, Vicky, Angela, Helen, and Jess, for sharing the ups and downs, and all the girly gossip. To my New Zealand family who always make me laugh after a long day. To my parents for always supporting me and trying to understand my work. To my husband for always being there for me and tolerating my occasional emotional break-downs. We made it!
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