

Uruuru Whenua: Using Cultural Symmetry to Rebalance Mātauranga and School Mathematics

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Abstract

Tērā te karanga i te rangi

He manu tuia, tui tuia

Tuia i runga

Tuia i raro

Tuia i roto

Tuia i waho

Tuia te here tangata

Ka rongo te pō

Ka rongo te ao

Haumi ē, Hui ē, Tāiki ē!

(Kingi et al., 2021; Massey University, 2022; Tapiata, 2019)

As Māori (Indigenous peoples) of Aotearoa/New Zealand, we invoke the words of karakia (oral chant) to bind ourselves to the environments we inhabit on land and at sea. The karakia presented here extolls the audience to take a bird's perspective to become fully aware and present in what is happening above, below, and all around them in both light and darkness. This way, spatial awareness and reasoning intertwine with Māori oral and visual narrative practices and daily rituals. To ensure the intergenerational transmission of mātauranga (Māori ways of knowing, being and believing), Māori aspirations for schooling should include living in a way that is recognisably Māori to Māori, while also accessing notions of academic success. However, due to the colonisation of Aotearoa/New Zealand by the British Crown, Māori have endured more than 100 years of education policy that invalidated mātauranga, including the exclusion of the Māori language from schooling. Māori communities, academics, policymakers and pouako (teachers) have agitated for change, causing considerable shifts in education policy in Aotearoa/New Zealand since the 1970s, including how curriculum is developed.

As part of national curriculum and assessment programmes, the NZ Ministry of Education now promotes epistemological parity for mātauranga (Ministry of Education [MoE], 2022a). However, tensions between Māori aspirations for schooling and colonial schooling ideologies continue to shape curriculum development, including pāngarau (mathematics in Māori-medium schools), which is the

particular focus of this thesis. This includes minimising mātauranga to promote the acquisition of curriculum achievement objectives imported from countries with colonial legacies, such as the UK.

By using the cultural symmetry framework, a Bourdieusian analysis of policy and curriculum development, and teaching as inquiry, this research explores the opportunities, challenges, and tensions for rebalancing mātauranga and school mathematics at macro and micro levels. In doing so, the illumination of Māori wayfinding and spatial reasoning practices was identified as an opportunity to support Māori aspirations for schooling through the pāngarau curriculum and classroom practices.

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Te Reo Māori in this Thesis

This thesis is written in English and includes narratives, chants, interview transcripts and student-created artefacts in Māori. The research was conducted primarily in te reo Māori (the Māori language), and translations or interpretations have been provided for Māori language data either alongside the presented data or within the main text. A licensed Māori language translator (Aatea Solutions) was contracted to provide translations for the interview data in Chapter 4. I provided translations for the classroom artefact excerpts in Chapter 6. In the main text, a translation for Māori language terms is provided in brackets following its first appearance in the text, and a glossary is provided in Appendix 1. The glossary is not intended to be definitive but to provide insight into how the Māori terms are used in the thesis. Where there are multiple dialectal or similar commonly used terms in the Māori language, an alternative has also been provided in brackets.

The primary audience for this thesis is Māori immersion schooling communities, policymakers, curriculum and assessment developers, and pouako (teachers). However, the decision was made to write the thesis in English to provide access to the findings, conclusions, and recommendations to Indigenous audiences beyond Aotearoa/New Zealand and members of the mathematics education field working alongside Indigenous researchers and practitioners. There is limited development in the academic language of the pāngarau research discipline at this time. This thesis further contributes to this development by providing definitions and examples of Māori terminology that could further expand how te reo Māori is used in pāngarau research. However, there is a danger in writing about pāngarau and mātauranga in English, as Māori terms could be inadvertently stretched to accommodate English-language terminology, and particularly mathematics terminology underpinned by Western scientific paradigms. Further discussion of this tension is offered in the literature review in Chapter 2 and in the analysis of interview data in Chapter 4.

1. He Whakatakinga | Introduction

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua

(Te Tāhūhū o te Mātauranga, 1996, p. 5)

This whakataukī (formulaic saying) is an excerpt from narratives about Tāwhaki, a Pacific ancestor from whom Māori, the Indigenous people of Aotearoa/New Zealand, descend. The personification of the knowledge seeker, Tāwhaki, ventures on a perilous journey to acquire valued expertise (Mead, 2013; Potae, 1928; Williams, 2016). This premise sets the scene for a series of exciting and memorable narratives. In addition to providing wayfinding, quantification, and measurement information (Trinick, 1999), the Tāwhaki narratives connect the past and present, influencing the way knowledge is valued in te ao Māori (the Māori world).

The developers of the first Māori language mathematics curriculum document, *Pāngarau i roto i te Marautanga o Aotearoa* (Te Tāhūhū o te Mātauranga, 1996), likened the aka matua (main vine) in the whakataukī to Māori aspirations for curriculum and schooling. The whakataukī also features in the current Māori-medium mathematics curriculum document, *Te Marautanga o Aotearoa* (Te Tāhūhū o te Mātauranga, 2017, p. 40). A third iteration of the pāngarau curriculum is currently in development. Nearly 30 years after the original pāngarau curriculum document was released, this thesis revisits the way the pāngarau curriculum has addressed Māori aspirations for schooling, over that time, through the following research question:

What are the opportunities, challenges, and tensions for further indigenising pāngarau curriculum and classroom practices?

The research question was examined through three data sources. The first data source presented in Chapter 4 is semi-structured interviews with mātanga pāngarau (pāngarau experts). The four mātanga interviewed were directly involved in the initial curriculum development or supported its implementation. The second data source is literature on Māori and Pacific wayfinding and spatial orientation practices. Finally, student and teacher-created classroom artefacts, demonstrating both pāngarau and wayfinding, were collected in four pāngarau classrooms at two different Māori language immersion kura (schools). By examining the primary research questions at both the macro (policy and curriculum) and the micro (classroom) levels, opportunities, challenges, and tensions were illuminated to advance Māori aspirations for schooling further. The study context, including the historical and socio-political landscape that has shaped Māori aspirations for schooling, is summarised in the

following sections. Finally, the rationale, researcher positioning, methodology, research sub-questions and methods are summarised in the remaining sections of the chapter.

Māori Aspirations for Schooling

Debated since the 1930s, Māori aspirations for schooling were revisited in 2021 by 24 prominent Māori academics and researchers (Smith et al., 2021). In a collaborative think piece informing the third New Zealand national curriculum development project, Smith et al. (2021) identified two complementary aspirations for Māori schooling. These were “to live as Māori and to be citizens of the world” (Smith et al., 2021, p. 10). Similarly, Durie (2003), a Māori prominent academic in both the education and health fields, asserted that education for Māori must provide preparation for participation in Māori society while also ensuring readiness to engage with the wider world. These goals were further defined in education policy as Māori achieving success *as Māori* (MoE, 2021). Therefore, this thesis further defines the *aka matua*, or Māori aspirations, for pāngarau curriculum and schooling, as enabling Māori to thrive as global citizens in an environment that takes for granted the value of *mātauranga* (Māori ways of knowing, being and believing).

Featured in academic literature since the 1970s and present in Māori manuscripts in the 19th century (Smith et al., 2016), *mātauranga* is a term that has been popularised as a way of addressing boundless and continually evolving Māori knowledge systems (Mead, 2013). Entirely present and without distinct boundaries (Cooper, 2012), *mātauranga* encompasses Māori ontologies, epistemologies, and axiologies (Smith et al., 2016). The term *mātauranga* is particularly relevant to the body of knowledge that draws on Pacific ancestry and has continued to evolve in Aotearoa/New Zealand (Royal, 2006). *Mātauranga* is underpinned by *whakapapa* (Nicholson, 2020), and “a genealogical narrative, a story told layer upon layer, ancestor upon ancestor up to the present day” (Te Rito, 2007, p. 1). *Whakapapa* (genealogical frameworks) is also a knowledge-creation tool (Royal, 1998) which can be used to explain new phenomena (Forster, 2019). Tapsell (2010, 2011), whose work focused on museums and museology, discussed the broad groupings of *mātauranga* as groupings of *whakapapa*. Therefore, *mātauranga* can be understood as the knowledge and applications of knowledge that have evolved through generations and are still applied, adapted, and have meaning for Māori communities today (Smith et al., 2016). To Smith et al. (2016), responses by Māori to colonisation are examples of *mātauranga*, such as the development of curricula in *te reo Māori* (McMurphy-Pilkington, 2004; Trinick, 2015) and the revitalisation of non-instrumental voyaging practices (Spiller et al., 2015).

Illuminating *mātauranga* in the national curriculum in support of Māori schooling aspirations exemplifies the process of decolonising indigenisation. This is an Indigenous-led process from which Indigenous peoples, their languages, and knowledges directly benefit (Gaudry & Lorenz, 2018). However, practical applications of *mātauranga* that could be illuminated in pāngarau curricula,

thereby further indigenising pāngarau, are not always easily accessible to curriculum writers. In Aotearoa/New Zealand, mātauranga has been historically devalued in schooling policy by its omission (Allen & Trinick, 2022; Mead, 2013). In the last 20 years, this has been partially justified by education policies and research prioritising Māori culture (Cooper, 2012). Through colonisation, Māori, alongside the peoples of the global south, have been positioned as producers of culture rather than knowledge (Cooper, 2012). For example, European academics researching Māori narratives embellished, altered, or discounted historical information to support their views and claims (Taonui, 2006). Alternatively, mātauranga, such as karakia, are sometimes conflated with Judeo-Christian prayers (Williams, 2016), limiting understanding of their other applications. A further example is the renaming of constellations such as the Southern Cross, which appears on the Aotearoa/New Zealand flag beside the Union Jack (a cross representation). Rather than performing the role of a personified ancestor in genealogical wayfinding narratives, Te Taki o Autahi (Moorfield, 2003) was interpreted by Europeans as a cross and named the Crux Australis (Wassilieff, 2016). This thesis argues that the Eurocentric reinterpretation of mātauranga as not ‘proper knowledge’ continues to be used to justify its exclusion from school mathematics curriculum and classroom practices beyond attempts to illuminate mātauranga in tokenistic ways.

Conversely, the whakataukī introduced at the beginning of this chapter provides practical wayfinding information for both Tāwhaki’s journey and those seeking to illuminate aspects of mātauranga that have been, and continue to be, invisibilised in school mathematics. In the whakataukī, Tāwhaki’s grandmother Whaitiri extolls Tāwhaki to grab hold of the aka matua and not the aka tāepa (loose side shoots) to ensure a successful ascent (Williams, 2016). Tāwhaki was tasked with recognising and grasping the climbing aid suitable for weight bearing. Often overlooked, the practical and often pedagogical characteristics of the Tāwhaki narratives could usefully inform pāngarau curriculum development and classroom practices. This thesis makes visible the pedagogical characteristics and practical applications of narratives like these as valued cultural capital that ‘holds weight’ in schooling for Māori. The following sections examine the role of curriculum, often the arbiter of what knowledge is valued in schooling.

Māori Experiences of Schooling

This section presents terminology and historical events that have defined school mathematics experiences for ākonga Māori (Māori students). There are several critical analyses of the early colonisation period in Aotearoa/New Zealand (see Hetaraka, 2022; Ka’ai-Mahuta, 2011; Simon, 1992) and contemporary Māori education policies (see Penetito, 2010; Smith, 2012). Therefore, this introductory chapter analyses literature focusing primarily on the positioning of te reo Māori and mātauranga in schooling policy and research and how this has influenced the school mathematics

curriculum and classroom practices. This literature provides the evolving context for Māori language schooling until the late 1970s and early 1980s, which shaped the pāngarau curriculum development project in the early 1990s. When the first missionaries and settlers arrived, Māori had a robust system for educating their children to ensure the survival of their communities in Aotearoa/New Zealand (Jones & Jenkins, 2011; Riini & Riini, 1993; Trinick, 2015). After 1840, more Pākehā (of European descent) settlers arrived and a British colony was created. As a result, European forms of government and schooling were established. Simon (1998) argued that the function of the missionary schools in the early 1800s was to provide a formalised context to assimilate Māori communities into European beliefs, attitudes, and practices, with the intent to ‘civilise’ the Māori population.

Maintaining an additive bilingual approach in schooling (May et al., 2004) was the desired goal of Māori communities (Jones & Jenkins, 2011). In contrast, the primary goal of colonial schooling was subtractive bilingualism designed to speed up the assimilation of Māori to European cultural norms (Ka’ai-Mahuta, 2011; May et al., 2004). For over 150 years, colonial policies intentionally disrupted the intergenerational transmission of te reo Māori and, thus, the dissemination of mātauranga (Cooper, 2012; Mead, 2013; Smith et al., 2016). These policies were both overt and covert. The *Tōhunga Suppression Act* 1907 served to suppress the dissemination of mātauranga by tohunga (experts), while the English-only schooling policy excluded the Māori language from compulsory schooling (Ka’ai-Mahuta, 2011). Covert policies such as English-only workplaces (Chrisp, 2005) further diminished the opportunity for Māori to achieve economic success *as Māori*. Throughout the period following World War II and into the 1960s, the urban migration of Māori significantly impacted the health of te reo Māori (May & Hill, 2005). This urban shift by Māori families led to a new generation of Māori being raised far from their local marae (meeting space). At that time, social policy advocated for the ‘pepper-potting’ of Māori households by interspersing them in English-speaking Pākehā communities, which resulted in Māori children losing their language (Benton, 1997; Ka’ai-Mahuta, 2011). By the 1960s, Aotearoa/New Zealand was one of the most linguistically homogeneous countries in the world (May, 2005). By the 1970s, less than one-fifth of the Māori community were fluent Māori language speakers (Benton, 1979); most of these were elderly, and less than 5% of Māori school children could speak te reo Māori. By the mid-1970s, Māori was recognised as an endangered language (Fishman, 1991) and threatened with possible extinction (Spolsky, 2003). It was against this background of rapid and significant language loss that the Māori community initiated bilingual schooling (May & Hill, 2005).

Until the 1970s, state-run schools had legislation in place to enforce English-language instruction (Hetaraka, 2022). In 1979, Richard Benton released a national report on the health of the Māori language, demonstrating its stark decline. Four communities were identified in Benton’s (1981) research that had maintained some degree of intergeneration language transmission despite declining

Māori language fluency elsewhere. Located in Rūātoki (Bay of Plenty), Tawera (Bay of Plenty), Hiruhārama (near Gisborne), and Ōmahu (Hastings) (Benton, 1984), these communities were in rural areas and had strong genealogical associations. Through the strength of their convictions, the four communities were able to persuade both the then Minister of Education and officials of the Education Department (renamed to Ministry of Education in 1991) to allow Māori language (bilingual) instruction in their schools (Benton, 1984). Consequently, four schools were given bilingual status in the late 1970s and early 1980s.

While no longer enforcing monolingual English-language instruction, the government continued to determine what could be taught to Māori in schooling through English-language syllabi, including *Mathematics for Junior Classes to Standard Four* (Department of Education, 1985) and *Mathematics: Forms 1 to 4* (Department of Education, 1987). There was no formal Māori language syllabus and limited Māori language resource materials to support teaching and learning mathematics in te reo Māori (Trinick, 2015). The responsibility for resource development, and planning what would be taught, fell to principals, staff, and school communities (Benton, 1984). The transitional nature of these initial bilingual education programmes has been criticised (May & Hill, 2005), as they were still intent on transitioning students eventually from Māori to English. However, these bilingual schools provided the impetus for developing curriculum terminology, including mathematics terminology, in te reo Māori (Barton, 2008).

Following these bilingual schooling reforms, Kōhanga Reo (early childhood language nests) were launched in 1981. Kōhanga Reo were initially run independently by parents motivated by the urgent need to stem the loss of te reo Māori (Benton, 1984). Kōhanga reo provided opportunities for native-speaking kaumātua (elders) to pass on their language to the youngest members of their communities (King, 2001). Through this independent education movement, whānau Māori (extended families) challenged the dominant educational ideologies and empowered themselves to make vital educational decisions for their tamariki (children) (Bishop, 2012).

Graduates from Kōhanga Reo entered the state primary school system, which at that time, still included questionable or non-existent Māori language instruction. Consequently, concerns emerged about state schools undoing the language revitalisation work of Kōhanga Reo (Smith, 1997). The poor response by the state to these concerns motivated Māori communities to further indigenise primary school education. Kura Kaupapa Māori (Māori immersion primary schools) were established in 1985 (Smith, 1997), also outside the state-school system. Kura Kaupapa Māori ensured the continuity of Māori language instruction for Kōhanga Reo graduates (Bright et al., 2013). These schools expanded rapidly throughout the 1980s, compelling the government to recognise Kura Kaupapa Māori as a formal alternative to English-language instruction. Kura Kaupapa Māori were incorporated into the state system in 1989 (McMurchy-Pilkington, 2004). Eventually, the growing number of Kura

Kaupapa Māori graduates prompted the development of wharekura (Māori immersion secondary schools) and wānanga (Māori tertiary institutions).

The founding document of kura kaupapa Māori, *Te Aho Matua*, centralised te reo me ōna tikanga Māori (Māori language and protocols) as the medium of instruction (Department of Internal Affairs [DIA], 2008; Tākao et al., 2010). *Te Aho Matua* also took for granted the value of Māori ways of knowing, doing and being. Arguably, the first education policy document to position mātauranga as taken-for-granted, *Te Aho Matua* also promoted separating the teaching and learning of subjects in the te reo Māori by space, time, and purpose. Therefore, if English was to be offered at Kura Kaupapa Māori, it must be offered as a second language (L2) subject and not as the primary language of instruction. Guided by *Te Aho Matua*, Kura Kaupapa Māori aimed to revitalise te reo Māori and mātauranga through an emancipatory schooling movement (Penetito, 2010; Smith, 2012). Because of this, the establishment of Kura Kaupapa Māori redefined Māori student experiences of schooling. However, there was still a need for Māori language curriculum documents to support the aspirations of Kura Kaupapa Māori and other emerging Māori language immersion schooling pathways.

Pāngarau Curriculum Development

This section examines the four curriculum development projects that have taken place in Aotearoa/New Zealand from the 1990s to today and their varying impacts on Māori experiences of school mathematics. There have been three national projects (one is currently underway) and one localised project. In order to preface the discussion, an overview of Māori immersion schooling terminology is offered.

In addition to Kura Kaupapa Māori and the remaining bilingual schools, today there is a range of Māori language instruction schooling options, such as Kura ā-Iwi (tribal area), Kura ā-Rohe (regional), and Māori immersion programmes located on English-language instruction school campuses. The term Māori-medium is used by Te Tāhūhū o te Mātauranga (MoE) to identify learning programmes where 51%–100% of instruction is in te reo Māori (MoE, 2022b). According to MoE (2022b) data, there are now 305 schools offering Māori-medium programmes, with 23,161 students aged from Year 1 (age 5) to Year 13 (age 18) enrolled in a Māori-medium learning programme (MoE, 2022b). The use of the term Māori-medium to denote the level of Māori language instruction has necessitated the categorisation of English-language instruction as English-medium (May et al., 2004), an example of the indigenising effect of Māori community aspirations for schooling on a previously monolingual English education system. This thesis uses the term Māori immersion schooling to denote 81%–100% of instruction delivered in te reo Māori and adherence to the language separation policy of *Te Aho Matua* (Level 1 immersion settings according to the MoE). In order to become state-

funded, all the various forms of Māori immersion schooling have been required to teach state-mandated school subjects, including mathematics.

Penetito (2010) suggested that it was a neoliberal ideology that gave support to the government's decision to integrate Kura Kaupapa Māori into the state-funded education system via the *Education and Training Act 1989*. The establishment of Kōhanga Reo and Kura Kaupapa Māori initially outside the state system was seen as supporting the argument for wider educational restructuring and change (Openshaw, 2014). The Tomorrow's Schools legislation, the key policy of the 1980s Labour Government, transferred the responsibility of school administration from the Department of Education to schools' boards of trustees (BOT) (Macpherson, 1989). Composed of elected representatives of the school community, the BOT was responsible for the governance of their local schools (Macpherson, 1989). Effectively, Tomorrow's Schools shifted the administrative burden from the state to the community (Macpherson, 1989). The devolution of authority from the state to the community provided an avenue for Kura Kaupapa Māori to gain access to state funding while retaining relative autonomy over the day-to-day running of the kura (school) (McMurchy-Pilkington, 2004).

The first curriculum development project began in Aotearoa/New Zealand in the early 1990s and was motivated by ideologies of neoliberal education reform (Openshaw, 2014). In the 1990s, New Zealand was following international trends in curriculum development and the United Kingdom (UK) had also completed a mathematics curriculum development project around this time (McMurchy-Pilkington, 2004). The then education minister, Lockwood Smith, believed that the Aotearoa/New Zealand document should be largely based on overseas examples, such as that of the UK, thus saving time and money. Thereby, ignoring Māori aspirations to live *as Māori*. Despite arguments about poor Māori achievement being used to bolster the need for a national curriculum, there was no recognition of the aspirations of Māori-medium schooling (Trinick, 2015).

The Māori-medium schooling sector was not represented in the key decision-making positions in the (re)development of the New Zealand curriculum at the time (McMurchy-Pilkington, 2004). Instead, Māori-medium teachers were simply tasked with translating the content of the English-medium curriculum into te reo Māori. To complete these translation tasks, teachers had to create specialised terms for subjects such as mathematics (Christensen, 2004b). This situation was taxing on the Māori-medium teacher workforce, often with dubious terminology outcomes, and needed remedying (Barton et al., 1998). The government assumed that Māori-medium schools would simply implement a translated version of the state-mandated English-medium curriculum (McMurchy-Pilkington, 2004) as had been done in the past for other school syllabi. After extensive lobbying by Kura Kaupapa Māori and other stakeholder groups, somewhat surprisingly, the Minister of Education agreed to develop a separate Māori-medium mathematics curriculum (McMurchy-Pilkington et al., 2013). However, the Māori-medium curriculum document still had to mirror the English-medium structure and have the

same mathematics content (McMurchy-Pilkington & Trinick, 2002), ignoring Māori aspirations for school curriculum informed by mātauranga.

The Māori-medium education community was divided on whether to continue participating in this first curriculum development project in Māori-medium education (McMurchy-Pilkington & Trinick, 2002). Some saw an opportunity to advance the linguistic goals of Māori language revitalisation via a national curriculum document, providing the impetus for language research supporting the systematic regeneration of te reo Māori in classroom discourse (Trinick & May, 2013). However, Kura Kaupapa Māori argued it was a continuation of colonising ideologies, albeit this time through a Māori language curriculum (McMurchy-Pilkington & Trinick, 2002). Ultimately, the decision had to be made to produce a translation of the English-language document or to implement the English-language curriculum (Trinick & Heaton, 2021).

A Māori-medium curriculum development project was thus agreed upon, and the MoE was then obliged to support teachers and schools in implementing the Māori-medium curriculum through a range of initiatives such as professional development, classroom resource development, and research (McMurchy-Pilkington et al., 2013; Trinick et al., 2015). Thus, the development of *Pāngarau i roto i te Marautanga o Aotearoa* (MoE, 1996) supported the elaboration of te reo Māori to communicate school mathematics concepts, thereby supporting wider language revitalisation aims while also facilitating the acquisition of school mathematics (Allen, 2015). Therefore, Māori aspirations for schooling were supported to some extent through the development of the first national pāngarau curriculum.

The requirement to translate the English version was removed for the second round of Māori-medium curriculum development in 2007–2008 (McMurchy-Pilkington et al., 2013). However, the basic structure of the 1996 curriculum was maintained. The reasons for this are not immediately obvious and are explored further in this thesis. Despite the increased capacity to develop Māori-medium curricula and the MoE's rhetoric being more accommodating to difference (McMurchy-Pilkington et al., 2013), the opportunities to explicitly illuminate mātauranga still appeared to be limited. In this way, mātauranga was excluded through the translation requirement of the first national pāngarau document and further invisibilised by omission in the second.

Perhaps the reason for the omission of mātauranga in the second round of pāngarau curriculum development was the MoE's increased support of schools localising (their) curriculum content (Trinick & Heaton, 2021). Schools and kura were encouraged to realise the school mathematics learning objectives of the national curriculum in a highly localised fashion, which created opportunities to incorporate localised pāngarau activities (Meaney et al., 2012). The assumption here may have been that mātauranga would be illuminated at the classroom level. However, this thesis

argues that explicit guidance on how to localise the national curriculum was lacking. The lack of guidance resulted in mātauranga being further marginalised, positioned merely as a vehicle for acquiring school mathematics.

The second Māori immersion curriculum development project, *Te Marautanga o Te Aho Matua*, completed by Kura Kaupapa Māori governing body Te Rūnanga Nui o Ngā Kura Kaupapa Māori o Aotearoa in 2015, has been categorised by the MoE as a localised curriculum (Trinick & Heaton, 2021). As observed by Trinick and Heaton (2021), *Te Marautanga o Te Aho Matua* identifies aspects of mātauranga and their connections to the principles in *Te Aho Matua* (DIA, 2008) with examples of practices and a sequence of activities exemplified through student work samples. However, as Trinick and Heaton (2021) concluded, the interaction of national school mathematics curriculum and mātauranga-based localised curricula in pāngarau classrooms remains unclear. This interaction will be further explored in this study.

At the time of writing (2022), the third national pāngarau curriculum development project has recently commenced. It is hoped that in this iteration, the developers may have further agency in illuminating mātauranga and that explicit guidance will be provided to deliver localised mātauranga. The MoE's guidance for localising the English-medium national curriculum has also been strengthened since the second curriculum development project (MoE, 2019). *Te Tāhūhū o te Mātauranga*, through the *Education and Training Act 2020*, now places responsibility on schools to ensure that their plans, policies, and local curricula reflect mātauranga. Therefore, schools must provide an environment where mātauranga is a normalised curriculum component. However, beyond “fostering learning-focused relationships within your school and the wider community and iwi” (MoE, 2019, p. 7), there seems to be little national guidance on how to ensure that classroom practices support Māori aspirations for schooling.

Māori communities, iwi (tribes), and hapū (sub-tribes) generally guide how mātauranga can be used and by whom at the local level (Moewaka Barnes et al., 2021). In *He Raukura mō te Mokopuna*, a strategy designed to enhance mātauranga across the curriculum, Rau and colleagues (2022) argued that mātauranga is thus best positioned at the localised school curriculum level, where the kura community has agency in determining the content and implementation. Again, Rau and colleagues (2022) provide very little detail on how this is to occur beyond presuming that local mātauranga experts will support and resource the increased presence of mātauranga in schools. This thesis adds further insight into the opportunities, challenges, and tensions for illuminating mātauranga through localised curriculum to support Māori schooling aspirations at both the macro-curriculum and policy levels and the micro-classroom levels.

Rationale

As exemplified across the four curriculum development projects, the challenges of illuminating mātauranga and te reo Māori, and not just viewing these as conduits to the promotion of school mathematics (essentially Western mathematics) (Meaney et al., 2013), is an ongoing challenge for pāngarau curriculum and classroom practices. Maintaining the status quo would continue assimilationist practices that invisibilise mātauranga. Ka'ai-Mahuta (2011) argued that despite the onus on Māori to agitate for change in Aotearoa/New Zealand schooling, the burden of providing education in te reo Māori underpinned by mātauranga still rests with the government. This is because Te Tiriti o Waitangi¹ and its English-language counterpart, The Treaty of Waitangi, signed in 1840, formalised a partnership between Māori and the British Monarchy (Orange, 2015) and paved the way for state-mandated schooling for Māori (Simon, 1998).

Article 3 of Te Tiriti o Waitangi guaranteed Māori equal rights and privileges with British subjects (Orange, 2015). While British descendants have been afforded a state schooling system in the English language underpinned by Western knowledge, the same right has not been equitably extended to Māori (Ka'ai-Mahuta, 2011). To rectify this, the state education system must actively promote and protect Māori rights under Te Tiriti o Waitangi (Education and Training Act, 2020). State agencies are also obligated to work with Māori to protect mātauranga in ways that support Māori rights and interests and enable Māori to derive collective benefit (Waitangi Tribunal, 2011).

However, the issue of ensuring that mātauranga is explicitly taught in schools continues to be contested. The responsibility to determine how mātauranga can be used and by whom is an issue that has been the subject of a claim to the Waitangi Tribunal (2011), which is the key legislative avenue for potential redress for Māori claims against the state. The Waitangi Tribunal (2011) determined that kaitiaki (stewards) have the agency to protect and limit the uses of mātauranga. Their report into claims on Aotearoa/New Zealand law and policy affecting Māori culture and identity identified kaitiakitanga (stewardship) as an obligation through perpetual kin relationships (Waitangi Tribunal, 2011). As signatories to Te Tiriti o Waitangi (or The Treaty of Waitangi), iwi and hapū Māori are the primary kaitiaki of the mātauranga with which they have enduring kin relationships. Therefore, curriculum writers, policymakers, pouako and kura may not be considered kaitiaki with authority to determine the illumination of mātauranga in curriculum and classroom practices.

¹ Te Tiriti o Waitangi and the Treaty of Waitangi was signed in 1840 by Māori leaders and a representative of Queen Victoria. While presented as two versions of the same document, the contents of Te Tiriti o Waitangi (in te reo Māori) and The Treaty of Waitangi (in English) are distinct enough that they are generally treated as two separate documents, with Te Tiriti o Waitangi given precedence by the United Nations Declaration of the Rights of Indigenous peoples.

Despite ongoing tensions about the illumination of mātauranga in education, much of the recent focus of Māori education research is based on a thesis of culture and identity, excluding any particular focus on Māori knowledge and epistemology (Cooper, 2012). This is attributed to the presumption that mātauranga is inferior to Western science (Smith, 2012). ‘Real knowledge’, such as mathematics, is often considered universal and culture free (Ernest, 1991). This is demonstrated in Aotearoa/New Zealand curriculum resources where Māori culture and language are promoted as vehicles to improve the teaching and learning of Western science (Allen & Trinick, 2022). Where Māori knowledge has been afforded some recognition, this has often been based on visual elements, the arts such as signs, images, and iconography, which are immediately recognisable as representing that culture (Allen & Trinick, 2022; Jahnke & Jahnke, 2003). For example, kōwhaiwhai (painted scroll ornamentation) have been used to exemplify transformation geometry (Allen & Trinick, 2022). However, the invisible aspects—values, relationships, problem-solving processes, and knowledge transmission practices that assisted Māori with meaning and sense-making have been ignored (Bishop et al., 2007; Meaney et al., 2021) or arguably invisibilised.

One of the arguments for the omission or tokenistic positioning of Indigenous knowledges in school curricula delineates the traditional (pre-European contact) and modern (post-contact) eras in colonised countries. This positions Indigenous knowledges as static, old or irrelevant (Smith et al., 2016) and is thus used to justify their exclusion from schooling. Concepts of pre-contact numeracy, particularly place value, are examples of mātauranga that have been erased through omission in national pāngarau and mathematics curricula in Aotearoa/New Zealand (Christensen, 2004b). The justifications for erasure or omission are further entrenched when success in the modern world is determined primarily by economic factors such as employability (Te Maro, 2018). According to Porsanger (2011), whose research makes Indigenous Sámi theories of knowledge visible, this traditional–modern dichotomy incorrectly presupposes that sophisticated Indigenous practices have been superseded. Citing reindeer ear tagging as an example of ongoing practical applications of Sámi knowledges, Porsanger (2011) argued for a research paradigm that omits the hostile application of linear timelines to Indigenous knowledges. In doing so, Porsanger (2011) invites us to position ancient and ongoing practices as omnipresent, aligning with the definitions of mātauranga presented in this introductory chapter (Cooper, 2012; Mead, 2013; Smith et al., 2016).

The inferior positioning of mātauranga has been refuted at the philosophical level in discourses about Māori *success as Māori*. For example, Durie (2003) believed that if students were to graduate from compulsory education having achieved success as Māori, this required learning about Māori practices and artefacts. This means being able to have access to te ao Māori, te reo Māori, and mātauranga. However, pāngarau curriculum development projects, thus far, have limited opportunities for this to occur in classroom practices. The issues are two-fold. Firstly, what is valued in pāngarau, as

expressed in curricula, and secondly, who influences this narrative (Te Maro, 2018). This current doctoral study identifies ways that mātauranga and interconnected Pacific knowledge systems are being maintained or expanded in ways that could support pāngarau curriculum development and classroom practice. For example, Indigenous peoples of the Pacific, in Micronesia, Hawaii, and the Marshall Islands have maintained or significantly revitalised their ocean-going travel and wayfinding techniques between islands (Goetzfridt, 2008). Wayfinding refers to using environmental cues to navigate from one location to another (Farr et al., 2012). Successful wayfinding depends on both environmental factors and human perceptions of these factors. Echoing Tāwhaki's need to identify the weight bearing aka matua, to navigate from one location to another successfully, wayfinders must understand the big picture of the environment they are traversing to determine the pertinent details (Spiller et al., 2015) or actions to take.

Māori and Pacific knowledge holders have contributed to several wayfinding research projects. For example, the University of Hawaii has offered an ethnomathematics programme since 2018. This programme draws on Hawaiian wayfinding as the basis of its STEM-focused curricula (Furuto, 2014). Wayfinding practitioners in Aotearoa/New Zealand, Te Matau a Māui Voyaging Trust, have constructed a physical wayfinding device, ātea a Rangi (star compass), and offer education programmes to schools (Ātea a Rangi Educational Trust, 2018). These programmes provide examples of how Māori wayfinding practices could be utilised in pāngarau curriculum and classroom practices. However, much of the literature and resources focus on wayfinding at sea. This study adds further discussion to this body of research by exploring how narratives and other spatial representations used for wayfinding between islands were transferred to wayfinding on land in Aotearoa/New Zealand by Māori, as discussed more fully in Chapter 5.

Researcher Positioning

According to Bishop and Glynn (1999), researchers guided by whakapapa “understand themselves to be involved somatically in the research process; that is physically, ethically, morally, and spiritually and not just as a ‘researcher’ concerned with methodology” (p. 170). This quote closely describes my positioning as a Māori researcher. I am a Kōhanga Reo graduate and was a student of Te Kura Reo Rua o Ōmahu, one of the original bilingual schools. In the early 1980s when I was born, the semi-rural community of Ōmahu still had several native speakers and, therefore, the opportunity for intergenerational language transmission (see Benton, 1979). The desire for Māori immersion schooling gained traction within a group of like-minded parents with enough children between them to justify hiring a specialist teacher. As a result, a Māori language immersion class was created at Ōmahu school, with my older brother of two years, one of its first enrollees.

I have since gained a Bachelor of Māori Development and become a Māori-medium teacher, teacher educator, researcher, and now a parent of a Māori-medium student. While I have some experience in English-medium schools as a student and teacher, most of my learning and working life has primarily been in spaces where te reo me ōna tikanga are taken-for-granted. As someone who has had access to these types of spaces and pathways from birth, I take for granted the value of mātauranga (including te reo Māori) and its contributions to my well-being.

For example, while participating in a research project involving Māori teachers and school-based marae, I disclosed that I generally always worked or learned in spaces with marae or that utilised marae nearby. At that time, I had no plans to work or learn in an environment that did not actively practice Indigenous customs and rituals as part of the educational and professional environment. Without Māori rituals and practices, for me, any perceived economic or professional benefits were essentially voided. This reflects the privileged mindset of being born into, and therefore having genealogical links to, the genesis of the Māori-medium education movement. I only became fully aware of this when I began researching pāngarau curricula development for my master's degree, and I have become even more cognisant of this worldview through completing this doctoral study. This is because I have had to engage with literature and structures that treat mātauranga as different and inferior. I have become more cognisant of those who are trapped by this rhetoric and are therefore denied the social, economic and overall well-being benefits that mātauranga provides.

My lived experiences affected the way that I approached this research project. I am an insider of Māori-medium and kaupapa Māori education at every level. However, I am also an outsider as a researcher tasked with making objective conclusions. Like Tāwhaki, I, too, must maintain hold of the aka matua by identifying and selecting the ideas that 'hold weight.' In this way, this thesis represents one part of my journey alongside Tāwhaki in seeking the knowledge that will ensure the ongoing indigenisation of Māori-medium education. My genealogical connections to this work also illuminate my ethical responsibility in maintaining objectivity while ensuring that the study supports the goals of Māori-medium education without inviting undue criticism from those who may seek its demise. This is a delicate balance, as appropriating the status and economic benefits of completing this work through academia could further alter concepts of how mātauranga expands.

Research Methodology

The following methodological approaches were used to guide the collection and analysis of data and its application in classrooms: cultural symmetry (Meaney et al., 2021), teaching as inquiry (MoE, 2020) and Bourdieu's (1992, 2010) theory of social reproduction. The cultural symmetry framework (Meaney et al., 2021) positions Indigenous languages, Indigenous knowledges, and mathematics as equally valued learning outcomes in schooling. This framework was used to examine the efficacy of

pāngarau curriculum and classroom practices in promoting or further marginalising the importance of te reo Māori, mātauranga and school mathematics. The framework was also used to highlight the risks and rewards of explicitly teaching mātauranga in pāngarau classrooms.

Therefore, school mathematics and te reo Māori are two of the three foci of the cultural symmetry framework. A considerable body of research argues for explicitly teaching the mathematics language as it simultaneously supports the acquisition of mathematical knowledge (Dowker et al., 2008; Hunter, 2005; Pimm, 1987; Pitvorec et al., 2011; Schleppegrell, 2007). Similarly, communicating mathematically using multiple representations has previously been shown to simultaneously support the acquisition of te reo Māori and school mathematics (Allen, 2015). In this study, the efficacy of encouraging communication through multiple representations was interrogated for its efficacy in encouraging the acquisition of mātauranga by ākonga pāngarau (pāngarau students), thereby addressing all three foci of the cultural symmetry framework.

Teachers can use a spiralling inquiry process to investigate aspects of their practice (Timperley, 2011) and the teaching as inquiry cycle (MoE, 2020) was used to guide the research conducted in pāngarau classrooms. The learning inquiry phase of the cycle was used to investigate the needs of the participant kura, including their localised aspirations for the illumination of mātauranga in pāngarau classrooms. Diagnostic school mathematics assessment data were also collected during this phase. The focusing and teaching inquiry phases of the cycle provided opportunities to discuss how lesson sequences could address all three foci of the cultural symmetry framework alongside the research informing pedagogical actions. Finally, the teaching and learning phase of the cycle provided opportunities to investigate the illumination of mātauranga alongside school mathematics in pāngarau classrooms and how this aligned with Māori aspirations for schooling.

The lens of Bourdieu's (1992, 2010) theory of social reproduction and the three interconnected methodological tools: field, cultural capital, and habitus were used to examine structural challenges limiting the transmission of mātauranga in pāngarau curriculum and classrooms. Bourdieu (1992) argued that positive revaluation of previously marginalised societal positions, such as the position of mātauranga in pāngarau curriculum and classroom practices, required the appropriation of economic and symbolic advantages. The establishment of Kura Kaupapa Māori, where te reo Māori and mātauranga are taken-for-granted as valued cultural capital and the appropriation of four curriculum development projects by Māori to further indigenise schooling are examples of a Bourdieusian analysis of historical events. Thus, Bourdieu's (1992, 2010) toolkit was used to gain further insights into the structures, agents and ideologies that have continually marginalised mātauranga in school mathematics.

Research Question

As discussed earlier, the main research question investigated in the study is:

What are the opportunities, challenges, and tensions for further indigenising pāngarau curriculum and classroom practices?

This research question was broken down into the following sub-questions:

1. *Which ideologies have influenced the indigenisation of pāngarau curriculum and classroom practices?*
2. *How can conflicts between mātauranga and school mathematics be addressed through teaching wayfinding?*
3. *How effective is the cultural symmetry framework in addressing tensions between mātauranga and school mathematics?*

Methods

The research drew on two primary data sources and one secondary source. The first was semi-structured interviews with mātanga pāngarau (pāngarau curriculum experts). The mātanga provided specific examples of uniquely Māori ways of communicating mathematically, utilising multiple representations, including mātauranga. The four mātanga interviewed were either involved in developing the first pāngarau curriculum document between 1992 and 1995 or were involved in supporting resource creation and initial implementation. Therefore, they were uniquely placed to discuss the ideological tensions that arise from teaching Western knowledge-based curricula such as pāngarau in Māori-medium schools. They also offered valuable commentary on what they perceived as the positive and negative impacts of this on Māori aspirations for schooling.

The second primary data source was pouako and ākonga-created classroom artefacts collected in two kura kaupapa Māori. Two classes and up to three teachers in each kura participated in the research. The classroom artefacts were analysed according to the phases of the teaching as inquiry cycle and the cultural symmetry framework as discussed earlier. The analysis of classroom artefacts provided opportunities to investigate the interplay of national pāngarau curriculum guidelines and localised aspirations for mātauranga and to address any conflicts between the two.

The secondary data source for this study was Māori and Pacific wayfinding literature (see Chapter 5). Archeoastronomical, anthropological, visual arts and ethnomathematics literature was used to supplement literature on Māori mathematics education to further inform the teaching of mātauranga in

pāngarau classrooms. This literature search provided insights into the process of researching mātauranga to support the implementation of localised mātauranga activities.

As the study was conducted during the COVID-19 pandemic, digital technology was utilised in both primary data collection phases. Further discussion on the increased reliance of pāngarau learning programmes on digital technology is also offered in Chapter 3. The affordances and limitations of digital technology for promoting mātauranga and te reo Māori through digital representations are also offered in Chapter 3 and explored further in Chapter 6 when considering the collected classroom artefacts.

Chapter Outlines

This introductory chapter outlined the context of this study, including the evolution of Māori-medium schooling, pāngarau research and the development of pāngarau curricula. While supporting the linguistic goal of language revitalisation, attempts to revive and promote mātauranga have been limited by ideological and structural factors—a consequence of colonial legacies. This includes the ongoing propensity to promote Western knowledge-based school mathematics in pāngarau curriculum and classroom practices at the expense of mātauranga.

Chapter 2 presents a literature review of approaches by Indigenous groups, both nationally and internationally, who have attempted to indigenise mathematics education. One of the gaps in the literature is how Indigenous groups use their agency to develop curricula that simultaneously promote Indigenous languages, Indigenous knowledges, and school mathematics.

Chapter 3 outlines the cultural symmetry framework (Meaney et al., 2021) and the teaching as inquiry cycle (MoE, 2020) that were used to investigate opportunities for simultaneously promoting te reo Māori, mātauranga, and pāngarau at the curriculum and classroom levels. As mathematics education is frequently a political concern, Bourdieu's (1992, 2010) theory of social reproduction was used to analyse and summarise the study data at the macro-curriculum and policy levels. The research contexts, participant information and data collection methods are also presented in this chapter.

Chapter 4 presents data gathered through semi-structured interviews with Māori-medium pāngarau expert practitioners. The interviewees included members of the writing group for *Pāngarau i roto i te Marauranga o Aotearoa* (MoE, 1996) or those who have taught, researched, and provided professional development for pāngarau teaching at the secondary school level in the three decades following. The interviews supplement the literature as there is a lacuna focused on indigenising pāngarau curriculum and classroom practices. The analysis of interview data illuminates the tensions, challenges, and opportunities for indigenising pāngarau curriculum (macro-level) and classroom practices (micro-level).

Chapter 5 presents literature on the characteristics of Māori and Pacific wayfinding devices. Opportunities for explicitly teaching Māori ways of coding and disseminating spatial information are identified in the literature alongside mnemonic and pedagogical uses of wayfinding devices throughout the Pacific.

Chapter 6 presents pouako and ākonga-created artefacts from two Māori-medium wharekura with students in Years 7–10 (11–14 years of age). The data collection and analysis in this chapter are guided by the teaching as inquiry cycle (MoE, 2020) and the cultural symmetry framework (Meaney et al., 2021). These data illuminate opportunities, challenges, and tensions in enacting the indigenisation of pāngarau curriculum at the micro-classroom level.

Chapter 7 presents a synthesis of the findings of the thesis. Bourdieu's (1992, 2010) thinking tools of field, habitus and cultural capital are employed to explore macro-curriculum and policy-level opportunities, challenges, and tensions for illuminating mātauranga.

Chapter 8 revisits Māori aspirations for schooling and highlights the agentic opportunities for rebalancing mātauranga and school mathematics through a dual-epistemic curriculum framework. Finally, the chapter discusses the study limitations and makes recommendations for further research.

2. Arotakenga Mātākōrero | Literature Review

This doctoral study examines the risks and rewards for further indigenising pāngarau curriculum and schooling for Māori. In this chapter, opportunities, challenges, and tensions for reconciling Indigenous knowledges and languages within contemporary mathematics curricula, such as pāngarau, are identified in the literature and critiqued. The first sections of this chapter identify epistemological, ontological, and axiological tensions impacting international curriculum indigenisation projects. Following this, indigenisation approaches employed by researchers within the mathematics education field and the influences of this on curriculum and compulsory schooling are offered. The chapter then examines the emerging literature discussing the risks and rewards of indigenising pāngarau curricula. Similarly, the pāngarau research literature is critiqued from ontological, epistemological and, to a lesser extent, axiological perspectives. This includes exploring how aspects of mātauranga have been incorporated into pāngarau teaching, learning and assessment programmes, or have the potential to be.

Indigenising Curriculum

Gaudry and Lorenz's (2018) definition of indigenisation is helpful in this study as it delineates the risks and benefits of three curriculum indigenisation approaches: Indigenous inclusion, reconciliation, and decolonial indigenisation. The focus on Indigeneity in this thesis is on the minority disempowered Indigenous groups whom external powers have colonised, such as Māori in Aotearoa/New Zealand. Within the context of tertiary curriculum, Gaudry and Lorenz (2018) positioned decolonial indigenisation as a transformative indigenisation programme that included teaching, research, and administration. The researchers characterised decolonial indigenisation as a process that directly benefits Indigenous languages, knowledge, and peoples. In contrast, Indigenous inclusion is a process of enabling teaching staff and students to integrate into existing structures. An example of Indigenous inclusion experienced by Māori in Aotearoa/New Zealand is the social housing policy known as pepper-potting. As discussed in Chapter 1, Māori families were pepper-potted in Pākehā dominant communities, furthering the assimilatory work of colonisation (Benton, 1997; Ka'ai-Mahuta, 2011).

Gaudry and Lorenz (2018) defined a third type of indigenisation, reconciliation, which is reflected in the Truth and Reconciliation Calls to Action in Canada (Government of Canada, 2022).

Reconciliation is oriented towards establishing a wider and more inclusive consensus on what constitutes knowledge (Gaudry & Lorenz, 2018). Gaudry and Lorenz describe the reconciliation approach as a process for determining what relationships should exist between the academy and Indigenous communities. This echoes the Aotearoa/New Zealand MoE's (2019) guidance to schools to develop learning-focused relationships with iwi to localise the national curriculum. However, the

approach seems more focused on making existing colonial structures more amenable to Indigenous peoples but does not seem to produce real structural change.

Gaudry and Lorenz (2018) have thus criticised both Indigenous inclusion and reconciliation approaches in education as ineffectual and merely positioning Indigeneity as an add-on to the existing structures and hierarchies of the coloniser. In these approaches, Indigenous knowledges and peoples are co-opted into providing access to Indigenous knowledges for the benefit of non-Indigenous peoples; therefore, assimilation into the dominant cultures and structures is the eventual outcome (Gaudry & Lorenz, 2018). The use of Indigenous languages, knowledges, and schooling to promote the acquisition of Western mathematics concepts (Allen & Trinick, 2022; Meaney et al., 2021) is a further example of Indigenous inclusion and reconciliation, thereby marginalising mātauranga in curriculum and classroom practices. This thesis argues against appropriating mātauranga into government-mandated curricula to make school mathematics more palatable to ākonga Māori (Māori students). Ākonga Māori may enjoy further success and satisfaction in pāngarau due to the illumination of mātauranga in pāngarau curriculum and classroom practices. However, if illuminating mātauranga in pāngarau curriculum and classroom practices are not positioned as goals, then mātauranga is likely to be included or reconciled with school mathematics and thus further marginalised.

Guerzoni (2020), in a review of literature on indigenising curriculum in higher education, provided a summary of the challenges Indigenous peoples face when participating in this process. Echoing the literature reviewed in this section, Guerzoni (2020) identified the tokenistic positioning of Indigenous knowledges and resources, or conceptualising Indigenous knowledges as merely contained in antiquities, as two such challenges. Repositioning Indigenous knowledges as merely one of many groups in a multicultural society has been another method for subjugating Indigenous knowledges to further promote dominant Western epistemologies (Antoine et al., 2018).

A further challenge to indigenising curriculum highlighted by Guerzoni (2020) is abrogation. Of particular interest to this study, abrogation can involve the wholesale replacement of Western knowledge with Indigenous knowledge or the hybridising of the two knowledge systems (Nakata, 2007, 2017). Both these approaches have been utilised in national curriculum documents for compulsory schooling in Aotearoa/New Zealand. Hybridising mātauranga and school mathematics through pāngarau curriculum may seem effective in reversing the abrogation of mātauranga. However, indigenisation resists abrogation or hybridisation and positions knowledge systems as distinct and complementary (Guerzoni, 2020; Morelli et al., 2013). As defined by Gaudry and Lorenz (2018), Indigenous inclusion could be seen as a form of hybridisation or a more gradual transition to abrogation by the dominant knowledge system. For example, the teaching of school mathematics through te reo Māori was negatively seen by members of the Māori community as stretching the

language and culture to include the ideologies of the coloniser (Christensen, 2004b; McMurchy-Pilkington, 2004; Meaney et al., 2012; Smith, 2012). As the first and current pāngarau curricula are based on an imported English-language document, this could be seen as promoting a delimited view of mātauranga through omission.

Ontological and Epistemological Pluralism

Indigenous academics working in Western academic institutions are adept at walking in two worlds and speaking to two audiences simultaneously—the academic audience and their own Indigenous communities (Galla & Goodwill, 2017; Ka'ai, 2008). Arguably, Indigenous academics are required, by the pluralistic expectations of these two audiences, to examine information and experience through multiple lenses. Ontologies are the 'cultural lenses' or orientations used to group information and experiences to create epistemologies (Mika & Stewart, 2017). Epistemology is the grouping of information and experiences to create understandings (Mika & Stewart, 2017). Mika and Stewart (2017), proponents of Māori philosophy, position epistemology and ontology not as separate but as a linked process where ontology precedes epistemology. For Mika and Stewart (2017), ontology provides the lens through which epistemological experiences are grouped and where new knowledge is created.

Ontological or cultural pluralism, as demonstrated by Indigenous academics, enables the co-occurrence rather than the hybridisation or abrogation of different cultural resources (Pöllmann, 2021). Indigenous knowledges provide interrelated ontological categories unlike those of Western science, which have been categorised as distinct (Govender & Mudzamiri, 2022). Hauser and colleagues (2009) argue that ontological pluralism underlies any meaningful co-existence of epistemologies in curricula, providing opportunities for Indigenous and Western ways of knowing and being to co-exist.

Miller and colleagues (2008), who theorised the benefits of interdisciplinary research for investigating complex issues, advocated a shift away from promoting one epistemology or theory of knowledge over another. Instead, Miller et al. (2008) promoted epistemological pluralism as a way of examining the tensions that exist between different knowledge systems. However, the authors also conceded that often well-intentioned efforts to value multiple knowledge systems equally in research still demonstrate a preference for one theory of knowledge over another. This is because epistemologies shape how questions are answered and how knowledge is validated and legitimised through assumptions and methodologies (Miller et al., 2008).

Andreotti and collaborators (2011), in their examination of the applications of epistemological pluralism in higher education, provided a two-step process for epistemological pluralism to occur.

Comparing the contextual and historical emergence of each knowledge system is the first step. An investigation of the potential contributions of each knowledge system to current issues is the second. Thus, epistemological pluralism moves beyond synthesising two epistemological lenses, likely privileging the dominant one, but promoting both simultaneously (Andreotti et al., 2011). Epistemological pluralism, as outlined by Andreotti and colleagues (2011), provides a way to regenerate mātauranga by examining the historical emergence of mātauranga in the Pacific and its contextualisation to Aotearoa/New Zealand.

Indigenous Mathematics Education Research

In contrast to epistemological pluralism, European anthropologists recorded what they considered representations of Indigenous mathematical practices from a singular and often pejorative view. For example, Best (1907) examined the numeration system of “neolithic Māori” and the “rudiments of modern science as observed in Māori usage” (p. 94). Crawford (as cited in Harris, 1987), in a very early publication, demonstrated his knowledge of Australian Aboriginal numbers by listing “the rudest numerals of the lowest savages of which we have any knowledge” (p. 30). Pre-European contact oral traditions such as stories and chants, used by researchers to explore Indigenous mathematical concepts (Chauvin, 2000), were also recorded in the 19th century by missionaries in parts of the Pacific. These records often omitted material that the missionaries found distasteful or conflicted with their religious views (Ruggles, 1999), such as sexual content (Taonui, 2015). Interpretations of mathematically related ideas in these early studies often reflected historical issues of Western political dominance (Trinick, 2015). Jahnke and Jahnke (2003) argued that the Eurocentric belief in the superiority of Western paradigms over Indigenous worldviews has not significantly diminished. The ongoing inferior positioning of mātauranga in the national curriculum provides evidence of this effect.

While a reasonable corpus of research focusing on Indigeneity in mathematics education exists, it generally positions Indigenous students as being “the other” (Jorgensen, 2011) because their achievement in mathematics is not the same as for non-Indigenous students. Using a Bordieusian framework, Jorgensen (2011) critiqued the experiences of Indigenous Australian students in mathematics schooling. Jorgensen (2011) provided examples of how mathematical practices that Indigenous students learn in their homes and communities are disregarded when they do not align with the unquestioned practices of school mathematics. Jorgensen (2011) argued that Indigenous students are further disadvantaged when taken-for-granted assumptions about learning are structured into curriculum documents, assessments, or pedagogical practices (including group work, streaming and textbooks).

Indigenous students in Australia are sometimes regarded as having advanced spatial awareness compared to their European peers due to the prevalence of formal directional language use, such as cardinal directions in their communities (Edmonds-Wathen, 2011). However, younger Indigenous students are often expected to unlearn more advanced skills in favour of more rudimentary ones (such as left, right, front and behind) as this is what is expected to be known by the majority non-Indigenous students of the same age (Jorgensen, 2011). If students fail to do this unlearning within the defined timeframe outlined in national curricula, teachers can then falsely assume that the Indigenous students are having difficulty learning the less advanced skills. In this way, colonial schooling practices create knowledge debts for Indigenous students (Aikenhead, 2018). When Indigenous student achievement does not follow expected guidelines, a lack of relevant knowledge is attributed to their supposed academic failure and related diminishing opportunities to advance (Aikenhead, 2018). Jorgensen (2011) argued that the majority, so-called “mainstream” practices and ways of knowing and doing, are valued over others by the school mathematics field. The expected knowledge and behaviours, or cultural capital, are often determined by curriculum structure and content (Jorgensen, 2011).

Ethnomathematics

During the 1980s and 1990s, growing resistance to deficit interpretations of Indigenous students’ learning emerged among teachers, mathematics educators, and researchers (Powell & Frankenstein, 1997) and the colonial prejudices underlying them (Bishop, 1990). Such critical work has attempted to show that other forms of mathematics exist beyond imported Western school mathematics (Gerdes, 1985). International mathematics education research groups, such as the International Congress on Mathematics Education topic groups and the Mathematics, Education and Society research group, have had contributors working from sociological, cultural, and linguistic rights perspectives. Forums such as these have created spaces to explore the indigenisation of curricula and to discuss what the indigenisation of the mathematics education field means in practice and theory.

Scholars concerned with Indigenous knowledges have challenged the mono-epistemic view that mathematics is universal and culture free (Barton, 1996; Ernest, 1991). Positioning mathematics education as acultural ignores the way school mathematics promotes the cultural practices of the majority non-Indigenous students and disregards the cultural capital of Indigenous students (Jorgensen, 2011). Bishop (1991) argued that mathematics is a cultural product of participation in various activities, such as counting, locating, designing, playing, measuring, and explaining. Bishop (1991) also characterised mathematics as creating relationships between people, their society, and their environments. For example, the seminal work of Gay and Cole (1967) and Lancy (1978) proposed a creative mathematical education that used Indigenous mathematics as the starting point. Ascher and Ascher (1981) studied informal mathematics outside the formal education system and its

potential applicability to school mathematics. Gerdes (1985) provided a critique of situations in which mathematical elements existed in the daily life of Indigenous populations during colonial occupation but were not recognised as mathematics by the dominant colonial group's ideology of Western mathematics supremacy. To raise the status of Indigenous peoples' mathematics, Gerdes (1985) endeavoured to reconstruct or "unfreeze" Indigenous mathematical thinking that was "hidden" or "frozen" in techniques such as basket making, which might stimulate cultural awareness in Indigenous learners of mathematics.

D'Ambrosio (1999) conceptualised the relationship between culture and mathematics as ethnomathematics. The ethnomathematics discipline, drawing on minority groups' rights perspectives, has been instrumental in creating space for research that promotes Indigenous knowledge systems. Owens (2014) asserts that mathematics education must provide opportunities for Indigenous students to explore spaces and places that have meaning and representation to them. Fyhn and colleagues (2011) highlighted the Sámi mathematical practices that can be used in contemporary Western mathematics classrooms, including measurement concepts and an understanding of ratios. Ascher (1995) documented Marshall Island navigators' representations of spatial information. Furuto (2014) has furthered this work by utilising Pacific wayfinding techniques and star compasses as the basis for geometry investigations. Barton (2008) and Trinick et al. (2015) added a language dimension to the ethnomathematics literature in the New Zealand context. Examining the interplay of language and mathematics learning is particularly relevant if most Indigenous knowledge systems have been eroded by colonial oppression. The Indigenous language can offer clues and cues to the nature of the Indigenous knowledge system (Trinick, 2015) because most of the culture is expressed in the language (Fishman, 1991).

However, many of the ethnomathematics studies listed previously are primarily concerned with Indigenous concepts that already align with Western mathematics paradigms, such as Bishop's (1991) six examples. Prominent ethnomathematics researchers, Ascher and Ascher (1981), cautioned that the knowledge base of Western researchers could limit interpretations of Indigenous practices and artefacts. According to Ascher and Ascher (1981) and Ascher (1995), this is particularly evident when the artefact or concept being explored is previously unknown to the researcher, as "even under the best circumstances, another culture is blurred as if seen through heavy gauze" (Ascher & Ascher, 1981, p. 4). Ruggles (1999) observed that this is also evident when the practices contained in Indigenous language records, such as oral narratives and chants, are interpreted by non-language experts relying on coincidental dictionary definitions. Therefore, ensuring that Indigenous knowledges and languages are promoted in ethnomathematics studies as equally valuable to the mathematics being studied is an ongoing challenge for the indigenisation of mathematics education research.

Cultural Symmetry: Indigenising Mathematics Education Research

To address the challenge of valuing the tripartite outcomes of language, culture, and mathematics through their cultural symmetry framework, Meaney et al. (2013) proposed a three-step approach whereby research and teaching foci provide cultural, linguistic, and mathematical stimuli. As Meaney and colleagues (2021) outlined, the first step is acknowledging the Indigenous knowledges connected to the processes and artefacts under study using the appropriate Indigenous language. The second step is examining the designs used to create the different artefacts and discussing them from a range of perspectives to see how these design elements are used in other cultural artefacts. The third step is discussing the design elements concerning mathematical principles and showing how mathematics can add value to understandings about the artefacts without detracting from other cultural understandings.

There are a growing number of applications of the cultural symmetry framework in Indigenous mathematics education literature. For example, Trinick and collaborators (2017) examined Māori spatial orientation concepts and the importance of symmetry in wharehau (Māori meeting houses), while Meaney and colleagues (2021) reviewed applications of cultural symmetry in Norway and Aotearoa/New Zealand. As exemplified in these studies conducted *by and with* Indigenous peoples, research in Indigenous contexts is shifting away from research *on* Indigenous peoples (Jorgensen & Wagner, 2013). Through examining the dual-epistemic positioning of mātauranga and school mathematics in pāngarau classroom practices, this doctoral research adds further insight into the use of the cultural symmetry framework to inform research conducted by Māori.

The Emergence of Pāngarau Education Research

Pāngarau education research gained significant momentum with the development of *Pāngarau i roto i te Mārauranga o Aotearoa* (MoE, 1996). To produce the translation of the English-language document *Mathematics in the New Zealand Curriculum* (MoE, 1993), significant elaboration of te reo Māori was required and research projects were conducted to support the development (Barton & Fairhall, 1995; Barton et al., 1998; Trinick, 1999). The pāngarau education research agenda has since grown to include the contributions of pāngarau education at the macro-level of Māori language and cultural revitalisation efforts and policies (Meaney et al., 2012; Trinick, 2015; Trinick & May, 2013), as discussed in the previous chapter. Some studies have also addressed the ontological impacts of pāngarau education on Māori autonomy and identity (Barton & Fairhall, 1995; McMurchy-Pilkington, 2004; Te Maro, 2018). A further area of research examines pāngarau professional development and resourcing policies and how these have impacted micro-level classroom challenges such as the pedagogical content and language knowledge of pouako and ākongā achievement (Allen, 2015; Christensen, 2004b; Hāwera, 2011; Trinick & Stevenson, 2006).

A smaller section of pāngarau education research focuses on Māori epistemology and the illumination of mātauranga in pāngarau classrooms. Earlier studies tend to draw on the work of teachers in the early bilingual schools or by elders (Ohia, 1995; Rikihana, 1990; Trinick, 1999) and use discipline titles that preceded pāngarau such as matematika (mathematics), mahi whika (working with figures) or tātai (setting in order, calculation, reciting genealogy). A small number of more recent studies outline the development and utilisation of the cultural symmetry framework at the micro-level, introduced earlier in this chapter (Meaney et al., 2012, 2021; Trinick et al., 2015, 2017). By examining the illumination of mātauranga in pāngarau curricula and classrooms through the tripartite focus of the cultural symmetry framework, this thesis further expands this emerging section of pāngarau education research on micro pedagogical challenges. The following sections identify the challenges and opportunities for illuminating mātauranga in pāngarau curriculum and the impacts of the macro-policy and curriculum on pāngarau classroom practices, including linguistic and ontological tensions. Finally, the epistemological tensions, challenges, and opportunities for illuminating mātauranga in pāngarau classrooms and curricula are identified within the wider pāngarau education literature.

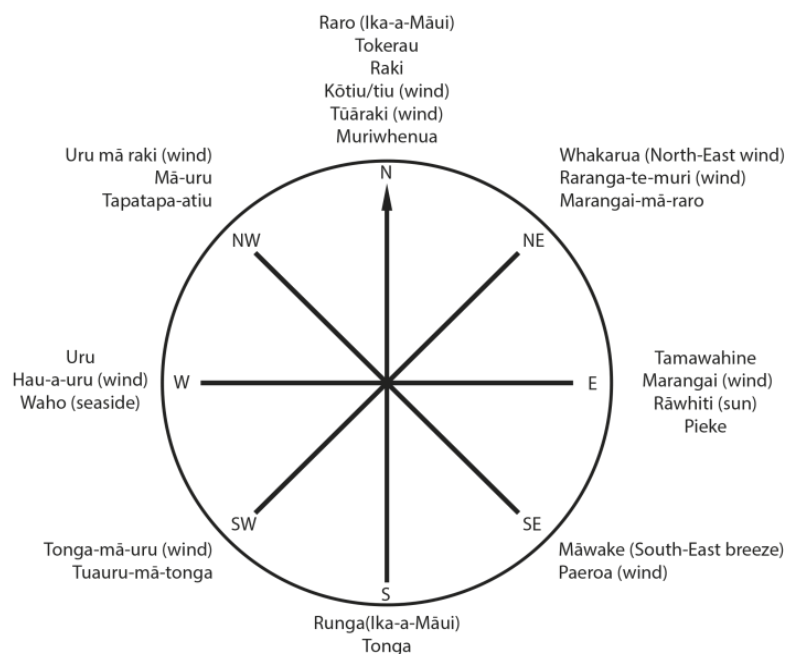
Mitigating the Ontological Risks of Language Elaboration

Pāngarau curriculum and classrooms are a unique interface between knowledge systems and ways of knowing, joined in many ways by the specialised pāngarau language or register. Cummins (2000), one of the leading authorities on bilingual education and second language acquisition, characterises a linguistic register as the language required to function in a particular context or task. Similarly, Halliday (1978), who wrote extensively on the role and features of the mathematics register, describes a register as the set of meanings pertaining to a particular language function along with the words and structures required to communicate those meanings. The pāngarau register can be seen as a specialised set of meanings used to communicate mathematical ideas within the unique context of Māori-medium classrooms. The development of the pāngarau register has required a significant Māori language elaboration project to support the communication needs of both pouako and ākonga in pāngarau classrooms (Trinick, 2015). Therefore, the pāngarau register has been elaborated in tandem with the development of pāngarau curricula and classroom resources. As a result, te reo Māori has undergone vocabulary expansion and grammatical change over the last three decades in this new Māori-medium curriculum context (Trinick, 2015).

The pāngarau register, as exemplified in classroom resources, includes several mathematical representations that draw explicitly on mātauranga. Mathematical representations can visually or tangibly communicate mathematical ideas and relationships (Goldin, 2014). Therefore, the pāngarau register also includes visible and tangible forms of communication, such as written words, diagrams,

physical models, mathematical expressions, formulas, and equations. For example, the *kāpehu hau* (wind compass) appears in the curriculum in spatial orientation resources (see Figure 1). *Kōwhaiwhai* have also been used to illustrate concepts of transformation geometry in *pāngarau* classroom resources. Further, illuminating *mātauranga* in *pāngarau* curricula and classrooms will increase the ways *mātauranga* informs representations of *pāngarau*. It is important to understand whether this will strengthen or further erode concepts of Māori ontology, diminishing the opportunity for students to experience success *as Māori*. Therefore, the experiences of curriculum developers and classroom practitioners as recorded in the literature are presented in this section and inferences are drawn for the potential illumination of *mātauranga*.

Figure 1. *Kāpehu Hau* | Wind Compass (Trinick et al., 2015, p. 424).



Barton and Fairhall (1995) discussed the impact of *pāngarau* register elaboration strategies on Māori ways of knowing and being. Trinick (2015) also discussed concerns that were raised about how the *pāngarau* register development introduced new styles of meaning (i.e., representations and graphs), new ways to tell stories (i.e., mathematical number stories), new syntactic structures (i.e., for multiplication), and combining existing grammatical features into new combinations during the elaboration. Recognising the dangers of ‘modernising’ the Māori language along these lines, Barton and Fairhall (1995) further warned against the uncritical adoption of new terminology by teachers and the potential erosion of the ‘Māori ethos’ of the language. They queried:

... has the European techno-rational ethos been embraced, Trojan-like, as an inevitable part of modern life, and thereby destroyed the conceptual basis of Maori culture? (p. 1)

Mika and Stewart (2017), whose work explored concepts of Māori philosophy and ontology, asserted that ideas from a Māori perspective are vulnerable to human intrusion and that the fabric of an idea is provided by what it relates to and has past and future connections to other ideas and entities. This sentiment is echoed by Trinick (2015), one of the leaders of the elaboration process:

Changing the language used to describe the experience, whether it is a change in natural language, such as English to te reo Māori—or a change in the register within a language—will inevitably have an impact on how that experience is described and therefore what is valued in that experience. (p. 194)

However, not all contributors to the pāngarau register development agreed that the elaboration process would irreversibly change the Māori worldview contained within the language. Rikihana (1990) advocated for the ongoing expansion and growth of te reo Māori and believed that the language possessed the linguistic flexibility to be both descriptive and scientific. According to Rikihana (1990), the expansion of te reo Māori to include mathematics and scientific terms strengthened, rather than eroded, te reo Māori. This argument is pertinent to the revitalisation of mātauranga and the potential for mātauranga to be further expanded and strengthened through pāngarau curricula development.

Similarly, Ohia (1995), one of the early pioneers of the pāngarau research discipline, positioned pāngarau as a subset of the Māori world. Ohia (1995) argued that the elaboration of te reo Māori to support pāngarau curriculum development made explicit links to mātauranga. Therefore, Ohia (1995) believed that Māori teachers and students could take ownership of the pāngarau curriculum as their own. This seems to echo Smith and collaborators' (2016) argument that agentic responses to colonisation, such as teaching school mathematics in te reo Māori, expand concepts of Māori ontology and, therefore, mātauranga. However, neither Rikihana's (1990) nor Ohia's (1995) arguments explicitly addressed the risks of hybridising mātauranga and school mathematics. Evidence supporting both sides of these linguistic and ontological arguments, and the strategies employed to mitigate any negative consequences, are identified in the remainder of this section. Both Rikihana's (1990) and Ohia's (1995) arguments are further explored in the interviews with mātanga pāngarau in Chapter 4 and the analysis of classroom data in Chapter 6.

An ideology adopted by pāngarau register elaborators to guard against the hybridising of te reo Māori was linguistic purism. Throughout the 1980s and 1990s, Te Taura Whiri i te reo Māori (the Māori Language Commission) advocated for linguistic purism to provide a criterion for the choice of new vocabulary (Harlow, 1993). Purism often involves returning to (or searching for) linguistic authenticity (Annamalai, 1979). Purism can take the form of removing elements from the language (usually vocabulary) that appear to be foreign or corrupt or lacking in true authenticity in the linguistic culture in question (Annamalai, 1979), as was the case with te reo Māori. The linguistic

purism ideology and strategies employed by pāngarau register elaborators to achieve it seemed to support the concept of ontological pluralism where the mathematics and pāngarau registers became distinct and complementary, rather than a single hybridised register. Linguistic purism seems to exemplify a resistance to the hybridisation of languages, ensuring the Indigenous language does not become a pidgin or hybrid language and maintains its distinctive qualities. According to Trinick and May (2013), linguistic purism ensured that strategies were employed to avoid assimilating naturally occurring innovations (such as transliterations and words borrowed from English) into subject register dictionaries. This contrasts with hybrid or pidgin languages such as Bislama in Vanuatu, a European-lexifier pidgin language, where borrowed English words are pronounced with Indigenous phonology (sounds) (Baldauf & Kaplan, 2006). Purging transliterations from Māori-medium curriculum documents promoted the revitalisation of native terminology (Trinick, 2015).

Although the linguistic purism ideology was adopted to limit the erosion of te reo Māori, the use of te reo Māori to express school mathematics concepts has changed the way non-mathematical ideas are expressed. An example of this is how newly coined language terms in the pāngarau register are making their way outside the pāngarau classroom to be used in everyday speech. For example, the term 'ake' used to denote a positive integer and is now used in some cases to describe a positive feeling (Trinick, 2015). To avoid this happening, the pāngarau register elaborators coined terms that did not lend themselves to everyday use (Barton et al., 1995a), increasing the conceptual distance between the pāngarau register and the existing non-mathematical meanings of Māori terms (Barton & Fairhall, 1995).

The term pāngarau was created to represent one of the big ideas in school mathematics—multiple relationships. In te reo Māori, rau means many and pānga means relationships (Barton et al., 1998). As discussed in Chapter 1, mātauranga is a relational knowledge system underpinned by whakapapa; therefore, the term pāngarau resonated with the curriculum developers (Barton et al., 1998). Pāngarau was chosen over other terms used at the time, such as the transliteration whika (figure) or the native term tātai (Trinick, 1999). However, not all language elaborators were convinced that a new term was needed. Tātai, which can be understood as measuring, arranging, setting in order, or reciting (genealogy) (Moorfield, 2003), was a term that could have been used for the curriculum title. Coining the new term pāngarau created conceptual distance between the new curriculum document and the existing practice of tātai. While pāngarau and tātai seem to communicate similar ideas, the term pāngarau distinguished the Māori language school mathematics curriculum and tātai practices, guarding against the potential negative consequences of hybridisation. However, tātai could also be consigned to obscurity through exclusion from the high-status pāngarau curriculum. Further interrogation of the negative consequences to tātai practices is presented in the analysis of mātauranga interview data in Chapter 4. The next section examines the challenges for pouako and ākongā who

were tasked with implementing the pāngarau curriculum and the newly elaborated pāngarau register in classrooms.

Challenges to Communicating Mathematically in Pāngarau Classrooms

Creating a specialised subject register at speed created pedagogical implications for pāngarau classrooms. The pāngarau register was elaborated significantly over a three-year period to support the pāngarau curriculum development project undertaken between 1992 and 1995 (McMurchy-Pilkington, 2004; Trinick, 2015). In comparison, the English-language mathematics register has evolved over hundreds of years and assimilated naturally occurring innovations over this time. As discussed previously, the language purism ideology adopted by the pāngarau register elaborators required the removal of naturally occurring innovations such as transliterations (Trinick, 2015). This became problematic in the two decades following the development of the first curriculum document because many Māori immersion ākonga and pouako in Māori immersion pāngarau classrooms were second language (L2) learners of the language of instruction (Christensen, 2004a; May et al., 2004; Meaney et al., 2012). This is a legacy of decades of covert and overt language erosion policies whereby te reo Māori usage became restricted to a few (non-formal) language domains (Trinick, 2015). In addition to acquiring the language of instruction—te reo Māori, pouako and ākonga were tasked with acquiring the new pāngarau register quickly to ensure the implementation of the new curriculum document. This would have been a challenging task in a first language. The removal of borrowed words from the English language and the language separation policy adhered to in many Māori-medium schools and kura meant that pouako and ākonga were thus unable to use their stronger language, English, to support the acquisition of the new pāngarau terms.

The challenge of learning the new pāngarau terms was even more acute at the upper-secondary levels where the specialised terms were used infrequently. Because of the challenges faced by pouako and ākonga in implementing the first pāngarau curriculum document, Trinick (2015) believed it would have been more efficient to transliterate (borrow) terms from English. Stewart (2007), who contributed to a subsequent pūtaiao (kaupapa Māori science) curriculum development project, questioned the time and resources being directed towards creating terms that were rarely used, particularly at upper-secondary levels of curricula. Both Trinick (2015) and Stewart (2007) seem to believe that the specialised terms used at upper-secondary levels of the curriculum were used so infrequently that borrowing terms from English was unlikely to have a significant negative impact on te reo Māori outside the pāngarau and pūtaiao classrooms. The risks and rewards of linguistic purism for the development and classroom implementation of pāngarau curricula are explored in subsequent sections of this chapter and Chapter 4.

The pāngarau register elaborators had to make an inevitable choice between utilising English transliterations or purging these under the linguistic purism ideology. This dichotomy seems to echo Porsanger's (2011) traditional–modern dichotomy, where purging transliterations leans towards maintaining the authentic traditional language at the expense of a more pragmatic solution for ākongā and pouako. As Porsanger (2011) exemplified in the Sámi context, both the traditional and the modern continue to exist simultaneously. Likewise, there is a mixture of transliterations, newly coined terms and native terms being used in pāngarau classrooms, as exemplified by the mātanga interview data presented in Chapter 4 and classroom data presented in Chapter 6. Further to this, the use of transliterations for pāngarau terms such as half (hāwhe), are often seen as identity markers for those who are native speakers or learned their language from native speakers prior to standardisation (Meaney et al., 2012), creating a further divide between those who learned their language 'traditionally' and those who did so through 'modern,' linguistically 'pure' curricula.

Beyond vocabulary, pouako and ākongā had to master the grammatical structures for communicating school mathematics concepts in te reo Māori (Trinick, 2015). Pimm (1987), a seminal writer on the relationship between language and mathematics learning, argued that students (and teachers) require a functional grasp of the mathematics register to communicate mathematically. A language function is the ability to use language for a specific purpose (Halliday, 1978). To provide for the functional needs of pouako and ākongā in communicating mathematically, the Māori syntax has been stretched, producing a combination of unfamiliar words and strange syntax (Barton & Fairhall, 1995). This has caused considerable difficulty for those tasked with using it in classrooms, particularly as L2 learners.

For example, one of the ways that meaning can be constructed in mathematics in English is through logical connectives—conjunctions that join ideas together that have a logical relationship (Fairhall et al., 2007). These relationships can be sequential (time) or about reason and purpose and are heavily embedded in Western mathematics (Trinick, 2015). These situations include similarity, contradiction, cause and effect and logical sequence (e.g., the hypothetical situation signalled by the connector “if” in the following sentence: “If I subtract 5 from 12, how many remain?”) (Trinick, 2015, p. 62). In adopting the pāngarau register, Māori teachers are charged with constructing syntactical structures to reproduce this phrasing in Māori. This can be done in a way that preserves Māori structures or can replicate English grammar using Māori words. Grammatical issues were generally mitigated by the abundance of logical connectors in te reo Māori. Mātanga 3, interviewed in Chapter 4, provided examples of uniquely Māori ways of communicating about mathematical operations to illustrate this point.

Terms such as “relations” are slightly more difficult to incorporate into pāngarau instruction. As Meaney and collaborators (2012) observed, the appropriate term varies to suit different contexts. Unlike English, where the term “relationship” can be used in various contexts, in te reo Māori, the

term “relation” can be translated as either whanaunga or pānga. Whanaunga implies a human relationship. Therefore, whānau terms are considered inappropriate to describe “relationships between mathematical ideas and objects” (Meaney et al., 2012, p. 88). If so used, it will cause a meaning change to the term and thus a change to the wairua or ‘spiritual essence’ (Meaney et al., 2012).

For pouako and ākongā grappling with new terminology and syntax, this was made even more difficult by the language separation policy adhered to by Māori immersion settings, including Kura Kaupapa Māori (Christensen, 2004a; May et al., 2004; Trinick, 2015) thereby removing the opportunity to use their stronger language, English, to support their acquisition of the new terms and syntax in Māori. As discussed in Chapter 1, *Te Aho Matua* (DIA, 2008), the founding document of Kura Kaupapa Māori, advocates the separation of instructional languages by time, place, and often, person teaching. Cummins (2000), a leading researcher in language acquisition, questioned this rigid separation and argued that there is little evidence to support pedagogical assumptions about the benefits of this approach.

Similarly, Jones (2009), a mathematics language researcher in Ysgol Cymraeg (Welsh language immersion schools), argued that code-switching or switching between Cymraeg (Welsh) and English could lead to internalised beliefs that Cymraeg was not capable of expressing mathematical concepts. Arguably, this is also applicable in Māori immersion classrooms and could further support the mono-epistemic argument that Māori are producers of culture and not knowledge. Practically, in English-language dominant countries such as Aotearoa/New Zealand, once the switch from Māori to English has occurred, there is no guarantee that pouako and ākongā will switch back (Trinick, 2015). Therefore, when ontological and epistemological arguments are considered, language separation seems to provide some resistance to hybridising the two languages.

A further challenge to the ongoing indigenisation of Māori immersion classrooms and curricula is how the pronunciation of te reo Māori is changing. The pronunciation of te reo Māori has also been greatly influenced by English over time (Watson et al., 2016). This issue is significant for Māori language instruction because slight changes in pronunciation affect the meaning being conveyed. For example, the term ‘pāngarau’ was coined to provide a specific term for referring to the curriculum content; however, when the macron is omitted, which can be an issue for L2 speakers, the term ‘pangarau’ now refers to many guesses or puzzles. As the incorrect pronunciation leads to a term that could reflect the beliefs of some ākongā about pāngarau, this shift could be cognitively calcified. Another example is the term paremata, a native term that refers to compensation. However, if a macron is added, this then becomes the transliteration pāremata (parliament). While Māori are engaged in the Waitangi Tribunal process of negotiating compensation for breaches of Te Tiriti o Waitangi, the idea that parliament provides compensation is not too far of a conceptual stretch.

A final example is changes in intonation when posing a question. Questioning is a pedagogical strategy that can aid the development of mathematical understanding (Fraivillig et al., 1999). To pose questions for ākongā to consider, pouako model the stress and intonation patterns of te reo Māori. While the intonation of a question in English raises at the question mark, the opposite is true for te reo Māori. As evidenced by professional voice actors who have been trained to perform in the Māori language, the intonation lowers at the question mark (M. Ngaropo, personal communication, September 9, 2018). The loss of this distinctive feature of te reo Māori supports arguments about erosion through hybridisation. This will be further interrogated through mātaunga interview data in Chapter 4 and the classroom data analysis in Chapter 6.

The literature reviewed suggests that the pāngarau register is a type of hybrid language—a grammatical mixture of Māori and English. There is growing evidence to suggest that the syntax of te reo Māori is also generally changing to be more like English (Harlow, 2001). Overall, the literature reviewed in this section seems to indicate that the creation of the pāngarau register has irreversibly changed the Māori language. In the late 1990s, Barton and colleagues (1998) asserted that the inclusion of mathematical discourses was changing the Māori language and its speakers. Therefore, developing a mathematics register in an Indigenous language, despite the emphasis on “authentic” vocabulary, can still become an unwitting vehicle for transforming the phonology and syntax of a language and thus providing a gentler pathway to abrogation, as described in previous sections. While the concerns of irreversible change and the eroding of ‘the Māori ethos’ of the language were consistently raised in the literature, strategies for mitigation were limited. As identified by Trinick (2015), perhaps one of the ironies facing the revival of the language is that to save te reo Māori, the language has had to become more like English.

Mātauranga in Pāngarau Curriculum and Classroom Resources

While the elaboration of te reo Māori to teach school mathematics has contributed to wider revitalisation efforts, the illumination of mātauranga has not followed the same trajectory. For example, an early example of a bilingual classroom teaching resource produced by Rikihana (1990) *Tikanga Tau, Tikanga Āhua, Tikanga Tātai, Tikanga Mahi, Tikanga Huinga* included pūrākau (narratives). Pūrākau preserve and disseminate ancestral knowledge, reflecting Māori ontological concepts, and portray the lives of tīpuna (ancestors) in creative, diverse, and engaging ways (Lee, 2005). Lee’s (2005) research on the use of pūrākau as a pedagogical device found that the skills of telling and deciphering pūrākau can be considered essential Māori cultural skills that need to be taught and learned. Without written records, tohunga were tasked with coding and conveying their observations through creative and engaging oral and visual narratives (Christensen, 2013; Mead, 2013). Memorising the information contained in narratives was a survival skill (Barton & Fairhall,

1995). Rikihana (1990) identified specific pūrākau and aspects of mātauranga that had synergies with school mathematics, including pūrākau concerned with tātai arorangi (celestial navigation). However, Rikihana (1990) did not provide further detail on how to teach pūrākau alongside school mathematics. It could be argued that in the absence of explicit instruction on the skills of constructing and decoding pūrākau, Western, likely Christianised, decoding conventions preferencing moralistic, spiritual, or disparaging interpretations may be unintentionally promoted. In addition to the literature referenced in chapter 1, further exploration of the dominance of Western and Christianised interpretations of pūrākau and karakia are explored in chapter 5. Similarly, the Tāwhaki narratives were discussed in the opening sections of *Pāngarau i roto i te Marautanga o Aotearoa* (Te Tāhūhū o te Mātauranga, 1996). Tāwhaki's characteristics of exploration, problem solving, and service highlighted the synergies between the pāngarau curriculum, tātai arorangi and tātai whenua (environmental genealogies), as exemplified in the pūrākau. However, no further information on what or how to teach was offered.

According to Lee (2005), a pūrākau pedagogical approach implores that the storyteller is skilled in creating and delivering the narrative, and the audience is attuned to listening and making meaning from the metaphors, analogies, and representations. Similarly, Archibald's (2008) research outlines the Indigenous storywork methodology and pedagogy, informed by Coast Salish and Stó:lō narrative experts in Canada, was implemented at both curriculum and classroom levels. Archibald (2008) highlighted the importance of oral language skills, body language, tone and intonation in transmitting Indigenous knowledges through retelling narratives. Building on the work of Archibald (2008), Galla and Goodwill (2017) discussed the importance of video for creating pedagogical resources that draw on Indigenous narratives, as Indigenous knowledges were conveyed largely through oral language before the introduction of writing technology. These researchers highlight that text-based classroom resources or curriculum guidelines are unlikely to fully convey the intricacies of Indigenous narrative practices such as pūrākau and, therefore, primarily text-based methods of illuminating mātauranga may be insufficient. Similarly, Treloyn and Dowding (2017) discussed the use of digital technology to support the intergenerational transmission of Indigenous practices. Further analysis of the use of digital technology to create, capture and perpetuate mātauranga is offered in Chapters 4 and 6.

Despite the lack of information about how to teach pūrākau about tātai arorangi alongside pāngarau, Rikihana's (1990) resource does provide further information about the characteristics of pūrākau and the way mātauranga is organised within them. According to Rikihana (1990), mātauranga is organised into three broad categories. Uruuru tau (te kete tuatea) comprises ancestral knowledge (Moorfield, 2003) acquired through intergenerational transmission, which includes theology and philosophy (Rikihana, 1990). Uruuru rangi (te kete tuarangi) contains knowledge pertaining to the patterns of energy that operate beyond human sense perception (Moorfield, 2003), which was transmitted through oral literature (Rikihana, 1990). The third category, uruuru matua (te kete aronui),

foregrounds embodied and empirical knowledge accumulated through experience (Moorfield, 2003), including technological innovations (Rikihana, 1990).

In Rikihana's (1990) teaching resource, Tāne, rather than Tāwhaki, is tasked with the journey of retrieving valued information and is also responsible for tātai arurangi. Interestingly, in Rikihana's (1990) retelling of the Tāne pūrākau, Tāne travels to Rangiātea to acquire the knowledge sought. Rangiātea is also a significant site for the regeneration of wayfinding and navigation practices in Tahiti. Tūpaea, who was sought out by Captain James Cook to help guide the journey of the Endeavour throughout the Pacific, was a tohunga (expert) from Rangiātea (Tapsell, 2010, 2011). Rangiātea is also considered a significant site of knowledge dissemination amongst navigators, reflected in the following whakataukī:

E kore au e ngaro, he kākano i ruia mai i Rangiātea (Spiller et al., 2015, p. 6).

[I will never be lost; I am a seed from Rangiātea – Translation by author]

Although the connection between pāngarau, tātai practices and the nature of mātauranga are presented in both *Pāngarau i roto i te Marautanga o Aotearoa* (Te Tāhūhū o te Mātauranga, 1996) and *Tikanga Tau, Tikanga Āhua, Tikanga Tātai, Tikanga Mahi, Tikanga Huinga* (Rikihana, 1990), no explicit pedagogical content knowledge was provided. Therefore, in the absence of a curriculum that provided explicit guidance on teaching Māori epistemology, pouako have made various attempts to create access to mātauranga for their ākonga (Allen & Trinick, 2022). One strategy adopted by pouako was to recontextualise mathematics in Māori sociocultural practices (Trinick et al., 2015), such as describing and applying mathematics to solve problems associated with hosting tribal events (McMurphy-Pilkington, 1995). Ohia (1995) discussed the place of Māori imagery and art forms as contexts for mathematics learning to create links between Māori knowledge and pāngarau curricula. While teachers and students identified the underpinning cultural values and beliefs, the focus of the lessons was essentially based on school mathematics.

For example, researchers such as Knight (1996) have noted that kōwhaiwhai (discussed in Chapter 1) have been utilised in pāngarau classroom resources. Kōwhaiwhai patterns involve combinations of transformations, including reflection, rotation, translation and reflection (see Figures 2 and 3), and shears. Knight (1996) analysed and classified kōwhaiwhai patterns according to geometrical properties and kōwhaiwhai were used extensively to support the teaching of transformation geometry in Aotearoa/New Zealand schools (Meaney et al., 2013).

Figure 2. Glide Reflection (Auckland Museum, 1998, p. 3).



Figure 3. Vertical and Horizontal Reflection (Auckland Museum, 1998, p. 3).



Attempts were made to connect Māori culture to mathematics by identifying Western mathematical representations found in Māori artefacts, such as symmetrical patterns (Knight, 1996). However, by the 1990s, only focusing on patterns as exemplars of Western mathematics representations was disparaged by many Māori as insufficient to count as revitalising Māori knowledge (Barton, 1993). Māori artist and academic Robert Jahnke (cited in Paama-Pengelly, 2010) cautioned against using Māori patterns and designs without addressing their cultural meanings. Relegating Māori art and design to the status of decorations or ornaments, devoid of meaning beyond aesthetics and geometric analysis, strips the Māori visual art forms of their spiritual and cultural significance (Paama-Pengelly, 2010). This potentially shifts cultural norms (Jahnke & Jahnke, 2003), thereby obscuring mātauranga from those who wish to learn it (Jahnke, 1998).

Whiting (cited in Christensen, 2013), a prominent Māori artist, teacher, and designer of whareniui, discussed the importance of narratives to the whareniui design process. The visual representations of carving and kōwhaiwhai often displayed on whareniui rafters can be considered visual representations of pūrākau (Paama-Pengelly, 2010). For example, kōwhaiwhai patterns express important cultural values, such as unity, genealogy, and family interconnectedness (Witehira, 2013). One of the characteristics of kōwhaiwhai patterns is the use of a continuous line representing an unbroken genealogical line. Therefore, line breaks are not commonly seen (Riini & Riini, 1993). The patterns can differ from tribe to tribe, many having kōwhaiwhai unique to their geographical areas, defining the environment where the tribe exists (Adsett et al., 1992).

Employed by the Department of Education's Advisory service in 1961 to introduce Māori art into the school syllabus, Sandy Adsett is particularly renowned for perpetuating the practice of kōwhaiwhai or painted scroll ornamentation. When asked to articulate the defining characteristics of Māori visual practices such as kōwhaiwhai, Adsett and colleagues (1996) asserted that they must identify Māori to Māori. Therefore, using kōwhaiwhai as a vehicle for promoting the acquisition of school mathematics concepts could potentially shift the ontological norms of Māori visual narrative practices. This is not to criticise pāngarau teachers, as it was the reality they were working in and was based on their schooling and education experiences, often in English-medium schools (Trinick, 2015).

Areas for Further Research

Barton and Fairhall (1995) identified three areas of research that could better inform decision-makers at the policy and classroom levels to stem further unwanted language and (potentially) cultural change in Māori-medium education contexts. The first is the need to analyse the use of the pāngarau register by students and teachers in classrooms (Barton et al., 1995b). Literature that examined the grammatical (Harlow, 2001) and phonological (Watson et al., 2016) strength of the Māori language in the face of English-language interference more generally concluded that changes have occurred. However, there is minimal research on how daily classroom practice supports or resists these changes (Allen, 2015; Trinick, 2015) and to what extent the uniquely Māori worldview has remained intact or has been changed due to language elaboration (Barton & Fairhall, 1995). Chapter 6 further investigates the use of the pāngarau register by ākongā.

Barton and Fairhall (1995) also suggested anthropological research that could rediscover and revitalise historical language features that have fallen into disuse. Addressed to some extent by researchers such as Trinick (1999) to aid the pāngarau curriculum writers, this work continues into the present. Through the efforts of He Kupenga Hao i te Reo (n.d.) and Moorfield (2003), digital dictionaries have been created, ensuring new and native terms are easily accessible to pāngarau teachers. However, nearly 25 years after Barton and Fairhall (1995) questioned changes in the Māori language and worldview due to pāngarau curriculum development and language elaboration, Te Maro (2018) posed a similar question about pāngarau schooling. Conversely, there is little discussion on whether mathematics is being changed through mathematical discussion in Māori (Iseke-Barnes, 2000). Iseke-Barnes suggested further research into how Māori and Indigenous knowledge systems and school mathematics are evolving in relation to each other. The assumption could be that Western knowledge-creation practices and frameworks for organising and disseminating information are positioned as normative (Jorgensen, 2011) and therefore continually eroding mātauranga. The tokenising of kōwhaiwhai in mathematics classrooms and resources seems to provide evidence to this effect. Chapter 5 adds further discussion to the anthropological area of research suggested by Barton

and Fairhall (1995), while data analyses across Chapters 4-6 provide further insight into the shifts occurring for both mātauranga and school mathematics due to their confluence in pāngarau curriculum and classroom practices. Before the data are presented, the following chapter discusses the research methods and methodology selected for this doctoral study.

3. Ngā Tikanga me ngā Tukanga Rangahau | Research Methods and Methodology

This chapter justifies and critically evaluates the methodologies and methods chosen to address the illumination of mātauranga in curriculum and classroom practices in supporting Māori aspirations for schooling. Included in this chapter is a discussion on how the research design adopted for this doctoral research accommodated the COVID-19 pandemic throughout 2020 and 2021. A flexible methodology and data analysis framework was required to address the breadth of macro (political) pāngarau curriculum development projects and the micro (curriculum) outcomes of enactment at the classroom level. Bourdieu's (1992, 2010) theories of social reproduction were used to investigate the macro-level impacts of curriculum development on micro-level classroom practice. The cultural symmetry framework (Meaney et al., 2021) guided the identification of pedagogical opportunities for simultaneously promoting te reo Māori and mātauranga within objectively structured boundaries of school mathematics curricula. The teaching as inquiry cycle (MoE, 2020) guided the classroom research design and data analysis. In this way, the data sources and methodological tools were triangulated (Patton, 1999) to sensitise readers to the opportunities, challenges, and tensions for rebalancing mātauranga and school mathematics in pāngarau curriculum and classroom practices.

Sensitising Readers to Further Indigenise Pāngarau Research

Rather than presenting a definitive step-by-step guide for pāngarau teachers, curriculum and assessment writers or policymakers, this doctoral research identifies tensions, challenges, and opportunities for rebalancing mātauranga and school mathematics. In the ontological sense, this study presents a view shaft from which to examine the efficacy of actions taken by actors holding interrelated roles (Blumer, 1954). While definitive quantitative studies that draw on a positivist research paradigm define concepts or actions by pre-determined criteria, dictating what is to be seen (Pöllmann, 2021), sensitising qualitative studies that draw on an interpretivist paradigm can make visible what has been obscured (Blumer, 1954). As argued in the literature review, mātauranga has been devalued, disrupted, and dismantled through Aotearoa/New Zealand's colonial education and research agendas over time. By considering the contexts (Stake, 2010) from which mātauranga and school mathematics epistemologies have emerged, interpretivist research can bring to light what was previously disregarded (Reichert, 2014).

The following broad features can characterise interpretivist approaches:

1. They position participants as authoritative informants who possess valued knowledge about the research topic and context (Smyth & Whitehead, 2007) rather than passive recipients of positivism
2. They treat reality as subjective and consider the environment, human perception, and sociocultural factors (Willis, 2007).

By using an interpretivist research design, this study formulates and interprets sensitising concepts to inform research, policy, curriculum development and teaching.

To ensure Māori ways of knowing and being are explicitly promoted, this study presents findings through the lenses of mātauranga and school mathematics, interrogating the epistemological contexts and circumstances from which they emerged. Positioning different epistemological trajectories as distinct and complementary provides an opportunity to examine the unique challenges and opportunities this co-occurrence provides through multiple ontological lenses (Hauser et al., 2009; Pöllmann, 2021). Addressing the ways that epistemologies seem to compete or conflict (Miller et al., 2008) and how each epistemology could contribute to contemporary challenges (Andreotti et al., 2011) is a significant challenge for pāngarau curriculum and classroom practices.

Kaupapa Māori research methodologies similarly make visible Indigenous ways of knowing and being in the Western academy, founded on colonial ideologies that were, and arguably still are, hostile towards Indigenous peoples and knowledge systems (Jahnke & Jahnke, 2003; Smith, 2012).

Therefore, kaupapa Māori research methodologies provide a lens for viewing research by and with Māori. This includes sensitivity towards Indigenous communities' wariness of outsider research due to many decades of having research done *to them* instead of *by them* or *with them* (Smith, 2012).

Flawed scientific arguments have been, and continue to be, used by colonial governments to justify the denial of linguistic, cultural, land and other human rights to Indigenous peoples (Smith, 2012).

Academic research often positions Indigenous knowledges as antiquated or irrelevant rather than continuing to expand in the present (Porsanger, 2011; Smith et al., 2016). To address concerns for the representation of Māori ontologies and epistemologies, kaupapa Māori researchers have argued that Māori ways of knowing and being should be taken-for-granted (Smith, 2000) in the same way that science takes for granted its own epistemological assumptions as a universal position against which all other knowledges are assessed (Cooper, 2012; Smith, 2012). Kaupapa Māori researchers have thus created space for Māori and other Indigenous scholars to utilise the research process in forming and answering their own research questions, thereby responding to the entrenched coloniality in so much existing (Western) academic research (Cooper, 2012; Smith et al., 2016)

Similarly, this study creates space to consider the impact of pāngarau curriculum and classroom practices on Māori aspirations for schooling and to recognise these aspirations as ontologically pluralistic. Māori success *as Māori* (MoE, 2020) is interpreted in this thesis as living in a way that is recognisably Māori to other Māori while also accessing notions of Western academic success. Therefore, this thesis positions further indigenising pāngarau curricula as a process of balancing both mātauranga and school mathematics in a way that enables both Māori and Western academic knowledge-creation practices to be explored simultaneously. However, as Te Maro (2018) argued, presenting mātauranga alongside school mathematics through academic research could (still) be seen as an attempt to legitimise mātauranga by linking it with the powerful academic field of mathematics. Similarly, Mika and Stewart (2017) asserted that ‘Māori-fying’ a Western approach does not make it anything other than the original Western approach. Neither legitimising mātauranga nor Māori-fying school mathematics are aims of this study. To address both the epistemological—the knowledge that is valued, and the ontological—accepted ways of being and seeing in rebalancing mātauranga and school mathematics, the terminology used in this thesis has thus been carefully considered.

For example, the term ‘traditional knowledge’ is purposefully omitted from the data analyses and conclusions offered in this thesis. This echoes the position of Porsanger (2011), who critiqued the linear Western timeline that Indigenous knowledge-creation practices are often forced to accommodate. Indigenous knowledge is often positioned as traditional or antiquated compared to modern Western science. Porsanger (2011) argued that categorising Indigenous knowledge along a linear timeline between tradition and modernity is hostile to Indigenous epistemology. Therefore, the data analysis in this thesis positions both tradition and modernity as omnipresent and inevitably intertwined (Porsanger, 2011).

The potentially hybridising terms of ‘Indigenous mathematics’ or ‘Māori mathematical practices’ are also omitted from the data analysis and conclusions of this study. Matamua (2019) discussed the importance of Māori retelling narratives about Māori beliefs, culture, and ways of thinking in a manner that is acceptable and appropriate to Māori. Mātauranga is a term recognised as Māori, by Māori, and for Māori. It is therefore used in this thesis to identify practices, language, and representations that draw on what can only be termed in the broadest sense Māori knowledge systems, ways of knowing, being and believing.

The Field, Habitus and Cultural Capital of Pāngarau Curriculum and Classrooms

Bourdieu’s (2010) conceptual thinking tools of field, habitus, and capital are useful in examining educational inequality, cultural reproduction, and social positioning (Costa & Murphy, 2015). In this research, field, habitus, and cultural capital are used to make the norms of mathematics education

visible and show how these norms constrain the indigenisation of pāngarau curriculum and classroom practices. Mathematics education can be conceptualised as a field that promotes particular (culturally embedded) ways of knowing and doing (Jorgensen, 2011). Examining mathematics education, which includes compulsory schooling through Bourdieu's concept of field, provides a vantage point from which to critique its rules and boundaries. Bourdieu (2010) argued that while our decisions may be our own, they are guided and restricted by the fields in which we exist. This is because fields of action are sites of capital acquisition (Costa & Murphy, 2015). The objective structuring practices of a field determine a social group's ability to amass capital, such as cultural capital, social capital, and economic capital, and convert capital from one form to another (Bourdieu, 2010). These various forms of capital determine position and place within a social structure (Costa & Murphy, 2015). For example, cultural capital can be amassed through education and converted into economic capital and social capital through career advancement.

Curriculum documents and assessment practices objectively structure the field of mathematics education (Jorgensen, 2011) by determining what is important to know. Excluding explicit guidelines on teaching mātauranga from the pāngarau curriculum can be considered an example of an objective structuring practice by the mathematics education field. When Indigenous students internalise the effects of objective structuring practices, they can subjectively internalise a particular learner identity and misrecognise their own mathematical abilities (Jorgensen, 2011).

Internalised realities form the underlying logic from which people conduct their lives and were termed habitus by Bourdieu (2010). The habitus provides a model for organising and articulating the position that a particular group occupies in social space relative to the positions occupied by other social groups (Costa & Murphy, 2015). Conceived as a three-dimensional space, the habitus encompasses the volume and composition of capital that an individual or group possesses and the potential to increase or decrease their capital (Bourdieu, 2010). Therefore, the habitus is interrelated with the acquisition, composition, and conversion of capital from one form to another, and thus determines the trajectory of an individual or social group (Bourdieu, 2010).

Bourdieu's toolkit offers a model for examining the social reproduction and (potential) transformation (Pöllmann, 2021) of pāngarau curriculum and classroom practices in relation to the mathematics education field. Pāngarau curriculum and classroom practices have successfully challenged the English-language-only instruction norm that has existed in mathematics education for more than 100 years in Aotearoa/New Zealand (Trinick, 2015). Pāngarau curriculum and classroom discourse have debunked arguments about the limitations of Indigenous languages for communicating school mathematics concepts (Trinick, 2015). English-medium mathematics education is one of two government-funded language instruction options in what is now known as a dual-medium education system, evidenced by the bilingual title of mathematics/pāngarau often used in Aotearoa/New Zealand

mathematics education policy and research (see Anthony & Walshaw, 2007; MoE, 2021). However, the requirement for the first pāngarau curriculum to mirror the imported Western knowledge-based school mathematics curriculum is a constraint that still exists in practice. The purpose of the pāngarau curriculum and classroom practices has not yet been expanded to explicitly include the dissemination of mātauranga. The reasons why are further interrogated in Chapters 4–6.

The appropriation of a Western epistemological and methodological approach for examining the illumination of mātauranga in pāngarau curriculum and classrooms rightfully requires critique. The argument here is that the methodological lenses of kaupapa Māori and indigenisation provide the methodological guidance required to examine the research questions and analyse data. This may be true. However, Bourdieu's (2010) notion of cultural capital has been usefully appropriated by Māori education researchers to argue against the assimilatory social reproduction function of compulsory schooling for Māori. Echoing Bourdieu's (2010) theory of the social reproduction of high-status cultural capital through education, authors such as Bishop and colleagues (2009) argued that the education system in Aotearoa/New Zealand is designed to serve the interests of the dominant, largely monocultural, Pākehā elite. Aikenhead (2018) likewise likened the poor performance of mathematics schooling for Indigenous students to a knowledge debt. Using Bourdieu's (2010) lens, a knowledge debt is thus a debt in cultural capital. Beyond schooling, cultural capital debts can limit access to social and economic capital (Bourdieu, 2010). Forcing Indigenous peoples to learn the knowledge, history, and traditions of others while denying them the opportunity to learn their own further creates an Indigenous knowledge debt and, therefore, a debt in Indigenous cultural and social capital.

As noted by Durie (2003), “if after twelve or so years of formal education, Māori youth are totally unprepared to interact within te ao Māori (the Māori world), then, no matter what else had been learned, [their] education would have been incomplete” (p. 199). Durie (2003) further argued that Māori achieving educational success as Māori [means] being able to have access to te ao Māori, the Māori world – “access to language, culture, marae ... tikanga ... and resources...” (p. 199). The absence of mātauranga in pāngarau curriculum and classroom practices objectively, and likely subjectively, structures the learning experience of ākongā Māori and therefore influences the habitus of ākongā Māori. This potentially limits both the Māori and school mathematics cultural capital ākongā can convert into social and economic capital beyond schooling.

Further to the cultural capital debt school mathematics ascribes to Indigenous students, the symbolic and physical violence of colonisation continues to limit Indigenous peoples' access to social and economic capital (Pöllmann, 2021). Racist government policy, such as pepper-potting social policy (Benton, 1997; Ka'ai-Mahuta, 2011), discussed in Chapter 2, has been and continues to be detrimental to Māori society (Allen & Trinick, 2022). Disadvantages in economic, health, education and criminal justice outcomes have been experienced by Māori from birth and have worsened in the last three

decades (McIntosh, 2005). Ministry of education policy further defines the role of schools and kura as supporting “learners/ākonga to see the connection between what they’re learning and the world of work” (MoE, 2020, p. 2), through collaboration with industry, employers, and tertiary education providers. This removes from Māori communities the determination of how cultural capital can be converted into social and economic capital, instead affording more control to those who may or may not choose to employ or further educate them. The objective structuring practices of mono-epistemic school mathematics curricula perpetuate socio-economic or symbolic violence towards ākonga Māori. Therefore, the marginalisation of mātauranga in curriculum and classroom practices inflicts ākonga Māori with cultural capital debts that are likely to become social and economic capital debts beyond schooling. The cultural symmetry framework conceptualised by Meaney et al. (2013) provides a counterbalance to the continued promotion of Eurocentric cultural capital in schooling at the expense of mātauranga, and so it is to this framework that I now turn.

The Cultural Symmetry Framework

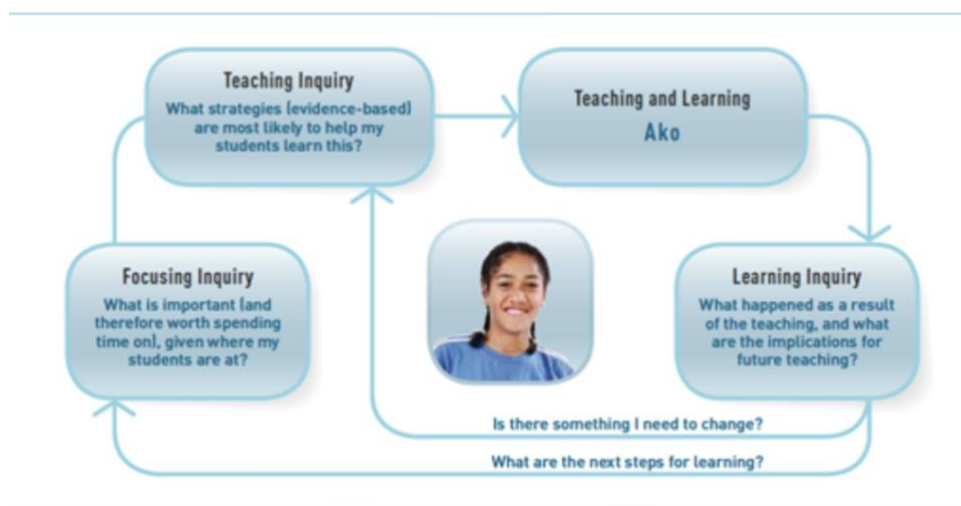
To ensure that Indigenous knowledges and languages are valued in the mathematics classroom, Meaney and collaborators (2013) argued for a pedagogical framework that promoted both the visible and invisible elements of Indigenous culture. In response, the cultural symmetry framework now appears in pāngarau teaching and assessment policy (Rau et al., 2022). The elevation of the framework to the policy level could attempt to address a key challenge for pāngarau curriculum and classrooms in simultaneously valuing mātauranga and te reo Māori, and not just viewing these social constructs as mere conduits to the promotion of school mathematics (Meaney et al., 2013). The cultural symmetry framework was used in this study as a methodological lens for data analysis. This lens ensured that mātauranga, te reo Māori, and school mathematics concepts or content were addressed explicitly in the data analysis of mātanga interviews (Chapter 4) and classroom artefact data (Chapter 6).

While the cultural symmetry framework is designed to address te reo Māori and mātauranga separately, and this was also the intention of the study design, the reality of doing this in Māori-medium pāngarau classrooms was a challenge. As Stewart (2007) noted, language and knowledge are not easily divided. This was evident in the data collected in Māori-medium classrooms where te reo Māori is a taken-for-granted language of instruction and recognised as a component of mātauranga. The tension created by the cultural symmetry framework in delineating te reo Māori and mātauranga is further explored in Chapters 4–6 and critiqued in Chapter 8 alongside the study recommendations and conclusions.

Teaching as Inquiry

The teaching as inquiry cycle, which appears in both Māori-medium and English-medium curricula in Aotearoa/New Zealand (MoE, 2020; Te Tāhūhū o te Mātauranga, 2017), promotes genuine curiosity by pouako about the impacts of pedagogical decision-making. The cycle is also designed to guide the analysis of practical applications of theory to inform practice and further reflection (Timperley et al., 2007). As it is currently considered, the teaching as inquiry cycle (see Figure 4) is designed to support teachers in identifying areas of professional learning that could improve their capability in designing high-quality learning experiences for students (Timperley, 2011). The cycle was utilised in this study to guide data collection and structure the analysis of classroom artefact data.

Figure 4. Teaching as Inquiry Cycle (MoE, 2020).



The phases of the teaching as inquiry cycle and their application in this study are outlined below:

1. The learning inquiry phase involved learning about the Kura Kaupapa Māori contexts. This included the learning needs and aspirations of the kura communities for both mātauranga and school mathematics.
2. The focusing inquiry phase involved collaborative reflection between kura principals, teachers, and me, as the researcher, on the data collected in the learning inquiry phase. Decisions were made in this phase on the foci of the subsequent teaching and learning sequences and the classroom data that would be available for collection for research purposes.
3. In the teaching inquiry phase, the agreed foci were further supplemented by both practical (local experience) and formalised theory (Timperley, 2013) to support the intended pedagogical actions.

4. The teaching and learning phase comprised implementing the planned lesson sequences and pedagogical actions agreed upon. Teacher and student artefacts were collected and analysed during this phase of the inquiry cycle.

The learning inquiry phase of the cycle is presented in the research design section of this chapter, as it was used to guide the collaborative design of the classroom research. The remaining focusing inquiry, teaching inquiry and teaching and learning phases of the cycle are presented alongside the data collected for each phase in Chapter 6.

Aligning with the study foci, the teaching as inquiry cycle provided an opportunity to highlight the professional learning needs of Māori-medium teachers, a challenge and tension in illuminating mātauranga classroom practices. This included identifying challenges to pedagogical content knowledge in both mātauranga and school mathematics and pedagogical language knowledge in te reo Māori (see Allen, 2015; Christensen, 2004b). Pedagogical content knowledge includes the ways of representing and communicating about a subject to make it understandable to others (Shulman, 1986). Professional development priorities for the education system are often determined at the national level by the needs of English-medium teachers and the assumption that the needs of Māori-medium pouako are the same (Allen & Trinick, 2020; Allen, 2015; Murphy & Reid, 2016). However, Māori-medium pouako are often L2 learners of the language of instruction and have completed their education in English-medium Western knowledge-based schooling (Christensen, 2004a). Therefore, extra support is required to develop pedagogical language knowledge alongside pedagogical content knowledge.

One of the criticisms of the teaching as inquiry cycle is the lack of focus on the aspirations and goals of Indigenous communities and challenges posed to Indigenous aspirations by the schooling system, such as Indigenous knowledges not being explicitly present in national curricula. However, this current study utilised the learning inquiry phase of the teaching as inquiry cycle to scan the wider learning environment and gain a better picture of the participant kura community's aspirations for mātauranga. Thus, the teaching as inquiry cycle was employed to understand how classroom learning is influenced by the community (Timperley, 2013), and the focus of the inquiry thus shifted from merely focusing on quantifiable student outcomes (Timperley, 2015) directed by national curricula.

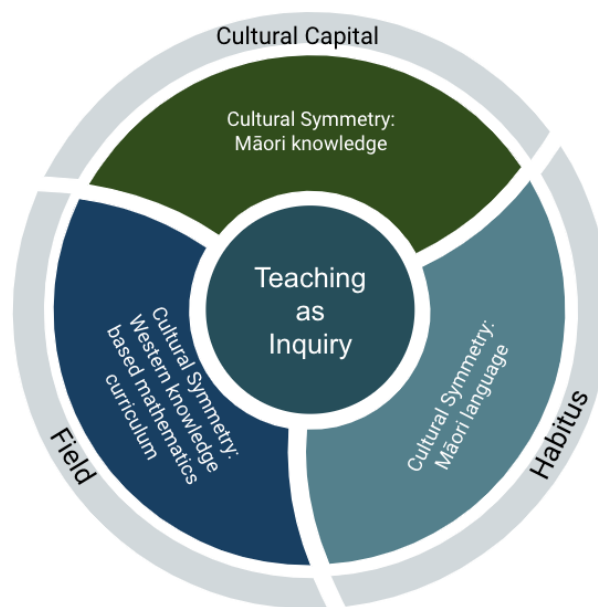
A collaborative approach was used to ensure local Māori community aspirations were promoted in utilising the teaching as inquiry cycle for classroom data collection and analysis. Participant tumuaki (principals), pouako, and I identified mātauranga learning aspirations and resources within the community. This process led to the identification of digital assets created by mātauranga experts, both within and outside the immediate school community, that could be utilised by pouako and ākonga to further their understanding of localised mātauranga. Therefore, opportunities to analyse the impact of pedagogical choices on the learning of mātauranga, and the associated reo Māori required to

communicate it, were not obscured by the tendency to focus on quantifiable school mathematics curriculum-driven achievement outcomes alone.

Diagram of Methodological Framework

The three methodological approaches have been drawn together in Figure 5. Figure 5 positions Bourdieu's (2010) toolkit of field, habitus, and cultural capital in the outer ring as these tools are used to synthesise the macro-level challenges and opportunities for further indigenising pāngarau curricula and classrooms. The cultural symmetry framework with its tripartite foci of te reo Māori, mātauranga and Western mathematics-based curriculum form the middle ring, as they are used as pedagogical aids and a methodological tool to delineate the analysis of data within the three data sources. Finally, the teaching as inquiry cycle is positioned in the centre of the diagram as this is used to analyse the enactment of pāngarau curriculum at the micro-classroom level.

Figure 5. Methodological Framework Diagram.



Research Design and Data Collection Methods

The main research question investigated in the study was:

What are the opportunities, challenges, and tensions for indigenising pāngarau curriculum and classroom practices?

The research question was examined by triangulating three data sources (Patton, 1999): 1) semi-structured interviews with mātanga pāngarau, 2) literature on Māori and Pacific wayfinding practices and representations, and 3) student and teacher-created classroom artefacts. These data sources and the research sub-questions they examine are presented in the following sections.

Semi-Structured Interviews with Mātanga Pāngarau

In addition to the literature reviewed in Chapter 2, semi-structured interviews with mātanga pāngarau were used to answer the research sub-question:

Which ideologies have influenced the indigenisation of pāngarau curriculum and classroom practices?

Presented in Chapter 4, the semi-structured interview data supplemented the paucity of the literature identified in the Chapter 2 literature review that focused specifically on pāngarau curriculum and classroom practices. Four mātanga pāngarau were interviewed and pseudonyms of mātanga 1–4 were assigned. The pool of mātanga pāngarau is small; therefore, they have been described here as a group to maintain confidentiality. All have been, or are, pouako in wharekura of pāngarau. They have also all been involved in curriculum and resource development since the emergence of pāngarau education research. Therefore, they are in the unique position of being able to discuss the opportunities, challenges and tensions for illuminating mātauranga in pāngarau curricula and classrooms.

The mātanga interviews lasted 45–60 minutes, were semi-structured, and three of the four interviews were conducted in Māori. A list of four key questions (see Appendix 2) was used to ensure consistency between the collected interview data. However, follow-up questions were asked as required, and mātanga were encouraged to elaborate on their answers. The interviews were conducted and recorded by me via Zoom due to the COVID-19 pandemic context. The use of technology as a data collection tool to mitigate the impacts of COVID-19 on this study is further discussed in the final section of the study design. I transcribed the interviews and each response was numbered. Longer responses were divided into parts so that each data item was approximately two to four sentences in length. Mātanga were given an opportunity to correct their transcriptions.

Literature on Māori and Pacific Wayfinding Devices

A search of the literature on Māori ways of communicating spatial information through oral and visual narratives and their Pacific origins was used to answer the research sub-question:

How can conflicts between mātauranga and school mathematics be addressed through teaching wayfinding?

There is a paucity of literature pertaining to Māori wayfinding practices, including representations of spatial information, particularly on land. While wayfinding at sea could provide valuable learning experiences for ākonga, not all may have access to these experiences during their compulsory schooling. However, ākonga are finding their way between land-based destinations daily. Learning about the wayfinding devices and the wayfinding practices of their tīpuna (ancestors) could provide valuable practical information as well as strengthen their connections to their whakapapa. However, few Māori wayfinding resources are designed for use in classrooms. The examination of anthropological studies to support the illumination of te reo Māori and mātauranga in pāngarau curriculum and classroom practices was suggested by Barton and Fairhall (1995). The mātanga interviewed in Chapter 4 further reiterated the need to identify aspects of mātauranga in the literature that could be illuminated in pāngarau curricula. It is not possible within the scope of this current study to conduct an exhaustive anthropological search designed to illuminate and reconstruct all the components of the shattered Māori knowledge systems underpinning wayfinding. Pragmatically, the study thus focused on the characteristics of Māori and Pacific wayfinding devices on land and at sea. Anthropological texts were supplemented with those from Pacific archeoastronomy, ethnomathematics and Māori visual and oral narratives.

Pāngarau Classroom Data

Finally, the following research sub-questions were explored in conjunction with the pedagogical choices and challenges faced by Māori-medium pāngarau teachers. Teacher-created artefacts collected for the study included lesson sequences, assessment data and teaching resources. Student-created artefacts were also collected.

1. *How effective is the cultural symmetry framework in addressing tensions between mātauranga and school mathematics?*
2. *How can conflicts between mātauranga and school mathematics be addressed through teaching wayfinding?*

Classroom data collection took place in two different Māori immersion pāngarau settings (assigned the pseudonyms Kura Tahī and Kura Rua). Both settings were Kura Kaupapa Māori, Level 1

immersion settings with 81% or more te reo Māori instruction. Kura Tahī is in an urban environment and Kura Rua is in a rural environment. There were two classes involved in the research from each setting, totalling 55 ākonga, five pouako and two tumuaki. The shape of the research was negotiated with tumuaki and pouako as elaborated in the following section.

The Researcher's and Teachers' Roles

The research was designed in collaboration with the tumuaki and pouako at both participating kura to ensure that the experience of participating in the research was mutually beneficial to both the participant kura and the research project. It was important that participation in the research did not increase pouako workload and that there was a benefit to pouako from participating in the research. Therefore, a professional learning and development component accompanied the research. This arrangement was intended to create continuity across the pedagogical actions beyond the research project. The research design also included the participation of a specialist pāngarau teacher who was also knowledgeable in wayfinding. Mentoring and modelling by an expert peer can play an important part in teacher professional development (Timperley, 2013, 2015); however, there are few specialist pāngarau teachers (discussed further in Chapter 4). To provide professional development for pouako and direct access to expertise for ākonga, a specialist pāngarau teacher participated in the study and was assigned the pseudonym Pouako Pāngarau. Pouako Pāngarau worked alongside the pouako of the kura and me to design and deliver a sequence of pāngarau lessons.

The kura were also interested in learning how to strengthen the use of digital technology. This was partly in response to periodic school closures due to the COVID-19 pandemic context necessitating remote learning. Providing this mentoring, as requested by the kura, was an important part of the project. I was tasked with modelling the use of digital technology to support the continuity of the pedagogical approaches introduced through the research, and primarily responsible for facilitating data collection to ensure that this extra workload did not burden the pouako of the kura. This included collecting diagnostic assessment data at the beginning of the research and facilitating ākonga use of Android tablets and screencasting software to capture their learning artefacts. The workload for teachers was an important consideration of the study design due to the added pressures of school closures and disruptions resulting from the COVID-19 pandemic (see Allen & Trinick, 2021).

The data presented in the classroom data in Chapter 6 are grouped into two categories. The first category is teaching artefacts, which includes researcher notes collected during collaborative planning meetings with pouako; curriculum documents; and curriculum support materials, including assessment materials, planned lesson sequences, and classroom teaching resources (both digital and physical). The second category is student-created artefacts. These are student assessment responses,

digital presentations captured on tablets with screencasting software, and pen and paper-based responses to teaching stimuli.

The Learning Inquiry

The teaching as inquiry cycle used to guide the collection of classroom data in this study is a tool that can be used by teachers to guide reflective and ongoing improvements in pedagogical actions and decision-making (Timperley et al., 2007). In this current study, improvement is defined as rebalancing the mātauranga and school mathematics aspirations of the participant kura in classroom practices. The challenge for pouako is addressing both outcomes within the existing time and resources. The learning inquiry phase aimed to gather information from various sources, such as assessments and national curriculum and policy guidelines, to further understand the learning that had already taken place before the study commenced to inform further pedagogical decisions.

The Aotearoa/New Zealand Ministry of Education describes the learning inquiry phase as how “teachers monitor their students’ progress towards the identified outcomes and reflect on what this tells them” (MoE, 2020, para. 4). This study critiques this approach as lacking a particular emphasis on kura community aspirations. The study purposefully includes kura community learning aspirations for mātauranga as articulated in kura documentation, which may not be assessed directly. The following section presents an overview of each of the learning settings, student numbers and year levels, and the kura aspirations for the teaching and learning of pāngarau curriculum content. While te reo Māori is a taken-for-granted language of instruction in participant kura, diagnostic information on ākonga ability to articulate pāngarau ideas through multiple representations of the pāngarau register was also collected in this phase.

Kura Research Contexts

Kura Tahī is in a culturally diverse urban centre and could be considered pan-tribal or serving students from several different tribal areas. Kura Tahī also has a contributing Kōhanga Reo on-site. In contrast, Kura Rua is in a rural, predominantly, and increasingly Māori community (Statistics NZ, 2018). A large proportion of ākonga at Kura Rua have genealogical links to common ancestors. A total of 55 ākonga in Years 7–10 (11–14 years of age), five pouako, and two tumuaki participated in the research across the two kura. Table 1 outlines the numbers of ākonga and pouako participants at each kura along with their assigned pseudonyms.

Table 1. Participants and Pseudonyms from Each Setting.

Kura pseudonyms	Ākonga pseudonyms	Student year levels (number of ākonga)	Pouako pseudonyms	Tumuaki pseudonyms
Kura Tahī	Ākonga 1–19	Year 7–8 (10) Year 9–10 (9)	Pouako 1–3	Tumuaki 1
Kura Rua	Ākonga 20–55	Year 7–8 (15) Year 9–10 (21)	Pouako 4–5	Tumuaki 2
At both settings	Pouako Pāngarau			

Kura and Community Aspirations for Mātauranga and Pāngarau

An examination of Kura Tahī’s website and discussions with Tumūaki 1, Pouako 1–3 and Ākonga 1–19 revealed that the community’s mātauranga aspirations have a particular emphasis on Māori martial arts, genealogical knowledge, and Māori performing arts. At Kura Rua, discussions with Tumūaki 2, Pouako 4–5, and Ākonga 20–55 revealed that Kura Rua was implementing a learning programme for local genealogical narratives.

Kura Tahī’s aspirations for pāngarau curriculum content-based teaching and learning programmes, as discussed with Tumūaki 1 and Pouako 1–3, emphasised numeracy skills. This may reflect school numeracy achievement reporting requirements. Discussions with Tumūaki 1 revealed an immediate need for teachers to address numeracy learning concerns. The instant recall of basic facts had been identified by Tumūaki 1 as an area of concern for students in Year 7/8, while mental subtraction strategies needed strengthening for Year 9/10 students. Tumūaki 1 also asked that both the use of physical manipulatives and digital technology be exemplified for participating teachers as part of the research project. Tumūaki 2 of Kura Rua had similar concerns regarding number knowledge and mental strategy learning. Both tumūaki wanted the school’s numeracy data updated by conducting Te Uiuī Aromatawai, discussed in the next section, at the beginning of the research project.

Tumuaki 1 and Tumūaki 2 were also interested in using digital tools to ensure continuity of learning during intermittent school closures due to the COVID-19 pandemic (see Allen & Trinick, 2021). Further discussions with Kura Rua pouako revealed enthusiasm for the affordances of digital technology to support the tripartite goals of supporting te reo Māori, mātauranga, and pāngarau curriculum content teaching and learning. Digital technology is a ubiquitous and powerful tool whose affordances can be utilised to meet specific Indigenous language (and Indigenous knowledge) revitalisation goals (Galla, 2016). In addition, the use of digital technology has long been a strategy for overcoming linguistic and geographical isolation for Māori-medium schools, particularly for curriculum areas such as pāngarau, which have very few specialist teachers (see Allen & Trinick,

2020). Ākonga desire to use digital tools for reinforcing numeracy concepts was also discussed by pouako and tumuaki at both kura.

Diagnostic Assessment Data

The study was designed to support the participant kura to meet pāngarau learning and assessment requirements alongside mātauranga learning aspirations, as advocated by the cultural symmetry framework. Te Uiui Aromatawai was the diagnostic interview used to collect numeracy data for this study. Ākonga were presented with problems to solve mentally and respond with their solution and solution method orally. Problems ranged from simple addition problems, such as $5 + 6$, designed to test ākonga ability to instantly recall number facts, to more complicated word problems that required ākonga to first decode the problem and then identify relevant mathematical information (see Appendix 3 for diagnostic interview scripts).

Te Uiui Aromatawai is part of a suite of resources developed for the Numeracy Development Projects, Te Poutama Tau for Māori-medium, consisting of a range of government-funded professional learning and development programmes for teachers. The projects aimed to improve student outcomes in mathematics and pāngarau (MoE, 2007). Te Poutama Tau was developed as a response to the need for pāngarau teachers to access professional learning and development relevant to the Māori-medium teaching context (Christensen, 2004a). Te Poutama Tau provides all teaching and assessment resources in te reo Māori and supports the use of Māori language as the waka (vehicle) that carries the teaching of numeracy concepts (Te Rōpū o Te Poutama Tau, 2011). Therefore, Te Uiui Aromatawai is useful in gaining insights into ākonga mathematical thinking and their ability to communicate their understanding through the pāngarau register, which can be affected by the number of years ākonga have been in Māori immersion education (see Table 2). While the project is no longer funded directly, resources such as Te Uiui Aromatawai are still widely used to inform pāngarau teaching and learning programmes.

Table 2. Curriculum Levels and Years in Māori Immersion Schooling (He Kupenga Hao i te Reo, 2010).

Curriculum level	Number of years in immersion education
1	1–2
2	3–4
3	5–6
4	7–8
5	9

Kura Tahī and Kura Rua used Te Uiui Aromatawai and other Poutama Tau resources as part of their regular teaching and learning programmes. The diagnostic data collected for this study through Te Uiui Aromatawai was used to inform the collaborative planning with tumuaki and pouako, of a sequence of lessons designed to support the acquisition of pāngarau curriculum content, mātauranga and te reo Māori.

Te Uiui Aromatawai was administered by Pouako Pāngarau, a specialist pāngarau teacher and I at both kura to ensure consistency. During this process, ākongā were prompted to use manipulatives or pen and paper to respond to questions if they could not clearly articulate their thinking orally. It became evident that some ākongā were still developing the language required to articulate their mathematical thinking, indicating that they had not yet spent the necessary time in immersion education for the diagnostic data to be a valid measure of their mathematical thinking (see Allen, 2015). The limited time students are immersed in te reo Māori, and therefore their need to learn the language of instruction alongside the curriculum content, was identified as a challenge by tumuaki and pouako and is discussed further by the mātaunga in Chapter 4. To complete Te Uiui Aromatawai, ākongā were provided with other methods for showing their understanding. However, the data collected would need to be supplemented with further assessments over time for any judgements to be made about mathematical understanding.

While Te Uiui Aromatawai is challenging for L2 learners of the language of instruction, it does provide an opportunity to assess ākongā ability to articulate thinking through the specialised language of the pāngarau register. This information was used to design lessons to further support the language development of participating ākongā. Rather than providing the raw data collected through Te Uiui Aromatawai, a summary of the learning needs identified through the interview process is given in Chapter 6 as part of the focusing inquiry stage of the teaching as inquiry cycle. Providing a summary of the data aligns with my ethical responsibility to support Māori aspirations for schooling, by investigating Māori immersion determined research priorities while limiting undue scrutiny, likely underpinned by historically colonial pejorative ideologies.

Classroom Research Plan

Tumuaki 1 requested weekly visits by Pouako Pāngarau to model the teaching of numeracy knowledge and mental strategies over two terms (20 weeks total). Tumuaki 1 was also interested in the researcher facilitating weekly sessions with ākongā on capturing their mathematical understanding using digital devices. Tumuaki 1 expressed a desire to ensure that there was continuity of learning despite ongoing intermittent school closures.

Tumuaki 2 requested site visits when possible, considering the distance of the kura from the urban centre where Pouako Pāngarau and I were both located and the potential for regional travel restrictions due to the COVID-19 spread. Pouako Pāngarau and I visited the kura three times to support teachers' professional development and complete data collection for the research. The first visit was to conduct the requested workshop on digital technology use with Kura Rua pouako. The second visit was to conduct Te Uiui Aromatawai and update Kura Rua numeracy data. The third visit involved modelling the teaching of local genealogical narratives alongside pāngarau curriculum content, thereby supporting Kura Rua's existing planning for this to occur.

Data Analyses

The study applied a pragmatic approach to analysing the literature, semi-structured interviews with mātanga pāngarau, and classroom data. For the semi-structured interviews, a deductive approach was initially used, where theory was applied to the data (Reichertz, 2014) in the form of the pre-determined codes of the cultural symmetry framework: te reo Māori, mātauranga, and school mathematics. A Google Form was designed with drop-down boxes for the pre-determined codes, the question number, and the assigned mātanga pseudonym. A further drop-down box was used to label responses pertinent to specific pāngarau curriculum content (strands). Once this initial deductive process had been completed, whereby the cultural symmetry framework foci were applied to the interview data, a more inductive approach was used. This allowed for like responses to be grouped thematically (Reichertz, 2014; Stake, 2010). Four text entry boxes were included in the Google Form to allow for freeform descriptive labels to be assigned to a key idea, context, theory, or author being discussed. Through this inductive process, thematic codes emerged from the semi-structured interview data as it was being analysed. Google Sheets was then used to generate a spreadsheet of transcribed text and assigned codes. Presented in Chapter 4, the semi-structured interview data was analysed according to the cultural symmetry framework foci with the inductive themes as sub-section headings.

Literature on the characteristics of Māori and Pacific wayfinding devices was also analysed according to the tripartite foci of the cultural symmetry framework. The framework was supplemented with the findings from the mātanga interviews and the theoretical perspectives of ontological and epistemological pluralism. Therefore, a deductive approach was initially followed. The lenses of mātauranga and mathematics wayfinding were defined in the literature, and these definitions were used to guide the analysis of wayfinding data. Where appropriate, inferences were made between similar terms used across the Māori and Pacific wayfinding literature to gain a deeper understanding of the context and emergence of Māori wayfinding epistemologies. An inductive approach was then utilised to infer how the findings from the literature review (Chapter 5) could be extended (Reichertz,

2014) to inform pāngarau curriculum development and classroom practices. These inferences were then analysed further in Chapter 6.

The original intention for the classroom data was to group it according to the cultural symmetry framework foci, as was done for mātanga interviews and wayfinding literature. However, the collaborative research approach, designed with the participating kura, lent itself toward organising data according to the teaching as inquiry cycle phases. While still deductive in applying an existing theory and set of codes to the data, this better illuminated the collaborative journey the research project took in both settings. An inductive approach was then applied to individual classroom artefacts, and these were grouped and analysed according to either the mātauranga or school mathematics foci being taught. The collection of both teacher-created and student-created classroom artefacts also provided an opportunity for language error analysis (Houia, 2002). While this approach may seem unnecessarily negative, errors are a natural part of language learning (Houia, 2002). According to Houia (2002), language error analysis provides both theoretical and practical outcomes. The theoretical outcome is identifying whether or not the concerns expressed around language change in the literature and semi-structured interview data were evident in the classroom and, therefore, may require systemic intervention. The practical outcome provides opportunities for classroom practitioners to identify and correct errors before they become fossilised (Houia, 2002) via various pedagogical interventions, including explicit modelling (see Allen, 2015).

COVID-19 Pandemic Context

The main data collection period for this research coincided with the arrival and considerable disruption of the COVID-19 pandemic in Aotearoa/New Zealand in February 2020 and the two years that followed. Therefore, the study design had to adapt to the circumstances of the pandemic, which included extended periods when kura were physically closed and learning was conducted remotely. Due to the legacy of assimilatory education policies, the COVID-19 pandemic exposed and further exacerbated the deep education inequities that already existed for Māori-medium education (Allen & Trinick, 2021). Despite the structural support pāngarau education has received because of the high-status of mathematics, the study still encountered structural challenges.

Access to digital tools, such as digital pāngarau teaching and learning resources, was a barrier to the continuity of learning and research in both contexts. To address this challenge, Tumuaki 1 and 2 requested that digital technology and exemplifying its use for pouako was woven into the research study design. This was achieved in two ways. Android tablets with screencasting applications were supplied to both settings for the duration of the research, and professional development sessions were provided to pouako at both settings prior to the research commencing. Pouako were shown how to

utilise the affordances of screencasting software to promote the acquisition of te reo Māori and mātauranga alongside pāngarau curriculum content.

While not the primary focus of the study, the pandemic context highlighted the ubiquity of digital technologies and the increasing reliance of Māori-medium schools and learning programmes on digital strategies to overcome geographic and linguistic isolation (Allen & Trinick, 2020). Galla's (2016) research into the use of digital technology for revitalising Indigenous languages (and the Indigenous knowledges conveyed by the language) found that the selection and use of digital solutions must be tailored to the linguistic, cultural, social, environmental, economic and technological context. The increased reliance on digital tools for ensuring the continuation of teaching and learning programmes during school closures due to the COVID-19 pandemic was an important factor in the selection and use of digital technology in this doctoral study. According to Galla (2016), digital technology provides opportunities for Indigenous communities to create and publish their own language (and knowledge) artefacts. Therefore, the affordances of digital technology for visually capturing and sharing the student-created mātauranga-focused digital assets beyond the classroom as desired by each kura also influenced the mātauranga foci of the study. Māori ways of coding spatial information and appropriating spatial data science technologies (e.g., Global Positioning System [GPS], geographic information system [GIS], Google Maps) were explored and are exemplified in Chapter 6. The use of digital technology to create shareable teaching and learning resources in te reo Māori was also discussed by the mātanga interviewed in Chapter 4.

Ethical Considerations

As a Māori-medium graduate, teacher, and parent, I am an insider or member of the group being researched and an outsider conducting the research who is not staff or whānau of the kura. Insider positioning provided access to Māori-medium research contexts and a responsibility for the way the research was used, influencing the way the content of this study was communicated. Galla and Goodwill (2017), both Indigenous researchers who conduct research in their own languages in their own communities, identified that, “our languages encode a knowledge system and perspective that are unique to our Indigenous communities and express particular concepts and experiences that may not be easily understood or represented through colonial languages or by non-community members” (p. 71).

Therefore, my positioning as an insider of Māori-medium education, both linguistically and through my own education experience, has strengthened the analysis and interpretation of findings.

Furthermore, the outcomes of protecting and strengthening Māori-medium schooling necessarily pervade the analysis, conclusions, and recommendations. Characterising the study as by Māori-medium for Māori-medium also implies that Māori are the primary audience. However, the intended

audience has been extended to support Indigenous efforts to indigenise education for Indigenous students worldwide. Therefore, the global audience of this thesis helps create a sense of objectivity for the analyses and conclusions. This includes communicating the findings in English to ensure access to a wide range of Indigenous educators and researchers.

The research design also recognised and problematised the role of academic research in documenting Indigenous knowledges, which is an ongoing tension for Indigenous academics globally (see Porsanger, 2011). The ethical dilemma here is the problematic appropriation of academic research as a vehicle for regenerating mātauranga, therefore strengthening the positioning of academic research as normative in mātauranga creation practices (Smith et al., 2016). If academic institutions are being appropriated by Indigenous scholars to disseminate Indigenous research, ongoing indigenisation of the institution is also required. Therefore, this study aims to further sensitise readers to the role of academic research in appropriating mātauranga to further Western science research objectives, thereby ignoring questions and problems posed by Māori.

Universities are engaged in transdisciplinary or interdisciplinary studies that seek to integrate community and Indigenous knowledge systems with science disciplines such as mathematics. This is often done under the seemingly altruistic commitment to address the complex challenges of society (Organisation for Economic Co-operation and Development, 2021). Moewaka Barnes and colleagues (2021) noted that Indigenous knowledge holders are rightly wary of transdisciplinary research. Mātauranga has been, and continues to be, a resource for economic exploitation by non-Māori (Jahnke & Jahnke, 2003) rather than a source of benefit for Māori. As Miller and colleagues (2008) observed, the rigidity of academia can result in further conservation of established behaviour patterns despite espoused commitments to equally value two (or more) knowledge systems. This provides further evidence that Eurocentric beliefs in the superiority of Western paradigms over Indigenous worldviews remain deeply embedded within European societies (Jahnke & Jahnke, 2003). Those concerned with mātauranga must remain aloof from these concerns and focus on the issues important to Māori, despite institutional pressures to the contrary (Cooper, 2012).

Similarly, there is a real danger that mātauranga will be further appropriated in curriculum and classroom practices solely to promote the acquisition of school mathematics by ākonga Māori. Mathematics education researchers recognise the limits of Eurocentric mathematical viewpoints, particularly in promoting global citizenship through mathematics education—primarily for non-Indigenous students (Owens, 2014). However, the reciprocity for appropriating Indigenous knowledges to further the goals of the mathematics education field is primarily framed as transitioning Indigenous students from Indigenous knowledges to school mathematics (Sinclair et al., 2016), thereby providing opportunities for employment and further education in Western knowledge (Aikenhead, 2018; Meaney & Trinick, 2020; Te Maro, 2018). The danger here is that the findings of

this current study are utilised to gain access to mātauranga to further the aims of the mathematics education field *for* ākonga Māori without considering Māori aspirations for their own schooling. Often, Māori education discourses are focused on pathologising Māori underachievement and marginalising the agency of Māori communities in determining and addressing their own needs (Smith, 2012; Te Maro, 2018). This highlights the predominance of non-Māori researchers investigating Māori education, causing distrust and wariness of the research process (Smith, 2012). The research methods of this study thus sought to empower Māori to conduct and build on their own research by inquiring into their own needs for pāngarau curriculum and classroom practices. The following data chapters explore this approach through semi-structured interviews with four mātanga pāngarau, a literature search on the characteristics of Māori and Pacific wayfinding devices, and finally, a collaborative research project in two kura.

4. Ngā Uiui Mātanga Pāngarau | Expert Interviews

This chapter presents the findings of semi-structured interviews with four mātanga pāngarau. The mātanga interview data are used to examine the ideologies influencing the indigenisation of pāngarau curriculum and classroom practices. Assigned pseudonyms Mātanga 1–4, the mātanga are all experienced pāngarau teachers and curriculum and resource developers. Because of this, the mātanga are in the unique position of being able to discuss the ideologies influencing pāngarau register elaboration and whether similar opportunities, challenges and tensions exist for illuminating mātauranga at both the macro (policy and curriculum) and micro (classroom) levels. The mātanga also shared insights on the perceived impact of their contributions to indigenising pāngarau curriculum and classroom practices in support of Māori aspirations for schooling.

Interview Questions and Data Coding

The following questions guided the semi-structured interviews with the four mātanga:

1. Ka taea e koe te tautohu i ētahi o ngā rauemi o te reo Māori ka whakatauiria i te Māoritanga o ngā whakawhitinga whakaaro? He tauira o te reo ā-waha, o te reo ā-tinana hoki pea.

[Can you identify resources within the Māori language that exemplify uniquely Māori ways of communicating? These examples could be both verbal and non verbal.]

2. Mā te aha ka ahurei ēnei tauira ki te reo Māori?

[What makes these examples unique to the Māori language?]

3. Tautohua ētahi o ngā rauemi o te reo pāngarau ka huawaere i te tirohanga Māori i ngā whakawhitinga whakaaro pāngarau?

[Can you give some examples of resources within the pāngarau language that facilitate mathematical communication through a Māori cultural lens?]

Questions were provided to mātanga in both te reo Māori and English prior to the interviews. The questions were asked in te reo Māori and the mātanga chose the language of response. Discussion and clarification were offered in English when prompted by the mātanga. Mātanga were asked to elaborate on their answers, or follow-up questions were asked about the ideologies influencing their responses.

The interview data in this chapter were coded and are presented according to the three cultural symmetry framework foci. I designated the codes of mātauranga, te reo Māori, and school

mathematics to each of the mātanga interview responses based on the core idea they were expressing. The mātauranga label was used to code data that explicitly discussed opportunities and challenges for illuminating mātauranga in pāngarau classrooms via curriculum and assessment. The reo Māori code was used to group responses that identified both positive and negative benefits of elaborating the pāngarau register for pāngarau curriculum and classroom practices or the wider project of Māori language revitalisation. The reo Māori code was also used to group responses that provided evidence of both beneficial and detrimental language change in pāngarau classroom discourse, curriculum and resources. Finally, the school mathematics label was used to group responses that identified opportunities, challenges and tensions for the co-occurrence of school mathematics and mātauranga in pāngarau curriculum and classroom practices. These data are presented in the body of this chapter in three corresponding sections: 1) Illuminating mātauranga in pāngarau curriculum and classroom practices; 2) The impacts of pāngarau register elaboration; and 3) Balancing school mathematics and Māori aspirations for schooling.

Illuminating Mātauranga in Pāngarau Curriculum and Classroom Practices

The first data set in this chapter addressed the Indigenous knowledge foci of the cultural symmetry framework. In this section, all four mātanga identified uniquely Māori ways of expressing ideas and information, both verbally and physically, through discourse and practices. Mātanga 1 discussed Māori narratives used to communicate spatial information on land and sea. Mātanga 2 identified the use of whakapapa as a cognitive framework for organising knowledge and explaining the world, and Mātanga 3 discussed the concept of te whakatangata (personification) and its potential use as a pedagogical aid. Mātanga 1 further described uniquely Māori ways of locating and orientating. Finally, Mātanga 4 discussed personal experiences of the challenges and opportunities for communicating about mātauranga in pāngarau classrooms.

Communicating Spatial Information Through Narratives

Mātanga 1 identified the use of narratives as a uniquely Māori way of communicating spatial information that has synergies with school mathematics. Mātanga 1 gave the use of uruuru ao, narratives for navigating between islands, used by Māori and Pacific ancestors. According to Mātanga 1, these narratives created cognitive maps:

The resurrection of mātaunga Māori which is often historically memories of Hawaiki such as the Tāwhaki [narratives]. One form of stories is the uruuru ao which are still practised in Micronesia and parts of the Pacific...

...they are boundary markers by themselves, they are routes by themselves ... they are often in the form of navigation from island to island. It must be that some of the islands are so far

away that the [islands are beyond the] horizon ... so they have to use the stars... but they are essentially chase stories, catching fish stories. They lay down navigation routes.

Mātanga 1 also discussed how spatial information is expressed in te reo Māori, using direction and location as examples. An exploration of uruuru ao seemed to present opportunities to further exemplify Māori ways of expressing direction and location in pāngarau curriculum and classrooms:

There's also the whole discussion around the directional ... maths uses a lot of directional language. Spatial orientation language for all sorts of things. And then Māori when it changes from ... a position to a direction the 'whaka-' prefix pops into it ... kids could not... understand the difference between 'ki mua' and 'whakamua'.

Well one is a position, and one is a direction. So, whereas in English they are both the same. I am going to the front. So, what designates the front. So that there's the front of the house and I'm going to the front... But in Māori you can't do that because grammatically it's different. So, if I want to go to the front. I could say, "Kei te haere au ki mua o te whare". But if I wanna go forward in a forward direction ... "ka haere whakamua". So, whereas "Kei te haere au ki mua". I am going to that location. I don't necessarily mean I am going forward. You can say, "Kei te haere whakamuri au ki mua o te whare".

When Māori became settled in the much bigger land mass of Aotearoa/New Zealand, the characters in the stories changed, and according to Mātanga 1, the land-based narratives became known as uruuru whenua. Mātanga 1 used the narrative of Te Wheke o Muturangi (Grace, n.d.), initially a sea orientation uruuru ao, as an example of this shift:

The one that we have in NZ that is still part of our narrative is Te Wheke o Muturangi where they chase the wheke (octopus) from Hawaiki to Aotearoa. Then it becomes an uruuru whenua. It becomes a land story. So, it transitions from a sea one to a land one. So as the wheke goes around the New Zealand coast, it names landmarks etcetera.

Mātanga 1 shared three other examples of localised uruuru ao and uruuru whenua that are part of the oral and now written records of Māori tribal groups. These are the stories of the moki (a type of fish), Pania of the Reef, Ngātoroirangi and Tūhoronuku:

Moki

When the moki came to Aotearoa from Hawaiki, one of the stories involved the fishing grounds and it's a story about how the fishing grounds get named, but its more than that they are navigation aids so in the story they are not the bearings but the landmarks you use to

locate these grounds. So, the story is not a chase story, but it is a story about this man who goes from grounds to grounds and does all these amazing things and as he does so he lays down navigation markers... how to get from one to the other.

Pania of the Reef

Now you have the one that always springs to mind is the... it's a sea marker... in that it is laying down that reef out to sea.

Ngātoroirangi

The Ngātoroirangi story where he goes around White Island, Ngauruhoe, all those sorts of things and he lays down these boundary markers.

Tūhoronuku

Now remember one of the sons Taiamai... There's actually a monument up there now... and it's called an uruuru whenua.

Similar to the description of the exploration narratives uruuru ao and uruuru whenua provided by Mātanga 1, Taonui (2015), whose doctoral thesis described the characteristics of Māori narratives, noted the journeys of Ngātoroirangi and Tia as a series of journeys projected back onto the two figures. Taonui (2015) stated that the purpose of exploration narratives is to describe boundary markers and routes used over generations. The characteristics of exploration, boundary and route-marking narratives are explored further in Chapter 5.

Mātanga 1 also shared two further catalysts for the change in the way Māori practised uruuru ao and uruuru whenua. The first is the use of modern technology:

They've kind of become, you know, with the use of modern technology they kind of got lost.

The second catalyst for change discussed by Mātanga 1 was the Christianisation of karakia. The pseudonyms Kaumātua 1 and Kaumātua 2 are used here to protect the confidentiality of the kaumātua named in the transcript excerpt:

I have been with [Kaumātua 1] and [Kaumātua 2] on trips and they would often do a karakia. Over time it's become Christianised, it's become like a Christian karakia. But I suspect in the old days they were not. They were ways of establishing ownership. They were ways of protecting yourself when you went into a new area.

Mātanga 1 identified uruuru ao, uruuru whenua, and karakia as ongoing narrative practices used to disseminate routes to important resources and to establish boundary markers. Mātanga 1 also identified two challenges to the ongoing practice of uruuru ao and uruuru whenua. The first is the Christianisation of karakia resulting in alternative meanings and practices being attached to the narratives. This obscures the spatial information contained in karakia, posing a challenge for its illumination in pāngarau curriculum and classroom practices. Dual ontological lenses could be used to sensitise researchers, curriculum developers, pouako and ākonga to the Christianisation of karakia. Karakia could be explored through mātauranga and Christian lenses, aligning with an ontologically pluralistic approach and the cultural symmetry framework. By using this approach, similarities and differences in the characteristics of each practice could be examined, providing opportunities for epistemological pluralism (Andreotti et al., 2011; Miller et al., 2008). This dual-epistemic approach to the illumination of mātauranga is explored further in the literature reviewed in Chapter 5. The second challenge identified by Mātanga 1 is the advent of digital technology, as some of the practical uses of uruuru ao and uruuru whenua have been superseded by GPS or GIS. Acknowledged as a challenge to the perpetuation of mātauranga here, the affordances of digital technology to support the revitalisation of te reo Māori and mātauranga are further explored in subsequent sections of this chapter and Chapter 6.

Communicating Through Personification and Genealogical Frameworks

According to Mātanga 2, whakapapa, a defining characteristic of mātauranga, has synergies with school mathematics:

Ko tētahi āhuatanga o te ao ... āhuatanga nui, ko tēnei mea te whakapapa. Ko tā te whakapapa, he tūhono i te tangata ki tōna ao. Ki ngā tāngata kei ōna taha, ki te ao tūroa, ki te hītori. Ki ngā wā o uki. Nō reira ko tēnei mea te whakapapa, he honohono kia mōhio ai te tangata ki ōna pānga. Ki tērā āhuatanga, ki tērā āhuatanga o te ao.

[One of the features of this world... important features, is genealogical descent. The role of genealogy is to link a person with their world. To the people they stand alongside, to the natural world, and to history. To ancient times. Hence, it is through whakapapa, that a person can understand their relationships. To each and every aspect of the world.]

Mātanga 3 discussed a link between mātauranga and pāngarau curriculum through the alternative title of tātai, which can be used to express practices of measuring, arranging, setting in order, planning, and reciting genealogy. The meaning of tātai has also been expanded as part of the pāngarau register elaboration process to denote calculation (Barton et al., 1998; Trinick, 2015). While the argument for using the title of pāngarau for the curriculum was presented in Chapter 2, Mātanga 3 shared a clear preference and argument for the native term tātai:

Ko te kupu tino Māori rā, ko te kupu tātai. He whakapapa tōna. Pāngarau, he mea tito noa nei. Heoi anō kotahi anō te puka, ka puta ka pāngarau.

[The more Māori word is the word ‘tātai’. It has a whakapapa. Pāngarau, is just a made-up word. However, there is just the one book, and the word used is pāngarau.]

Interestingly, whakapapa is stated as the reason for Mātanga 3’s preference for the term tātai. Mātanga 3 believed that the term has whakapapa. In this case, the term whakapapa seems to be used by Mātanga 3 to indicate epistemology, demonstrating the use of whakapapa to connect ideas, practices, and experiences, not just people. Tapsell (2011) asserted that whakapapa frameworks could be conceptualised as broad categories of knowledge, similar to the way kete (baskets) or uruuru are used in the Tāne and Tāwhaki narratives, respectively (as discussed in Chapter 2).

Mātanga 4 also discussed the use of whakapapa, genealogical connections between people, by Māori elders as a lens for solving problems and providing for the community:

Kōrero ki ngā kuia me ngā koroua. Kāre e whāiti te titiro. Kei te tirohia ngā whānau katoa o te hapū o te marae me ngā pānga ki a tātou katoa. Kātahi ka whakairo ... oh arā te karangatahi rā, te whanaunga rā, arā, māna tēnei. Ko te tiki atu i ngā rauemi tangata me ō rātou pūkenga nē? I whai hua. I whai hua ai te hapū katoa, te marae. Koira tētahi tino rerekētanga.

[Talk with the elders. They do not have a narrow viewpoint. All the families of the hapū and the marae are considered, and the impacts on all of us. Then they will think ... oh, that cousin, or that relative, they can do this. Fetching the human resources with their skills, perhaps? It worked. The entire hapū benefitted, and the marae. That is one of the big differences.]

Similar to the discussion of Mātanga 1 around the characters used to enliven uruuru ao and uruuru whenua, Mātanga 3 advocated engaging characters to enliven pāngarau curriculum and resources. Using the example of Pacific ancestor Māui, Mātanga 3 discussed the pedagogical aid of te whakatangata to bring pāngarau to life:

Ko te whakauru i ngā mea o Māui. Kia kaua a Māui e huri hei taputapu tātai me kī. Engari... ko taku hiahia ko te whakatangata i tēnei mea te tātai.

[Inserting those things about Māui. Māui shouldn’t be turned into what you might call a calculating device. But... I would like to humanise this thing, mathematics.]

Further insights into the use of genealogical narratives and personification to enliven important information and make it more memorable are explored in the literature reviewed in Chapter 5. Further discussion of the use of whakapapa as a way of connecting ideas is also discussed in subsequent

sections of this chapter. Both uses of whakapapa seem to present opportunities to illuminate mātauranga in pāngarau curriculum and classrooms.

Challenges to Teaching Mātauranga in Pāngarau Classrooms

Mātanga 4 discussed the role of pāngarau teachers in transmitting mātauranga. Drawing on personal experience, Mātanga 4 attested that this was not always done consistently:

Nōku pea te hē, kāore i āhei te tiki atu i ngā horopaki i ngā wā katoa e hāngai ana ki tō rātou ao Māori.

[Perhaps this is my own fault, I can't always come up with a context that aligns with their Māori world.]

Based on this discussion, I posed more questions, including, “Can we be sure that ākongā pāngarau are learning mātauranga? If not, what is the purpose of pāngarau schooling for Māori?” Mātanga 4 responded by saying that some students will have a deep understanding of mātauranga, but most will have a general understanding. For Mātanga 4, it was easier to be confident about students' mathematical understanding than any depth of mātauranga:

Mō ētahi [ākongā] āe. Mō te nuinga kāore pea. Engari, e kōrero ana au, kāore pea e tino mārāma ki te hōhonutanga o te whakaaro Māori. Engari te nuinga kei te mārāma ki te ariā pāngarau. Engari kāore pea au e kaha te kī ko te nuinga e mārāma ana ki te whakaaro Māori.

[For some [ākongā] yes. For the majority, perhaps not. But I'll say this, perhaps they don't entirely understand the deeper Māori concepts...but most understand the mathematical concept. But, perhaps I shouldn't really say that the majority understand the Māori way of thinking.]

Mātanga 4 discussed the importance of drawing on mātauranga within the community, including physical and visual representations such as those contained in whareniui when seeking to address questions about the purpose of pāngarau schooling for ākongā Māori. This reflects similar work by Trinick and colleagues (2017) who examined symmetry in whareniui:

Kei ngā whare tupuna o tēnā marae o tēnā marae ngā kōrero nērā? Ngā poupu whakairo, i ngā tukutuku, i ngā kōwhaiwhai. Me te tika o tērā pātai. Kei te mōhio a koutou tamariki ki ērā kōrero? Ki te kore, he aha ngā hua o te kura?

[The stories are in the ancestral houses of all our marae aren't they? In the carved and the woven panels, and in the painted designs. And the pertinence of that question. Do your children know those stories? If not, then of what value is the school?]

Mātanga 4 identified some of the efforts that had been made to connect mathematics concepts to mātauranga in national assessment projects, such as construction techniques, design, hosting large events, and performing arts:

... i āwhina i Te Mana Tohu Mātauranga, NZQA ki te hanga i ngā ngohe aromatawai. Anā, ko tētahi o māua tino whāinga ko te kimi i ngā horopaki Māori. Kua tiki atu i te whare tūpuna, te maihi, ngā amo, hei āwhina ki te hanga i ngā pātai e hāngai ana ki te ine me te āhuahanga. Arā, ko ngā pūkenga o pākoki... te mea i te rōanga me ngā koki o roto... ko te mahi hāngī tētahi ... he nui te pāngarau i roto i tērā mahi. ... ngā tunu kai, mehemea he hākari nē?

[... [we] helped Te Mana Tohu Mātauranga, NZQA, to create the assessment tasks. Indeed, one of our major tasks is to find Māori contexts. We've brought in the ancestral houses, the bargeboards and supporting uprights to help construct questions that align with measurement and geometry. That is, the skills of trigonometry... to find the lengths and angles involved... cooking a hāngī (earth oven) is another ... there is lots of mathematics in this work. ... cooking food, if it is a feast, eh?]

One of the assessment contexts Mātanga 4 identified involved comparing the performing arts and speech competition judging criteria to that used at the Olympics:

E titiro ana au ko tētahi o ngā aromatawai kua tiki atu mātou, ko Te Matatini me ngā kōrero, arā, he mahi nama, he mahi tauanga. Ko te mahi o ngā kaiwhiriwhiri, ngā kaiwhakawā. Tēnei tikanga o te tokorima. Anā, ... e ai ki te taumahekeheke o te ao, te oromipia, ka tango te tau mōkito me te tau mōrahi, anā, ka tiro i te tauwaenga o ngā mea e toru o waenga. Ko te whakawhitiwhiti whakaaro ki ngā taiohi, ko tēwhea rawa te inenga tauwaenga pai, ko te tau ahuwaenga pai, ko te tauwaenga rānei, ko te tau toharite rānei. Ērā momo. Ko tētahi o ngā kaupapa ka aroa ko ngā mahi o te kapa haka, ā, kua kōrero hoki mātou mō te... Ngā Manu Kōrero, hei āta tūhura i ngā hua ka puta....

[I'm looking at one of the assessments we retrieved, on Te Matatini along with the descriptions, it is numerical work and statistics. The job of the panel, of the examiners. The rule of five. It's like this, ... in the Olympic Games, you remove the lowest number and the highest number, then you look at the average of the three in the middle. Exchanging views with young people, which is the best measurement of the average, the best number for the middle, the median or the mean? Those kinds of things. One of the topics to consider is the kapa haka (Māori performing arts), the manu kōrero (speech competitions) too, to deeply explore the outcomes...]

While the intention expressed by Mātanga 4 was to illuminate mātauranga in the assessment examples given, the content being assessed is school mathematics. While an understanding of mātauranga could be implied through engaging in the assessment tasks, there did not seem to be specific assessment criteria in these examples for mātauranga. As identified in the pāngarau literature, often well-meaning attempts to draw on mātauranga in pāngarau assessments and resources seem to inadvertently promote internationally recognised mathematical practices (Allen & Trinick, 2022). This seems to reflect the resignation of Miller and colleagues (2008), that even well-meaning attempts to promote two knowledge systems simultaneously often privilege one knowledge system over the other. However, Mātanga 4 also made a prediction about the future of pāngarau curriculum and resources. Within 20 years, Mātanga 4 believed that pāngarau and the associated teaching resources would better reflect mātauranga:

Me taku matapae i roto i te 20 tau anō e heke mai nei ka whanake ake ērā, ka Māori ake ērā rauemi.

[And my prediction is that in the next 20 years, those will develop further, those resources will become more Māori.]

As identified by Mātanga 4, teaching mātauranga in pāngarau classrooms is not as simple as identifying mātauranga that has synergies with school mathematics. Teachers need guidance in utilising mātauranga in pāngarau classrooms to ensure that mātauranga is acquired by students as intended and not further appropriated to promote the acquisition of school mathematics (see Allen & Trinick, 2022). Mātanga 4 concluded that it is difficult to say that Māori knowledge is being acquired by students in pāngarau classrooms in any depth, believing it was much easier to make judgements about students' mathematical understanding than their understanding of mātauranga. This is unsurprising, as the development of pāngarau curriculum up until this point, has not explicitly addressed mātauranga, as argued in this thesis. Mātanga 4 discussed community resources and the role of elders in modelling the practice of mātauranga, such as utilising whakapapa as a resource for problem solving. The tensions, challenges, and opportunities identified here are further explored in Chapter 6.

The Impacts of Pāngarau Register Elaboration

The lessons learned from pāngarau register elaboration likely apply to the illumination of mātauranga in pāngarau curriculum and classrooms. In this section, the views of four mātanga on the benefits and challenges of curriculum development and language elaboration for pāngarau curriculum and classroom practice are discussed. This includes the ideologies underpinning elaboration, the tensions between wider language revitalisation movements, and the realities of pāngarau classrooms. While

Māori schooling aspirations include te reo Māori, teaching school mathematics in te reo Māori has been likened to a Trojan Horse (Barton & Fairhall, 1995), furthering the work of colonisation (McMurchy-Pilkington, 2004). This second interview data set addressed the Indigenous language foci of the cultural symmetry framework. In the following sections, the mātanga offer advice on addressing the ideological tensions of pāngarau register elaboration and the practical challenges of implementing the register in classrooms for pouako and ākongā. As part of the data analysis, inferences were drawn between the opportunities, challenges, and tensions of pāngarau register elaboration and the potential illumination of mātauranga in pāngarau curriculum and classrooms.

Bilingual Schooling and the Need For Pāngarau Register Elaboration

To give further context to the pāngarau register elaboration work, Mātanga 2 discussed the early beginnings of bilingual pāngarau education. The small number of schools and, therefore, few teachers, meant that each teacher was creating words and resources as needed in the classroom:

Ka hoki aku mahara ki ngā tau 80, i au e mahi kaiwhakaako ana i tētahi kura tuarua. Ka tīmata.... tētahi kaupapa reorua. E toru, e whā pea ngā kura i taua wā puta i te motu, he kura reorua, he akoranga, he karaehe, akomanga reorua rānei i roto i ngā kura auraki. Ko Ōmahu tētahi.

[I remember back in the 80s, when I was teaching in a secondary school. That's when.... a bilingual programme was started. Maybe three or four schools around the country at that time were bilingual schools, bilingual lessons or classes within mainstream schools. Ōmahu was one.]

Mātanga 4 discussed the non-existent resourcing for those who taught in bilingual schools compared to today:

Ko tētahi kōrero nui mōu mō tāu tamaiti e kuraina ana i tēnei wā, he pai kē atu ngā rauemi i tēnei wā i tā te wā i te kura koe. Nē? I aua tau rā ko [Kaumātua 3] mā e hangahanga ana i ngā rauemi i taua wā tonu.

[One thing you should know about your child's education these days, the resources are so much better than they were at the time when you were at school. Don't you think? In those times, [Kaumātua 3] and the others would be creating resources on the spot.]

Mātanga 2 also discussed the prevalence of borrowed words in the pāngarau register prior to the 1990s and how this was not perceived as a threat to the integrity of te reo Māori at that time:

...I te wā pea, i te taenga mai o te reo Pākehā ki tēnei whenua, he māmā noa iho te kapo atu i tētahi kupu mino, tētahi kupu whakawhitinga oro mai i te reo Pākehā ki te reo Māori. Hei whakaahua mā te Māori i ngā āhuatanga hou pēnei i te whika, te hāwhe i ērā atu āhuatanga. I aua wā rā, he kaha tonu, he pakari tonu te reo Māori. Nō reira, kāhore ērā whakawhitinga kupu, kāhore ērā kupu mino i whakamōrea, i whakararu rānei i te reo Māori. He pai noa iho. Ka pakari tonu te reo.

[...When the English language first arrived in this country, it was not a problem to borrow a word, transliterating the sounds from English into Māori. So that Māori could explain new concepts like numerals (whika), or half (hāwhe) and those kinds of things. In those days, the Māori language was still strong and robust. Hence, those cross-over borrowed words did not pose a problem or any threat to the Māori language. It was alright. The language was still strong.]

Mātanga 1 also observed that the pāngarau register was naturally expanding through borrowing words from the English language prior to pāngarau register elaboration:

... actually, it wasn't our determination to use just the native terms... in the 80s a lot of schools, a lot of communities, a lot of teachers used transliterations. It was a common strategy to bring words, through social interaction, into the vernacular so schools just carried on using that strategy.

As discussed by the mātanga in this section, bilingual school teachers in the 1970s–1980s were tasked with translating English-language school mathematics resources in order to teach mathematics in their classrooms. In the literature reviewed in Chapter 2, Christensen (2004a) recognised the increased workload for teachers in translating English-language resources into Māori. Barton and collaborators (1998) argued that the translation efforts often produced dubious terminology outcomes. Parallels can be made here between the bilingual school teachers' attempts to create resources to teach mathematics in Māori and similar attempts to teach mātauranga, without national curriculum guidance. As discussed in Chapter 2, kōwhaiwhai has been used in pāngarau classrooms and resources. However, kōwhaiwhai have been criticised as not meaningfully addressing mātauranga but rather utilising it to illustrate school mathematics concepts (Allen & Trinick, 2022). In the case of kōwhaiwhai, these are generally transformation geometry concepts (Knight, 1996).

While huge strides have been made in Māori language resourcing for pāngarau classrooms in recent decades, as observed by Mātanga 2, this is not the case for all levels of pāngarau schooling. At senior secondary levels, both Trinick (2015) and Stewart (2007) believed that borrowing words from English that are used infrequently would not have a detrimental impact on te reo Māori. Perhaps this is a

reason less attention has been given to resourcing senior secondary pāngarau classrooms with Māori language resources. As discussed by Mātanga 4, at the senior secondary level, pouako are still translating English-medium school mathematics resources for use in their classrooms:

...ka whakaaro au mō ngā tau iwa tae atu ki ngā tau tekau mā toru. Ko te nuinga o ngā kaupapa whakaako he whakamāori i te kaupapa Pākehā i te nuinga o te wā...

[...I think for Years 9 right up to Year 13. Most of the teaching topics are translated from the English in most cases...]

Considering the realities of senior secondary pāngarau classrooms, the illumination of mātauranga in pāngarau curriculum needs careful consideration. Even if promoted in pāngarau curriculum, mātauranga could still be excluded from senior secondary pāngarau classrooms if dedicated resourcing is not provided to teachers. This is because mātauranga cannot be borrowed and translated from English-medium resources the way that school mathematics can.

Arguments For and Against Standardising the Pāngarau Register

The arguments for and against standardising the pāngarau register are presented here, and the implications of the arguments for illuminating mātauranga in pāngarau curriculum are discussed. Mātanga 2 and 3 provided key arguments for the coordinated and purposeful elaboration of the pāngarau register. As discussed in the literature and in the previous section, up until the early 1990s, the pāngarau register expanded ad hoc in pāngarau classrooms by borrowing words from English and terms created by pouako. Mātanga 3 argued the importance of utilising whakapapa, a transparent system for connecting ideas and, thereby, supporting students' conceptual development. Mātanga 3 presented the example of fraction terms:

Nō reira, he hononga ki waenganui i ēnei momo mea, hautanga. Nō reira, me whai whakapapa me whai pūnahatanga, te whakaingoa i ēnei āhukatanga. Nō reira, i puta ai te haurua, te hauwhā. Mehemea e mārama ana te tamaiti he aha tēnei mea te haurua, me te hauwhā, ka māmā noa iho tana kapo atu i te hautoru, te haurima, te hauono, te haurau. He āhukatanga nui ērā hautanga i roto i te pāngarau.

[Hence we see the connections among these things like fractions. So there needs to be a genealogical link and a system that is followed for naming these features. Hence we have the haurua (half) and hauwhā (quarter). If the child understands the word for half (haurua) and quarter (hauwhā), it will be easy for them to grasp the hautoru (third), haurima (fifth), hauono (sixth) and haurau (hundredth). Those fractions play an important role in mathematics.]

Mātanga 2 echoed this argument and believed that terms were created in the elaboration process in a way that gave cues or clues to the underlying concept:

Kei roto pea i ngā kupu hou ētahi tīwhiri ki te tikanga o te kupu.

[The new words might include some hints on the meaning of the word.]

Mātanga 2 also discussed the difficulties for students moving between schools, or even classrooms, if there was no standardised vocabulary. The issue raised by Mātanga 2 here is that if individual teachers created their own terminology, pāngarau discourse would become specific to each classroom.

Mātanga 2 believed that if pāngarau is taught in multiple settings, students should be able to move between them without difficulty, therefore, standardisation of the pāngarau register was required:

Mehemea ka neke te ākongā mai i tētahi kura ki tētahi, ā, he rerekē katoa ngā kupu pāngarau o taua kura, i te kura i mua rā, ā, ko te tamaiti tonu te papa. Ko te kaupapa tonu te papa. Nō reira, e whakapae ana mātou, e kaha tohe ana mātou mō ēnei āhuatanga hou e kōrerohia nei i roto i te reo Māori, kia kotahi tātou, ā, ki ēnei kupu nā.

[If the child moves from one school to the next, and all the mathematics words are different from the previous school, then the child is disadvantaged. The whole movement [kaupapa] is harmed. So, our hypothesis, we strongly argue for these new features of the language we have discussed, we should be united in using these new words.]

Mātanga 2 also argued that standardising academic registers is a way of addressing new technologies and provided examples from the hangarau matihiko (digital technology) register, which is currently being elaborated at pace to catch up and keep up with technological advances:

...he hua nui mehemea ka āhua kotahi te reo Māori e pā ana ki ēnei kaupapa hou o te reo. Pēnei i hangarau matihiko. Kārekau he hua... mēnā e rima, e ono, e whitu ngā kupu Māori mō tēnei mea te modem. He aha te hua o tērā ki tō tātou reo. Ēnei āhuatanga hou o te ao, kia kotahi rā te kupu, kia rua rānei. Hei painga mō te reo Māori. Hei painga hoki mō ngā ākongā i roto i ngā kura.

[...there are great benefits if we can more-or-less standardise the Māori language used for these new features of the language. Like digital technology. It will be unproductive... if there are five, six or seven Māori words for modem. Of what benefit is that to our language? For these new features of the world, there should be just one or two words. That would be best for the Māori language. It would be better for students in schools as well.]

The data presented in this section shows that the linguistic challenges for pouako in Māori-medium and kura are not the same as those of their English-medium colleagues. In a naturally developing language such as English, with a large international pool of native speakers, terms borrowed from other languages, neologisms, newly coined words, or alternative meanings for existing terms are constantly being invented on an ad hoc basis. As Stewart (2007) outlined, neologisms or newly coined terms have become widely accepted by their widespread use. For example, technical terms such as ‘tweet’ used to denote a post on the social media platform Twitter (Merriam Webster, 2022). In this way, the meaning of the existing word ‘tweet’ has been extended due to many people choosing to use this term over other possible alternatives such as ‘Twitter post’. Similarly, Mātanga 1 discussed the importance of the pool of speakers for register elaboration and curriculum development:

So, we’ve had to talk them into existence, and I think that’s what we have done more than in the written part. It’s been useful because it’s provided us with a model that the written doesn’t. The written exemplars of resources don’t by themselves create the register, the Māori register, it’s talking it.

For the English-language mathematics education register, this speaking-into-being process is completed by an international community of educators and experts. The small number of people engaged in delivering and researching pāngarau means this process requires a disproportionate amount of time from the few doing this work. Stewart (2007) posed the question regarding pūtaiao curriculum, which faces the same issues as to whether this is indeed the best use of the limited time and resources available. Therefore, further burdening the same small group of people with creating standardised mātauranga curriculum content and resources may prove unsustainable.

Again, not all mātanga were supportive of standardising the pāngarau register. To produce a register that followed consistent word creation principles, naturally occurring innovations such as transliterations were purged from the register. Using the term hāwhe as an example, Mātanga 2 discussed some of the resistance by pouako to purging transliterations from the pāngarau register:

E kōrero nei koe, tāua, mo te kupu nei te hāwhe. Kei te hoki aku mahara ki tētahi wā i te āwhina au i tētahi kura, ngā mahi pāngarau i tētahi kura, anā, i te kōrero mātou mō tēnei āhuratanga te hautanga, fractions. Anā, ko tā tētahi o ngā kaiako i mea mai, “Ā, hei aha noa ngā kupu hou nei. Pai noa iho te hāwhe, te koata, pai noa iho ērā kupu ki aku tīpuna. Nō reira, he pai noa iho ki a au”. Āna, ko tāku ki a ia, ka pai ko te hāwhe te hāwhe. Te koata, te koata. He aha rā tō kupu mo te third? Āna, ka rakuraku ia i tōna māhunga. Nā wai, nā wai, ka kī mai, “Ā, ko te hāwhe toru”. Āna, i reira ka raru.

[You were talking, we were, about this word hāwhe (half). I remember back when I was helping at a school, with the mathematics programme at a school, we were discussing the topic of fractions (hautanga). Then, one of the teachers exclaimed, “Oh, what good are these new words. It is fine to use hāwhe (half), koata (quarter), those words were good enough for my ancestors. So, they are good enough for me.” No, my response to them was, “It is OK, hāwhe for half. Koata for quarter. But what is your word for a third?” Then, they scratched their head. After a while, they said, “Ah, hāwhe toru (half three).” Behold, there lies the problem.]

Mātanga 1 also addressed the apprehension of fluent speakers to potential language change caused by the pāngarau register elaboration and the intergenerational language tensions that could be created:

There is always a concern about the new words. When I listen to the next generation, I don't even know what they mean either. But that happens anyway. That intergenerational language misunderstanding is common.

Mātanga 2 provided one of the strategies employed by language elaborators to ingratiate the Māori immersion schooling sector to the quickly expanding education registers such as pāngarau:

Ko tētahi āhuetanga e rata ai te tangata ki te kupu, mēnā e mōhio ana te tangata ki te takenga mai o te kupu hou. Nō reira, koira tētahi āhuetanga o Paekupu. Mēnā he kupu hou, ka āta whakaatu i te takenga mai o te kupu. Pērā i a tāua e kōrero rā mō te haurua. Mehemea e mōhio ana e mārāma ana te kaiako, te tangata rānei, ki te takenga mai o te kupu hou, ka rata atu ia, ka āhua rata atu ia ki te whakamahi i taua kupu ki roto i ana rerenga kōrero.

[One thing is that a person will warm to a word, if they know the origin of that word. Consequently, that is one of the features of Paekupu (online Te Marautanga o Aotearoa dictionary). If it is a new word, the origin of the word is prominently displayed. Like, for example, our discussion for the word haurua (half). If the teacher or the person knows and understands the origin of the new word, they will appreciate it, and are disposed to use that word in their sentences.]

Mātanga 2 further discussed the perceived negative impacts of standardised education registers on regional dialectal variations of te reo Māori:

Heoi anō, ka tohe mai ētahi ki tērā. Ā, he kupu tā tēnā iwi mō te modem. He kupu anō tā tēnā iwi mō te modem. Anā, e mea ana au, kei te tangata tonu, kei te iwi rānei tonu te mana i te mutunga o te rā mō ngā kupu e whakamahia ana e ia. Engari he mea nui, kia āta whakaaro te tangata, kia āta whakaaro te iwi rānei, i te whai hua o ēnei... o ngā kupu.

[However, some dispute that. And that tribe has a word for modem. And that tribe has a word for modem. Well, I would say, each individual, or each iwi has their own right (mana) at the end of the day over which words they use. But it is important that the person, or the iwi (tribe) carefully consider the benefits of these... of the words.]

Ki a au nei, kāhore. Karekau he tautohetohe. Ki te pakari te reo ā-iwi o te tangata, ka pakari tonu. Ahakoa tana tiki atu i ngā kupu hou hei whakaahua i ngā āhuatanga o te ao hou. Ka mau tonu tana reo ā-iwi.

[In my opinion, no. There is no cause for debate. If the tribal dialects of the person are strengthened, then indeed they are strengthened. Even when they utilise new words to describe the features of the new world. They will retain their own tribal dialect (reo ā-iwi).]

One of the main arguments supporting pāngarau register standardisation provided by the mātanga was ensuring that the pāngarau register had whakapapa, a system of clearly connected concepts, that supported ākongā conceptual development of school mathematics. The arguments against standardisation from the Māori immersion schooling sector and community seemed to communicate resistance to change. Resistance to the removal of transliterations from the register could have been associated with those generations of speakers who grew up with and therefore identified with more of a hybrid or pidgin version of te reo Māori as discussed in Chapter 2. Removing transliterations from the register also necessitates significant unlearning and relearning by pouako, ākongā, and the community about how to communicate school mathematics concepts. As new terms can only be validated by widespread use, the opposition to standardisation of the register could have resulted in a refusal to use the new terms. Therefore, the curriculum document risked sitting on shelves unused. Arguably, this has already happened in some cases, as identified by Mātanga 3 in the subsequent section on pāngarau resources.

Another challenge to standardisation was the perceived negative impacts on regional dialects of te reo Māori, which also has implications for the illumination of mātauranga in national curricula. Mātanga 2 argued that ākongā and pouako who moved between schools or classes should not need to learn new pāngarau registers, necessitating standardisation. However, an easy resolution could not be found for the issue of using different terms for the same concept in different regions to support and strengthen regional dialects. It is likely that an argument about maintaining mātauranga ā-rohe (regional), ā-iwi (tribal), and ā-hapū (sub-tribal) would emerge in opposition to illuminating mātauranga in a national curriculum document. Like te reo Māori, mātauranga has local, regional, or even whānau variations (Waitangi Tribunal, 2011). Therefore, there are two challenges here for the illumination of mātauranga in pāngarau curriculum. The first is gaining agreement that nationalised, and therefore, standardised concepts of mātauranga are possible and desirable. If this is not agreeable or desirable,

the second challenge is resourcing regions or smaller groups to create localised curriculum guidelines for teaching mātauranga at specific schools, which could reproduce the challenge for pouako and ākongā of needing to learn new concepts and terminology if they move localities or schools.

The Benefits of Pāngarau Register Elaboration

According to Mātanga 2, the systemic elaboration of the pāngarau register ensured that pouako and ākongā were equipped with the terminology to discuss school mathematics in depth. As discussed in Chapter 1, communicating mathematically through a specialised register has learning benefits (Pimm, 1991).

Ki te kore he kupu mō ngā āhuatanga hōhonu o te kaupapa pāngarau, hangarau aha atu rānei, ka ngaro te hōhonutanga o taua kaupapa rā. Ka mutu he whāiti noa iho tā te ākongā whai i aua kaupapa rā. Ka mutu ko tana hinengaro anō te papa, ā tona wā. I runga anō i te whakaaro ko te reo te tāhūhū o te hinengaro. Kia hihiri anō te hinengaro. Ki te kore e pakari te reo, ki te kore he kupu, ka noho ko te hinengaro hahaka noa iho, koretake noa iho i roto i ēnei tū kaupapa.

[If there is no word for the deeper features of mathematics, technology or other topics, the intricacies of the topic may be lost. In the end, the student will only achieve a narrow understanding of the topic. With the result that their intellect will eventually suffer. With the understanding that language is the ridgepole of the intellect. In order for the intellect to be energised. If the language is not strong, if the words are not available, the intellect will stay shallow and ineffective in these kinds of subjects.]

Mātanga 3 discussed the importance of the pāngarau register elaboration process for supporting the wider project of revitalising te reo Māori, despite objections from language revivalists about using pāngarau curriculum as a vehicle for revitalisation. Mātanga 3 believed that concessions had to be made as the Māori language was not prevalent in the community (see Benton, 1997):

Anā, ko tāku... he huarahi tino pai te pāngarau hei whakapakari i te reo... i te whakamahi i te tātai i te pāngarau hei whakapakari i te reo. Oh, i raruraru pai ai au i reira i tētehi i mea mai, “Kāo, kāre i te whakamahi i tēnei hei huarahi hei pakari ai te reo. Me ārahi te reo i ngā mea katoa.” Engari, ko taku mōhio i ngā ākongā, i ngā tamariki e whakaako ana au. Kāre i a rātou te reo, pakari rawa, kāre i o rātou mātua. Anā, te nuinga o rātou kare i o rātou kaumātua.

[Indeed, I'd say... mathematics is a great road towards strengthening [the Māori] language... using mathematical calculations to strengthen language. Oh, I got into some serious difficulties there with one who said, “No, don't use this as a path for strengthening [the Māori] language. We need to involve the language in everything.” On the contrary, in my

experience of students, of children I have taught, they don't have strong language, their parents don't. Even, for most of them their elders don't either.]

As discussed by Mātanga 3, the opposition to pāngarau classrooms as a suitable site for language revitalisation has implications for the illumination of mātauranga. It is likely that the same argument would be used to oppose the positioning of pāngarau classrooms as a site for the illumination of mātauranga due to ongoing concerns about its subjugation to school mathematics. Highlighting the positive impacts of the pāngarau register elaboration process on wider language revitalisation objectives could help address concerns for mātauranga.

Mātanga 1 discussed the positive impacts of the pāngarau register elaboration process in expanding the breadth and depth of meanings carried by individual terms. This process thereby reverses some of the negative impacts of language shift on te reo Māori (see Fishman, 2001):

In English medium... the linguistic landscape is English in New Zealand. Even if you go to a city like Rotorua that has a high Māori population, the linguistic landscape is still English.

What happened over the last hundred years, some terms of some aspects there's been an atrophy of Māori, of the range of meanings. Some words have a huge range of meanings in the development of the schooling language, we've broadened some of the meanings. We've taken them back to their original meanings and regained some of their use.

Similarly, Mātanga 1 outlined how the elaboration process produced opportunities to resurrect terms that had previously fallen into disuse:

The whole idea of resorting to the native stock was to resurrect or revitalise those terms that had fallen out of use anyway in general Māori language or to resurrect them...

Mātanga 2 gave an example of the unique prevalence of passive structures in te reo Māori and how this could be utilised in pāngarau discourse, which in turn supports Māori language revitalisation:

Ko tētahi āhuatanga o te reo Māori. Ko te kaha kōrerohia o te reo hāngū. E tino hāngai ana te pāngarau ki te whakamahinga o te reo hāngū. Pērā i te tāpiria, te tangohia, hei tohutohu i te tamaiti, ā, kō tā tātou mahi he tāpiri i tēnei me tēnei... He mea nui ki a au kia mau i tā te Māori whakatakoto i aua momo renga. Kāua tātou e tāhuri ki tā te Pākehā whakatakoto i te kupu. Toru tāpiri whā. E whai noa iho, ā-kupu noa iho, i tā te reo Pākehā 'three plus four', 'toru tāpiri whā'. Pai ake ki a au kia mau ki tā te Māori whakatakoto. Tāpirihia te toru me te whā, ka whitu.

[One of the features of the Māori language is the frequent use of passive language constructions. In mathematics the use of passive constructions is particularly relevant. Like, ‘tāpiria,’ ‘tangohia’, to show the child, yes, what we are doing here is adding this to that... I think it’s very important to retain the Māori structures for those kinds of expressions. We should not turn to English sentence structures. Toru tāpiri whā (three plus four). That follows, word for word the English ‘three plus four’, ‘toru tāpiri whā’. It is much better in my opinion to retain the Māori structure. Tāpiria te toru me te whā, ka whitu (add three to four, to give seven).]

Ko te hāngūtanga tētahi āhuatanga o te reo ka tino whakamahia i roto i te pāngarau. Nō reira e mea ana ko ngā mahi pāngarau i roto i ngā kura hei whakapakari anō i te reo Māori o ngā ākonga. Te tūmanako tērā.

[The passive structure is a language feature that is extensively used in mathematics. Hence, I think mathematics are used in schools to strengthen students’ Māori. That’s the hope.]

Mātanga 3 agreed that pāngarau discourse was particularly useful in strengthening the use of passive structures:

Heoi anō, kei te hoki atu ahau ki te reo hāngū, he wāhi tino pai. He marau tino pai nē, hei whakapakari ... a kawa nei... Me whakaaro mō te hanga o te reo Māori ake.

[So, I’ll return to the passive voice, it is a good place. It is a good subject, isn’t it, to strengthen ... in a formal fashion... We should think about the structure of the Māori language itself.]

Ka kōrero i tēnei mea add two... Tino reo hāngū tērā. Nō reira tāpiritia te whā ki te toru, ā, ka whitu i reira. Nō reira me hāngū te reo ki te kore e hāngū kua hē te reo.

[We are speaking about “add two...” In fact it is in the passive voice. Hence, “tāpiritia te whā ki te toru, ā, ka whitu” (add four to three, to obtain seven). Therefore, passive voice should be used; if it is not, the language is wrong.]

Nō reira, mehemea ka kite i te mea ... te whitu te tohu tango me te rima ka rua nē? ...ko te whitu tangohia te rima, ka rua. Engari, ko te reo Māori kei te hiahia te kī... tangohia te rima i te whitu ka rua.

[Hence, if you see this ... a seven, the takeaway operator, and a five, you get two, right? ... seven, takeaway five, leaving two. But for the Māori language, we want to say... takeaway five from seven, leaving two (tangohia te rima i te whitu ka rua).]

Mātanga 2 asserted that te reo Māori has shown itself capable of communicating new topics. This proves the capacity of te reo Māori to communicate mathematical concepts, which was one of the aims of pāngarau curriculum development (Trinick, 2015), and disproves pejorative ideologies towards the Indigenous language (Harlow, 2005):

E taea ana e te reo Māori, tō tātou reo Māori, te whakaahua i aua āhuatanga hou.

[The Māori language can, our Māori language can, articulate those new features.]

The data presented here shows that there have been many positive outcomes for te reo Māori through pāngarau register elaboration. However, as Mātanga 2 warns, these benefits are predicated on maintaining the uniquely Māori essence expressed through the grammatical features of the language:

Engari, ko te mea nui kia mau tonu te hā o te reo, kia mau tonu ki tā te Māori whakatakoto i te reo. Nō reira e ora pai tonu ai te reo. Ahakoa te maha o ngā kupu hou.

[But, the most important thing is to maintain the essence of the language, to maintain the Māori way of laying down the language. In that way, the language was still strong. Despite the many new words.]

Overall, Mātanga 1 believed that te reo Māori possesses the linguistic flexibility to withstand the pāngarau register elaboration process without undue detrimental impact:

I think the language is flexible enough. We've been able to create a pāngarau language without making too many unnecessary, unwanted changes.

This section draws inferences between the benefits of pāngarau discourse for revitalising te reo Māori and the potential benefits for mātauranga. For example, Mātanga 1 gave examples of meanings of terms that had previously atrophied and were restored and how native terms that had fallen into disuse were resurrected. However, as Mātanga 2 argued, the benefits to te reo Māori of pāngarau register elaboration and classroom discourse are predicated on the unique grammatical features of the language remaining intact. As discussed earlier in this chapter and in Chapter 1, whakapapa is one of the defining features of mātauranga (see Nicholson, 2020; Royal, 1998). Therefore, illuminating mātauranga in pāngarau curriculum and classrooms would need to be done in a way that does not erode or unnecessarily change concepts and understandings of whakapapa.

Mātanga 1 argued that the linguistic flexibility of te reo Māori has allowed it to assimilate the new pāngarau terms without too much erosion of the unique grammatical features. Could the same be said for mātauranga? According to Smith and fellow researchers (2016), mātauranga naturally assimilates new concepts. Similarly, Mead (2013) and Cooper (2012) both asserted that mātauranga is boundless

and ever-evolving. These assertions suggest that, like te reo Māori, mātauranga possesses the conceptual flexibility to withstand its illumination in pāngarau curriculum and classroom practices. In the next section, the mātanga discuss the negative impacts of pāngarau register elaboration on te reo Māori. Again, inferences are drawn for mātauranga.

The Potential Negative Impacts of Pāngarau Register Elaboration

Mātanga 1 discussed the inevitability of change and the importance of intentionally being able to distinguish between a change that is desirable or undesirable:

I think it's a hybrid language. But I guess the question that we always must ask ourselves. First, change is inevitable. It's going to happen. So, the question then becomes ... Is the change desirable or not? Now, sometimes it is and sometimes it isn't.

Mātanga 1 discussed the meanings of some Māori words becoming narrowed through being incorporated into education registers as an example of undesirable change:

Some of the words like 'whakamārama' or 'mārama', we've kind of narrowed its meaning to the English meaning of understand or to explain where it's got quite a broad meaning. So, in some cases, we have narrowed its meaning to the English-medium equivalent...

Similarly, Mātanga 1 also cautioned that adapting the language to express school mathematics can result in language interference:

I've gotta say that [the Māori language] is changing syntactically to be more like English because that's how people think. So that implies that the Māori cultural lens is changing to be more like English. Because we have been forced... as new concepts have come into the language that didn't exist before.

Mātanga 1 gave the example of positive and negative numbers, which do not seem to have a corresponding concept and, therefore, vocabulary for Māori, and how this issue was addressed:

...we struggled for a long-time with the concept of positive and negative... Because if you try and translate the English word you run into a whole lot of linguistic problems. And the concept didn't exist in te reo Māori, so it took us a while and I think we are okay now with 'tōrunga' and 'tōraro' [respectively], even then that's a little bit problematic because it's not always up and down. They could be left or right... But it's problematic in English anyway.

Mātanga 3 also shared concerns about ongoing language change and, like Mātanga 1, emphasised the importance of making language choices based on which changes are desirable or undesirable.

Mātanga 3 pointed out that this is particularly pertinent in a context where the colonial language is taken-for-granted as the default language:

Te mea hoki i kite au, hei kaiako tātai pāngarau, ka haere ngā tau, ka pahure ngā tau, ka rerekē haere te reo.

[One thing I've seen as a mathematics teacher, through the years, as years pass, the language continuously changes.]

Āe, engari ko te mea nui... kei te mārama, kei te mōhio tātou ki tā tātou e whiriwhiri nei.

[Yes, but the important thing is... we understand, we know what it is we decided.]

Me tino tūpato tātou, i te mea, ko te hanga ... whakahokihoki, arā, te mea default o tēnei ao, ko te mea Pākehā.

[We need to be very careful, because the construction... that is replicated, that is the 'default' in this world, is the English.]

Mātanga 3 also drew on the metaphor of the Trojan Horse (Barton & Fairhall, 1995) discussed in Chapter 2 to illustrate the potential for language interference for English speakers and as a warning against complacency in language choice:

Hoki ana ki tērā hoiho toroihana... i ngā wā katoa ... Ka mahi te mahi engari kia kaha te tangata ki te āta noho me te whakaaro, kei te aha ahau i konei? Me pēhea te haere? He aha ano ngā mea hei whakatikatika ake? I ahu mai ērā tauira i hea? Tērā whakamārama i hea?

[Returning to that Trojan Horse... time after time ... It works, but a person needs to make the effort to sit down and carefully consider, what am I doing here? How should this go? What else needs fixing? Where are those precedents from? Where does that explanation come from?]

Mātanga 3 used a grammatical feature of the Māori language to further exemplify this point and cautioned against losing sight of Māori aspirations for schooling, such as the revitalisation of te reo Māori and mātauranga:

He mahi a te tangata. He mahi hoki o te tangata. Ko te tikanga o tērā, ka mahi te tangata i wāna mahi, anā, kare i te whakaaro.

[It is what people just do. It is also what people purposefully do. The meaning of that is, a person does their work, but without actually thinking about it.]

Anā, nō reira, mehemea kei te hiahia tātou kia Māori rawa ... te wāhi, te kawa, ngā tikanga...
o wā tātou tamariki, me whakaaro i ēnei mea i ngā wā katoa.

[So, therefore if we want for it to be truly Māori ... the place, the context, the practices... of
our children, we should always carefully consider these things.]

Using Language Purism to Identify Undesirable Changes to Te Reo Māori

In the previous sections, Mātanga 1, 2, and 3 cautioned that any changes made to reo Māori must be intentional to protect the uniquely Māori essence of the language and support language revitalisation. As discussed by Mātanga 1 below, Te Taura Whiri i te Reo Māori, with the support of the MoE, directed the pāngarau register elaborators to adopt the linguistic purism ideology to purge undesirable innovations from the pāngarau register (Trinick, 2015):

It was the Taura Whiri really who, particularly when curriculum development started, they wanted consistency across the resources. We wanted consistency for mathematical reasons. Taura Whiri wanted to eliminate undesirable language change. So, we all had three different motivations to adopt linguistic purism.

...there are language elements that are not so problematic in English and there are other things that are.

The Ministry of Education wasn't concerned about linguistic purism. They were concerned about creating resources with a standardised vocabulary. We were concerned about ensuring mathematical integrity etcetera wasn't lost. The Taura Whiri were concerned about unwanted linguistic change ... and [wanted] to resurrect obsolete words that had fallen out of use and to purge transliterations.

I'm fine with linguistic purism because it has... expanded the Māori vocabulary while it's resurrected a lot of terms that had fallen out of use. So, I think the positives far outweigh the negatives.

Similarly, Mātanga 2 further emphasised the uniqueness of the Māori language and that this uniqueness ought to be protected, despite the necessity of language elaboration for communicating new topics:

He motuhake te reo Māori. He me nui anō hoki, ki a au, otirā ki a tātou te ao kōrero Māori, kia mau tonu te hā o te reo, kia mau tonu te tūturutanga o te reo, te Māori o te reo Māori. Ahakoa ngā kaupapa hou o te ao hou e noho nei tātou, pēnei i te pāngarau, te hangarau, te aha, te aha.

[The Māori language is unique. It is an important thing, for me, and indeed for our Māori speaking world, to hold onto the essence (hā) of the language, to maintain the authenticity of the language, the Māoriness of the Māori language. Despite the new imperatives of this new world we live in, like mathematics, technology, and so on.]

Echoing the sentiment of Stewart (2007) about the implications of purging transliterations from Māori-medium curriculum registers, Mātanga 1 identified challenges that linguistic purism poses for pouako and ākonga, particularly at the secondary school level:

I am of two minds about it. I think that there are probably terms particularly in the upper secondary that you use so rarely that maybe would have been better for us to just transliterate. They are few you know they are words that are rarely used outside of even outside of that specific maths class activity on that day.

And so, the problem I think about linguistic purism is when... students are transitioning from say a wharekura to an English-medium university or polytech. How easy is the transition when you have got completely different language? You have been learning in a completely different language. So would it have been easier for some of those upper-secondary words to have been just transliterations so that the students would have known straight away.

Rather than assimilating naturally occurring innovations into subject register dictionaries as discussed by Mātanga 1 and 2, the language purism ideology spurred the purging of transliterations from the pāngarau register. Māori-medium curriculum developers have therefore created subject registers from strategies used by other Indigenous groups to elaborate their languages (Trinick & May, 2013), such as creating new compound terms or reviving native terms, which have had flow-on effects for pouako and ākonga. The new pāngarau register was elaborated extensively during the three-year development project for the first pāngarau curriculum document between 1992 and 1995 (McMurphy-Pilkington, 2004; Trinick, 2015). This had pedagogical implications. To communicate mathematically, students and teachers require a functional grasp of the mathematics register (Pimm, 1991). Pouako were therefore tasked with acquiring the new terminology and teaching it to their students quickly to maintain mathematical discourse in their classrooms.

Similarly, illuminating mātauranga in pāngarau curriculum and, most importantly, classrooms, will require pedagogical shifts. It is thus also likely to have a similar detrimental effect on pouako workloads in the short term. It is possible that this new learning may also detrimentally impact ākonga school mathematics learning for some time. The question is whether Māori-medium and kura are willing to make these sacrifices in the short term for the potential long-term gains of further indigenising pāngarau schooling. In the next section, mātanga discuss the challenges of pāngarau

register elaboration and curriculum development on pouako, particularly L2 learners, and the mitigation strategies employed to alleviate any negative impacts.

Teaching and Acquiring the Pāngarau Register Simultaneously

Mātanga 2 acknowledged the burden on pouako in acquiring multiple recently elaborated education registers, such as pāngarau and hangarau matihiko, but also asserted that this is a professional responsibility:

Nō reira, he mahi nui tā te kaiako. Hāunga te pāngarau, arā atu anō ngā kaupapa. Pērā i te hangarau, te hangarau matihiko. E hia kē nei ngā kupu hou e pā ana ki te hangarau. Kāre e taea te huri tuara ki ērā kupu. Mehemea kei te whakaakona te hangarau ki roto i o tātou kura reo Māori me ako ngā kupu.

[Hence, there is important work for teachers. Beyond mathematics, there are other areas. Such as technology, digital technology. There are so many new words related to technology. We cannot turn our backs on those words. If technology is taught in our Māori language schools the words need to be learnt.]

Mātanga 3 argued that acquiring new terminology is necessary for promoting discourse around new topics:

E tika ana hoki, kia maha ngā kupu hou. Nā te mea, he kaupapa hou e kōrero nei tātou.

[It is proper too, that there should be many new words. Because it is a new area we are talking about.]

Mātanga 4 discussed the challenges for those teachers who had been educated in English and are now teaching pāngarau in Māori:

I te mea, ko tōku reo Māori, ko tōku reo tuarua. He mea ako nō taku pakeketanga kātahi anō au ka whai i tōku reo. Nō reira, I taku kaingākautanga ki ngā kaupapa pāngarau i ahu mai i taku tupuranga reo Pākehā. ... nō reira te nuinga o aku akoranga. He mea whakawhiti i ērā akoranga i roto. I aku tau pakeke nei e kimi haere i ngā horopaki e Māori ai te titiro i ērā pūkenga pāngarau.

[Because my Māori language is my second language. Something I learned as an adult. Only then did I pursue my [Māori] language. My love of mathematics topics stems from my growing up with the English language... which was the source of most of my learning. These have been translated. As an adult I have sought to find contexts with a Māori viewpoint of those mathematics skills.]

Ahakoia i tīmata ahau i te kura tauwiwi, reo Pākehā nei. I reira kē te koingo te hiahia ki te whakaako i roto i ngā kura Māori hei āwhina i ngā tamariki Māori.

[Even though I started out in a non-Māori school, in the English language. I still had a yearning to teach in Māori schools, to help Māori children.]

Mātanga 4 further discussed the challenges of acquiring the pāngarau register as an L2 language learner, including the difficulty in distinguishing between the newly coined terms and native terms:

Mō ngā kupu hou... Ko te nuinga o ngā kupu i roto i aku tau, he kupu hou māku ko te kimi haere, i te mea kāre i tupu reo Māori mai. He tini pea ngā kupu kua mau kua ako i roto i ngā tau. Kāre au i te tino mōhio mehemea ko ērā kupu ka kīia he kupu nō neherā. He kupu tawhito nō te ao Māori...

[About the new words... The majority of the words I've come across over the years were new words to me. Searching [for words], because [I] did not grow up in the Māori language. There are countless words I've acquired or learned over the years. I don't really know whether those words can be said to be from ancient times (nō neherā). Old words from the Māori world...]

Mātanga 4 also identified the importance of having language models and mentors for pouako to successfully make the shift from learning school mathematics in English to teaching it in Māori. Mātanga 4 named kaumātua and mātauranga experts, such as Mātanga 3, who had supported this journey over the years:

Otirā nō te whakawhitiwhiti kōrero ki te momo ... i tupu Māori mai. Mai i te tamarikitanga, tae noa mai. He māramatanga anō, he māramatanga anō i tērā tupuranga.

[But indeed [they are] from discussions with the kind [of people] ... who grew up Māori. From childhood and since. It is a different understanding, a different understanding that generation has.]

He tino mātanga whakaako pāngarau mā te whakaaro Māori. Mā te mātauranga Māori. Mā te reo Māori ... kei a ia ērā mātauranga. [Mātanga 3] Ko ia taku taura tuatahi. Ko ia taku kaiarahi tuatahi. Ki te whakaako i te ao Māori.

[A true expert in teaching mathematics from a Māori perspective. From a Māori knowledge system. Through the medium of the Māori language ... they have that level of knowledge. [Mātanga 3] who was my first exemplar. They were my first guide. On how to teach in the Māori world.]

Mātanga 4 also highlighted the importance of dictionaries such as *Te Reo Pāngarau* (He Kupenga Hao i te Reo, 2004) in supporting teachers who are L2 learners:

Ngā kupu, he ātaahua. Ko ētahi o ngā kupu, tērā rauemi... te reo pāngarau pea te ingoa, he tino rauemi. He kaha nōku, kei te mārama au ki te ariā pāngarau reo Pākehā nei. Arā, kimi haere ana au i te whakamāoritanga o ērā whakaaro.

[The words are so beautiful. Some of those words, that resource... te reo pāngarau (the language of mathematics) is its name, is an important resource. Through my efforts. I understand the mathematical concepts in the English language. Then, I seek ways to translate those ideas.]

As discussed by Mātanga 2 earlier in this chapter, Mātanga 4 identified the systematic development of new terms as valuable support for L2 learners. Mātanga 4 used the example here of te tupu pū (exponential), and the clear links the term has to the underlying concept and similar terms:

Kāre au i tino mōhio ki te kupu exponential i te reo Māori ... Kāre e tohua te pūrua, te pūtoru, te pūwhā rānei. He tupu pū tērā. Ko te āhua o te kauwhata, he huringa whakarunga. Kei te tere te tupu.

[I didn't really know what the word for exponential should be in Māori ... pūrua (square), pūtoru (cube), pūwhā (power of four) aren't suitable. That was tupu pū. The graph curves upwards. It increases rapidly.]

Mātanga 4 also discussed the use of dictionary resources such as *Te Reo Pāngarau* (He Kupenga Hao i te Reo, 2004) for furthering reo Māori development beyond fluency:

He ātaahua ngā kupu i roto i tērā rauemi. Ko te painga, ehara i te whakamāori noa i te kupu, pēnei i tāu i kōrero mō te ūnahi ko te rere o ngā whetū, ko te kiri o te ika. Kua pērā, he nui ngā kupu o roto i te reo pāngarau kua tiki atu i te tino māramatanga o tērā kupu. Te ahunga mai o tērā whakaaro. He tino rauemi tērā. Kaha nōku te whakamahi i taua rauemi i aku tau tekau tuatahi pea i au e whakaako pāngarau ana mā te reo Māori. He nui ngā akoranga o roto mōku hei pakeke...

[The words in that resource are beautiful. The good thing is that they are not just translated, like your talk about the parabola (ūnahi) being the path of the stars, the scales [or skin] of fish. It is like that. There are many words in te reo pāngarau, that carry the powerful insight of that word. The origin of those ideas. It is a really great resource. I often used that resource, in my

first 10 years maybe, when I was teaching mathematics through the medium of the Māori language. There was much in there for me to learn as an adult...]

As highlighted by Mātanga 4, many pouako are learning te reo Māori as a second language as adults. Therefore, they are simultaneously teachers and learners of the language of instruction (Meaney et al., 2012). Acquiring the newly coined terms of the pāngarau register at speed would have been difficult enough for fluent speakers of te reo Māori. As discussed in previous sections, at wharekura level, these teachers are often still tasked with translating English-language resources into Māori for their students, further increasing the difficulty of teaching pāngarau.

The reality of being a pouako in a pāngarau classroom needs to be considered carefully regarding illuminating mātauranga in pāngarau curriculum. Significant professional development may be required to equip pouako with the necessary pedagogical content knowledge to deliver any mātauranga curriculum content. Resourcing also needs to be considered. For the pāngarau register, Mātanga 4 discussed the usefulness of the dictionary or reference resource of *Te Reo Pāngarau* (He Kupenga Hao i te Reo, 2004), and similar reference resources may need to be created for mātauranga concepts included in pāngarau curriculum. Reference material does exist for learning more about mātauranga, such as uruuru ao and uruuru whenua, as introduced by Mātanga 1 in the first section of this chapter. However, these are often dense academic texts, recorded in English by Europeans and often in a disparaging tone, and therefore not suitable for Māori-medium classroom use. However, adapting these texts for classroom use could be a strategy for supporting pouako in illuminating mātauranga in pāngarau classrooms. Given this possibility, a selection of these Māori and Pacific oral and visual narratives is reviewed and presented in Chapter 5.

The Use of Language Models to Support Language Acquisition

The development of resources that model the vocabulary and grammar required to facilitate pāngarau discourse in classrooms was a strategy used by curriculum and resource developers to alleviate the learning challenges for pouako (Trinick, 2015). Mātanga 1 gave an example of a professional development project, Te Poutama Tau, in which resources were designed to support teachers in acquiring the new terms and syntax of the pāngarau register (see Christensen, 2004b):

...it needed to be more than just exemplifying terms. That's not sufficient. It needed to exemplify syntax and ways of talking using... language functions... of describing, synthesising, evaluating ... mathematical language functions. More than just the puna kupu.

Mātanga 1 explained that these resources included scripted examples of classroom pedagogical language:

They are deliberately structured around what the teacher says.

So, they are unique in that respect ... they have the teacher dialogue...

And that's because ... we felt that providing a script, which is what these resources are, would be useful language models...

There is a view that providing the types of resources that are scripted are not desirable, but they are often not considered from a language perspective.

...to do the two things really to help exemplify the syntax, which can arguably come from the puna kupu ...language functions... and language decoding strategies... We deliberately have some rephrasing examples... say it one way and then say it is another way.

I think we also tried to exemplify... where the verbal markers, the 'ki', 'kua', 'e', 'ka'. How they differ.

Initially, these resources were provided bilingually to aid second language acquisition, but Mātanga 1 explained that there was opposition to this strategy, and eventually, they were only produced in Māori:

... in the first iteration of the Poutama Tau materials, one side of the page was in English and then one side of the page was in Māori. They were mirrors of each other.

Now at the time we felt that having the bilingual resource would be more useful for the teacher to make sense of the new structure of the framework, the new ideas ... but there was also a belief held by some that it was better to ... delete the English-language examples because teachers would default to them all the time ... and just have the te reo Māori ones.

The removal of the English versions seems to show the strength of the language separation ideology discussed in Chapters 1 and 2. This could also be a point of contention for mātauranga. Resources may be required to support the teaching of mātauranga such as uruuru ao and uruuru whenua and providing these bilingually may support language acquisition for pouako. This would also make the resources available for pouako in English-medium teaching contexts, improving access to mātauranga for ākonga Māori in English-medium schools. However, as discussed throughout this chapter, there is the potential for further undesirable change to both mātauranga and te reo Māori if bilingual resources are not created and debated by experts. Archibald (2008) also discussed the detrimental impact that implementation of Indigenous knowledge by non-Indigenous non-expert teachers could have on the learning experiences of Indigenous students. Therefore, regardless of the language of instruction, expert guidance is required when implementing Indigenous knowledge resources in classrooms (Galla & Goodwill, 2017).

Mātanga 4 further discussed the resources available today for teaching pāngarau and how important it is to have Māori language resources even if the way they are developed does not explicitly promote mātauranga:

He tini ngā rauemi i ēnei rā, reo Māori nei. He rawe hoki. Ahakoa pea kua tīmata ki te pūkenga pāngarau me te tiki atu i te horopaki. Ko te hua i te mutunga ko te rauemi ko ngā pukapuka kei te tipu hei pānuitanga [mā] a mātou tamaiti i ēnei rā. He Māori te whakaaro o roto. Ahakoa ko te wā i hangaia mai, he rerekē pea te raupapa... I roto i te puka kei reira te horopaki, ngā pūkenga pāngarau me te whakaaro Māori...

[There are a multitude of resources these days, in the Māori language. Which is great. Even though perhaps, they started from the mathematics skills and selecting a context. The outcome in the end is the resource. The number of books [for] our children to read is growing these days. Where the thoughts inside are Māori. But due to the era during which it was created, perhaps the order is different... Within the book, is the context, the mathematics skills and the Māori way of thinking...]

Mātanga 3 gave a reminder of language models provided in early pāngarau (mahi whika or tātai), such as *He Puka Whika Hei Ako Mā Ngā Tāngata* (Taratoa, 1858). Mātanga 3 believed that returning to these writings periodically could help limit and reverse unwanted language change:

Anā, ki ahau nei, me hokihoki tonu tātou ki ērā tuhinga tawhito. Ka pānui, ka pānui, ka pānui kia mutu. Kia tae atu ki tua atu o ngā kupu i runga i te whārangi kia noho ki roto i te manawa, i te hinengaro, i te ngākau. Anā, ki reira kōhuahua ai. Arā, kātahi anō ka puta. Hika! He pēnei kē te kōrero i tēnei. Me pēnei kē te aro atu ki tēnei. Me pēnei kē te whai.

[Indeed, in my opinion, we should continue to return to those old writings. Read and read, and read through to the end. Until we can get beyond just the words on the page. They should be embedded in the heart, mind and soul. Indeed to bear fruit therein. Then, to emerge. Goodness! It is expressed this way instead here. We should approach it this way instead. And follow it this way instead.]

Similarly, having texts written in Māori exemplifying the teaching of mātauranga in classrooms is necessary for the illumination of mātauranga in pāngarau curriculum. However, finding older texts written for this purpose may prove more difficult.

Mātanga 3 concluded that although there were more resources today because of pāngarau curriculum development and language elaboration, there were still too few opportunities to engage with the pāngarau register, either in schools or the community:

Engari paku nei ngā mea ... kei te mōhio koe ki tērā nē. Tērā pea ko [Mātanga 1] e kōrero ana i ngā mea o Te Poutama Tau i tērā wā. Kua kite ahau i tētehi mea paku nei o ētehi kaiako i roto i te akomanga pāngarau i te pouaka whakaata. Engari i te kōrero mō te toru tekau hēkona. Nō reira... he paku rawa ngā huarahi e kite atu ai i te reo ā-tinana i roto i ngā mahi tātaitai, pāngarau nē? Nō reira, ko te nuinga o ngā mea kua kite au, kua puta mai hei pepa, pukapuka. He rapanga i te taunga ipurangi o tērā, o tērā.

[But only a few things ... you know that don't you? Perhaps [Mātanga 1] is talking about things in Te Poutama Tau of those times. I've seen one small issue with some teachers in mathematics lessons on video [television]. But, just a quick discussion, only 30 seconds. Hence... there is only a tiny opportunity to see the body language in calculating, or mathematics isn't there? Hence, most of the things I've seen are things released as paper or books. Things found on internet sites, here and there.]

Extending the discussion of the accessibility of high-quality language models and resources, Mātanga 4 outlined the importance of digital resources for supporting pāngarau teaching and learning programmes. Mātanga 4 asserted that digitising existing resources will ensure the longevity and accessibility of the resource creation efforts that have been completed to date:

Koira te ao e heke mai nei. Āe. E rua [ngā] painga o te rauemi matihiko. Ko te mea tuatahi ko te āhei o te whakaako tawhiti ki te kāinga ... Otirā, ka noho he puna rauemi mō ake tonu atu.

[That is the world of the future. Yes. There are two benefits to digital resources. The first thing is the ability to teach remotely to homes ... Ultimately, there is an enduring wellspring of resources.]

Mātanga 4 reflected on the use of video conferencing to support Māori-medium schools that do not have dedicated pāngarau specialist teachers, which could also have utility for teaching mātauranga at distance. This technology allows language and pāngarau expertise to be shared across multiple kura (see Allen & Trinick, 2020). As the illumination of mātauranga in pāngarau curriculum and classrooms would add further required expertise to the role of pouako, the use of distance learning technology could help support nationwide implementation, exemplifying the need to consider resource longevity and not just creation. Mātanga 4 lamented the lost opportunity to create a catalogue of video resources and language models from the video conferencing project:

... i whakaako pēnei [video conferencing] ki ngā kura o te motu, mō ngā tau tata ki te 15 ngā tau. Engari kāore ahau i te hopu i tētahi o ēnā ... he moumou tērā. He rauemi pai mehemea i hopu i a mātou akoranga. ... Arā he rauemi mō te rangatahi e tupu ake nei e hiahia nei te whakaako pāngarau reo Māori. Arā he rauemi mō... he raumei hei āwhina.

[... [I] taught this way [video conferencing] to schools throughout the country, for almost 15 years. But I did not record any of those ... that's a waste. It would have been a good resource if our teaching had been recorded. ... That is a resource for the young people growing up who wish to teach mathematics in Māori. That is a resource for him/her... a helpful resource.]

Mātanga 4 likened the video conferencing experience to the remote teaching necessitated by school closures during the COVID-19 pandemic and the immediate importance of creating digital resources and language models to support the Māori immersion schooling sector. However, in this instance, Mātanga 4 argued that the importance of capturing digital resources and creating a catalogue for future use had been recognised:

Koinā tētahi o aku kitenga i ēnei wiki e ono i te noho taratahi. I te whakaako au i tēnei momo ataata, ka kite au i te painga mehemea i hopukina aku akoranga katoa, he rauemi. He raumei ēnei. Kia whakaarohia ngā momo pūkenga, tāngata o te ao. Ahakoa pūtaiao, pāngarau, reo rangatira pea. Kei mate atu ēnei tāngata ēnei pūkenga kāore e mau ki ētahi o ō rātou akoranga pēnei, ataata nei, matihiko nei.

[That was one of my observations in these six weeks of self-isolation. I taught using this kind of video. I saw the benefits if I had recorded all of my lessons, resources. They need to address the kinds of skills, people of the world. Whether it be science, mathematics, or even te reo rangatira. Lest these people die and the skills are lost. These kinds of lessons have not been recorded on video or digitally.]

Mātanga 3 also discussed the importance of video resources to model non-verbal communication alongside verbal communication:

...pēhea rā te kite ai, i tēnei mea te reo ā-tinana? Te reo whakahaere, te reo tohutohu i waho atu o ngā rauemi kei te tukuna mai. Anā, me kite i te tangata, me rongo i te tangata. Nō reira, ko au hoki tētehi e whakaaro ana, pai kia whakarite kiriata o ngā tangata e pakari rawa ana te reo, e pakari ana hoki te mea... ngā tikanga tātaitai aha nei i roto i te pāngarau... Ehara i te mea me ki me, "Oh me pērā te mahi". Kāo... he mea hei tīmatanga. Hei wetewete, hei tātaritari i ngā hanga, i te reo... āe. Koinā hei tīmatanga.

[...how do we get to see, things like body language? Management language, instructional language, beyond those resources that are being released. That's right, we should see and hear the person. Hence, I am one of those people who thinks, it is a good idea to prepare videos of those who have robust language. And are also strong in... computation and other techniques of mathematics... It is not that we are saying, "Oh you should do it like this". No... it is a

starting point. To unpack and analyse the structures, the terminology... yes. That would be a start.]

As outlined in Chapter 2, there are uniquely Māori intonation patterns for posing questions that have suffered language interference from English. Television presenters who deliver content in Māori are tasked with modelling uniquely Māori intonation patterns (M. Ngaropo, personal communication, September 9, 2018), thereby helping reverse English-language interference. Correct intonation is particularly important for teachers who use questioning as a pedagogical device. Video and audio recordings, as discussed by Mātanga 3, are particularly important for modelling the unique non-verbal aspects of te reo Māori, which contribute to its Māori essence. Similarly, Archibald (2008) and Galla and Goodwill (2017) identified the importance of video resources for capturing and disseminating Indigenous knowledge-creation practices such as narrative practices, as text-based resources may not fully capture the nuances of primarily oral practices.

Similarly, Mātanga 3 lamented the prevalence of paper-based texts and resources, as these are not always referred to in the classroom but rather left on shelves. Mātanga 3 also identified a lack of understanding about how the language contained in the resources was utilised in classrooms, which is explored further in Chapter 6:

Ko te mate kē o ngā mahi pāngarau ahakoa te aha ... I te nuinga o te wā kei noho i runga i te pepa i roto i te pukapuka, anā, tērā pea he mate hoki tō te reo Pākehā. Anā, ka noho kōhatu rā ngā kōrero. Mehemea kei te tiaki tērā i te hanga o te reo i roto i te akomanga, kāre i te tino mōhio ki tērā.

[The unavoidable problem with mathematics ... Most of the time, that it be based on paper, or in books; this seems to be a problem in English as well. So, the content is fossilised. Whether that will ensure the development of the [Māori] language in the classroom, I don't know for sure.]

Mātanga 1 also addressed the need for online resources such as *Paekupu* (He Kupenga Hao i te Reo, n.d.), which are now available to support pāngarau teachers:

Now as well there is the dictionary, the papakupu, that has ... examples of communicating. Now the other thing that is good about them is when you click on them in the electronic form, in the Paekupu, it says it orally...

Mātanga 2 discusses the intentions of the *Paekupu* (He Kupenga Hao i te Reo, n.d.) web dictionary design. According to Mātanga 2, encouraging awareness of the terms and their origins and, in turn, encouraging depth of understanding of curriculum area concepts are two intentions of the digital

resource. Mātanga 2 also argued that easy and quick access to new terms supported teachers in acquiring curriculum registers such as pāngarau:

...ko tā Paekupu he āwhina. He āwhina i te tangata kia āta whakaaro ki ngā kupu, ki te tikanga o ngā kupu me te whakamahinga o te kupu.

[...the role of Paekupu is to help. To help people to think carefully about the words, the meaning of the word and the use of the word.]

Āe, koira te whāinga nui o Paekupu, ā, kia māmā ake ai te tiki atu a ngā pouako i ngā kupu e hiahia ana rātou. Kia ruku ai i te rētōtanga, i te hōhonutanga o ngā wāhanga ako o te marautanga.

[Yes, that is the main purpose of Paekupu, and to make it easy for teachers to access the words they need. To plunge into the depths, the deeper knowledge of the curriculum subject areas.]

He āwhina i te kaiako kia tere tiki atu i te kupu e rapu ana ia. Hei āwhina i a ia ki te whakaako i ērā wāhanga ako o te marautanga.

[To assist teachers to quickly access the words they are looking for. To assist them in teaching those curriculum subjects.]

A further design element of *Paekupu* (He Kupenga Hao i te Reo, n.d.) discussed by Mātanga 2 includes multiple uses of the same term as different parts of speech to support classroom discourse:

Koira anō tētahi āhuatanga o Paekupu, he whakatauirā i te whakamahinga o te kupu i roto i te rerenga kōrero kia āta kite ai mēnā he tūmahī, he tūingoa, he tūāhua. He aha rānei te momo kupu. Me pēhea rānei te whakamārama i roto i te rerenga kōrero. Tōnā hāngūtanga. Ērā atu āhuatanga. He āwhina, engari, kei te tangata tonu, i te mutunga o te rā, kei te tangata tonu te mana ki āna kupu.

[That is another aspect of Paekupu, it shows examples of how the word is used in sentences, so it is clear to see whether a word is a verb, a noun or an adjective. Or what kind of word it is. And how it should be explained in a sentence. Its passive voice form. And other aspects. It is a help, but it is up to the individual, at the end of the day, the individual has the authority decide their own words.]

Mātanga 3 also pointed out that the English language dominates digital and web-based resources, and this often directs the development of Māori resources:

te nuinga o ngā mea... kua tuhia i roto i te ipurangi hoki, te nuinga kei te kite ko te reo Pākehā kei te taraiwa tonu i te pahi... Kia kua tāua e ohore i tērā, i te mea, koinā te reo kei te tino kōrerohia. koinā hoki te huarahi i tae mai ai tēnei mea te pāngarau te tātai i tāwāhi...

[for most of these things... that are written on the internet as well, for most of them you can see the English language is still driving the bus... Let us not be shocked by that, because that is the language that is most widely spoken. That is also the avenue by which mathematics and calculation arrived from overseas...]

In this section, all four mātanga discussed the importance of digital resources for supporting classroom practices. Each gave examples of both digital and paper-based resources and the design elements that they believed made these resources useful for pouako and ākonga, particularly L2 learners. As the pāngarau curriculum has been implemented in Māori-medium schools and kura for nearly three decades, the still relative lack of high-quality digital resources is disheartening. This was noticed acutely by Mātanga 4 during the COVID-19 pandemic when many schools were closed for extended periods, and ākonga were expected to learn in isolation (see Allen & Trinick, 2021). A significant investment in digital resourcing will be required to illuminate mātauranga, and design features such as longevity, ease of access, and opportunities for language modelling need to be considered.

Balancing School Mathematics and Māori Aspirations for Schooling

The data presented in this section addressed the school mathematics foci of the cultural symmetry framework. Tensions, challenges, and opportunities created by the often-competing aspirations of living as Māori and accessing Western notions of academic success are examined within this framework foci. Mātanga 2 discussed the interwoven nature of the pāngarau register, pāngarau curriculum content, and the Māori language revitalisation goals of Māori-medium schooling:

Hei whakatairanga i te reo, hei whakatairanga anō i te mātauranga pāngarau. Ka haere tahi. Ki a au nei, ka haere tahi ērā mea e rua i roto i ngā akoranga pāngarau i roto i ngā kura, ahakoa, he akoranga pāngarau, ki a au nei, he akoranga reo anō hoki.

[To promote the [Māori] language, but also to promote mathematical knowledge. They go together. In my opinion these two things go together in mathematics lessons in schools, even if it is a mathematics lesson, in my opinion it is a language lesson as well.]

Reflecting the concept of ontological pluralism (Hauser et al., 2009), Mātanga 2 also asserted that pāngarau is a valuable tool for Māori to thrive in the wider world, both now and in the future, if pursued alongside te reo Māori and Māori identity:

Nā, e whakapono ana ahau ko te pāngarau tētahi o aua peka mātauranga, e whai oranga ai a tātou tamariki mokopuna i roto i ngā mahi nui o te ao hou, o te ao e noho nei rātou, me te ao onamata. Ehara i te mea, mehemea ka eke taumata te ākongā i roto i te pāngarau, ka pareā ki rāhaki tōna ao Māori. E kāo. E taea ana ngā mea e rua. E eke taumata ia i tōna ao Māori, i tōna reo Māori, i tōna ahurea Māori. Kia eke taumata anō ia i te mātauranga pāngarau. Ehara i te mea he wehewehe ngā mea e rua.

[Now then, I believe that mathematics is one of the major branches of knowledge, that can provide well-being for our children and our grandchildren involved in the great works of the modern world. Of the world they live in, and the ancient world. It is not as if, if the students were to achieve great things in mathematics that their Māori world would be pushed aside. No. Both things can be accomplished. That they can make outstanding achievements in the Māori world, in the Māori language and Māori culture. And also achieve outstanding levels of mathematical knowledge. It is not as if those two things are mutually exclusive.]

Echoing discussions around the whakapapa of ideas explored in previous sections, Mātanga 2 identified that mathematical knowledge could be organised based on the concepts' pānga (relationships) to each other. Mātanga 2 asserted that this is a synergy between Māori ways of knowing and school mathematics, supporting the simultaneous acquisition of both:

Ko tā te pāngarau he āta ruku, he āta kimi i ngā hononga o tētahi āhuatanga ki tētahi. Pērā i te hononga o tētahi tau ki tētahi tau. He rearua pea, he haurua, he aha ake atu rānei. Te hononga o tētahi āhua ki tētahi āhua. Te hononga o tētahi raraunga ki tētahi raraunga. Ērā atu āhuatanga. Nō reira, ko tērā tētahi āhuatanga ki a au nei he āhua ōrite te ao Māori, te ao pāngarau.

[The role of mathematics is to dive into, to carefully seek the relationships between one property and another. Like the relationship between one number and another. Perhaps a double, a half, or some other relationship. The connection between one shape and another. The connections between two data items. And other properties. Hence, that is one aspect, in my opinion, there is a similarity between the Māori world and the world of mathematics.]

Nō reira, he hononga tērā, ā-hinengaro nei o te ao Māori me ngā mahi pāngarau.

[Hence, there is a link, an intellectual link between the Māori world and mathematical practice.]

Similarly, Mātanga 3 gave an example of one of the earliest pāngarau (matematika) textbooks by Taratoa (1858) and how this illuminates an intersection between Māori ways of communicating and mathematics:

I ētahi wā ka hoki ahau ki ngā tuhinga a Henare Taratoa, te pukapuka tuatahi i tuhia eia, anā, ko tāku i kite ai i roto i tana pukapuka, i tana hiahia kia whai tonu i ngā tikanga o te tātai Pākehā. Ka puta ngā kupu pērā me te matematika, ērā momo kupu. Engari, ko wana kupu whakamārama i te taha, tino Māori nei te hanga. Anā, ki ahau nei me hoki anō ngā tāngata ki te tiroiro ki tērā pukapuka me te āta whiriwhiri he aha anō te wāhanga i ahu mai i te ao Māori i ngā whaakaro Māori me te whakahāngai i ērā ki tana e whiriwhiri ai kia hāngai tonu ai ki te tātai pāngarau tuku iho me kī, ki te ao Pākehā, Arapi, Īnia, he aha rānei.

[Sometimes I return to the writings of Henare Taratoa, the first book he wrote, well, the impression I got from the book of his desire to pursue the practices of Pākehā mathematical theory. Words like matematika appear, those kinds of words. But his accompanying explanations, are very Māori in their structure. Indeed, in my opinion people should review that book, and carefully consider which parts are derived from the Māori world, from Māori thought, and to align those parts with their own needs. So, to speak, to align the traditional mathematical calculations with the Pākehā tradition, the Arabic, Indian and others.]

However, Mātanga 3 highlighted the way knowledge is sequenced linearly in school mathematics and, therefore, taught in schools. He contrasted this to mātauranga, which requires comprehension of the whole before examining each of the parts:

I roto i te reo Māori, anei te pikitia nui, anā kātahi anō ka tiro ki ngā tini wāhi. I roto i te reo Pākehā, ka tīmata ki ngā ititi, ngā pitopito kōrero, hei hanga i te mea nui.

[In the Māori language, this is the big picture, and then to look into the many different sections. In the English language, you start with the smallest snippets, to build the larger whole.]

Ko te mate kē pea i roto i te kura, ko tēnei mea te hīkoi, hīkoikoi nei. Rua, tahi, tēnei, kātahi ko tēnei, kātahi ko tēnei. Ka ngaro tērā hanga o te titiro whānui. Anā, engari, ko te pai o te reo Māori me pērā. Ka kōrero i te pikitia nui kātahi anō, ka kōrero i ngā wāhanga ititi, rikiriki i roto i tērā.

[The problem, perhaps, in schools is the steps, the progressions. Two, one, this then that, and then this. So you lose the ability to see the big picture. But then, perhaps the strength of the

Māori language, you need to [see the big picture]. You describe the big picture, then you talk about the smaller components within that.]

Mātanga 4 also contrasted a Māori worldview and that of school mathematics. Mātanga 4 argued that the Māori worldview is holistic, where tools such as pāngarau can be useful, while Western science sees individual disciplines such as mathematics as worthy theoretical pursuits with practical applications being secondary:

Ko tērā pea te rerekētanga o te pāngarau [o te] ao Māori me te pāngarau o te ao tauwiwi...

[That is perhaps the difference between mathematics in the Māori world and mathematics of the non-Māori world...]

Ko te tirohanga Māori, anei taku ao, taku horopaki, he aha te pānga o tēnei ao... me tiki atu au i te pāngarau hei āwhina i tōku ao. Koinā pea te tino rerekētanga o te tirohanga Māori i tō te tirohanga tauwiwi. Tauwiwi tīmata ki te pūkenga kimi i te horopaki. Māori kei a tātou te horopaki tiki atu te pūkenga. Ko te ao Māori tērā.

[The Māori viewpoint, this is my world, my context, what is the influence of this world... I need to use mathematics to assist my world. That is perhaps the biggest difference between the Māori viewpoint and that of non-Māori. Non-Māori: begin with the skill, and then find a context. Māori: we have a context, seek the skills. That is the Māori world.]

Mātanga 3 provided an insight into the multifaceted nature of Māori concepts of quantification that are not always explicitly present in school mathematics curriculum:

Kei te kōrero mō te marau. Te marau-ā-kura. Nō reira i roto rā ētehi mea mō te tātai hoki. Inā, hoki ki tēnei mea te tatau... he maha ngā mea tatau i roto i te ao Māori... memeha kei te whakaraupapa tangata, ka kōrero mō tuatahi, tuarua, tuatoru, tuaiwa, ana mehemea kei te whakaora i te a kupu tuangahuru ka pai, engari, ki te kore, ko te tekau. Ko te kotahi tekau, arā, ērā momo tikanga o te reo. Arā kei te āhua pērā mēnā kei te kōrero mō te maha o ngā tāngata nē. Kotahi i te nuinga o te wā, anā, tokorua, tokotoru, tokoiwa...kotahi tekau.

[I'm talking about the curriculum. The school curriculum. So there are some things in there for calculations. If we return to counting... there are many examples of counting in the Māori world... if you are lining people up, you will be talking about: first, second, third, ninth. And if you want to revive the word tuangahuru (tenth – archaic form), that's good. But, if not, it is tekau (tenth). Or kotahi tekau (literally one ten), that is, those kinds of language structures. That is more-or-less how it is when you are talking about numbers of people, isn't it? One

(kotahi), most of the time, then, two, three, nine... ten (tokorua, tokotoru, tokoiwa...kotahi tekau).]

If the words and phrases presented by Mātanga 3 are not explicitly present in the curriculum and associated resources, they may not be used in classrooms as they are not direct translations of English-language curriculum content. Therefore, it is important for curriculum developers to consider not just enabling mathematical communication in te reo Māori via translation of English-language documents but also what ways of communicating or mātauranga are missing from translated documents. Mātanga 3 gave an example of the purposeful use of the term tuangahuru as an example of a reo Māori term for quantification that could be omitted from translated documents, such as the pāngarau curriculum and resources. Mātanga 3 gave further examples of how quantification can be expressed in uniquely Māori ways. The following example draws on the language of describing groupings of people in relationship to the speaker.

Te hanga hoki i roto i te reo. I kōrero tēnei mō... ko rātou, ko rāua, ko tā māua mahi, ko Pita nē? Ērā momo hanga o te reo e noho ai tētehi... ngā whakamāramatanga ki te muri, ki te taha. Ērā momo āhuatanga o te reo. He mea tātai.

[And the constructions used in the language. This is talking about... ‘rātou’ (those plural), ‘rāua’ (those two), our (Peter’s and my) work, isn’t that so? Those kinds of language constructions, where the... explanations lie behind, or alongside the language used. Those kinds of aspects of the language. They are [also] aspects of arithmetic.]

Mātanga 3 provided further examples of how counting terms can differ depending on context such as the use of the verbal particle ‘ka’ used to indicate cardinality while counting (Meaney et al., 2012). As explored further in Chapter 6, the particle ‘e’ seems to have become the commonly used term for indicating cardinality, potentially erasing this specific use of ‘ka’ over time:

Nō reira, mehemea kei te tataua haere. Kare i te tataua i tētehi mea. Kei te takitaki haere i nga tau, ki ahau pai noa, tahi, rua, toru, whā, rima, ono, whitu, waru, tekau. Engari, mehemea kei te kōrero e hia ētehi mea. Me tataua nei, ka tahi, ka rua, ka toru, ka whā, anā, oh, kua tahi tekau ngā āporo [he tauira tino Pākehā nei] kua tino mahi...

[So, if you are counting. Not counting individual items, but reciting the sequence of numbers, in my opinion it is fine to go: tahi, rua, toru, whā, rima, ono, whitu, waru, tekau (one, two, three, four, five, six, seven, eight, ten). But, if you are saying how many there are of some items, You should count this way, ka tahi, ka rua, ka toru, ka whā, anā, oh, there is one ten of apples [a real Pākehā example] that is frequently used...]

If students know and use the different ways of counting and indicating cardinality in te reo Māori it could be concluded that they are sufficiently illuminated in pāngarau curriculum, classroom resources and classroom practices, and therefore, they will be perpetuated. How ākongā use the range of quantification terms in te reo Māori is explored in the classroom data presented in Chapter 6.

Mātanga 3 was particularly interested in the way quantification information was coded in narratives, pointing out that this quantification information is not always immediately apparent. Mātanga 3 believed it is important to examine narratives that are not concerned with quantification explicitly. To Mātanga 3, the characteristics and features of the narrative are just as important:

Nō reira, me titiro tātou ki te reo ake... Ehara i te mea me rapu noa i nga mea tātai branded, waitohungia ki te mea tātai ... Whaitiri..., tatau i ngā kūmara, i ngā taro. Āe, tua atu o tērā, titiro ki te hanga o te kōrero, te takitaki haere, i tētehi kōrero.

[Therefore, we need to look at the actual language... It is not as if we need to seek out those things ‘branded’ tātai (Māori mathematics) carrying the tātai logo ... [For example the ancestress:] Whaitiri... counting kūmara (sweet potato), or taro (root vegetable). Yes, beyond that, look at the kinds of speech, or recitation in a narrative.]

In this section, mātanga provided examples of synergies and differences between the nature of mātauranga and school mathematics. Mātanga 2 likened the many interconnected mathematical concepts with Māori concepts of relationships among phenomena, echoing previous statements about whakapapa. However, Mātanga 3 described how mathematics is taught in schools as a ‘piece-by-piece’ approach to disseminating knowledge, rather than an interconnected whole. Mātanga 4 also described how school mathematics reflects the disciplinary nature of Western science and conflicts with a more holistic Māori worldview. Finally, Mātanga 3 provided examples of Māori ways of quantifying and counting that are not explicit in school mathematics. These examples could provide opportunities to explicitly highlight these synergies and differences between knowledge systems, reflecting ontological pluralism (Miller et al., 2008). Further exploring the benefits of each approach for solving contemporary problems would also align with epistemological pluralism (Andreotti et al., 2011). In this way, positioning mātauranga and school mathematics as distinct and complementary through incorporating ontological and epistemological pluralism in pāngarau curriculum and classrooms provides an opportunity to further indigenise both.

The Benefits of Learning School Mathematics in Māori

In this section, mātanga highlight examples of the benefits of learning school mathematics in the Māori language, providing a unique lens for examining mathematics concepts. In this first example,

Mātanga 1 discussed how some of the features of the Māori language directly support the acquisition of school mathematics conventions and concepts:

There's a recursive thing that goes on. Mathematics kind of pushes you in the direction of precision... You're always generalising in maths, but the generalising is not the same as generalising in everyday language... Because it requires a degree of precision, it also makes you be more precise with your Māori language. Which means that you then gotta have a good understanding of the manner particles.

Manner [particles], you know like the 'tinos' the the 'rawes' all those words that give some qualification to words.

I think one of the areas is the huge array of logical connectors that the Māori language has. All the 'ki's, 'i's, 'nā's, 'nō's, 'mā's. ...makes it, not easy but easier than it might have been if we did not have all that variety of logical connectors... they are the things that connect one idea to the other idea and they are useful in that respect... they're all the prepositions, all the conjunctions, all those sorts of things. So, they're a good example of a resource in the Māori language that facilitates mathematical communication.

Mātanga 1 then discussed the importance of negation as a mathematics convention for describing attributes, further illustrating distinctions between te reo Māori and English:

...The way Māori language negates differs according to what is being negated... If you are negating something that is being counted it's an 'e' and a 'kāore'. If you are negating something that is being described, it's an attribute, it's an 'ehara'.

Mātanga 3 exemplified the advantages of acquiring school mathematics concepts through te reo Māori including the holistic approach to problem solving this provides:

Ahakoā kāre i te tino rite ki tā te Pākehā: seven takeaway 5 nē... te kōrero, ā, tangohia te rima i te whitu, ka rua. Ka rite te tangohanga o te rima i te whitu... Ina pakari te hinengaro ki tērā momo kōrero, ka tahi, ka tino pakari te reo, ka rua, ka titiro whānui ki te mahi tātai kei mua i a koe.... Ehara i te mahi, te kōrero a kawa nei.... Kei te titiro ki te katoa o te mahi i te wā ōrite.

[Even though it is not exactly the same as in the English seven takeaway five, to say "tangohia te rima i te whitu, ka rua" (take 5 away from 7, to get 2). It is still taking five away from seven... If these kinds of ideas are properly instilled in the mind, then they will be instilled in the language, and secondly will give breadth to the understanding of working with

numbers... The task is not just the formal language... It is to see the whole of the task at once.]

Mātanga 2 gave the example of comparative terms within the Māori language, which provide a better understanding of school mathematics concepts:

Tētahi anō momo reo i roto i te pāngarau e kaha kitea ana ko te whakatairite. Ko te whakatairite i tētahi tau ki tētahi tau. Ko te whakatairite i tētahi koki ki tētahi koki. Nō reira, kei reira anō taua momo reo. Te reo whakatairite. He nui ake. Ko tēhea te mea nui rawa ake? He nui ake tēnei i tērā. He iti iho tērā i tērā. He rerekē tērā i tērā. Ko tērā te reo whakatairite.

[One other kind of mathematics language that is often seen is comparisons. Comparing one number to another. Comparing one angle to another. So, that kind of language is there as well. Comparison language. Greater than (he nui ake). Which is the largest one (ko tēhea te mea nui rawa ake)? This is larger than that (He nui ake tēnei i tērā). That one is smaller than that one (he iti iho tērā i tērā). That is different from that (he rerekē tērā i tērā). That is the language of comparisons.]

Mātanga 2 discussed the unique advantages for students in acquiring mathematical concepts through the recently elaborated pāngarau register. For example, the process used to create new compound terms such as 'haurua' (hau-rua) ensured that the meaning of two parts was implied. In contrast, the English term 'half' does not directly provide this cue to students. Mātanga 2 also cited a similar example for naming two-dimensional shapes such as a pentagon or taparima (five edges):

I ētahi wā, he māramatanga kei roto i ngā kupu hou ki te tikanga o te kupu. Ko te hau tētahi wāhanga o tētahi mea. Haurua e rua ngā wāhanga. Nō reira kei roto i te kupu tētahi māramatanga tētahi tīwhiri ki te tikanga o te kupu. Arā kē ngā momo, ngā kupu mō ngā momo āhua, pērā i te taparima, te tapaono...

[Sometimes, there are hints within the word as to the meaning of the word. The 'hau' is a part of something. Haurua, means there are two (rua) parts. Hence, there is insight, a hint within the word as to the meaning of the word. There are other examples, the words for different kinds of shapes, like the pentagon (taparima) and hexagon (tapaono)...]

Mātanga 2 gave another example of how the systematic process of elaborating the pāngarau register directly supports the acquisition of pāngarau concepts, citing the term tauwehe (factor):

Mehemea tātou ka whakaaro ki te kupu... i te reo Pākehā ko te 'factor'. Kāre au e kite ana i tētahi tikanga i roto i taua kupu Pākehā hei āwhina i a au kia mārama ki te tikanga o tēnei mea

te ‘factor’. Engari ko te reo Māori, ko te tauwehe. Kei konā tētahi tūwhiri nui ki te tikanga o tēnā kupu te tauwehe te ‘factor’ rānei. Anā, ko te tau ka taea e koe te whakawehe ki roto i tētahi kupu. Ko ngā tauwehe o te ono, ko te tahi, te rua, te toru me te ono. Nā te mea, ka taea e ērā kupu te whakawehe ki roto i te ono.

[If we think about the word... the English word ‘factor’. I can’t see any meaning within the English word to help me understand the meaning of a ‘factor’. But for the Māori word ‘tauwehe’ (factor). There is a big hint to the meaning of the word ‘tauwehe’ as compared to the ‘factor’. That is, one of the numbers that you are able to divide another [number] by. The factors of six are: one, two, three and six. Because those numbers can be used to divide the number six.]

Mātanga 2 also gave a probability example:

Ko te reo anō o te tūponotanga. Kei reira ētahi kīanga Māori. Hei whakaatu i ngā momo tūponotanga; ākene pea, kore rawa, kāhore e kore. Ērā tū rerenga ā te Māori ake. Ka whakahāngaitia ki te kaupapa pāngarau, te tūponotanga.

[The language of probability is a whole language again. For which there are Māori idioms. For describing the kinds of probability: ākene pea (probably), kore rawa (never), kāhore e kore (in all likelihood). Those kinds of very Māori sentences can be applied to the mathematical topic of probability.]

Ko tā te pāngarau he hoatu i tētahi hautau hei whakaatu hautanga. Anā, mēnā ko te ira rima te [hautanga]... te tūponotanga he ōrite te tūponotanga ka eke tētahi āhuatanga ki te tūponotanga kāre e eke taua āhuatanga. Kei waenganui, ko te kore ira rima tērā. Ko te haurua. Ki te taha o tērā ko ngā rerenga kōrero pēnei te: ‘ākene pea’, ‘kāre pea e puta’. Nō reira, āe, kei reira ngā āhuatanga pāngarau. Engari, kei reira hoki ngā āhuatanga o tō tātou reo ka haere i te taha.

[A role of mathematics is to provide fractions as a representation for proportions. For example, if the probability is point five. Then the probability that one outcome will occur is the same as the probability it will not occur. It is in the middle, it is zero point five. It is half. Alongside that are phrases like: ‘ākene pea’ (probably), ‘kāre pea e puta’ (may not occur). So, yes, there are features of mathematics there. But there are also features of our language that go together with that.]

In this section, Mātanga identified the distinct advantages for pouako and ākonga in examining school mathematics concepts through the lens of te reo Māori, particularly regarding the recently elaborated

terms of the pāngarau register. As discussed in previous sections, Te reo Māori provides a lens and way of communicating mathematically that is predicated on relationships between concepts. As discussed by Mātanga 2, the terms of the newly elaborated pāngarau register have been designed in a way that makes their relationality and meanings transparent. Ensuring these features of te reo Māori and the pāngarau register are explicitly addressed in pāngarau curriculum and classrooms provides further opportunities to examine the similarities and differences between mātauranga and school mathematics, and the benefits and challenges of each. In the next section, the mātanga discuss the challenging aspects of teaching school mathematics in te reo Māori.

The Challenges of Teaching School Mathematics in Māori

One of the challenges identified by Mātanga 1 when teaching school mathematics in Māori was the need for expertise in both pedagogical content knowledge and pedagogical language knowledge:

The problem is that a lot of teachers still struggle with the content, not just the mathematical content but also the pedagogical content and... the fact that schooling is pretty much the only language domain and maybe outside of the media where te reo Māori is reasonably strong. So, it's one of the other issues.

What you'll find is that one or the other problem presents itself to the speakers. So, a speaker might be reasonably fluent but doesn't understand the concepts. So, you've got two problems... and will they ever be overcome? Maybe over time as the generation of kids comes through who are educated in Māori-medium that have got good understanding of the concept.

Mātanga 1 also discussed the interdependencies of language acquisition and the acquisition of mathematical concepts:

There is a correlation between your ability to articulate, communicate mathematically, and your understanding of the content. They are not binary... They are interdependent. You really can't have one without the other. So, you've gotta have good language skills to articulate maths ideas. You've gotta have good maths knowledge to articulate. You need both.

Mātanga 1 further discussed the design of Te Poutama Tau professional development project resources (see Christensen, 2004a) and how these are designed to alleviate the issues outlined above:

... [It has] a section on what the teacher needs to know, and they have a complete section on how the teacher can articulate the content in te reo Māori.

Mātanga 1 also outlined the impacts of the pāngarau register elaboration alongside the use of oral assessments in pāngarau classrooms at the primary level:

So, all of the assessments are oral where they encourage children to communicate. But one of the problems of course is that the ability to orally illustrate sometimes lags what is in your thinking. So, you know what the answer is but you can't articulate it.

... teachers were judging kids, making judgements... about kids and their linguistic ability to articulate maths ideas rather than the ability or the understanding of the content.

The oral assessment, Te Uiui Aromatawai, used in this current study, was originally designed to alleviate the literacy burden of written assessments for English-medium students. However, this increased the difficulty of the assessment for L2 learners when answers were restricted to oral responses alone (Allen, 2015). This is an example of false assumptions about the transferability of English-language mathematics education research to Māori-medium contexts (Allen, 2015). In this study, the use of Te Uiui Aromatawai was informed by Māori-medium specific research. Ākonga used multiple representations to show and tell their mathematical thinking, thereby mitigating some linguistic challenges (see Allen, 2015; Christensen, 2004b).

Mātanga 2 discussed the importance of high-quality language modelling by teachers to ensure that students could access a depth of conceptual knowledge:

Me whakamahi te reo e hāngai ana ki te kaupapa. E hōhonu ai te ako. E ruku atu ai ngā ākonga ki te rētōtanga o te kaupapa. Kei riro te reo... 'me pēnei', 'me pēnei tō mahi', 'anei me pēnei'. Karekau he paku kōrero i te taha e mārama ai te ākonga he aha tāna mahi. He aha te hononga o tāna mahi ki mahi kē atu.

[We should use the language appropriate to the subject. So that the learning will be deeper. So the students can dive deeply into the subject. In case we end up with language like... "like this", "do like this", "like this here". Where there is no accompanying explanation at all for the student to know what they should be doing. Or what the connections are between his/her work and other topics.]

Mātanga 3 also discussed the importance of using the correct pāngarau register terms to ensure that students can better grasp pāngarau concepts:

Kia whakamahia e te kaiako ngā kupu e tika ana. Mā reira e eke ai te ākonga, te tamaiti ki ngā taumata o te pāngarau. I runga i te hinengaro mārama ki te kaupapa...

[The teacher needs to use the appropriate vocabulary. In order that the student, the child can achieve well in mathematics. With a high-level understanding of the subject...]

Mātanga 3 highlighted the lack of pāngarau specialists and how this contributes to the difficulties for teachers in finding appropriate support, mentoring, and critical debate:

Ko te mate hoki, ko te tokoiti kei te whakaako i te tātai (mathematics) i roto i te reo Māori. Anā, kei te huihui hoki ki te kōrero i ngā mea ka kōrero tāua i tēnei wā. Kia pai ai te tauwerowero o tētehi ki tētehi, te tauakiaki i tētehi ki tētehi me te tiakiaki o tētehi i tētehi. Kia haere whakamua kia kite i ētehi tauira hou. Pai tērā... I mua, te nuinga o tātou i te noho motuhake i tētehi kura. Anā ... pai ētehi. Engari, kāre i te kite, kāre i te rongō, kāre i te pānui i nga tauira hou. E oho ake ai ngā whakaaro. A me pēnei kē te haere... Āe... te pōuri hoki.

[There's also the issue that only a few are teaching tātai in the Māori language. And to meet to discuss these things we are talking about now. To enable us to challenge each other, encourage each other and to support each other. To move forward, to find new exemplars. That would be good... In the past most of us were isolated in our schools. Well... some were alright. But, they didn't see, hear or read the new exemplars. To wake up the ideas. So, it should be so... instead. Yes... it is sad.]

Tērā pea tētehi painga o te tokoiti. I te mea mehemea kei te ngana tonu kia whānui haere te ao tātai Māori rawa nei. Kāre i te kōrero mo te tatau i ngā kūmara me te whakaaro mō te harakeke. Engari, ... he wāhi tonu i runga i te waka mō nga kaihoe e tino kaha ana, e tino mārāma ana. Nō reira koutou kei runga tonu i te waka, kei te rapu hoa hou, kia pai te whiriwhiri i aua hoa.

[Perhaps that is the good thing about being just a few people. Because, if we stay determined to broaden the scope of the Māori way of mathematics. I'm not talking about counting kūmara, or thinking about working with flax. But, ... a place onboard the waka for strong paddlers, who are very knowledgeable. Therefore, those of you who are still aboard the waka, and are seeking new companions, choose your companions carefully.]

Mātanga 3 stressed the importance of ensuring pouako were aware of the significant changes that have been made to te reo Māori to accommodate school mathematics teaching so that new concepts were not too readily accepted as native terms:

Heoi, i mua rā, tahi rua tau i muri mai i te kōrero tētehi hunga mō te tātai... ko te kupu tuturu ko te pāngarau? Kāo i mea mai ngā kaumātua... koinā tētehi kupu tawhito. Pāngarau, ā, e hoa ma kei te hē. He mea hanga noa atu tērā.

[But earlier, one or two years ago, a group were discussing numbers... “Is ‘pāngarau’ the original word?” No. The elders said... “that is an old word.” Pāngarau, my friends, is wrong. It is just a made-up word.]

Similarly, Mātanga 3 stressed the importance of deliberate pedagogical decision-making, aware that teachers often revert to what and how they were taught:

Nō reira, ka tīmata te tangata me ōna whakaaro papai, āe anei, ko au kei te ārahi i ngā kōwhiritanga i roto i te mahi nei, nā wai rā, nā wai rā, ā, kua hoki ki ngā mea i ako ai te kaiako... ka hoki ki ngā huarahi i ako ai hei ākongā...

[And so, a person starts out with good intentions, yes, here I am, guiding the choices in this work. As time goes by, the teacher returns to those things they learnt... they return to the well-beaten path they learned as students...]

Mātanga 3 further asserted that pāngarau teachers must be competent in mathematics, te reo Māori, and mātauranga, but that Māori language competence was of utmost importance:

...ko te pakari hoki o te mōhio o te kaiako ki te tātai, ka tahi, ki te reo, ka rua, ki ngā tikanga, ka toru...Ēnā pērā pai noa. Pai noa.

[...the depth of the teacher’s knowledge of mathematics first, of the reo secondly, of tikanga, thirdly...if that’s it, that’s okay. It’s not a problem.]

Ko te pakari o te reo, o te kaikorero, o te kaiako... Engari, koinā, ki ahau nei, koinā tētehi wāhanga me pakari rawa. I te mea... ka puta ake te mātauranga tātai hoki i roto i tērā momo kōrero.

[The language ability of the speaker, of the teacher... But, that is, in my opinion one area, that needs to be very strong. Because... the mathematical knowledge will be conveyed by that kind of language.]

One of the reasons Mātanga 3 gave for promoting language ability as the most important skill of a pouako pāngarau was the ability to communicate mathematically in ways students could grasp:

Anā, e rua ngā mea i te kumekume i a au. Ko tētehi taha ko te mahi tātaitai ake, ka tahi. Ko tētehi mea hoki, ko te ngana kia kōrero kia mārāma ai ngā tamariki, ahakoa ngā ākongā, ahakoa ehara au, ehara hoki rātou i te tohunga ki te kōrero Māori, Māori tūturu nei... Me pēnei kē te kōrero, te hora nei i te kaupapa... te whakarite i tētehi reo ā-karaehe, ā-akomanga.

[So there are two things pulling me in different directions. On one side, the actual work of calculating, firstly. One other thing, endeavouring to speak so that the children can understand, no matter who the student is. Even though I am not and they are not expert speakers of Māori, classical Māori... It should instead be said, for the dissemination of this programme, the development of a classroom language, a language of learning.]

Mātanga 3 also stressed the importance of critical awareness by teachers of the contribution of their work to the wider project of indigenising schooling for Māori:

Engari, ko te kaiako Māori me whakaaro hoki i te noho o te Māori i roto i te ao Pākehā. Nō reira, e hara i te mea ko te marau anahe. Kāo. Ehara i te mea ko ngā tikanga e ako pai ai ngā ākongā. Ehara i te mea, koinā anahe. Me whakaaro hoki te kaiako Māori, anei au i roto i tēnei miru nui nei. Nō reira me aha au hei poipoi, hei tiaki, hei whakarauora hoki i te reo me te hanga Māori tawhito nei, hou nei o ngā tau kei te haramai?

[But Māori teachers have to consider the experiences of Māori living in the Pākehā world. Hence, it is not just the curriculum. No. It is not as though it is customs (tikanga) that ensure students learn well. It is not just that thing on its own. The Māori teacher needs to think, here I am in this big bubble. Therefore, what should I do to nurture, care for and revive the Māori language and ways, ancient and new, for the years ahead?]

Mātanga 3 argued that a lack of critical awareness could lead to teachers not recognising language interference in their own instructional language, further eroding the uniquely Māori essence of te reo Māori:

Anā, kātahi anō ka mutu te kōrero mō te kaiako e mea ana me āta whakaaro, me āta whiriwhiri ki tāna e mahi ai. Me pērā i ngā wā katoa. Kei mea, oh, anō anei ngā kupu. Ka pai ko te mea i roto i taku hinengaro, i taku mōhiotanga, he mea Pākehā, nō reira, ka tāpiri tēnei kupu ki konei, tēnei kupu ki konei, oh, arā, te kupu mō tērā, karekau he rerekētanga. He reo whakamāori. Ehara i te reo Māori.

[So, we have just finished talking about how teachers should carefully think and carefully choose what he/she does. They should be like that all the time. Lest we just say, here are the words. It's alright, what is in my mind, my knowledge, are Pākehā things. Hence, we put this word after that, this word here, oh, here's the word for that, but it is no different. It is a translated language. But not the Māori language.]

Mātanga 3 also asserted that the translated nature of the current pāngarau curriculum could potentially lead teachers to label familiar school mathematics concepts as Māori concepts. This could potentially lead students to reject less familiar Māori concepts as unimportant:

He marau whakamāori. Ehara i te marau Māori nē. Koinā te wero nui. Anā, engari tērā pea Piata, ka whaiwhai tonu i tērā hanga whakamāori i tēnei, i tērā, i tērā. Ā, hei te wā ka pōhēhē ko ēnā te mea Māori. Te mea kua waia.

[A translated curriculum. It is not a Māori curriculum is it? That's the big challenge. Well, perhaps, Piata, through repetition of that translation, in a variety of contexts. Eventually, the time will come where people believe that is the Māori way. The familiar way.]

As discussed by the mātanga in this section, the role of a pāngarau specialist teacher is multifaceted and balances the priorities of indigenisation and accepted school mathematics pedagogies. The priority of indigenisation requires a critical awareness of the changes that have been made to te reo Māori to facilitate the teaching of school mathematics in Māori. Articulating these changes in pāngarau curriculum may ensure that this critical awareness exists in classrooms. Several examples have been given in this chapter of pouako and Māori immersion schooling community members being unaware of the newness of a large proportion of pāngarau register terms, including the curriculum title of pāngarau. Illuminating the translated nature of the school mathematics content, alongside mātauranga in pāngarau curriculum, could provide further opportunities for epistemological pluralism. This chapter has thus far discussed several potential additions to the pāngarau curriculum to influence classroom practices. The following interview excerpts discuss how the existing content could be prioritised to accommodate these additions.

Of the range of concepts contained in the pāngarau curriculum, Mātanga 4 believed that three were being prioritised in Māori-medium—number, measurement, and algebra:

Ko te anga nui ki a waea ki a mātou te whakaaro ki ngā nama. I muri mai, ka whakahua mai ko ngā mahi ine. Kei runga ake i tērā, i te tau, ko te taurangi.

[The big objective is that we become accustomed to thinking about numbers. After that, comes measurement. Above that, numbers, is algebra.]

Engari... mōku nei, mehemea ka pātai mai te tangata o ngā whenu pāngarau e rima nei, ko ēhea pea ngā tino whenu, ko te tau [me te taurangi] me te tauanga.

[But... in my opinion, if someone were to ask which of the five strands of mathematics is the most important strand, I'd say number [and algebra] and statistics.]

Arā ... i roto i aku tau ko aku ākongā tino mātau ana. E tino eke i roto i ēnā mahi pāngarau, pakari ana ngā pūkenga tau. Ka tahi, ka rua, nā wai rā, pakari ana ngā pūkenga taurangi. Anā, e mōhio tāua ko te taurangi tērā e piki ai ki ngā rangi o te ao pāngarau.

[That is ... in my experience with my most capable students over the years, who achieved the best in those mathematics fields, they had strong number skills first. Second, eventually, they developed strong algebra. Well, you and I know that algebra takes us to higher skies of the mathematics world.]

Mātanga 4 discussed teaching statistical skills in interpreting the graphs showing the COVID-19 pandemic cases, the differences between different countries' strategies, and the metaphorical cues in the pāngarau register vocabulary for doing so:

Nā, hangarite te ūnahi, i te taha kotahi o te ūnahi ki te āhua o te tupu o te mate korona, huri noa i te ao, atu i Aotearoa. I Aotearoa anake, i tupu rārangi kē mai, te mate, anā, kua tāpapa nē? Kua heke te tupu. Ā, tērā kōrero, ā, kātahi ko te kōrero whakamutunga pea i tēnei wā, 95 ōrau o ngā tūrora kua ora. Kua whakamāui ake te kōrero. He kupu hou tērā ki ahau. Kua māui ake. Kua whakamānu ake rānei. Kua ora ō tērā 1,500, e mea ana ko te 5 ōrau noiho kei te pā tonu i tērā. He nui ngā kaupapa pāngarau i roto i tērā.

[So, the parabolic curve is the same, one side of the parabola matches the growth of COVID-19, throughout the world, apart from Aotearoa. Only in New Zealand, was there a different line for the disease, it lay flat didn't it? The growth went down. And, they say, the final word perhaps, at this stage, 95% of the victims have survived. The story is that recovery is underway. That's a new word for me. To recover (whakamāui ake). Or 'whakamānu ake' (refloat/recover). Of that 1,500, they say only 5 percent are still suffering from it. There are many mathematical topics in that.]

Ko te ika tērā ne. Ko te kiri o te ika tērā te ūnahi. Ka kite koe i taua āhua nē?

[That's the fish isn't it? The skin of the fish, that's the scales. Can you see that shape of it?]

In this section, the lack of a distinct cohort of pāngarau specialist teachers engaging in critical debate about what should be prioritised in pāngarau curriculum and classrooms was identified as a current weakness in pāngarau schooling by Mātanga 3. Mātanga 4 extended this idea to critical awareness in prioritising the pāngarau curriculum content that would most benefit Māori today, thereby creating room for the illumination of mātauranga. Establishing a pāngarau expert group to debate how mātauranga could be illuminated in pāngarau curriculum and classrooms could further support the

process. Rather than merely funding such a group for short-term curriculum development projects, a more long-term approach is likely required.

Summary of Mātanga Pāngarau Interview Data

An overarching tension exemplified in the mātanga interview data was the use of school mathematics to further language revitalisation. While mātanga gave examples of how pāngarau register elaboration, in tandem with curriculum development, has supported the revitalisation of te reo Māori, they were also very cognisant of the risks and concerns. Change was also considered inevitable by the mātanga. The critical analysis of what change is desirable and what change is undesirable seemed to be the primary tool for decision-making in pāngarau curriculum development and language elaboration over time. Linguistic purism and language separation ideologies, enforced by Te Taura Whiri i te Reo Māori and *Te Aho Matua* (DIA, 2008), also seemed to be used in decision-making. While there was consideration for the practical realities in the pāngarau classroom, especially for pouako who were also L2 learners of the language of instruction, the burden largely fell on pouako to learn and implement the register at speed. In this vein, the mātanga asserted that despite policymakers' intentions, pouako are charged with speaking the pāngarau register into being. However, the lack of specialist teachers engaged in robust and critical debate about pāngarau curriculum and classroom practices was lamented as limiting pāngarau schooling's ability to further realise Māori aspirations for schooling.

The mātanga also identified a range of unique aspects of te reo Māori and mātauranga that did not map easily onto the English language or school mathematics, and were therefore omitted from the pāngarau curriculum. The ongoing existence of these uniquely Māori ways of communicating and coding information in pāngarau classrooms, despite omission from the pāngarau curriculum, will be explored further in Chapter 6. Meanwhile, an exploration of Māori and Pacific wayfinding devices, reflecting the assertion of Mātanga 1 that uruuru ao and uruuru whenua provide opportunities for pāngarau curriculum and classrooms to illuminate mātauranga, is offered in the next chapter.

5. Ngā Uruuru Ao me ngā Uruuru Whenua | Wayfinding Representations

This chapter presents literature on wayfinding representations used by Māori to communicate spatial information. The term ‘uruuru whenua’ was defined by Mātanga 1, interviewed in Chapter 4, as a narrative for wayfinding on land. Mātanga 1 further suggested exploring ‘uruuru ao’ (narratives for wayfinding at sea) and uruuru whenua as an opportunity to ameliorate perceived tensions in teaching mātauranga in pāngarau classrooms. In order to identify opportunities to explicitly teach uruuru ao and uruuru whenua in pāngarau classrooms, a literature search of Māori oral and visual narratives used for wayfinding was conducted, including the Pacific origins of the wayfinding devices or representations and associated navigation practices. This chapter thus presents anthropological, archaeoastronomical and ethnomathematics literature alongside the few examples of pāngarau literature where specific mention is made of wayfinding representations and practices. The following section provides a discussion of how the study methodology was used to guide the literature search.

Utilising Dual Lenses for Examining Wayfinding Devices

The dual lenses of mātauranga and mathematics are provided to contextualise the data in the literature from two ontological viewpoints. Aligning with ontologically and epistemologically pluralistic approaches (Andreotti et al., 2011; Hauser et al., 2009) and the cultural symmetry framework, the use of multiple lenses in interrogating the data avoids the tendency of mono-epistemic interpretations, predominantly through a dominant Eurocentric lens (Andreotti et al., 2011). Using dual lenses provides opportunities to position mātauranga and school mathematics epistemologies as distinct and complementary, and to investigate the affordances of each in addressing challenges that Māori face today. In this section, mātauranga and school mathematics’ literature on navigation, wayfinding, and spatial orientation epistemologies are presented. This provides dual lenses for exploring literature on Māori wayfinding representations and their Pacific origins.

The first lens used to examine the literature is Māori concepts of wayfinding representations and associated knowledge-creation practices. As Owens (2014) argued, Indigenous concepts of location can include the relationship between humans and their environments. According to Forster (2019), Māori used whakapapa to create knowledge about environmental phenomena. Therefore, for Māori, whakapapa or genealogical narratives connect Māori to the environment and connect personified environmental phenomena to each other (Best, 1899). However, concepts and uses of whakapapa have since generally become limited to the recording of human genealogy (Forster, 2019), aligning with mono-epistemic European interpretations of genealogical knowledge-creation practices. As demonstrated in the mātanga interview data in Chapter 4, mātauranga often organises information,

including spatial information, in personified genealogical narratives containing linked events and ideas. Therefore, valued environmental and wayfinding information can be organised and presented in genealogical narratives. For Māori, interpreting oral and visual narratives, including genealogical narratives that explain environmental phenomena, is a learned skill for successfully navigating from one place to another.

Pacific peoples, from whom Māori descend, systematically explored the Pacific Ocean (Ruggles, 1999). However, regular voyages of over 300 kilometres were no longer undertaken in Polynesia by the time of European contact. Despite this, Tahitian navigator Tūpaia, who assisted Captain James Cook in his exploration of the Pacific, knew the existence of and could give detailed instructions on how to sail to dozens of Polynesian Islands (Ruggles, 1999). Some islands were up to 1,931 kilometres from Tahiti (Taonui, 1994). The mass movement of people throughout the Pacific would have been enabled by similar longer exploration journeys, which provided the experience of the local patterns of stars, currents, winds, and birds, indicating routes to habitable islands (Ruggles, 1999). This chapter presents a range of wayfinding representations that navigators in Aotearoa/New Zealand and the Pacific used to code and disseminate this valued environmental information to navigate from one place to another.

The second lens used to analyse the literature is mathematical. For mathematics, navigating from one place to another utilises wayfinding and locomotion (Wiener et al., 2009). For mathematicians, finding your way to a destination using environmental cues is known as wayfinding (Farr et al., 2012) and can be considered a spatial problem-solving task (Casakin et al., 2000). Solving the task and successfully reaching the desired destination is dependent on both human and environmental factors (Farr et al., 2012). Spatial orientation is a human factor; utilising mental manipulations of environmental cues, people form cognitive maps to locate themselves (Casakin et al., 2000). Therefore, from a mathematics perspective, spatial orientation is a spatial reasoning skill which involves the mental manipulation of 2D and 3D objects along with the relations within and between objects (Lowrie et al., 2020). An example is imagining a location or object from various vantage points or perspectives. While spatial orientation involves identifying a static point in space, wayfinding is the process of reasoning your way *through* space. Navigation necessarily requires movement and wayfinding between points.

The cognitive challenge of wayfinding, which requires reasoning, planning, and decision-making (Wiener et al., 2009), can be alleviated using spatial representations that approximate real-world situations (Freksa et al., 2000). These wayfinding representations establish relationships between environmental information and verbal or visual abstractions (Casakin et al., 2000). Spatial representations alleviate the cognitive challenge of safely and efficiently moving from one location to another, particularly when the environmental conditions are unfamiliar or disorientating (Farr et al.,

2012). However, the approximate nature of spatial representations may require the user to perceive real-world information and adjust their path accordingly (Freksa et al., 2000). This is part of locomotion (Wiener et al., 2009). Examples are signage and oral instructions to guide large crowds through transport hubs or to the nearest emergency exit (Farr et al., 2012). Those following given spatial representations may still need to adjust their path due to obstructions or hazards (Freksa et al., 2000). Therefore, wayfinding representations can be thought of as externalised spatial representations used by humans to create internalised cognitive maps to find their way between locations. Successful navigation between pre-determined locations requires a combination of well-designed wayfinding representations and learned locomotion and wayfinding skills (Wiener et al., 2009).

The preceding two paragraphs drew together a small excerpt from the extensive catalogue of mathematical wayfinding and spatial reasoning literature that explores the design and use of wayfinding and spatial orientation representations in school mathematics and their real-world applications. Therefore, pāngarau curriculum developers and pouako can draw from this same catalogue to support the acquisition of school mathematics navigation, wayfinding, and spatial orientation concepts. Conversely, no comparative catalogue explains the design and use of Māori wayfinding representations, as the development of Māori wayfinding practices has been disrupted by colonial education and research agendas. The pool of kaumātua and kaitiaki who were practitioners of wayfinding and navigation has also considerably diminished. Literature that illuminates the characteristics of Māori oral and visual wayfinding representations is examined in the following sections. The literature on Aotearoa/New Zealand wayfinding representations is then supplemented with literature addressing genealogically connected Pacific wayfinding representations and practices.

Using Karakia and Pūrākau for Wayfinding

Karakia known as uruuru whenua were used by arrivals to a new area to ensure a safe journey (Moorfield, 2003; Smith, 1907) and could also involve placing an object (Best, 1901; Williams, 2002) or visual wayfinding representation. Karakia are oral narratives that are usually chanted and can be used to record and disseminate valued information about the environment and, therefore, can also be thought of as wayfinding representations. The recitation of karakia by tohunga ensured the permanence of what their pupils had learned (Smith, 1913, as cited in Mead, 2013), thereby ensuring the memorisation and ongoing dissemination of valued wayfinding information over time. Placing a visual location marker and devising or reciting karakia could be considered wayfinding practices assisting or warning subsequent groups traversing the area. Pūrākau, as discussed in Chapter 2, is a term used to describe a wide range of narrative practices. When a narrative or chant is commonly referred to as karakia, that term is used in this section. Where there is no clear demarcation of the narrative type, the broader term of pūrākau is used. As discussed by Mātanga 1 in the previous

section, a range of narrative types provide boundary and route markers or can code and disseminate valued environmental information. Mātanga 1 preferred the terms *uruuru ao* and *uruuru whenua* for narratives that contained boundary and route markers. This section further investigates the distinctive characteristics of these narrative types.

The Waitangi Tribunal is tasked with hearing submissions from Māori regarding breaches of Te Tiriti o Waitangi, such as land, language, and cultural knowledge loss unrightfully perpetuated by the Crown. Evidence presented to the tribunal to support claims includes *karakia* and *pūrākau* containing observations of natural phenomena, effectively placing the claimants' ancestors in place and time. As Taonui (2015) observed, naming significant geographical features for significant ancestors could indicate the expansion of geographical territory. Therefore, spatial and wayfinding information contained in *pūrākau* and *karakia* has been presented by claimants to the tribunal to support occupation rights. I assisted my grandfather in approving a translation of a *pūrākau* featuring Ngāti Kahungunu (the people of Kahungunu) ancestor Tamatea Pōkaiwhenua (sometimes referred to as Tamatea the circumnavigator) to support a Waitangi Tribunal claim.

Taonui (2015) discussed the Tamatea narratives as an example of tracking territory expansion by naming geographical features and the accompanying narratives. A ridge near Porangahau, Hawkes Bay, and the accompanying narrative is perhaps one of the most well-known landmarks in the Tamatea narratives, as it is one of the world's longest place names—*Te-taumata-whakatangihanga-kōauaua-a-Tamatea-Pōkaiwhenua-piki-maunga-horonuku-turipūkākā-ki-tana-tahu* (Whaanga, 2017). Taonui (2015) traced the emergence of this place name and accompanying narrative to a ridge northeast of Hawkes Bay in Tūranga (Gisborne), where Taonui asserted that Ngāti Kahungunu occupied and subsequently expanded southwest into the geographical area of Rangitāne. According to Rangitāne, the well-known ridge name and narrative accompanying the ridge near Porangahau is named after their own eponymous ancestor. Therefore, Taonui (2015) argued that exploration narratives and their geographical features could be used to track the territory expansions and contractions of Māori over time.

Taonui (2015) also identified that *pūrākau* use the convention of supernatural travel to create a coherent journey that omits large regions that do not seem to have significance to the journey, thereby truncating the narrative and aiding the memorisation of significant geographical information. Taonui (2015) provided the example of Pacific ancestor Rongokako leaping from one location to another to speed up the journey from Wairarapa on the East Coast of Central Hawkes Bay to Murihiku in the North. While Rongokako touched the ground at significant sites, the journey became truncated due to the omission of the geographical regions in-between leaps (Taonui, 2015). This characteristic of *pūrākau* seems to reflect the guidance of Whaitiri to Tāwhaki presented in the introductory paragraph of this thesis to hold on to the *aka matua*, not the *aka tāepa*.

Further to tracking significant routes, boundaries, and geographical occupation rights, karakia can acknowledge environmental phenomena and the physical circumstances of the practitioner, as do other forms of uruuru ao and uruuru whenua. Supreme Court Justice Sir Joseph Williams, who served as chief judge of the Māori Land Court and deputy chairperson of the Waitangi Tribunal, recounted the popular karakia *Whakataka te Hau*, presented below, providing an example of how environmental and spatial information is coded and disseminated through karakia (Williams, 2019, p. 1).

Whakataka te hau ki te uru

Whakataka te hau ki te tonga

Kia mākinakina ki uta

Kia mātaratara ki tai

Kia hī ake ana te atakura

He tio, he huka, he hau hū

Tuturu o Whiti whakamaua kia tina! [Tina!]

Haumi e

Hui e

Taiki e!

Based on the environmental observations in the karakia, Williams (2019) argued that its author was a recent arrival to Aotearoa/New Zealand who had practised the karakia in preparation for a fishing expedition. Williams (2019) interpreted the karakia as a series of observations about the differences between a warmer Pacific Island of origin and the west coast of Aotearoa/New Zealand, where the karakia was likely being delivered. The first two lines end with the directional terms (now used in place of cardinal coordinates in pāngarau curriculum): uru (west), and tonga (south). In Aotearoa/New Zealand, the western wind is the prevailing wind, while the southern wind is always cold (Williams, 2019). The next two lines describe the conditions on land and at sea. The karakia continues to describe the time of day, the quality of light, and the various indicators of cooler temperatures. Williams (2019, p. 2) translated the lines as:

The west wind has changed

The south wind has fallen silent

...the land bristles

...the sea has goosebumps

...the first rays of a red dawn pierce the night

...revealing snow, ice, and frost (from the mountains to the sea).

Williams (2019) also pointed out that the last lines of the karakia do not contain a request to a higher power for a successful fishing expedition and safe return to shore, which distinguishes karakia from Judeo–Christian prayer traditions. Williams (2019) explained that despite the rather intimidating account of the bristly and icy winds blowing over the sea and land in the relative darkness, the karakia demonstrates a commitment to being completely present, mindful, and bound to operating within the novel environmental conditions that were being observed in this new place. Williams (2019) concluded that this information was likely valuable to other inhabitants of the island and other recent arrivals. This seemed to support the premise that karakia were used to code and disseminate valued environmental and spatial information that could be used to form a cognitive map of the local area.

Visual Wayfinding Representations in Wharenuī

As discussed by Mātanga 4 in the previous chapter, wharenuī have become storehouses of mātauranga (Trinick et al., 2017), reflecting the systematic exploration and habitation of the Pacific and the adaptation of existing technologies to the conditions of varying island locations (Matamua, 2019; Treadwell, 2017; Williams, 2019). Sea navigation skills and technology were repurposed for land-based endeavours, whereby boat building techniques, such as post-tensioning, were adapted to construct large, enclosed wharenuī (Treadwell, 2017). Significant navigation constellations, such as Matariki (Pleiades), were used to guide planting, harvesting, hunting, and fishing cycles (Matamua, 2019). The oral and visual narratives associated with te whare kōkōrangī (the space of Māori astronomical knowledge) were embedded over time into uniquely Māori knowledge-creation practices (Matamua, 2019).

In their exploration of the characteristics of the visual representations of mātauranga in wharenuī and how these could be taught in pāngarau classrooms, Trinick and collaborators (2017) identified the use of symmetry and ordered pairs in coding and disseminating valued information. In Ascher’s (1991) examination of Māori mathematical practices, she noted that this concern with symmetry is so predominant that it can be called the “organising principle ... in much of Māori stories, religion, social life, and economics” (p. 171). Frequently deliberate, indistinct disruptions were made to the symmetry in visual artefacts (Witehira, 2013). While it is not exactly known why Māori used design elements to disrupt the symmetry of bilateral structures, the consistent use of this design principle

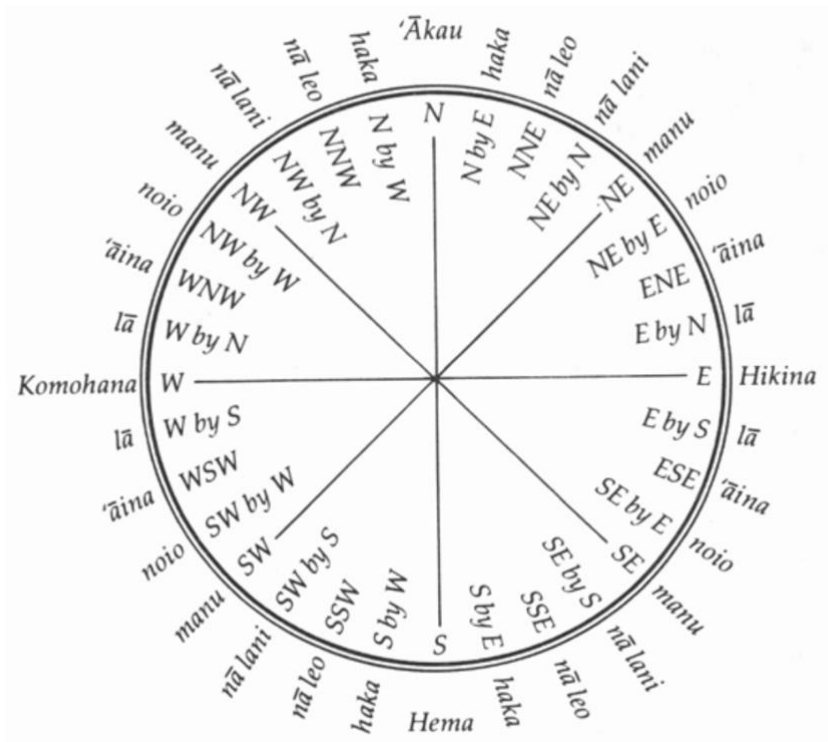
demonstrates that it was significant. Hanson (1983) believed disrupted symmetry in Māori art reflected tensions in the real world.

Due to the lack of literature on the specific characteristics of Māori oral and visual wayfinding representations used to navigate across land in Aotearoa/New Zealand, the remainder of this chapter explores the Pacific origins of pūrākau, karakia, and visual wayfinding representations. The revitalisation of navigation and wayfinding practices throughout the Pacific draws on genealogical links between practices from neighbouring islands, including Aotearoa/New Zealand (Furuto, 2014; Spiller et al., 2015). As a starting point, Goetzfridt (2008) provided a succinct summary of the literature on navigating in the Pacific and included key research on wayfinding representations. Goetzfridt's (2008) review included oral and visual representations of physical locations in space used by Hawai'i, Marshall Islands, and Caroline Islands navigators. These Pacific wayfinding representations are explored further in the following sections.

Hawaiian Wayfinding Representations

Pivotal to the reinvigoration of Pacific navigation and wayfinding research was the construction of the double-hulled vessel, Hōkūle'a, its journey from Hawai'i to Tahiti in 1976 using non-instrumental navigation techniques, and subsequent circumnavigation of the globe (Chauvin, 2000; Furuto, 2014; Goetzfridt, 2008; Polynesian Voyaging Society, n.d.). The work of Finney and collaborators (1976, 1986) in documenting the Hōkūle'a journeys, and the characteristics of the wayfinding representations used by Hōkūle'a navigators, featured prominently in Goetzfridt's (2008) literature review. Under the tutoring of the 1976 Hōkūle'a journey navigator Mau Pilaung from the Marshall Islands, Hawaiian navigator Nainoa Thompson devised a star compass for Hōkūle'a's round trip between Hawai'i and Tahiti in 1980 which is presented in Figure 6 (Finney et al., 1986).

Figure 6. Nainoa Thompson Star Compass (Finney et al., 1986, p. 54).



In addition to referencing the rising and setting points of stars in determining direction, Thompson also used environmental cues such as swell, wind and bird migration patterns as instructed by Mau Pilaung (Finney et al., 1986). Interestingly, Thompson utilised two additional techniques to determine the latitude and spatial orientation at sea. The non-standard measuring technique of the thumb and index finger was used to determine the angle of the star above the horizon by Thompson (Finney et al., 1986). Thompson also utilised pairs of stars that appeared close together and lay approximately north and south of each (Finney et al., 1986). Thompson was able to determine latitude by calculating the difference between the observed and memorised angular height (when viewed from the equator).

In separate studies, archaeoastronomists Ruggles (1999) and Chauvin (2000) examined Hawaiian oral narratives and identified symbolic relationships between the spatial characteristics of human activity and landscape topography or astronomy. Through this process, both Ruggles (1999) and Chauvin (2000) connected oral narratives with spatial information and made connections between topographical artefacts and the astronomical knowledges of Kānaka Maoli (Indigenous people of Hawai'i). Kānaka Maoli oral narratives were the responsibility of highly ranked individuals and were carefully learned and handed down from generation to generation (Ruggles, 1999). However, Ruggles (1999) conceded that the suppression of Kānaka Maoli knowledge systems through selective recording by missionaries, alongside the fusion of ancient and modern practices, made English interpretations of oral narratives less reliable. Without *ōlelo Hawai'i* (Hawaiian language) expertise,

Ruggles (1999) relied on knowledgeable informants to provide evidence of important elements of Kānaka Maoli and wider Pacific navigational and calendrical traditions.

The *Kumulipo* is presented as a genealogical narrative that organises environmental information by Chauvin (2000). Five astronomical reference points begin the *Kumulipo*, the earth, heavens, sun, moon, and the Pleiades constellation, Makali'i (Chauvin, 2000) or Matariki, as it is known by Māori. Written by the ancestors of Queen Liliuokalani and translated by her Majesty into English in 1897, the *Kumulipo* traces the origin of Queen Liliuokalani's family through genealogical pairs back to the first appearance of life in the universe. With its conception in te pō (the darkness), the *Kumulipo* outlines the formation of an ordered world from the pairing of opposites—day and night, light and darkness, male and female. According to Chauvin (2000), using a series of births (genealogy) as a framework for observed biological processes aligns with the evolutionary linking of mankind to the pre-human world. The author also identifies genealogical pairing (symmetry) as a mnemonic device. The *Kumulipo* is 2,000 lines long (Chauvin, 2000) and would have been committed to memory before the adoption of writing in Hawai'i. Therefore, the *Kumulipo* may provide valued environmental information through a mnemonic framework that supports its acquisition and retention.

Consistent with Māori narratives, in the *Kumulipo*, Papa and Wakea (spelled Wākea in some texts) are personifications of the earth and the sky, respectively (Chauvin, 2000; Taonui, 2006). Therefore, the *Kumulipo* and associated narratives could be considered wayfinding representations, as exemplified by the inclusion of astronomical referents (Chauvin, 2000):

Papa, the wife of Wakea begat a calabash; this included a bowl and cover. Wakea threw the cover upward and it became the heavens. From the bowl, he made the land and sea, from the juice he made the rain, from the inside meat he made the sun and moon, and from the seeds he made the stars. It must have been sometime later that, to give life on earth sufficient light and space to flourish, Earth Mother and Sky Father, once tightly bound together, became (in Tahiti and New Zealand myth) separated (p. 92).

On investigating the physical representation of the Papa and Wakea narrative, and the sky dome represented by a calabash bowl, Chauvin (2000) concluded that the narrative, alongside the physical representation, was used by navigators as a pedagogical device. In the narrative, the inverted bowl of the calabash was birthed by Papa and thrown into the sky by Wakea:

The dome was divided into spaces or zones and an assortment of astronomical circles and reference points – including the meridian, equator, ecliptic, tropics, and zenith ... the sky-dome rested on the rim of the earth where it intersected the latter to form the circular horizon. (Chauvin, 2000, p. 94)

Another narrative investigated by Chauvin (2000) personified aspects of the sky dome, such as Wakea's piko (naval) as the horizon, while other narratives describe the importance of Makali'i (the Matariki/Pleiades constellation) as pivotal to commemorating Hawai'i-loa's first arrival in the Hawaiian Islands (Chauvin, 2000).

A difficulty in making the characteristics of wayfinding representations such as the *Kumulipo* and associated narratives visible is the tendency of researchers to present mono-epistemic interpretations of narratives such as these and their associated visual representations. As discussed by Ascher and Ascher (1981), interpretations of Indigenous artefacts are often limited by the researcher's knowledge base. For example, Chauvin (2000) termed the *Kumulipo* a creation chant. This definition seems to align with Christian creation stories and potentially obscures the wayfinding applications of the *Kumulipo* and associated narratives. Chauvin's (2000) commentary also points out the recognition of Papa and Wakea as 'ancestors' in human genealogies. The apostrophes on the word ancestors appear in the Chauvin (2000) text and imply a mono-epistemic interpretation labelling Pacific ancestors as 'gods' and likening their genealogical groupings in the narrative to European pantheon constructions. Despite Chauvin (2000) characterising the genealogical framework as 'fanciful', the *Kumulipo* provides valuable insights into the use of ordered pairing in genealogical narratives to aid the memorisation of large amounts of environmental information that could be useful for wayfinding. However, the likening of Indigenous narratives by researchers such as Chauvin (2000) to European myths, legends or religious beliefs, could also obscure the environmental information from those who could utilise it, including teachers of Indigenous students.

The Ruggles (1999) and Chauvin (2000) texts reviewed in this section provide further evidence of the importance of ontological and epistemological pluralistic approaches to illuminating Indigenous epistemologies, and ensuring Indigenous ways of knowing and being are promoted equally alongside Western ontologies, epistemologies and axiologies. Taonui (2015) observed that while Māori narratives featuring conflict, romance, mass migration, and the genealogy of natural phenomena suffered reinterpretations that aligned them more closely with European belief systems or research agendas, Māori exploration narratives seemed to be less edited. The examination of the characteristics of Hawaiian wayfinding representations showed similarities between the use of whakapapa and personification to organise information and make it more memorable, as expressed by the mātanga interviewed in Chapter 4. The use of ordered pairs, as identified by Trinick and colleagues (2017) in the visual representations of wharenuī, and by Ascher (1991) as being pivotal to Māori knowledge organisation, was also prominent in the Hawaiian representations. The next section further examines wayfinding narratives used in the Puluwat atoll.

Puluwat Wayfinding Narratives

Anthropologists Gladwin (1970) and Riesenberg (1972) documented the navigation practices of the people of the Puluwat Atoll in the Central Caroline Islands. Gladwin's (1970) work explored the Puluwat navigation system as a whole, recognising it as a dead reckoning system. Similar to techniques employed by Mau Pilaung (Finney et al., 1986), navigating by dead reckoning means that the navigator's position and direction are determined by the distance and direction travelled from the previously known location (Gladwin, 1970). Gladwin (1970) observed the use of a star compass and mnemonic techniques, including narratives for memorising specific stars and information related to journeys from one point to another. Providing further insight into Gladwin's observations, Riesenberg's (1972) work illuminated the mnemonic nature of oral narratives used by navigators to commit valued information to memory. Known as *aāruwóow* (uruuru ao in Māori), like the *Kumulipo*, the narratives were used to organise and memorise large amounts of valued information (Riesenberg, 1972). Oral narratives documented by Riesenberg (1972) were premised on chasing a person or sea creature from one location to another. These chase stories sometimes resulted in circuitous routes between islands, providing a cognitive map of the relationships between landmarks (Riesenberg, 1972). Like Chauvin (2000), Riesenberg (1972) noted the prominence of pairing as a mnemonic device in the narratives. Riesenberg (1972) presented examples of whales with two heads or two tails and islands that were presented and named as pairs.

Riesenberg (1972) was also interested in variations between narratives of the same name when told by different navigators. He initially perceived this flexibility in the narrative style as a lack of regard for scientific accuracy that seemed unpragmatic in navigation practices—a mono-epistemic interpretation. However, through discussion with his Puluwat informants, Riesenberg (1972) learned that, generally, only non-crucial information was subject to creative licence and that navigation information was rarely changed. Riesenberg (1972) gave the example of an informant changing the purpose of navigating from one island to another. Rather than retrieving food, the informant decided that the new purpose of the journey was retrieving stone. Knowing that the new information was likely inserted due to a previous conversation about retrieving stone the two of them had, Riesenberg (1972) questioned the change. The informant said he preferred the idea of retrieving stone. Riesenberg (1972) realised that this new content was congruent and more specific to the island location, as stone retrieval was practised on the island, whereas retrieving food was less specific.

Reflecting the guidance of Whaitiri to Tāwhaki in the whakataukī shared in Chapter 1, the navigators interviewed by Riesenberg (1972) seemed to adhere to the *aka matua* (main ideas) that needed to be retained and disregarded the *aka tāepa* (loose side shoots/less important information). The navigators updated the narratives without interfering with the integrity or the purpose of the narrative—to successfully navigate between islands. When this characteristic of narrative flexibility is applied to

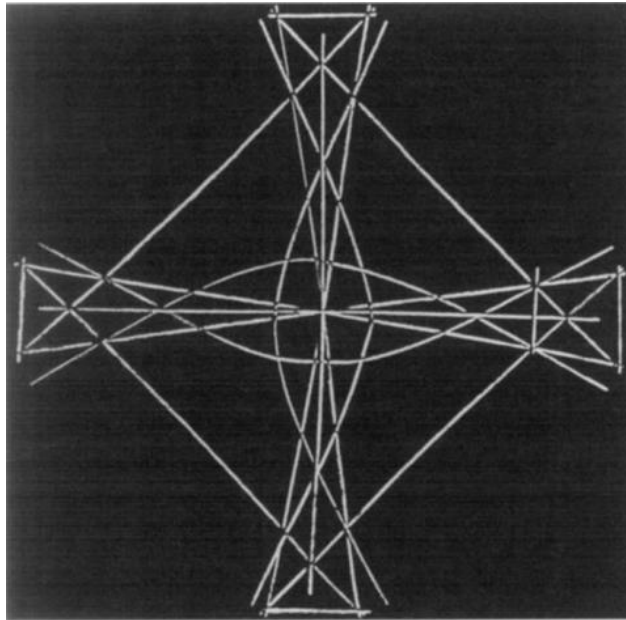
uruuru ao and uruuru whenua, which may have been altered due to colonial interference or creative licence by practitioners, it would be interesting to examine if the spatial information had been altered or not. There is potential that the characters and premise of the narratives have been altered in English-language texts due to European ontological, epistemological, and axiological determinations. However, the practical wayfinding information could have remained intact.

Marshall Islands Maps

Similar to the dual use of oral and visual representations as mnemonic devices by Kānaka Maoli navigators, those from the Marshall Islands also used a combination of oral and visual narratives to learn their craft. While at sea, navigators did not need physical representations of navigation routes, instead defaulting to oral narrative (including song form) (Ascher, 1995). However, on land, visual representations were sometimes used by Marshall Islands navigators as pedagogical aids. Ascher (1995) documented a visual mnemonic representation that she termed stick charts for teaching navigation skills in the Marshall Islands.

Ascher (1995) was primarily concerned with devising a mathematical interpretation of the woven planary representations. The Marshall Islanders constructed formalised and standardised models by weaving sticks together to create geometric representations. Ascher (1995) also observed the use of symmetry and curvilinear representations to show relationships between phenomena. Rather than being concerned with the distance or location of landforms in relationship to each other, the Marshall Islanders' woven maps depicted wind directions and waveforms in relationship to landmasses (see Figure 7). Therefore, the use of malleable sticks woven to create straight lines and curves conveyed the information that the navigators needed to successfully complete their journeys between islands.

Figure 7. Marshall Islands Woven Map (Ascher, 1995, p. 352).



The narrative and visual representation structures and practices continued by the Indigenous peoples of the Caroline, Marshall, and Hawaiian Islands all use narrative, personification, and symmetry or pairing to aid the memorisation of wayfinding information. Further exploration of the connection between pairing or symmetry in narratives and their physical representations would be required to make the function of the symmetry clearer.

Summary of Wayfinding Literature

The aspects of mātauranga gleaned from the texts analysed in this chapter can be divided into two categories: those that reflect Western concepts and those that are omitted or disparaged in the literature for not doing so, as identified in the discussion. For example, oral narratives such as stories and chants recorded in the 19th century by missionaries in parts of the Pacific, such as Hawai'i and Aotearoa/New Zealand, often omitted material that the missionaries found distasteful or conflicted with their religious views (Ruggles, 1999). If anthropological texts are to be used to inform the positioning of mātauranga in the curriculum, a process for mitigating the undue influence of Eurocentric belief systems must be followed. A methodology for identifying desirable and undesirable changes is required, similar to how linguistic purism was adopted by pāngarau register elaborators. In the next chapter, the teaching of Māori wayfinding representations alongside school mathematics is examined through teacher and student-created artefacts from four pāngarau classrooms across two Kura Kaupapa Māori.

6. Ngā Raraunga Akomanga | Classroom Data

This chapter explores conflicts that arise from rebalancing mātauranga and school mathematics in pāngarau classrooms through the teaching of uruuru whenua. Student and teacher-created artefacts were collected in four pāngarau classrooms across two kura kaupapa Māori that were assigned pseudonyms of Kura Tahī and Kura Rua. Both participant kura are government-funded Level 1 immersion settings and adhere to the language separation policy of *Te Aho Matua*, outlined in Chapter 1, whereby 81% or more of the language of instruction is te reo Māori. As discussed in Chapter 3, Māori aspirations for schooling could be considered aspirations for ontological pluralism (see Hauser et al., 2009), providing two (or more) lenses through which to experience and examine dual-epistemic curricula and classroom practices. This realises the policy rhetoric of Māori success *as Māori* (Durie, 2003; MoE, 2021), or living in a way that is recognisably Māori to Māori while also accessing academic notions of success. Therefore, access to mātauranga in schooling for Māori supports these aspirations. However, as existing pāngarau curricula do not explicitly promote mātauranga, such as uruuru whenua, conflicts between curriculum-based priorities for pāngarau teaching and learning and the promotion of mātauranga will necessarily occur.

These conflicts are analysed through the tripartite foci of the cultural symmetry framework and the teaching as inquiry cycle discussed in Chapter 3. The cultural symmetry framework outlines a high-level rationale for rebalancing mātauranga, te reo Māori, and school mathematics in classrooms by explicitly addressing each. However, the framework does not provide specific guidance to teachers on balancing these sometimes competing priorities in practice, which is particularly challenging for pāngarau teachers who work at the interface between historically colonial state-mandated expectations for pāngarau achievement and Māori aspirations for schooling. The teaching as inquiry cycle provides a process for analysing the efficacy of pedagogical choices and actions. Therefore, the phases of the teaching as inquiry cycle have been utilised in this study to analyse the efficacy of pedagogical choices and actions in rebalancing mātauranga and school mathematics. The learning inquiry phase of the cycle was presented as part of the research design in Chapter 3. For the learning inquiry, diagnostic data were collected through Te Uiui Aromatawai oral diagnostic assessment, and the mātauranga and school mathematics learning aspirations for both Kura Tahī and Kura Rua were identified. The remaining phases of the teaching as inquiry cycle are presented in this chapter. The following section presents the focusing and teaching inquiry phases. Following this, the final phase of the teaching as inquiry cycle, the teaching and learning phase, is presented, which includes the analysis of student and teacher-created artefacts.

The Focusing and Teaching Inquiry Phases

The focusing inquiry phase included a series of collaborative discussions with principals and pouako to determine the focus areas for the teaching and learning phase of the cycle. These discussions were conducted via Zoom, collaborative Google Docs and face-to-face meetings. Through this focusing inquiry process, mātauranga, te reo Māori, and school mathematics foci were decided. The teaching inquiry process included identifying pedagogical approaches in the literature that would support the teaching of the three cultural symmetry framework foci. The teaching inquiry literature is presented at the end of the mātauranga, te reo Māori, and school mathematics subsections that follow.

Teaching Uruuru Whenua Through Screencasting

A mātauranga focus of Māori wayfinding devices, such as uruuru whenua was utilised as this aligned with the mātauranga learning goals of both kura, which included local geographical knowledge and whakapapa. As identified in Chapter 5, uruuru whenua utilises whakapapa and personification to connect geographical locations, thereby providing spatial orientation and wayfinding information. A list of appropriate uruuru whenua resources was compiled for use with students, most of which featured the geographical areas of the kura within a larger narrative. Pouako 4 (a Year 10 teacher) at Kura Rua also supplied an uruuru whenua about the wider area where Kura Rua was located that was already being utilised by the kura to support the teaching of pūrākau and whakapapa.

A student inquiry project was also devised where students could research an uruuru whenua relevant to their own hapū, iwi, or local geographical area. Using the characteristics and narrative features of their researched uruuru whenua as a guide, students at both kura could write their own uruuru whenua based on significant landmarks in their own lives. The students could then represent the researched or created uruuru whenua through a drawn or digital map. Information about each location named in the uruuru whenua could be presented via annotations or captured via video or audio recording. To complete the uruuru whenua inquiry, ākongā would have developed the skills to communicate about location, direction, and distance by utilising the specialised language of the pāngarau register, including visual representations. The ākongā inquiry project could have been expanded to include uruuru ao, narratives for wayfinding between islands. However, ākongā may not get to experience voyages on waka such as Hōkūle'a or Te Matau a Māui, as discussed in Chapter 5, or other open ocean-going vessels during their schooling. That said, it would be advantageous if all ākongā in Māori-medium and kura could complete a voyage between islands and be included in the pāngarau curriculum as a learning outcome.

To illustrate the benefits of a wider uruuru ao and uruuru whenua inquiry project, this study focused on the immediate benefits for ākongā of being able to draw on mātauranga to understand wayfinding in their everyday geography, thereby providing dual ontological lenses and epistemologies for ākongā

to draw on in their everyday lives. As highlighted by the COVID-19 pandemic, not all ākonga have ready access to smart devices (see Allen & Trinick, 2020) with GPS and digital map functions that they could use to navigate from place to place. In addition, the cheaper versions of these technologies often have unreliable network coverage and battery life. Therefore, utilising mātauranga to understand wayfinding processes could have immediate benefits for ākonga navigating through unfamiliar or disorientating environments, including at night. Investigating local or ancestral ururu whenua could also provide a sense of connection for ākonga with their ancestors as they complete their daily journeys. The ancestral connection could also be strengthened for ākonga who learned the Māori names of significant landmarks in their local area, which could have been renamed or commonly referred to in English.

Literature on the affordances of digital technology for showing and telling mathematical reasoning was reviewed to support digital technology use by ākonga and pouako digital pedagogies, as requested by both principals. The use of digital technology in the study also ensured the continuity of teaching and research during intermittent school closures due to the COVID-19 pandemic. As discussed in Chapters 2 and 3, utilising multiple representations to communicate mathematical thinking has learning benefits, particularly for L2 learners of the language of instruction (see Allen, 2015). Show-and-tell or screencasting apps provide opportunities for students and teachers to capture their mathematical communications using multiple representations and then share these with others (Williamson-Leadley & Ingram, 2013). These types of apps typically have a whiteboard feature that students can either draw on or type onto, the ability to capture or upload images, and the ability to record audio or video explanations. Unlike pen and paper or real-time show-and-tell presentations, student-created video presentations provide opportunities to watch students work through problems, giving more insight into their mathematical thinking processes (Larsen et al., 2018). Therefore, both kura participating in the research were supplied with Android tablets and screencasting software. Mapping software (Google Maps) was also used during the study to support ākonga spatial reasoning through visual representations. Students at both Kura Tahī and Kura Rua were also provided with exercise books and stationery packs to capture their written and drawn artefacts as appropriate to the activity or based on their own representation choices.

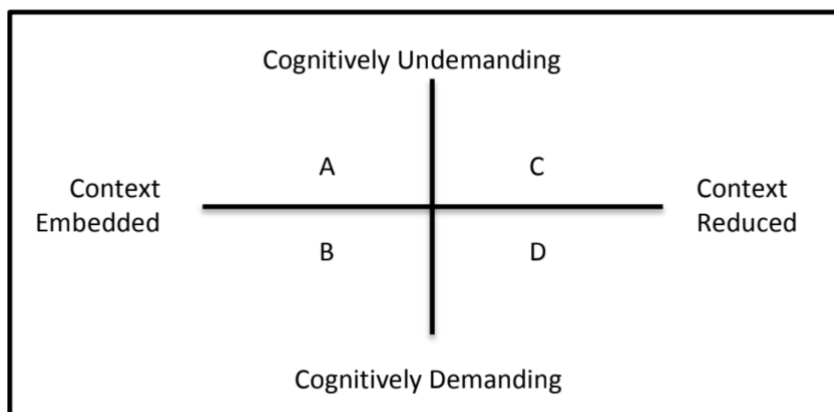
Teaching the Language of Numeracy and Spatial Reasoning

The diagnostic data collected at Kura Rua indicated that ākonga were still developing their ability to communicate numeracy knowledge through the specialised language of the pāngarau register. These data indicated that some ākonga had joined immersion education within the last 1–2 years, a period of significantly disrupted learning due to the COVID-19 pandemic. Therefore, pāngarau language development activities were identified for inclusion in lesson sequences. In Chapter 4, mātanga provided specific examples of grammatical and lexical features of the Māori language that express

uniquely Māori ways of quantifying, counting, locating, and problem solving. Of note was the range of contextual terms for quantification, the dependencies of locative terms on the position of the speaker, and the preference for passive sentence structures. While not explicitly included in pāngarau curriculum achievement objectives, teachers could model and reinforce these Māori ways of communicating information through pāngarau discourse. Opportunities for modelling to occur were also included in the lesson sequences. The mātanga interviewed in Chapter 4 also emphasised using narratives to code and disseminate quantification and spatial orientation information. The narrative devices of personification and symmetry were identified in the interview data and wayfinding literature as mnemonic tools, aiding information retention. Utilising these narrative techniques in pāngarau classrooms, where story or word problems are often employed to represent and contextualise mathematical concepts, could thus provide opportunities to strengthen te reo Māori and rebalance mātauranga and school mathematics in pāngarau classroom practices.

Story-based proportional activities supplemented with visual representations were discussed by me and the participant pouako at both kura to provide opportunities to develop language alongside pāngarau concepts. Story or word problems have a three-part structure: the set-up, a premise involving characters and location; the mathematical information needed to solve the problem; and a question (Gerofsky, 1996). Through the characters and location utilised, word problems provide a bridge between the mathematics classroom and activities practised in everyday life (Barwell, 2009). Word problems can be considered context-embedded mathematical tasks, as outlined in Cummins' (2000) well-known framework of academic language in Figure 8.

Figure 8. Framework for Cognitive and Contextual Demands (Cummins, 2000, p. 68).



The cognitive demand for word problems is generally increased or decreased by adjusting the difficulty of the mathematics. For L2 learners, the word problem context can be difficult to decipher (Barwell, 2009) because mathematical algorithms can be 'dressed up' in words. Therefore, word problems can be cognitively demanding context-embedded mathematical tasks, regardless of the

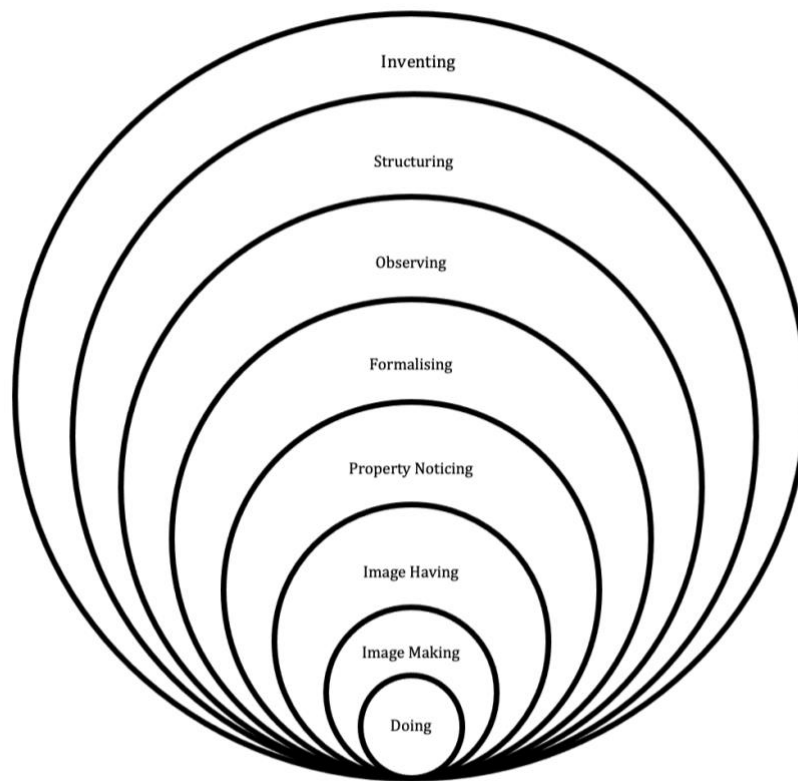
mathematics. Mathematical information can be represented visually or by using materials alongside the words to reduce the cognitive challenge of the word problem context. Illustrating the mathematics and the word problem context with visual and physical representations supports ākongā in simultaneously acquiring the mathematics and the language (Allen, 2015). Kaiako 4 at Kura Rua, for example, was interested in connecting real-world activities to algebraic concepts using word problems. Examples of this are presented in the teaching and learning inquiry section of this chapter. Using word problems also provided opportunities to examine teacher and student-created artefacts for occurrences of the uniquely Māori grammar and lexical features described in the literature and by the mātanga interviewed in Chapter 4.

Teaching Spatial and Proportional Reasoning Through Recursive Activities

The diagnostic interview data showed that students at both kura struggled to express proportionality orally or in diagrams. Therefore, Pouako Pāngarau and I suggested incorporating activities that reinforce proportional reasoning concepts. Presenting fractions and decimals using a variety of representations (spoken, written, drawn, symbolic, physical, and digital manipulatives), including continuous, discrete, and equivalent representations, can help students acquire this understanding (Allen, 2015). Multiplicative thinking was also identified as an area of difficulty, in particular, the range of strategies that can be used in division problems (see Appendix 3 for symbolic representations of the strategies expected at each stage of Te Uiui Aromatawai diagnostic interview). For example, huri kōaro (inverse operation strategy) could be used to convert a division problem to multiplication.

The Pirie and Kieren model (1989), a recursive theory of mathematical understanding, underlies the Numeracy Development Projects through which Te Uiui Aromatawai was created (Allen, 2015). According to Pirie and Kieren (1989), students can transition to more cognitively demanding activities by making connections between known facts and new knowledge. Sometimes referred to as the P-K theory (see Figure 9), this model illustrates the process of acquiring mathematical understanding from doing or performing learned actions to inventing or creating new knowledge. This process occurs through a series of recursive layers progressing from routine activities to those more cognitively demanding and providing opportunities to link known facts and new knowledge. When applied to the lesson foci of this study, multiplicative thinking supports the understanding of proportional reasoning which involves a multiplicative comparison between quantities and algebra (Hughes, 2002). Proportional reasoning problems are often represented spatially (Mulligan et al., 2018). Therefore, the focus on reinforcing proportional reasoning skills could also be used to introduce spatial representations, further preparing students to engage with uruuru whenua narratives.

Figure 9. P-K Model of Numeracy Acquisition (Pirie & Kieren, 1989, p. 8).



Based on the learning, focusing and teaching inquiries, the agreed school mathematics foci at both kura were proportional reasoning and spatial reasoning. The pāngarau register focus utilised multiple representations including word problems, numeracy and spatial information, and the mātauranga focus was uruuru whenua, narratives for wayfinding on land. Once these tripartite foci were decided, lesson sequences were collaboratively planned with pouako and tumuaki at both kura. The collaborative planning process ensured that the lesson sequence design addressed ākongā and pouako learning priorities. Professional development workshops focusing on using screencasting software were also conducted at both kura to support the development of pouako digital pedagogies, as requested by tumuaki.

The Teaching and Learning Phase

The first 10 weeks of the research at Kura Tahī involved weekly visits to the learning setting. The pouako at Kura Tahī were assigned the pseudonyms Pouako 1–3. Pouako Pāngarau, a specialist pāngarau teacher who supported the research, spent one hour with each of the two classes modelling the delivery of pāngarau curriculum content, the use of the specialised language of the pāngarau register, and the use of multiple representations to communicate pāngarau ideas. In the second 10 weeks of the programme, Pouako Pāngarau focused on modelling the delivery of spatial reasoning concepts and uruuru whenua narratives. Pouako 1 and Pouako 2 were then tasked with building on

these sessions in-between research visits. I spent one hour with each of the two classes during each of the weekly visits working alongside Pouako 1 and Pouako 2 to facilitate the use of the Android tablets and screencasting technology for capturing ākonga responses to the tasks assigned by Pouako Pāngarau. The final seven sessions were conducted and recorded via Zoom as the school site was physically closed due to COVID-19 pandemic restrictions. The Zoom sessions followed the same format where Pouako Pāngarau presented the pāngarau curriculum and mātauranga-based activities, which the students responded to using Zoom and digital whiteboard software. I supported Pouako 1 and 2 to facilitate the student response sessions. Kaiako 3 also provided digital technology assistance for these sessions.

Due to the distance of Kura Rua from the university, the research was conducted over three day-long visits. The first visit comprised the delivery of a professional development workshop for all teachers at Kura Rua focused on screencasting and other digital pedagogies for teaching pāngarau. The second visit was used to collect diagnostic assessment data. The final visit was towards the end of the proportional reasoning-focused lesson sequence. During this last visit, 15 Year 9/10 students took part in the learning session facilitated by Pouako Pāngarau, and I facilitated the use of the Android tablets and a range of software applications, including screencasting, to capture student responses. Zoom and Google Docs were utilised to collaborate on the sequence of lessons with Kaiako 4 and Kaiako 5 and provide activity ideas and resources for teaching in-between visits. The following section presents the pouako and ākonga-created artefacts collected during the research. The analyses include a critique of the efficacy of planned pedagogical actions in mitigating the conflicts that arose from explicitly addressing mātauranga, te reo Māori, and school mathematics in the pāngarau classrooms.

Ākonga and Pouako-Created Artefacts

Two types of ākonga-created and pouako-created artefacts were collected for the research. The first types of student-created artefact were word problems (both paper-based and digital) that utilised multiple representations of the pāngarau register. These artefacts addressed the proportional reasoning school mathematics focus and the pāngarau register focus. One of the considerations of the research was the use of multiple representations by pouako and ākonga in communicating mātauranga and pāngarau curriculum content, thereby supporting the simultaneous acquisition of pāngarau language, mātauranga and school mathematics (see Allen, 2015). In the first data set, ākonga selected an equation or algorithm and translated this into a word problem. Ākonga also represented their word problems visually. The combination of representations showed the ākonga abilities to represent an equation or algorithm using multiple representations of the pāngarau register. The second set of ākonga-created data involved the same process extended to include uruuru whenua, thereby addressing mātauranga, pāngarau register and spatial reasoning school mathematics foci. The artefacts collected in the second ākonga-created data set demonstrated ākonga abilities to code and decode

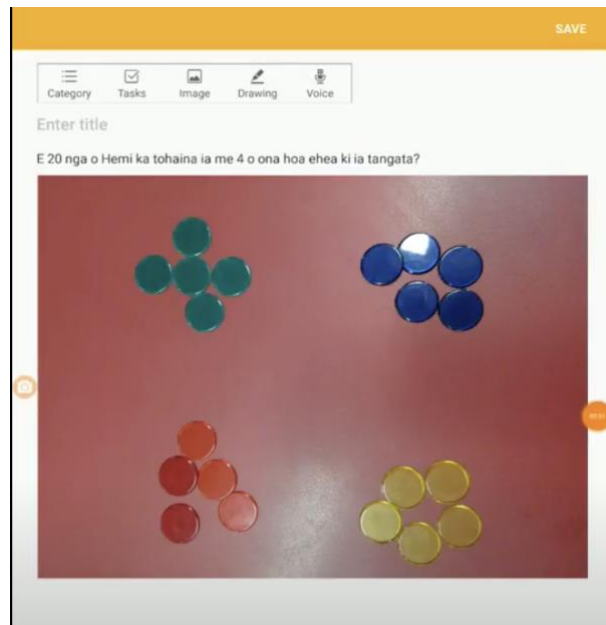
spatial information utilising multiple representations of pāngarau language. The analyses of ākongā-created artefacts are presented according to the tripartite foci of te reo Māori, mātauranga and school mathematics. The pouako-created artefacts included the lesson sequences for the proportional reasoning and ururu whenua lesson sequences and associated resources. These are presented in Appendix 4 so that they are easily accessible to pouako who may want to follow a similar process in illuminating mātauranga in their own classrooms.

Data Set 1: Proportional Reasoning Classroom Artefacts

In this section, ākongā-created artefacts for the proportional reasoning sequence of lessons are presented and analysed according to the tripartite foci of the cultural symmetry framework. The ākongā generally worked in pairs or small groups and used pen and paper or the memo application on the provided Android tablets to collate the different representations they produced. A memo app allowed ākongā to type word problems and equations, handwrite and create diagrams, record audio, and insert images. The ākongā then used the screencasting application on the tablets to record videos explaining the representations they had created or to capture videos of materials being manipulated, representing a problem and the solution method. I provided transcriptions and translations for ākongā-created artefacts.

The ākongā-created artefacts presented in the following sections illustrate ākongā abilities to represent multiplication, division, and proportional reasoning problems using multiple representations. The ākongā-created artefacts are presented according to the year levels they were in (e.g., Year 7/8 or Year 9/10). The sequence of samples for each class shows the different types of multiplication/division or proportional reasoning problems that ākongā decided to represent and capture using the Android tablets and apps. Initially, ākongā were encouraged to select simple problems that they were confident in solving. This ensured that ākongā understood the task and how to use the technology. This also provided an opportunity to identify areas of the pāngarau register that required further modelling by pouako. The ākongā then devised more complicated word problems as their confidence and accuracy increased, reflecting the recursive nature of the P-K pedagogical model (Pirie & Kieren, 1989). In addition to analysing written language choices and how mathematical thinking is communicated, the use of representations is also discussed. Particular attention is given to the analyses of grammatical and lexical aspects of the Māori language identified as uniquely Māori in the literature (Trinick, 1999, 2015) and mātanga interview data.

Figure 10. Ākonga-Created Artefact 1 – Kura Tahī Year 7/8.



In this work sample (Figure 10), four groups of five counters are arranged on the table surface. Each group of counters is a different colour.

Word problem 1:

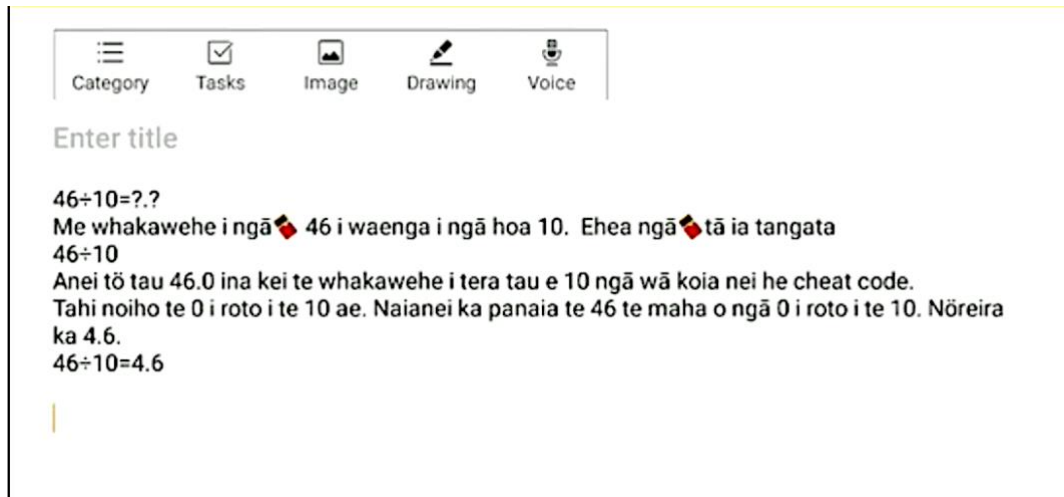
E 20 ngā o Hēmi; ka tohaina ia me 4 o ona hoa; ehea ki ia tangata?

[Hēmi has 20; he shares (distributes) these with 4 friends; how many for each person?]

In this word problem, some information is missing from the set-up. The word problem identifies 20 items, but the item is not named. Some of the key information needed to solve the problem is implied rather than stated explicitly. The main character Hēmi has 20 items and is sharing or distributing these with four friends. It is unclear in the word problem if Hēmi will retain any of the 20 items for himself. Terms in the pāngarau register that could be used to clarify the key information could be the use of the logical connector 'ki' instead of 'me'. This would indicate that the items are being distributed to a group of four friends rather than 'with' a group of four friends, as implied by 'me'. Mātanga 1 identified in Chapter 4 that there are many logical connectors in te reo Māori which can be used to make mathematical communication more explicit. This ākonga-created artefact shows that further modelling of the selection and use of logical connectors by pouako is required. The final part of the word problem is the question to be answered. In this case, 'ehea [sic] ki ia tangata', 'how many for each person' is asked. The material representation clearly shows four groups, so the combination of the word problem and the material representation is required in this case to understand ākonga thinking fully.

The passive term ‘tohaina’ is used in this example to denote sharing. The preference for passive constructions in problem solving was identified in mātanga interviews, although the term ‘e’ is usually used to denote the actor in a passive sentence. Therefore, to improve the grammatical use of the passive construction, ‘ka tohaina e ia’ could be used. However, the language choices made by ākonga in creating the word problem indicates that passive terms are being used in this classroom for problem solving, despite the preference for active structures for mathematics taught in English.

Figure 11. Ākonga-Created Artefact 2 – Kura Tahī Year 9/10.



In the ākonga-created artefact (Figure 11), ākonga provided an equation and illustrated the problem with chocolate bar emojis. The use of visual representations, such as emojis or counters, could alleviate the linguistic challenge of finding the correct words for the set-up, highlighting the superfluous nature of the set-up in many word problem contexts and potentially illuminating the underlying word problem structure for ākonga.

Word problem 2:

Me whakawehe i ngā (chocolate bar emoji) 46 i waenga i ngā hoa 10. Ehea ngā (chocolate bar emoji) tā ia tangata.

[(You) divide 46 (chocolate bar emoji) between 10 friends. How many (chocolate bar emoji) belong to each person?]

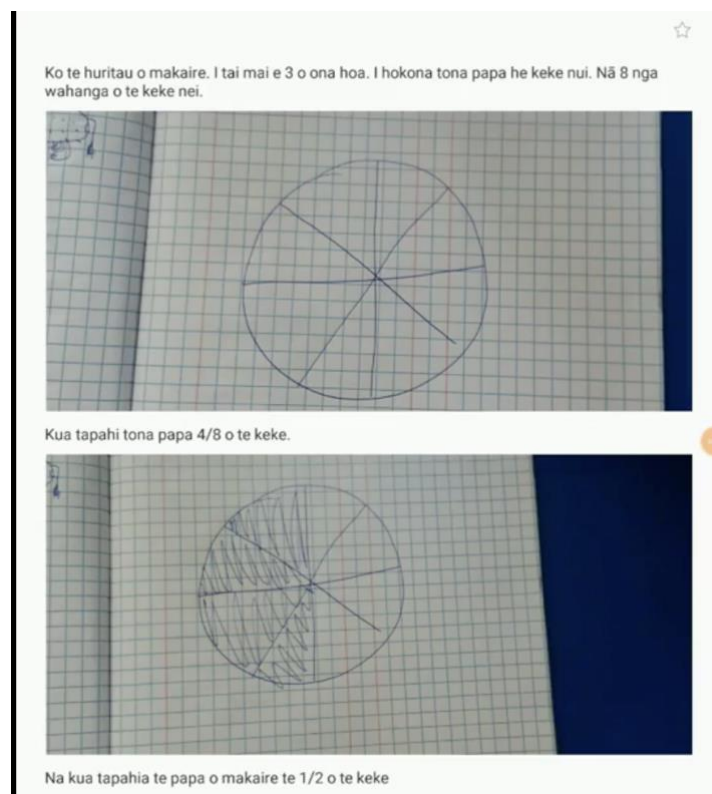
The language in this word problem is more sophisticated than the first. This artefact shows the preference for passive sentence structures in te reo Māori. The word problem begins with the passive command ‘me’ (which is different to the conjunction ‘me’, which can be used like ‘and’ or ‘with’ in English). The use of the passive command implies that the reader is the character in the word problem, and this encourages the reader to imagine themselves dividing 46 chocolate bars between 10 friends. The set-up also uses the term ‘whakawehe’ (to divide) and the words ‘i waenga i ngā hoa’,

indicating that the chocolate bars are to be divided amongst the 10 friends, thereby excluding the reader. After the set-up, numbers and operations are given, and a full stop is used before the question appears. This creates a clear separation between the mathematical information needed to answer the question and the question itself—indicating an understanding that word problems require information to work with and a question to answer.

Interestingly, the possessive term *tā* (belonging to) is used instead of the term *mā* (for). So, rather than asking “How many (chocolate bar emojis) *for* each person?” or “How many each person will get?”, the question asks “How many chocolate bars *belong to* each person?”. While this is not mathematically inaccurate, like the use of the passive command ‘me’, it is different to more common ways that word problem questions are generally posed in English. This could show that the student is still developing accuracy in using the *pāngarau* register or creatively using the language they have to express themselves.

The *ākonga* then provided a type of explanation of an algorithm or ‘cheat code’ for dividing by 10, which is shifting the decimal point to the right of the number of place value columns containing the place holder ‘0’ in the dividend. Again, the *ākonga* addresses the reader directly and provides step-by-step instructions on how to use the ‘cheat code’ to solve the problem.

Figure 12. *Ākonga*-Created Artefact 3 – Kura Tahī Year 9/10.



In Figure 12, ākongā represent the concept of equivalent fractions with a word problem and a drawn diagram of a cake cut into equal parts. The sophistication of the language used has increased further.

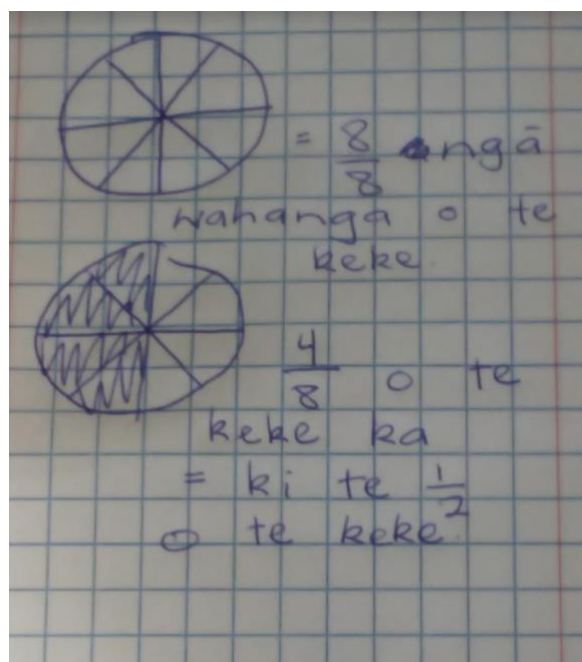
Word problem 3:

Ko te huritau o Makaire. I tae mai e 3 o ona hoa. I hokona tona papa he keke nui. Nā 8 nga wahanga o te keke nei. Kua tapahia tona papa $\frac{4}{8}$ o te keke. Na kua tapahia te papa o makaire te $\frac{1}{2}$ o te keke.

[It is Makaire’s birthday. Three of their friends have arrived. Their father has bought a large cake. There are 8 pieces of the cake. Their father has cut $\frac{4}{8}$ of the cake. Therefore, Makaire’s dad has cut $\frac{1}{2}$ of the cake.]

The set-up has been split into three sentences providing detailed information about the birthday party with three guests. The number of guests and the host implies the need for four pieces of cake. The quantities of 8 equal pieces and $\frac{4}{8}$ are then provided in the following two sentences. Finally, instead of a question, the ākongā state that by cutting four-eighths of the cake, Makaire’s father has cut half of the cake.

Figure 13. Ākongā-Created Artefact 4 – Kura Tahī Year 9/10.



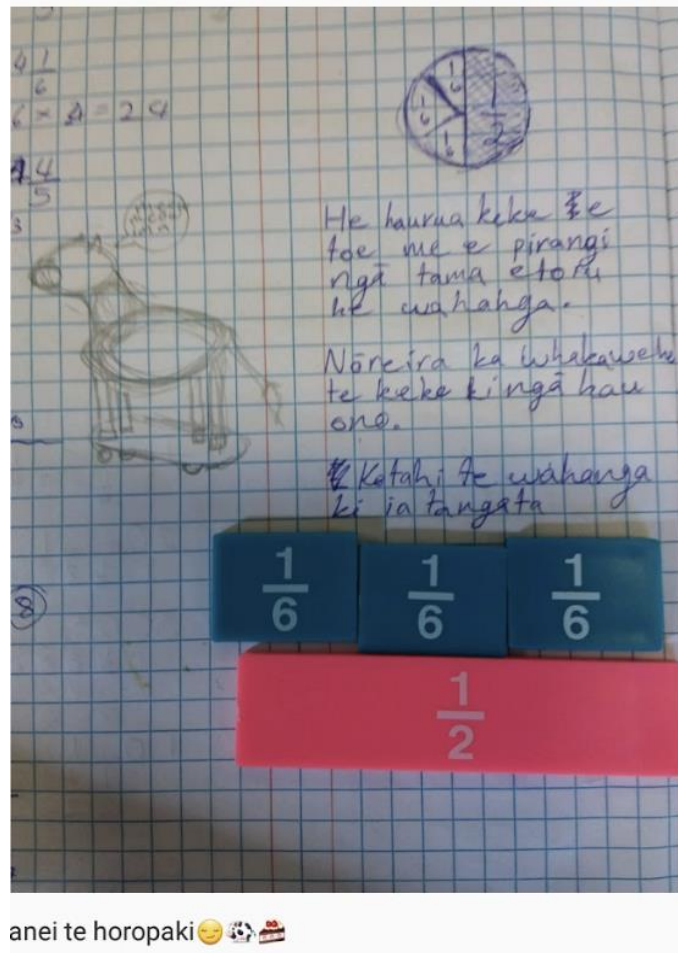
Word problem 4:

$\frac{8}{8}$ ngā wahanga o te keke. $\frac{4}{8}$ o te keke ka = ki te $\frac{1}{2}$ o te keke.

[$\frac{8}{8}$ pieces of cake. $\frac{4}{8}$ of the cake is = to $\frac{1}{2}$ of the cake.]

This fourth ākongā-created artefact (see Figure 13) combines a diagram representation with written detail interspersed with symbolic representations. Interestingly, the symbolic representations do not disrupt the grammar of the word problem, and the ākongā have represented the mathematics succinctly, drawing together three different representations.

Figure 14. Ākongā-Created Artefact 5 – Kura Tahī Year 9/10.



Word problem 5:

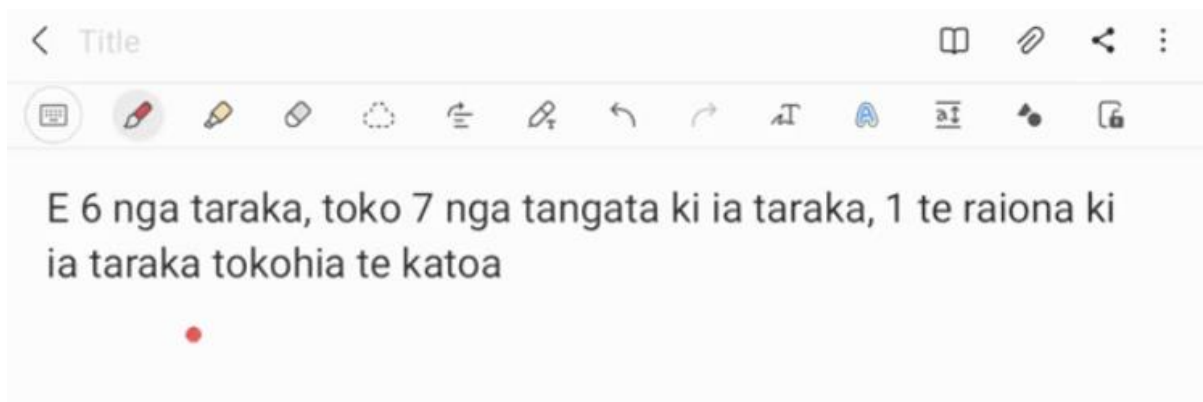
He haurua o te keke e toe mai me e pirangi ngā tama e toru he wahanga. Nō reira ka whakawehe te keke ki ngā hau ono. Kotahi te wahanga ki ia tangata.

[There is half a cake left and three boys want a piece [each]. Therefore, the cake is divided into sixths. One piece for each person.]

In word problem 5 (Figure 14), ākongā applied the concept of equivalent fractions to the division problem of $\frac{1}{2}$ divided by 3. Three different representations have been used to do this. A drawn diagram

of a cake cut into halves and then one of the halves divided into three equal parts. A word problem thus applies this to a real-world situation and the use of physical manipulatives, showing that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$. While the quantification term 'tokotoru' could be used here to denote three people specifically, this word problem uses the quantification term 'e toru'. This is not incorrect and still shows the use of the 'e' to distinguish between the term toru (the counting number 3) and the term. In English, counting and cardinal numbers are the same. Along with previous examples, the evidence across both classrooms at Kura Tahī indicates that despite the prevalence of the English language outside the school setting and the potential for language interference, uniquely Māori ways of quantifying and problem solving using passive structures are evident in spontaneous student language choice in both classrooms. However, the verbal particle 'ka', preferred by Mātanga 3 for indicating cardinality when counting was not present in ākongā-created artefacts, could indicate that 'e' has become the preferred term in classroom discourse or has continued to be spoken into being.

Figure 15. Ākongā-Created Artefact 6 – Year 9/10 Kura Rua.



Word problem 6:

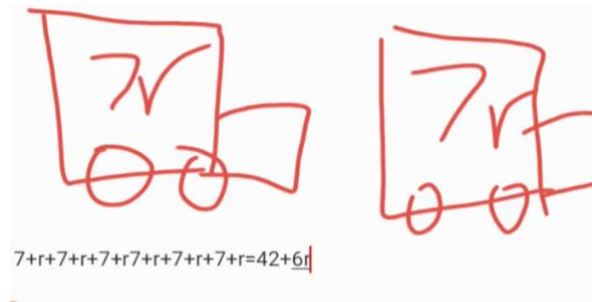
E 6 nga taraka, toko 7 ngā tangata ki ia taraka, 1 te raiona ki ia taraka tokohia te katoa

[There are 6 trucks, there are 7 people in each truck, there is one lion in each truck how many (people) altogether?]

At Kura Rua, Kaiako 4 worked with students to begin applying their multiplication and division strategies to algebraic contexts. This word problem shows the ākongā trying to represent different quantities in a word problem that requires multiple steps to solve (see Figure 15). In this problem, the use of 'tokohia' in the question implies an answer that specifically totals the number of people. As discussed previously, the prefix 'toko-' denotes a quantity of people in Māori. This specificity is not present in the English language. However, the ākongā drawn diagram and equation in Figure 16 indicate that they are asking for the number of people and the number of lions. The student used the transliteration 'raiona' as there is no native or recently created term for this animal. It is not possible

to make a judgement on whether students consider their animal companions as people. However, the importance of animal companions in Māori narratives such as uruuru whenua investigated in the literature and mātanga interviews indicate the potential that this problem is being communicated through a uniquely Māori worldview. The appearance of the question term ‘tokohia’ indicates that uniquely Māori ways of communicating about quantities are also present in the spontaneous language of students in this class at Kura Rua.

Figure 16. Ākonga-Created Equation and Diagram – Year 10 Kura Rua.



Challenges to Creating Digital Word Problems in Māori

There were some issues with using the memo app on the Android tablets to create word problems in Māori. One issue was the device auto-correcting the ākonga typed Māori language word problems by inserting English words. Once the ākonga identified this issue, I taught the ākonga how to turn this function off. Also, not all ākonga knew how to add macrons to typed Māori words and this was also modelled by me and ākonga who already knew how. However, not all typed ākonga-created artefacts contain macrons. Because of this, no comments have been made about missing macrons in ākonga-created word problems. A further challenge for analysing ākonga-captured work samples is the quality of images. Screen capture software diminishes the quality of images as they are formatted for easy electronic sharing. If the students had not captured clear images, the lowered quality of the screen-captured videos exacerbated any initial blurring. This has lowered the number of images of suitable quality to publish in the written format of this thesis. However, due to the high volume of ākonga-created and captured artefacts, this has not been an issue in selecting a range of ākonga-created artefacts that illustrate students’ learning of mātauranga, te reo Māori, and pāngarau curriculum content.

Summary of Findings for Data Set 1

A significant proportion of the research project on strengthening the teaching and learning of numeracy could be seen by the challenges pāngarau teachers face in creating space for mātauranga in classrooms. However, the request by tumuaki for a numeracy focus provided opportunities to examine aspects of grammar and terminology for quantification that were identified in the literature and by mātanga interviewed in Chapter 4 as challenging for both pouako and ākonga. The tumuaki concerns

about numeracy, while likely connected to student achievement reporting, also seemed to be motivated by the COVID-19 pandemic context. There had been considerable disruptions to ākongā learning during the school year preceding the research, including government-enforced school closures (see Allen & Trinick, 2021; Hunia et al., 2020). The research was conducted during the 2021 school year, which was also significantly disrupted by COVID-19. The indicative post-test data for the three ākongā at Kura Tahi showed that some shifts in understanding of proportional reasoning had occurred during the proportional reasoning school mathematics foci sequence of lessons. The three ākongā tested volunteered to go first and seemed very eager to show what they knew and how their understanding had improved since the first round of diagnostic testing. The three diagnostic interviews also took longer than anticipated, as ākongā provided extended explanations for their strategies and solution methods. Unfortunately, the post-testing for the rest of the ākongā at both settings could not proceed during the research period due to ongoing school closures. Despite this, the three ākongā improved their understanding of proportional reasoning when retested according to Te Uiu Aromatawai. Improvements in their utilisation of the pāngarau register to explain their thinking and solution methods were also observed during the testing.

While the proportional reasoning activities may not have explicitly focused on aspects of mātauranga, the use of word problems provided interesting insights into how ākongā viewed their worlds, such as the use of the prefix toko- to quantify people and animals. Toko- is generally used to quantify people. Some would argue that within the Māori worldview, animals or pets hold the same level of significance as family members, and therefore the use of the term is warranted. It would be interesting to explore this further with other ākongā to see if they also hold this view. The use of the passive command 'me' to invite the reader to work through the word problem was also an interesting language choice that could be explored further.

The sequence of work samples, which are presented sequentially from Year 7/8 through to Year 9/10, show increasing sophistication in the language choices used for the word problems. This seems to show that the students are developing more sophisticated ways of using the pāngarau register to communicate mathematical ideas as they progress through year levels. This development was particularly evident in the data collected at Kura Tahi. The role of pouako in modelling and scaffolding this language use is evident in the data collected. The same progression could not be shown for data collected at Kura Rua due to the one-day ākongā-created artefact collection period which was a study limitation. However, the same language development is likely to occur over time.

Also present in the ākongā-created word problems were uniquely Māori ways of using the pāngarau register, as identified in the literature and interviews with mātanga pāngarau. These aspects of the register do not have English equivalents. There was concern in the literature (Barton & Fairhall, 1995) and from the mātanga that this language would be lost through English-language interference. For

example, pāngarau discourse prefers passive structures when communicating about problem solving. However, the equivalent discourse in English-language mathematics preferences active structures. When this is coupled with the high number of second language learners who are also teachers of pāngarau, there is a danger that these uniquely Māori ways of communicating mathematically are being lost. For example, the verbal marker ‘ka’, used to indicate cardinality when counting, was not present in the ākongā-created artefacts. Therefore specific illumination of the use of ‘ka’ in curriculum and classroom resources might be required to perpetuate its use in classroom discourse. There is also anecdotal evidence, as discussed by the mātanga, of pouako translating English sentences into Māori in classrooms and foregoing Māori grammar altogether. However, ākongā-created artefacts collected at both kura showed that a range of uniquely Māori terms and structures were being used spontaneously by ākongā. This could indicate that these ākongā are thinking about quantifying and problem solving in Māori. While not definitive, this is encouraging and also helps further validate the assertions of the pāngarau register elaborators about the syntactical strength of te reo Māori.

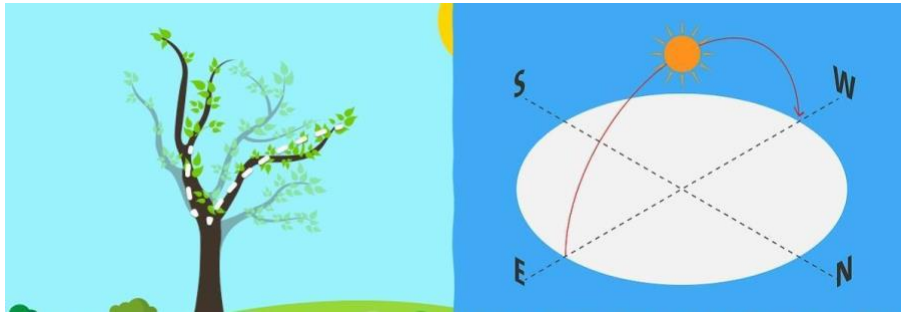
Data Set 2: Uruuru Whenua Classroom Artefacts

The data presented in this section demonstrate the ways teaching uruuru whenua addresses conflicts between mātauranga and school mathematics in pāngarau classrooms. A sequence of lessons focusing on uruuru whenua and underlying spatial reasoning concepts was delivered as part of the weekly sessions with both the Year 7/8 and Year 9/10 classes at Kura Tahi. A one-day session was conducted at Kura Rua where the Year 9 and Year 10 students participated. Following this learning sequence, both learning settings experienced government-mandated COVID school closures. During this period, Pouako Pāngarau and I supported both learning settings with advice on delivering learning via Zoom. In the case of Kura Tahi, Pouako Pāngarau and I modelled the continuation of the uruuru whenua inquiry project through Zoom delivery and spatial reasoning activities that could be completed at home.

Uruuru Whenua Resource Examples

As the teaching of uruuru whenua is not explicitly included in pāngarau curriculum and classroom resourcing, a selection of resources and activities is discussed here and further exemplified in ākongā-created artefacts. During the uruuru whenua lesson sequence, the ākongā and pouako discussed a range of environmental indicators for direction, location, and orientation, such as how trees grow away from the prevailing wind and the path from east to west (see Figure 17). Specific terms that exemplify mātauranga were also discussed. For example, muri (back or behind) and raki (northerly wind) are both terms used to indicate north, implying the direction of migration of Māori ancestors from the Pacific. Cardinal direction terms and representations were also discussed and examined.

Figure 17. Tree Growth Animation and Path of the Sun Diagram Indicators of Direction (The Natural Navigator, 2021).



A range of digital representations of uruuru whenua was also explored. These included animations, live drama, and map representations (ArcGIS or Google Maps). Figure 18 shows screenshots of digital representations of the journey of Poutini and Waitaiki (see Appendix 4 for a list of further resources).

Figure 18. Animated Video (Maipi, 2022) and The Legend of Poutini Map (Google Maps, n.d.) of Poutini Uruuru Whenua Poutini.



The following series of student-created artefacts illustrate the types of activities completed by the ākongā and their developing understanding of uruuru whenua and spatial reasoning. The students were instructed by Pouako Pāngarau to create their own uruuru whenua which utilised landmarks that were significant to them. Students were also encouraged by Pouako Pāngarau to connect these landmarks through the main narrative structures utilised by uruuru whenua. This gave the students the opportunity to develop their understanding of the mnemonic features of uruuru whenua that their ancestors utilised. As with the proportional reasoning ākongā-created artefacts, I supported ākongā to represent their understandings of uruuru whenua via the Android tablets and apps. For this second sequence of lessons, ākongā were shown how to identify, mark and elaborate on significant landmarks contained within uruuru whenua by dropping pins, creating lists, and adding notes to the landmarks in Google Maps.

Ākonga-Created Uruuru Whenua Artefacts at Kura Tahī

During the last session before school closures at Kura Tahī, students were instructed by Pouako Pāngarau to create a diagram of their location in reference to significant landmarks that align with cardinal directions. The first ākonga-created artefact, Figure 19a, shows four distinct geographical locations. The ākonga-created artefact, Figure 19b, shows nine locations. The cardinal directions have also been added to the diagrams. Students used the diagrams and Google Maps to create their own uruuru whenua or chase story, as characterised by Mātanga 1 in Chapter 4 and the literature on Puluwat navigators in Chapter 5.

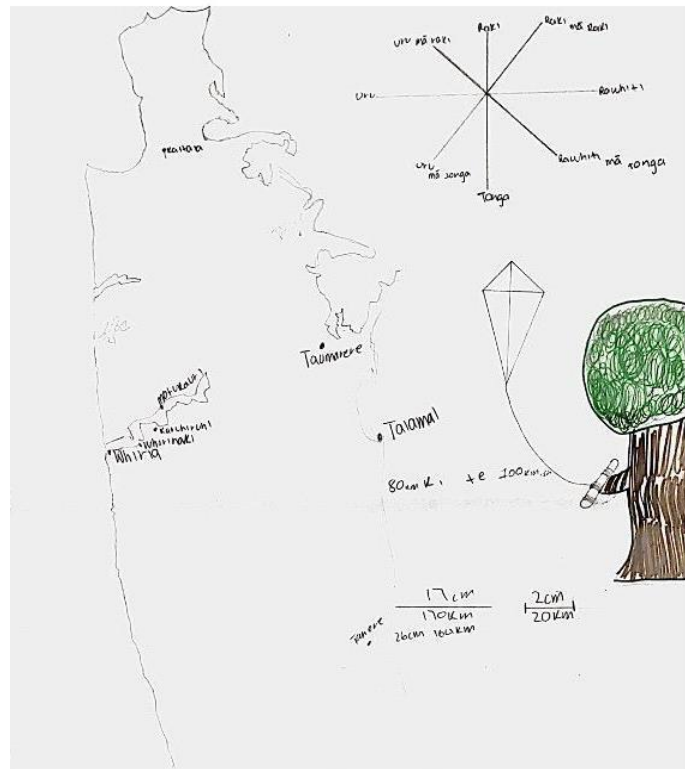
Figure 19. Ākonga Created Diagrams of Landmarks Indicating Cardinal Directions.



Ākonga-Created Uruuru Whenua Artefacts at Kura Rua

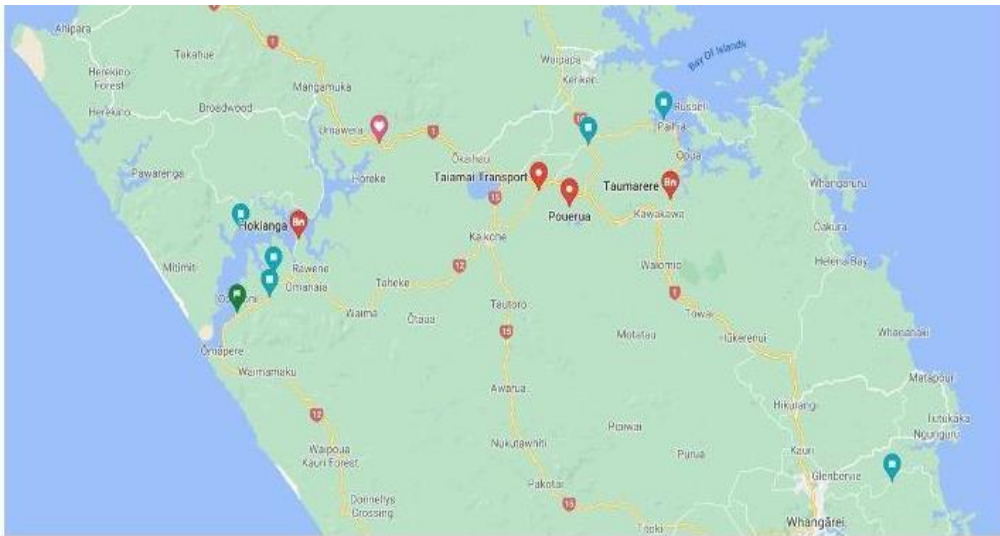
Ākonga at Kura Rua examined the characteristics of the uruuru whenua about Tūhoronuku which outlines the geographical boundary markers of two adjoining regions. This was supplied by Kaiako 4. Ākonga had prior knowledge of this uruuru whenua and familiarity with some landmarks in the wider geographical region. During the one-day session at Kura Rua, ākonga were instructed by Pouako Pāngarau to represent the significant landmarks of the uruuru whenua in a hand-drawn map (see Figure 20). Ākonga were also instructed by Pouako Pāngarau to try and create a scale and cardinal directions on their hand-drawn maps. Creating a scale required the ākonga to use the proportional reasoning and multiplicative thinking they had been developing in the preceding lesson sequence. Ākonga checked their hand-drawn map against Google Maps to ensure their estimated distances and directions between geographical features were accurate. If this was not the case, ākonga could adjust these accordingly. The students could also refer to Google Maps for any locations they were unfamiliar with.

Figure 20. Hand-Drawn Map Representing Significant Uruuru Whenua Landmarks.



In the next activity, I supported ākonga to represent the uruuru whenua digitally by creating a list of pinned locations on Google Maps (see Figure 21) and annotating these with notes of the location's significance. However, the list of Google Map locations did not always reflect those in the uruuru whenua. This was because the Māori names for significant locations in the uruuru whenua were not commonly used or official names. The use of Google Maps also caused difficulty when more than one location with the same name or a business name was identified instead of a landmark. In this case, expert guidance on the official or commonly used names of geographical points described in the uruuru whenua would have been beneficial. This prompted a discussion with students about who held this knowledge in the kura community. The students were able to name some of the kaumātua in their community that they could visit and ask for clarification on the landmarks that were more difficult to identify. Once completed, the digital maps could also be shared as learning resources for the kura community.

Figure 21. Student-Created List of Pinned Locations.



Summary of Findings for Data Set 2

The uruuru whenua activities provided a rich context for teaching mātauranga alongside school mathematics concepts of spatial reasoning. The students could represent their understanding of these narratives in diagrams, hand-drawn maps, and digitally using Google Maps. The students at Kura Rua (Years 9/10) were very engaged in the day’s activities about Tūhoronuku. A group of students stayed in the classroom during their lunch break to continue working on the hand-drawn map activity. When I asked why they did not take a break and come back after lunch, they said they really enjoyed doing ‘this stuff’. Ākonga at both kura were also encouraged to explore the mnemonic functions of the narrative genre by creating their own chase stories, which included landmarks that were significant to them.

Summary of Classroom Data

The data presented in this chapter shows examples of activities used to encourage the development of proportional and spatial reasoning while addressing conflicts between mātauranga and school mathematics. The student-created artefacts provided evidence of students communicating their understanding of multiplication, proportional reasoning, location, and direction alongside their reasoning strategies. As discussed earlier in the chapter, communicating mathematical thinking orally or in written words is difficult for students who are learning the language of instruction (Allen, 2015; Barwell, 2009). Therefore, exemplifying the specialised language of the pāngarau register and connecting this to real-world contexts through word problems and uruuru whenua could support students in acquiring te reo Māori, mātauranga, and school mathematics simultaneously. While the data presented here indicate that students gained further confidence in communicating and

representing both proportional reasoning and uruuru whenua concepts, the post-testing was not completed due to COVID-related school closures. Therefore, further research is required in this area.

One of the challenges identified for teaching and learning word problems and uruuru whenua in pāngarau classrooms was the linguistic limitations of digital technology. For example, the memo app autocorrected Māori words to English and had to be switched off. There was also difficulty locating Māori named landmarks or geographical points within Google Maps. The ākongā and I were able to strategise solutions to both problems. In the case of Māori names for landmarks, the ākongā suggested members of the kura community with expert knowledge. This is a significant consideration for this type of activity. Expert guidance is required when utilising Māori oral and visual representations in classroom teaching. These could be provided through curriculum guidelines and support from local knowledge experts and kaitiaki. If not, the aka tāepa could be promoted over the aka matua where things are termed ‘Māori’ because they are the most familiar, as asserted by Mātanga 4 in Chapter 4. This is particularly likely when the aspects of mātauranga that are well documented reflect Western science concepts because they have been documented primarily by non-Māori or non-expert authors.

The following chapter further theorises the data presented in Chapters 4–6 through the lenses of Bourdieu’s (2010) field, habitus, and cultural capital. This provides a macro perspective on the mātauranga interview data, wayfinding literature and classroom data, and the potential of the findings of this study to further indigenise pāngarau curriculum and classroom practices. Finally, Chapter 8 provides conclusions and recommendations. Included in Chapter 8 is a critique of the cultural symmetry framework when used as both a pedagogical tool and a methodology for researching the indigenisation of school mathematics curriculum and classroom practices.

7. Matapaki | Discussion

This study examined the opportunities, challenges, and tensions for further indigenising pāngarau curriculum and classroom practices. This chapter summarises the analysis of data from semi-structured interviews with mātanga pāngarau, Māori and Pacific wayfinding representation literature and pāngarau classroom artefacts. The data presented here were analysed through the lenses of the cultural symmetry framework (Meaney et al., 2021) and teaching as inquiry (MoE, 2020) in the preceding data chapters. In this chapter, Bourdieu's (2010) lenses of field, habitus, and cultural capital are used to further interrogate the macro (policy and curriculum) level implications of the study findings.

Bourdieu's (2010) habitus provides an interface between personal agency and external structures to enhance our understanding of how macro-structures and micro-agency interplay (Costa & Murphy, 2015; Jorgensen, 2011; Pöllmann, 2021). Bourdieu's (2010) theory of agency is an unsteady property that emerges through shifting interactions of capital, habitus, and field. Habitus is defined as the set of internalised behaviours, perceptions, and beliefs that individuals carry with them (Costa & Murphy, 2015). Misrecognition can occur when the habitus of an individual or marginalised group internalises the pejorative views of society towards them (Bourdieu, 2010). Jorgensen (2011) likened this to the internalised deficit beliefs of Indigenous students due to the marginalisation of Indigenous knowledges and ways of knowing in mathematics education. Marginalisation is achieved through objective structuring practices such as curriculum and assessment (Jorgensen, 2011). The marginalisation of Indigenous ways of knowing and being in curriculum and assessment promotes deficit beliefs about Indigenous knowledges (Jorgensen, 2011).

Subjective structuring occurs when Indigenous students and their teachers internalise deficit beliefs about Indigenous knowledges, often extending the deficit beliefs to students' mathematical abilities (Jorgensen, 2011). Teachers' low expectations of academic success have been linked to lower academic achievement for Māori students (Bishop et al., 2009; Turner et al., 2015), thereby ascribing a knowledge debt (Aikenhead, 2018). Similarly, this thesis argues that cultural debt is ascribed to Indigenous students through the objective structuring practices of schooling, such as the exclusion of valued Indigenous cultural capital from curriculum and assessment. Cultural capital is converted to social and economic capital beyond schooling (Costa & Murphy, 2015). Therefore, cultural capital debt limits Indigenous peoples' agency in converting valued cultural capital into necessary social and economic capital beyond schooling. In turn, Indigenous peoples' trajectories are negatively impacted, limiting economic and social agency throughout their lives.

Objective Structuring Practices Limiting Mātauranga in Pāngarau Curriculum

The data showed clear differences between how mātauranga and the pāngarau curriculum are structured. Barton and Fairhall (1995) discussed the term pāngarau and how it illuminates the relationality of mathematics concepts in synergy with Māori worldviews. Mātanga 2 and 3 explained that the meaning of the created term pāngarau, many relationships, provided a connection between mātauranga and school mathematics. Mātanga 2 discussed the importance of relational whakapapa frameworks used to organise mātauranga:

Nō reira ko tēnei mea te whakapapa, he honohono kia mōhio ai te tangata ki ōna pānga. Ki tērā āhuatanga, ki tērā āhuatanga o te ao.

[Hence, it is through whakapapa, that a person can understand their relationships. To each and every aspect of the world.]

However, the scientific ontological concern with definition and classification (Govender & Mudzamiri, 2022; Pöllmann, 2011; Smith, 2012) arguably still pervades the structure of pāngarau curriculum content. Mātanga 3 believed that the pāngarau curriculum structure, and the way pāngarau is taught in classrooms, followed a disconnected piece-by-piece approach in opposition to its pāngarau title:

Ko te mate kē pea i roto i te kura, ko tēnei mea te hīkoi, hīkoikoi nei. Rua, tahi, tēnei, kātahi ko tēnei, kātahi ko tēnei. Ka ngaro tērā hanga o te titiro whānui. Anā, engari, ko te pai o te reo Māori me pērā. Ka kōrero i te pikitia nui kātahi anō, ka kōrero i ngā wāhanga ititi, rikiriki i roto i tērā.

[The problem, perhaps, in schools is the steps, the progressions. Two, one, this then that, and then this. So you lose the ability to see the big picture. But then, perhaps the strength of the Māori language, you need to [see the big picture]. You describe the big picture, then you talk about the smaller components within that.]

The literature on the pāngarau curriculum showed that the curriculum development process was constrained by government structures, resulting in the pāngarau curriculum reflecting the structure of the English-language document (McMurchy-Pilkington, 2004; Trinick, 2015). However, the pāngarau curriculum development process also appropriated the funding and status of school mathematics. This appropriation of economic benefit and status (Bourdieu, 2010) enabled groups of Māori experts to debate and discuss desirable and undesirable changes to te reo Māori and mātauranga. For example, the curriculum writers and pāngarau register elaborators debated using the existing term of tātai as the title of the new school mathematics curriculum. Tātai was the original title of the school mathematics

curriculum (Trinick, 1999), and Mātanga 3 believed the term signalled a uniquely Māori cultural lens and this has continued to be a point of contention:

Ko te kupu tino Māori rā, ko te kupu tātai. He whakapapa tōna. Pāngarau, he mea tito noa nei. Heoi anō kotahi anō te puka, ka puta ka pāngarau.

[The more Māori word is the word ‘tātai’. It has a whakapapa. Pāngarau, is just a made-up word. However, there is just the one book, and the word used is pāngarau.]

...kei te ngana tonu kia whānui haere te ao tātai Māori rawa nei.

[...if we stay determined to broaden the scope of the Māori way of mathematics.]

Tātai practices exemplified by Rikihana (1990) in the bilingual classroom resource, *Tikanga Tau, Tikanga Āhua, Tikanga Tātai, Tikanga Mahi, Tikanga Huinga*, included whakapapa (geneological tables) and the associated personified narratives of environmental phenomena. Rikihana’s (1990) resource also discussed the broad categorisations of mātauranga into three uruuru (groupings). The wayfinding representation literature presented in Chapter 5 also discussed the use of whakapapa, personified narratives, and ordered pairs as pedagogical and mnemonic devices. Navigators learned the genealogical and narrative connections between constellations and other environmental phenomena to discern spatial orientation information (see Chauvin, 2000; Finney, 1986; Riesenberg, 1972). However, the first and subsequent pāngarau curriculum documents do not explicitly address uniquely Māori concepts and have instead simply mirrored the structure of the English-language mathematics document.

The meaning of tātai had been extended through common usage to include arithmetic, as demonstrated by Mātanga 3 in the phrasing ‘ngā tikanga o te tātai Pākehā’ (the algorithms of Pākehā arithmetic). The examples of Whaitiri quantifying and grouping kūmara in the Tāwhaki narratives and tauutuutu (alternating speakers at a formal gathering) were given by Mātanga 3 as routine practices of tātai, which had synergies with school mathematics arithmetic. Mātanga 3 clearly preferred tātai as the title for the first school mathematics curriculum, thereby retaining and expanding the meaning of the native term—a strategy used in the pāngarau register elaboration (see Trinick, 2015). However, after robust debate, the term pāngarau was coined and then selected by Te Taura Whiri i te Reo Māori, a state agency led by Māori experts, as the curriculum title (Barton et al., 1998).

Perhaps the decision to embrace the new term pāngarau as the curriculum title reflected the structural constraints imposed on the curriculum writers. It may not have been appropriate to title a school mathematics curriculum with achievement indicators imported from countries with colonial legacies, such as the UK (McMurphy-Pilkington, 2004), as tātai. Particularly when the wide array of tātai

practices beyond arithmetic were excluded by design. Where visual representations of environmental phenomena have been incorporated into pāngarau classroom practices, such as kōwhaiwhai (Allen & Trinick, 2022), there did not seem to be sufficient flexibility in pedagogical practices and curriculum achievement outcomes to address mātauranga explicitly. Instead, the use of kōwhaiwhai was only seen as an aid for the acquisition of the school mathematics concepts of transformation geometry, as explicitly stated in pāngarau curriculum.

A further structural constraint of school mathematics curriculum is that it is essentially static between iterations. Much of the focus of school mathematics is disseminating the universal truths of mathematics, such as arithmetic facts and Euclidean geometry (Bishop, 1990), and assessing whether or not students have grasped these concepts. When asked if ākongā in Māori immersion schooling were becoming proficient in mātauranga, Mātanga 4 concluded that it was challenging to make this judgement. Mātanga 4 believed that some students would deeply understand mātauranga, and most would have a general understanding:

Mō ētahi [ākongā] āe. Mō te nuinga ... kāore pea e tino mārama ki te hōhonutanga o te whakaaro Māori.

[For some [ākongā] yes. For the majority ... perhaps they don't entirely understand the deeper Māori concepts.]

However, for Mātanga 4, it was easier to be confident about students' mathematical understanding than any depth of mātauranga:

...te nuinga kei te mārama ki te ariā pāngarau. Engari kāore pea au e kaha te kī ko te nuinga e mārama ana ki te whakaaro Māori.

[... but most understand the mathematical concept. But, perhaps I shouldn't really say that the majority understand the Māori way of thinking.]

This is unsurprising as resources, guidance, and training are provided for assessing mathematical understanding, thereby objectively structuring what is considered fixed and universally valued as the cultural capital of school mathematics.

In contrast, mātauranga is continually evolving (Cooper, 2012; Durie, 2003; Smith et al., 2016). Mātanga 1 discussed two distinct changes in the practice of wayfinding narratives, uruuru ao, over time. The first change was the movement away from sea voyaging to land-based travel by Māori in the 18th century (Ruggles, 1999):

The one that we have in NZ that is still part of our narrative is Te Wheke o Muturangi where they chase the wheke from Hawaiki to Aotearoa. Then it becomes a uruuru whenua. It becomes a land story. So, it transitions from a sea one to a land one.

This spurred a shift in the context of wayfinding narratives but not necessarily a change in their characteristics. Taonui (2015) identified the characteristics of land-based exploration narratives as providing boundary markers and routes, marking geographical occupation. Taonui (2015) also observed the mnemonic functions of the exploration narratives genre. Whereby, the journeys of multiple ancestors were collapsed into one narrative or large geographical areas were omitted from a journey through supernatural movement. Both characteristics made the pertinent information, the *aka matua* as expressed by Whaitiri in the Tāwhaki narratives, easier to remember.

The second shift in the practice of uruuru ao and uruuru whenua identified by Mātanga 1 was the acquisition of new technologies post-contact with Europeans, such as writing (Jones & Jenkins, 2011), and now GPS and GIS:

...with the use of modern technology they kind of got lost.

This lessens the need to code and memorise wayfinding information in narrative form. However, these technologies do not necessarily lessen the value of the mnemonic and pedagogical characteristics of wayfinding representations, such as uruuru whenua.

The third shift discussed by Mātanga 1 was the Christianisation of *karakia*, whereby *karakia* are often likened to Judeo-Christian prayers (Williams, 2019) instead of wayfinding representations:

I have been with [Kaumātua 1] and [Kaumātua 2] on trips and they would often do a *karakia*. Over time it's become Christianised it's become like a Christian *karakia*. But I suspect in the old days they were not. They were ways of establishing ownership. They were ways of protecting yourself when you went into a new area.

As discussed in Chapter 5, uruuru whenua are recorded in texts by Smith (1907) and Best (1901) as *karakia* used by arrivals to a new area to ensure a safe journey and which often involved placing an object or marker. The Williams (2002) and Moorfield (2003) Māori language dictionaries also reflect the *karakia* definition of uruuru whenua. The alternative term of *tātai whenua* was used by Forster (2019) to describe similar practices of connecting occupants to geographical areas. Taonui (2015) referred to narratives marking routes, boundaries, and geographical occupation as exploration narratives and believed these types of narratives may have largely been spared reinterpretation through European ontological lenses.

Lack of consistent terminology made the analysis of these narratives difficult in the interview and wayfinding data. The term *uruuru whenua* was used to describe the narratives for wayfinding on land used in the collection of classroom artefacts; however, *pūrākau* was the term more commonly used by *ākonga* and *pouako*. As discussed in Chapter 2, *pūrākau* refers to a range of narrative practices. There is potential to standardise the language used to describe different narratives in the *pāngarau* curriculum, lessening the burden of selecting the right term for *pouako*. However, as Mātanga 3 highlighted, distinctions need to be made between native terms that appear in curriculum alongside newly coined terms. Mātanga 3 further asserted that created or borrowed terms that are familiar could come to be accepted as ancestral in the absence of critical examination. Mātanga 3 used the term *pāngarau* to illustrate this point:

...i mea mai ngā kaumātua... “koinā tētehi kupu tawhito”. Pāngarau, ā, e hoa ma kei te hē. He mea hanga noa atu tērā.

[...the elders said... “that is an old word”. Pāngarau, my friends, is wrong. It is just a made-up word.]

The summarised data presents two different types of shifts in *mātauranga* occurring over time. Technology and dominant European beliefs and research agendas are external factors contributing to shifts in *mātauranga*. Other shifts are spurred by Māori actors, such as the standardisation of the *pāngarau* register in curriculum and travelling to, or settling in, a new geographical area. Reflecting these ongoing changes in a relatively static and disconnected curriculum structure presents a challenge for *pāngarau* curriculum developers and policy writers. It seems that the last two *pāngarau* curriculum development projects have favoured scientific ontological assumptions about the categories, classifications, and fixed or universal nature of knowledge. In turn, the relational and evolving ways of organising information through illuminating *whakapapa* frameworks or narrative practices such as *uruuru whenua* have been excluded by omission.

However, the *mātanga* interview data highlighted the potential for irreversible change that could be caused by objectively structuring *mātauranga* into *pāngarau* curriculum and classroom practices. Mātanga 1 asserted that conscious choices need to be made about “which changes are desirable, and which are undesirable”. Critical engagement with literature and seeking a group of knowledgeable peers were identified as strategies by Mātanga 3 to stem unwanted change caused by the standardised nature of national curriculum documents. In particular, Mātanga 3 argued the importance of an expert group that could discuss and debate the desirable and undesirable impacts of *pāngarau* curriculum and classroom practices:

Kia pai ai te tauwerowero o tētehi ki tētehi, te tauakiaki i tētehi ki tētehi me te tiakiaki o tētehi i tētehi. Kia haere whakamua kia kite i ētehi taurira hou... E oho ake ai ngā whakaaro. A me pēnei kē te haere...

[To enable us to challenge each other, encourage each other and to support each other. To move forward, to find new exemplars... To wake up the ideas. So, it should be so... instead.]

Mentoring by an expert peer was identified by Mātanga 4 as a strategy for ensuring newer pouako had the support they needed to engage critically with new curriculum content and terminology:

He tino mātanga whakaako pāngarau mā te whakaaro Māori. Mā te mātauranga Māori. Mā te reo Māori ... kei a ia ērā mātauranga.

[A true expert in teaching mathematics from a Māori perspective. From a Māori knowledge system. Through the medium of the Māori language ... they have that level of knowledge.]

The discussions by Mātanga 3 and 4 about the importance of groups of experts and mentors in furthering the indigenisation of pāngarau curriculum and classroom practices are reflective of the work of the mathematics education field in supporting school mathematics. The next section presents findings about factors supporting and constraining the emergence of the pāngarau field, which could provide similar expertise to pāngarau schooling.

The Emerging Pāngarau Field

Based on the current study's findings, pāngarau has emerged as a research discipline and is arguably now also emerging as an academic field. According to Bourdieu's definition, a field is a site where agents acquire different forms of capital to gain a position or place in a social structure (Costa & Murphy, 2015). In this way, agents use the tools of the field to generate a particular kind of habitus (Costa & Murphy, 2015). As argued throughout this thesis, the cultural capital being disseminated explicitly in pāngarau classrooms, beyond te reo Māori (the language of instruction), is school mathematics. Therefore, te reo Māori appears to be the distinguishing factor between the tools offered by the pāngarau field and the mathematics education field in habitus creation.

Mātanga 3 asserted that while proficiency in school mathematics was important, te reo Māori expertise was the most important skill for pouako:

Ko te pakari o te reo, o te kaikorero, o te kaiako... koinā tētehi wāhanga me pakari rawa. I te mea, ka puta ake te mātauranga tātai hoki i roto i tērā momo kōrero.

[The language ability of the speaker, of the teacher... But, that is, in my opinion one area, that needs to be very strong. Because... the mathematical knowledge will be conveyed by that kind of language.]

One of the reasons provided by Mātanga 3 for this assertion was the linguistic flexibility required to ensure that the students understood what was being taught:

...ko te ngana kia kōrero kia mārama ai ngā tamariki, ahakoa ngā ākonga, ahakoa ehara au, ehara hoki rātou i te tohunga ki te kōrero Māori...

[...endeavouring to speak so that the children can understand, no matter who the student is. Even though I am not and they are not expert speakers of Māori...]

A further reason for prioritising language expertise for pouako was the lack of language use in the wider school community as illustrated by Mātanga 3:

...he huarahi tino pai te pāngarau hei whakapakari i te reo. Nō reira, i te whakamahi i te tātai i te pāngarau hei whakapakari i te reo, oh, i rarururu pai ai au i reira i tētehi. I mea mai, “Kāo, kāre i te whakamahi i tēnei hei huarahi hei pakari ai te reo. Me ārahi te reo i ngā mea katoa”. Engari, ko taku mōhio i ngā ākonga, i ngā tamariki e whakaako ana au. Kāre i a rātou te reo, pakari rawa, kāre i o rātou mātua. Anā, te nuinga o rātou kare i o rātou kaumātua.

[Indeed, I'd say... mathematics is a great road towards strengthening [the Māori] language... using mathematical calculations to strengthen language. Oh, I got into some serious difficulties there with one who said, “No, don't use this as a path for strengthening (the Māori) language. We need to involve the language in everything”. On the contrary, in my experience of students, of children I have taught. They don't have strong language, their parents don't. Even, for most of them their elders don't either.]

Therefore, the tools being offered by the field of pāngarau for amassing cultural capital and creating habitus are essentially school mathematics and the use of pāngarau discourse to support the revitalisation of te reo Māori.

However, as Te Maro (2018) found, students in Māori immersion schooling acquire deficit beliefs about their pāngarau abilities, similar to the findings of Jorgensen (2011) regarding Indigenous students in English-language schooling. Te reo Māori is an important factor for distinguishing pāngarau and mathematics education fields. However, the illumination of mātauranga in pāngarau curriculum and classroom practices could provide even further distinction and potentially reverse the internalised deficit beliefs being acquired by ākonga. This thesis argues that the purpose of the pāngarau curriculum and related classroom practices can be further expanded to include the

dissemination of mātauranga. This was exemplified by the pāngarau classroom teaching sequences that illuminated uruuru whenua alongside pāngarau curriculum content, discussed in the following section. The study also investigated Māori and Pacific wayfinding literature and pāngarau resources that could support the illumination of mātauranga in pāngarau curriculum and classroom practices. The wayfinding literature illuminated the mnemonic characteristics of genealogical frameworks for organising information using symmetry, ordered pairs, and personified narratives. Chase stories were observed by Riesenber (1972), who investigated the navigation practices of Puluwat navigators or aafuwóów. Like the uruuru ao discussed by Mātanga 1, the Puluwat narratives involved chasing sea creatures from island to island and provided navigation routes. Interestingly, the navigators used creative licences to alter the extraneous information in the narratives, but the important navigation information was not altered (Riesenber, 1972). This also corresponds with the findings of Taonui (2015), whereby extraneous information in Māori exploration narratives was truncated or omitted.

Symmetry and ordered pairing were observed across all the narrative and visual wayfinding representations, as presented in Chapter 5. The ordered pairing of twin islands and creatures with two tails or two heads were characteristics of the Puluwat narrative conventions (Riesenber, 1972). The use of star pairing to discern latitude was also observed in the navigation practices of Hawaiian navigator Nainoa Thompson (Finney et al., 1986). Ordered pairing was observed in the Hawaiian *Kumulipo* genealogical chant associated with the physical wayfinding representation of the sky dome (Chauvin, 2000). Ascher (1995) also observed symmetry in the physical wayfinding maps of Marshall Islands navigators. Trinick and colleagues (2015) observed symmetry and ordered pairing in the visual representations of Māori epistemologies in whareniui. Reflected in ordered pairing in whakapapa frameworks, Ascher (1991) asserted that symmetry is a defining characteristic of the organisation of Māori society. The current study argues that equipping ākongā with Māori ontological and epistemological tools such as whakapapa frameworks, ordered pairing, and wayfinding narrative conventions, both verbal and visual, provides further tools for constructing pāngarau habitus. Utilising the cultural capital of mātauranga for constructing ākongā habitus further differentiates the pāngarau field from mathematics education and could limit any negative subjective structuring experienced by ākongā via school mathematics.

The mātanga interview data identified some challenges to expanding the purpose of pāngarau curriculum and classroom practices to explicitly include mātauranga. Mātanga 4 stated that pouako need guidance and mentoring from those with expertise in mātauranga, particularly those who have grown up in a uniquely Māori environment:

Otirā nō te whakawhitiwhiti kōrero ki te momo ... i tupu Māori mai. Mai i te tamarikitanga, tae noa mai. He māramatanga anō, he māramatanga anō i tērā tupuranga.

[But indeed [they are] from discussions with the kind [of people] ... who grew up Māori. From childhood, and since. It is a different understanding, a different understanding that generation has.]

Mātanga 4 also discussed the role of kaumātau in determining and demonstrating practices such as whakapapa:

Ko te ao Māori tērā. Kōrero ki ngā kuia me ngā koroua. Kāre e whaiti te titiro. Kei te tirohia ngā whānau katoa o te hapū o te marae me ngā pānga ki a tātou katoa. Kātahi ka whakaaro...oh arā te karangatahi rā, te whanaunga rā, arā, māna tēnei ko te tiki atu i ngā rauemi tangata me ō rātou pūkenga nē? I whai hua, i whai hua ai te hapū katoa, te marae.

[Talk with the elders. They do not have a narrow viewpoint. All the families of the hapū and the marae are considered, and the impacts on all of us. Then they will think... oh, that cousin, or that relative, they can do this. Fetching the human resources with their skills, perhaps? It worked. The entire hapū benefitted, and the marae.]

The need for expert guidance in the teaching of uruuru whenua was also identified in the student-created classroom artefact data. While creating a digital representation of the significant sites in a local uruuru whenua, ākongā from Kura 2 had difficulty locating some of the named locations on Google Maps. In some cases, the mapping software directed the students towards a business with a similar name or offered more than one geographical location that shared the same name. However, ākongā could identify kaumātua within the community and ask for guidance on identifying the locations mentioned in the uruuru whenua. Therefore, the pāngarau field could be expanded to include those who have the expertise to support the illumination of mātauranga in pāngarau curriculum and classrooms, which may include kaumātua and kaitiaki.

The data summarised in the preceding paragraphs identify some of the resourcing and support required to illuminate mātauranga in pāngarau curriculum and classrooms and further distinguish the pāngarau field from mathematics education. Bourdieu (1992) argued that redefining the habitus or the position one group occupies in relation to others in society requires the appropriation of status and the economic advantages of dominant positions. As shown in the pāngarau literature (Barton & Fairhall, 1995; McMurchy-Pilkington, 2004; Trinick, 2015), pāngarau curriculum development provided the resources to expand the pāngarau register, support pāngarau discourse in classrooms, and contribute to the broader revitalisation of te reo Māori. Therefore, pāngarau curriculum development is an established vehicle for appropriating the status and resources of mathematics education to benefit the pāngarau field. The following section summarises findings from the semi-structured interviews and

classroom data about the benefits and ongoing tensions of appropriating pāngarau curriculum development for pāngarau classroom discourse.

The Cultural Capital of Pāngarau Classroom Discourse

Pāngarau discourse, one of the tripartite foci of the cultural symmetry framework, has contributed significantly to the revitalisation and maintenance of te reo Māori and its related linguistic cultural capital. This was shown across the literature discussed in this thesis, the semi-structured interviews, and the classroom data. Specific examples of Māori ways of communicating about quantities included a range of language choices that do not reflect English-language preferences in mathematics discourse. For example, in the student-created word problem below, ‘E’ is used to indicate a cardinality. The passive term ‘tohaina’ is also used to denote sharing.

E 20 ngā o Hēmi ka tohaina ia me 4 o ona hoa ehea [sic] ki ia tangata?

[Hēmi has 20. He shares (distributes) these with 4 friends; how many for each person? –
Translated by author]

In the semi-structured interviews, Mātanga 2 discussed the preference in pāngarau for passive structures rather than the active structures preferentially used in English-language mathematics discourse:

Ko tētahi āhuatanga o te reo Māori. Ko te kaha kōrerohia o te reo hāngū. E tino hāngai ana te pāngarau ki te whakamahinga o te reo hāngū.

[One of the features of the Māori language. Is the frequent use of passive language constructions. In mathematics the use of passive constructions is particularly relevant.]

A third example observed in the classroom data of a uniquely Māori way of communicating pāngarau concepts was using the prefix toko- to denote a quantity of people. There is no comparative prefix in English for differentiating between a quantity of people and any other quantity.

E 6 nga taraka, toko 7 ngā tangata ki ia taraka, 1 te raiona ki ia taraka tokohia te katoa.

[There are 6 trucks, there are 7 people in each truck, there is one lion in each truck how many (people) altogether. Translated by author]

Interestingly in this example, ākonga present the number of people and lions in each truck. The question term of tokohia implies that the student is either only concerned with the total number of people across the six trucks or is elevating the lions to the status of people. As discussed in Chapter 5,

some would argue that pets have equal status to people and there is no specific prefix denoting a quantity of pets; therefore, the use of *toko* is appropriate.

The summary of student *pāngarau* discourse findings showed evidence of these uniquely Māori ways of communicating mathematically. They were used spontaneously by *ākonga* in both participating *kura* of this study, showing that Māori worldviews influence how mathematics is being discussed. These findings begin to answer a query first raised by Iseke-Barnes (2000) and discussed in Chapter 2 about whether teaching school mathematics in Māori is changing school mathematics. Arguably, conceptualising school mathematics concepts through a Māori lens *does* alter the way school mathematics is perceived by *ākonga*. This provides the tool of *te reo Māori* for constructing *pāngarau* *habitus* and could limit any negative subjective structuring experienced by *ākonga*.

Despite the strides made in ensuring *pāngarau* classrooms have the Māori language resources to support *pāngarau* classroom discourse, some teachers are still translating English-language mathematics resources into Māori. As discussed by Mātanga 4, at the secondary school level, translating English-language mathematics resources into Māori is an ongoing task:

Ka whakaaro au mō ngā tau iwa tae atu ki ngā tau tekau mā toru. Ko te nuinga o ngā kaupapa whakaako he whakamāori i te kaupapa Pākehā i te nuinga o te wā...

[I think for Years 9 right up to Year 13. Most of the teaching topics are translated from the English in most cases...]

Mātanga 4 also highlighted the usefulness of dictionaries such as *Te Reo Pāngarau* (He Kupenga Hao i te Reo, 2004) in alleviating the task of translating resources and needing to find specific terms:

...he nui ngā kupu o roto i te reo pāngarau kua tiki atu i te tino māramatanga o tērā kupu. Te ahunga mai o tērā whakaaro. He tino rauemi tērā. Kaha nōku te whakamahi i taua rauemi i aku tau tekau tuatahi pea i au e whakaako pāngarau ana mā te reo Māori.

[...there are many words in *te reo pāngarau*, that carry the powerful insight of that word. The origin of those ideas. It is a really great resource. I often used that resource, in my first 10 years maybe, when I was teaching mathematics through the medium of the Māori language.]

The standardisation of the *pāngarau* register was a strategy used by the register elaborators to minimise the number of new terms that *pouako* and *ākonga* had to learn. One of the arguments raised by Mātanga 2 for standardising the *pāngarau* curriculum and register was lessening the occurrence of classroom-specific *pāngarau* registers created by *pouako*. However, there could be implications for *ākonga* who transition between schools or classrooms if *pouako* resource translations do not use standardised terminology.

As discussed by Mātanga 2 and 3, primary school teachers initially created or translated resources in bilingual schools in the 1970s and 1980s, as there was no coordinated approach to resourcing at that time. Many more supports are available now for pouako in creating and translating resources, as identified by Mātanga 4. However, the need for pouako to continue translating resources some 40 years later is disheartening. The following section discusses some of the realities of balancing mātauranga and school mathematics in pāngarau classrooms, along with the implications for pāngarau curriculum and policy.

Balancing Māori and School Mathematics Cultural Capital

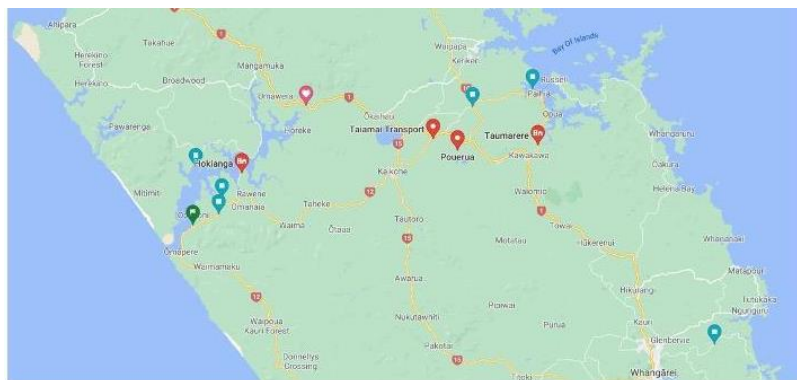
The cultural symmetry framework was used in this study as a methodological tool to address tensions between mātauranga and school mathematics, both in curriculum development and in pāngarau classrooms. This section summarises the data collected using the cultural symmetry framework in four pāngarau classrooms across two kura. The framework has found widespread use as both a pedagogical tool (Meaney et al., 2021) and a metaphorical lens for teaching and assessment policy (Rau et al., 2022). As shown in the classroom data chapter, the tumuaki and pouako of the kura who participated in this study were tasked with balancing aspirations for the teaching and learning of mātauranga, as defined by the community, with reporting on pāngarau achievement. According to Te Maro (2018), pouako are expected to ensure school mathematics achievement, often at the expense of mātauranga. The need to balance multiple and sometimes competing priorities was further exacerbated by the COVID-19 pandemic context, where continuity in learning during intermittent school closures also became a priority (see Allen & Trinick, 2021). Therefore, the classroom-based data collection in the current study was designed to support pouako in balancing the priorities of mātauranga and school mathematics achievement. This led to the classroom data collection being separated into two distinct but linked learning sequences.

The first learning sequence addressed kura needs to capture numeracy data and to strengthen number knowledge, number strategies—the valued cultural capital of school mathematics. This was achieved through designing tasks that modelled and required multiple representations of the pāngarau register—written, oral, diagram, words and symbolic. Ākonga were provided with digital tools such as Android tablets with notetaking and screencasting apps, and physical tools such as textbooks, stationery, and physical manipulatives to support their simultaneous acquisition of pāngarau concepts and the pāngarau register (see Allen, 2015 for further examples of this approach).

The second learning sequence focused on wayfinding narratives, termed uruuru whenua in this thesis. Again, students were supplied with a range of digital tools, such as Android tablets with mapping and screencasting apps, as well as the physical tools listed previously. The first sequence of lessons supported the second sequence by reinforcing and extending ākonga capacity to communicate using

the specialised language of the pāngarau register while drawing upon multiple representations simultaneously. Therefore, the design of the two distinct but complementary learning sequences reflected the pedagogical priorities of the participant kura. The lesson sequence design also exemplified epistemological pluralism (see Andreotti et al., 2011), where both epistemologies were addressed, and their applications to contemporary Māori concerns were explored. One contemporary concern is the perpetuation of uruuru whenua, despite differing travel patterns, modes of transport, and the introduction of digital wayfinding technologies such as GPS and GIS. Therefore, Māori must decide how to perpetuate the practice of uruuru whenua into the future. The student-created classroom artefacts highlighted in this study demonstrated the perpetuation of uruuru whenua through digital technology by representing significant locations and landmarks digitally, as shown in Figure 22.

Figure 22. Ākongā-Created Digital Uruuru Whenua Representation.



A further mechanism for continuing the creation and dissemination of uruuru whenua is Māori immersion school curriculum such as pāngarau. The following chapter revisits the study's aims and presents conclusions and recommendations for further research.

8. Ngā Whakapae me ngā Whakahau | Conclusions and Recommendations

Conclusions and recommendations for further indigenising pāngarau curriculum and classroom practices and ways to advance the pāngarau field are presented in this chapter alongside suggestions for further research. In the previous chapter, Bourdieu's (2010) framing of field, habitus, and cultural capital was used to synthesise data from three sources: semi-structured interviews with mātanga pāngarau, wayfinding representation literature, and pāngarau classroom artefacts. Data analyses identified opportunities, challenges, and tensions in illuminating mātauranga in pāngarau curriculum and classroom practices. Interviews with mātanga pāngarau and analysis of the relevant literature supported the notion that Māori want to access the cultural capital of school mathematics and the cultural capital of mātauranga simultaneously (see Durie, 2003; Smith et al., 2021), thereby providing two forms of cultural capital to convert into social and economic capital beyond schooling (Bourdieu, 2010). This final chapter provides conclusions and recommendations on how pāngarau curriculum and classroom practices can further support the realisation of these pluralistic Māori schooling goals. The conclusions and recommendations are presented according to the macro (national curriculum and policy) and the micro (Māori immersion schools and classroom) levels investigated in this doctoral research. Limitations of the study are discussed in the final sections, as are areas for potential further research.

Illuminating Dual Epistemologies in Pāngarau Curriculum

On order to support pluralistic Māori aspirations for schooling, an ontologically and epistemologically pluralistic approach to curriculum structure and content is required. Illuminating pluralistic epistemologies (Andreotti et al., 2011; Miller et al., 2008) of both mātauranga and school mathematics, without hybridising (Guerzoni, 2020) mātauranga or 'Māorifying' (Mika & Stewart, 2017) school mathematics, was a key challenge identified in this study for pāngarau curriculum and policy developers. The shift from the centralised model of state-mandated achievement objectives to a hybrid model where national and localised curricula are in operation in Māori-medium schools (MoE, 2019; Rau et al., 2022; Trinick & Heaton, 2021) provides a further avenue for illuminating the dual epistemologies of school mathematics and mātauranga in the pāngarau curriculum. Arguably, Kura Kaupapa Māori have always followed this practice through their drawing from the founding document *Te Aho Matua* (DIA, 2008), the localised curriculum guidelines *Te Marautanga o te Aho Matua* (Te Rūnanga Nui o ngā Kura Kaupapa Māori o Aotearoa, 2022; Trinick & Heaton, 2021) and the national pāngarau curriculum. This current study investigated the efficacy of this model in rebalancing mātauranga and pāngarau curriculum in four pāngarau classrooms across two kura.

One of the tensions evident in the relevant literature is the mathematics education field's overarching role in determining pāngarau curriculum content. Drawing its achievement objectives directly from the English-medium curriculum (McMurchy-Pilkington, 2004; Trinick, 2015), *Pāngarau i roto i te Marautanga o Aotearoa* (MoE, 1996) created a structural link between pāngarau and the norms of the arguably more powerful mathematics education field. Furthermore, the more recently refreshed pāngarau curriculum area of the Māori-medium curriculum document *Te Marautanga o Aotearoa* (Te Tāhūhū o te Mātauranga, 2017) retains a similar structure to the first document. Therefore, pāngarau education is still linked to the learning priorities and objective structuring of the mathematics education field.

The ethnomathematics literature reviewed in Chapter 2 showed that mathematics education has nonetheless created space for Indigenous knowledges in mathematics education research. However, the motivation for including Indigenous practices and artefacts in school mathematics seems to stem from a desire to promote the ideals of global citizenship (see le Roux & Swanson, 2021) by providing non-Indigenous students with opportunities to gain better understandings of diverse ways of mathematising. Global citizenship for Indigenous peoples, as presented in the mathematics education literature, seems to entail transitioning *from* ancestral knowledges *to* school mathematics (Sinclair et al., 2016).

Where Indigenous knowledges are afforded space in mathematics education research, the concepts are often those that map more easily onto existing school mathematics concepts. As identified in Chapter 5, concepts such as symmetry (Ascher, 1991), the pragmatics of locating and orienting oneself in relationship to land masses or landmarks (Owens, 2014) or Bishop's (1990) six (measuring, designing, locating, counting, explaining, and playing) are more likely to be promoted and maintained in the national curriculum as they have become part of the mathematics education field's epistemology. The appropriation of kōwhaiwhai to illuminate the school mathematics concepts of transformation geometry (Allen & Trinick, 2022) is an example of the promotion of school mathematics through subjugating mātauranga. The ongoing marginalisation of Indigenous knowledges in school mathematics curriculum is deemed necessary as school mathematics provides employment and further education opportunities in 'Western knowledge' (Aikenhead, 2018; Te Maro, 2018) for Indigenous students. This argument pervades the Aotearoa/New Zealand MoE policy requiring schools and kura to make connections between school learning and the world of work (MoE, 2022c).

The objective structuring practices of school mathematics curriculum, such as the exclusion of Indigenous knowledges, thus continue to encourage the internalisation of deficit beliefs by Indigenous students about their own mathematical abilities (Jorgensen, 2011; Te Maro, 2018). The resulting lowered expectations have been linked to lowered academic achievement (Bishop et al., 2009; Turner

et al., 2015) and subsequent knowledge debt (Aikenhead, 2018) for these ākongā. This thesis argues that school mathematics knowledge debt is akin to cultural capital debt. The exclusion of Indigenous knowledges from school curriculum further ascribes Indigenous cultural capital debt to Indigenous students. Debts in cultural capital limit social and economic trajectories of ākongā beyond schooling as they have less cultural capital to convert to economic and social capital. An opportunity for the emerging pāngarau field to distinguish itself and its curriculum priorities from the objective structuring practices of the mathematics education field is thus, as argued here, through explicitly illuminating mātauranga and providing guidance on its teaching in pāngarau classrooms. Therefore, ākongā can access the cultural capital of mātauranga when constructing internalised beliefs about their abilities as pāngarau learners.

Marginalised groups can change their educational trajectory by appropriating the structures and economic advantages of more privileged groups (Bourdieu, 1992). One of the key arguments highlighted in both the literature review and mātanga interviews in the appropriation of curriculum development for the revitalisation of te reo Māori, was the acquisition of status and economic advantages afforded to (and by) the mathematics education field. These advantages included funding for teaching, assessment, professional development, and resource creation, given school mathematics' relatively prestigious subject positioning. The funding of the pāngarau register elaboration project and curriculum resource development has thus alleviated the burden of word creation, translation, and resource creation from individual teachers (Christensen, 2004b; Trinick, 2015). However, the status and economic advantages of the mathematics education field have not yet been extended to the revitalisation and maintenance of mātauranga. This remains a tension for pāngarau curriculum writing, as highlighted in the mātanga interviews and lobbying by Kura Kaupapa Māori (McMurchy-Pilkington, 2004; Trinick, 2015; Trinick & Heaton, 2021). The restrictive requirement to mirror the English-language document imposed by the MoE on the first pāngarau development project was removed from the second; however, the structure of the second document remained unchanged. Mātauranga was again excluded by omission.

Whether the status and economic advantages of the mathematics education field will be an affordance to the revitalisation and maintenance of mātauranga in the current third national pāngarau curriculum development project is yet to be seen. Policy denoting who can determine the contributions of mātauranga to school curriculum and how (in what ways), shows a clear preference for vesting the responsibility for mātauranga in local school communities (Education and Training Act, 2020; Rau et al., 2022) through localising curriculum. Similarly, The Waitangi Tribunal (2011) has vested the care and responsibility for mātauranga with kaitiaki. Those who have enduring kin-based relationships with areas of mātauranga are generally understood to be iwi, hapū and Māori communities. The classroom data from the current study identified opportunities for the localised delivery of mātauranga

in two kura. Further discussion of the limitations of this approach is offered in subsequent sections. The following section provides conclusions on the challenges identified in the study for illuminating mātauranga in the national pāngarau curriculum.

Challenges to Illuminating Mātauranga in the National Pāngarau Curriculum

To better understand the arguments for and against the documentation, standardisation, and sequencing of mātauranga via the national curriculum, the arguments for and against standardising te reo Māori via the pāngarau register and pāngarau curriculum development were identified in both the literature review and Mātanga pāngarau interview data. In the literature, pāngarau register developers and curriculum writers highlighted the compromises and perhaps irreversible changes made to te reo Māori through the pāngarau curriculum development process (Barton & Fairhall, 1995; Meaney et al., 2012; Trinick, 2015), both positive and negative.

All four mātanga pāngarau interviewed for this study asserted that overall pāngarau discourse has supported Māori language revitalisation. One of the strategies used by curriculum register elaborators for revitalising te reo Māori was resurrecting terms that had fallen into disuse, often expanding their meanings in the process (McMurchy-Pilkington, 2004; Stewart, 2007; Trinick, 2015). Reservations were expressed in the literature about the syntactic strength of te reo Māori and the possibility of eroding uniquely Māori ways of communicating (Barton & Fairhall, 1995). Questions were also asked about the potential for communicating mathematically in te reo Māori to change school mathematics (Iseke-Barnes, 2000). The classroom data in this current study showed that uniquely Māori ways of communicating mathematically were being used spontaneously by participating ākonga across both kura, despite the potential for interference by the discourse norms of English-language school mathematics.

The standardisation of te reo Māori through the pāngarau register and curriculum was highlighted as both positive and negative in both the literature and mātanga interview data. The interview data showed that positive benefits for pouako were the removal of the burden of translating resources into Māori or alleviating this challenge through providing standardised dictionaries. Likewise, the burden for ākonga of learning multiple classroom or school-specific pāngarau registers throughout their schooling was also removed (Christensen, 2004b). The negative outcomes of pāngarau register elaboration and curriculum development for te reo Māori identified in the literature and mātanga interview data were changes in syntax (Barton & Fairhall, 1995) and the marginalisation of dialectal variations. The following sections further explore both these positive and negative benefits for mātauranga through documentation, standardisation, and sequencing of mātauranga via the national pāngarau curriculum.

Reflecting the universality of school mathematics' concepts (Bishop, 1990) and ontological assumptions of Western science about classification and hierarchy (Govender & Mudzamiri, 2022), the pāngarau curriculum objectively structures pāngarau classroom practices. Mātanga 3 identified the constraining nature of pāngarau curriculum and classroom practices as disseminating concepts, piece by piece. This current study contrasted the findings about the fixed nature of pāngarau curriculum content with the evolution of Māori and Pacific wayfinding representations and Māori worldviews.

Semi-structured interview data and Māori and Pacific wayfinding literature demonstrated that whakapapa frameworks, the use of symmetry and ordered pairing, exploration narratives and wayfinding representations are continually evolving in the face of external structures and Māori agency. External structures were identified in the literature, mātanga interviews, and classroom data as new technologies and dominant Western worldviews. Agentic actions by Māori included the systematic elaboration of the pāngarau register and settling or exploring new geographical areas. Māori worldviews and wayfinding practices were described by the mātanga interviewed and the wayfinding representation literature as inherently relational, open to innovation or creative licence and preferencing the 'big picture' before discerning the important details. Therefore, the seemingly oppositional ontological assumptions of mātauranga and school mathematics present challenges for pāngarau curriculum and policy writers. The following section explores the use of the cultural symmetry framework as both a methodological and pedagogical tool for addressing tensions between mātauranga and school mathematics at both the macro-curriculum (policy) and micro (school and classroom) levels. Limitations of the study are also discussed.

Limitations of the Application of the Cultural Symmetry Framework

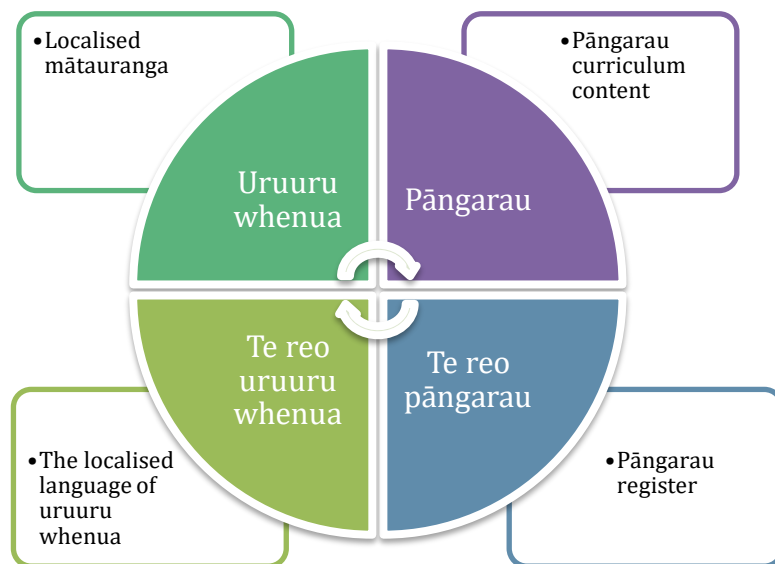
The application of the cultural symmetry framework in this study aligned with existing notions that the framework outlines a high-level rationale for rebalancing te reo Māori, mātauranga, and pāngarau curriculum content by explicitly addressing each (Meaney et al., 2021). However, it does not provide specific guidance to curriculum writers, policymakers, school leaders, or pouako on how to balance these sometimes competing priorities. The current study explored proportional reasoning and spatial orientation school mathematics foci, wayfinding representations, such as uruuru ao and uruuru whenua, as the mātauranga foci, along with the benefits of, and difficulties in, teaching and learning the pāngarau register as the reo Māori foci. Mātanga interview data and classroom artefacts were analysed accordingly.

However, the reo Māori foci could have been divided further to better illuminate the opportunities, challenges, and tensions in teaching uruuru whenua in pāngarau classrooms. In Māori immersion pāngarau classrooms, the language required to communicate the Indigenous knowledges and school mathematics concepts is the same—te reo Māori. As asserted by Mātanga 2 and 3, promoting

uniquely Māori ways of communicating that are not specific to pāngarau is just as important as using the correct pāngarau terminology. Therefore, an opportunity was lost in the current study to collect and analyse examples of spontaneous use of reo Māori for communicating about wayfinding that is not currently contained in the standardised pāngarau register. Separating the reo Māori foci into basic interpersonal communication skills and cognitive academic language proficiency (see Cummins, 2000) would have provided a theoretical lens beyond mathematical communication (Pimm, 1991) through the pāngarau register for data analysis.

While there would likely be an overlap between the language required to communicate wayfinding concepts contained in uruuru whenua and the pāngarau register, there may have been opportunities to illuminate the uniquely Māori ways of communicating about uruuru whenua concepts beyond the pāngarau register. For example, dialectal variations of uruuru whenua terms, such as direction terms or time markers that describe the environmental indicators of a geographical region (Matamua, 2020; Trinick, 1999), not currently featured in the standardised pāngarau register, were not analysed. The symmetrical approach, presented in Figure 23, provides opportunities to illuminate the concepts and practices of uruuru whenua alongside the uniquely Māori ways of communicating about the concepts and practices beyond the pāngarau register.

Figure 23. Reconfigured Cultural Symmetry Framework.



Thus, the cultural symmetry framework has been reconfigured as an ontologically pluralistic lens for examining the dual epistemologies of mātauranga and school mathematics alongside the language of mātauranga and school mathematics.

This doctoral study explored the potential for the localised delivery of mātauranga in two kura. However, the circumstances of a pāngarau researcher and pāngarau expert supporting delivery may not reflect the circumstances of most kura. Furthermore, the delivery was also limited to two school

terms and had to be truncated further due to the COVID-19 pandemic. As such, further research is required to better understand how kura can actualise the delivery of mātauranga in pāngarau classrooms through localised curriculum.

Recommendations for Further Research

In order to support the illumination and delivery of mātauranga in both national and local curricula, further research is required into the aspects of mātauranga that can be illuminated appropriately in the pāngarau curriculum. The literature on Māori and Pacific wayfinding representations, reviewed in Chapter 5, were often highly technical manuscripts, and some historical accounts were written in a disparaging colonial tone. The review of the literature further demonstrated that some narratives had been altered to reflect European beliefs or research agendas. However, exploration narratives seemed to suffer the least editing (Taonui, 2015) in this regard and could thus provide a (more) reliable source for illumination in pāngarau curriculum and classroom practices alongside narratives that have suffered alterations. Further research into the characteristics of exploration narratives and their potential illumination in pāngarau curriculum and classrooms is thus required.

Currently, there are few text resources designed to aid the illumination of mātauranga in pāngarau classrooms. The examples reviewed in this thesis did not contain guidance on how to simultaneously address mātauranga and school mathematics (see Rikihana, 1990). A range of terminology was also present in the classroom data for different types of narratives. *Karakia*, *pūrākau*, *uruuru ao* and *uruuru whenua* were all terms identified in the literature and *mātanga* interviews for narratives that represented boundary markers, routes, and valued environmental information. Further research is required into the pedagogical content and pedagogical language knowledge required to illuminate these narratives in pāngarau curriculum and classrooms going forward.

Without significant research and classroom resource creation, the illumination of wayfinding representations and narrative practices could fall to pouako. As was seen with the use of *kōwhaiwhai* in mathematics classrooms (see Allen & Trinick, 2022), this can be counterproductive, as reinterpretations by those without significant expertise can lead to changes in cultural norms (Jahnke & Jahnke, 2003). As argued in this thesis, any changes to Māori ways of knowing, being and believing resulting from pāngarau curriculum and classroom practices must be debated, critiqued and guided by experts.

Appendix 1: Glossary

Maori term	Use in thesis
Ā-hapū	Subtribal
Ā-iwi	Tribal
Aka matua	Main idea
Aka tāepa	Extraneous information, loose side shoots
Ake	Upwards, in an upwards direction, greater
Ākene pea	Probably
Ākonga Māori	Māori students
Ākonga pāngarau	Pāngarau students
E toru	The cardinal number 3
Hā	Essence
Half	Hāwhe
Hangarau matihiko	Digital technology
Hapū	Sub-tribes
He nui ake	Greater than
Huri kōaro	Inverse operation strategy
Iwi	Tribes
Kāhore e kore	In all likelihood
Kaitiaki	Stewards, those with kin relationships to mātauranga
Kaitiakitanga	Stewardship
Kanaka Maoli	Indigenous people of Hawaii
Kapa haka	Māori performing arts
Kāpehu hau	Wind compass
Karakia	Oral chant
Kāre pea e puta	May not occur
Kaumātua	Elder

Maori term	Use in thesis
Kete	Baskets
Kōhanga Reo	Early childhood language nest
Kore rawa	Never
Kotahi	One
Kotahi tekau	One ten (literal)
Kōwhaiwhai	Painted scroll ornamentation
Kura ā-iwi	Tribal school
Kura ā-rohe	Regional school
Kura kaupapa Māori	Māori immersion primary schools
Mā	For
Mahi	Work
Mahi whika	Working with figures
Mana	Power, Authority
Manu kōrero	Speech competitions
Māori	Indigenous peoples of Aotearoa/New Zealand
Marae	Meeting spaces
Mātanga pāngarau	Pāngarau curriculum experts
Mātauranga	Māori ways of knowing, being and believing
Mātauranga ā-rohe	Regional ways of knowing, being and believing
Matematika	Mathematics
Matua	Main vine
Muri	Back or behind
Nō neherā	Ancestral times
Paekupu	Online Te Marautanga o Aotearoa dictionary
Pākehā	Of European descent
Pānga	Relationships
Pāngarau	Mathematics in Māori-medium schools

Maori term	Use in thesis
Piko	Naval
Pouako	Teachers
Puna kupu	Vocabulary lists
Pūrākau	Narratives
Pūrua	Square
Pūtaiao	Kaupapa Māori science curriculum
Pūtoru	Cube
Pūwhā	Power of four
Raki	Northerly wind
Rātou	Those, plural
Rāua	Those two
Tā	Belonging to
Tamariki	Children
Taparima	Five edges
Tātai	Setting in order, calculation, reciting genealogy
Tātai arorangi	Celestial navigation
Tātai whenua	Environmental genealogies
Tauutuutu	Alternating speakers at a formal gathering
Te ao Māori	The Māori world
Te pō	The darkness
Te reo Māori	The Māori language
Te reo pāngarau	The language of mathematics
Te Taura Whiri i te Reo Māori	Māori Language Commission
Te whakatangata	Personification
Te whare kōkōrangi	The space of Māori astronomical knowledge
Tekau	Ten
Tikanga Māori	Māori language and protocols

Maori term	Use in thesis
Tīpuna	Ancestors
Tohunga	Expert
Tokotoru	Three people
Tonga	South
Toru	The counting number 3
Tuangahuru	Tenth
Tumuaki	Principals
Tupu pū	Exponential growth
Tūranga	Gisborne
Ūnahi	Parabola
Uru	West
Uruuru	Grouping of practices, rituals, understandings
Waka	Vehicle
Wānanga	Māori tertiary institutions
Whakamānu ake	Refloat/recover
Whakamāui ake	Recover
Whakapapa	Genealogical frameworks
Whakataukī	Formulaic saying
Whakawehe	To divide
Whānau	Extended families
Wharekura	Māori immersion secondary schools
Wharenuī	Māori meeting houses
Whika	Figure

Appendix 2: Interview Questions

Rārangi pātai uiui/Schedule of interview questions

Tapanga Rangahau: Tau Kē: Te whakamahinga hangarau kawekawe hei whakawhanake i ngā ākonga ki te whakawhitiwhiti whakaaro pāngarau i ngā akomanga pāngarau arareo-Māori.

Project Title: Tau Kē: The use of mobile technology to support students to communicate mathematically in Māori-medium pāngarau contexts.

Kaiwhakahaere Matua/Principal Supervisor: Toihuarewa Stephen May

Kaiwhakahaere Tūhono/ Co-Supervisor: Toihuarewa Tūhono Tony Trinick

Kairangahau Ākonga/Student Researcher: Piata Allen

Ko tētahi o ngā whāinga o tēnei kaupapa rangahau ko te tautohu i ngā rauemi o te reo Māori (ngā āheinga, ngā āhuatanga, ngā hanga hoki o te reo ā-waha me te reo ā-tinana) ka huawaere i ngā whakawhitiwhiti whakaaro pāngarau mā te tirohanga Māori ahurei. Ko Piata Allen te kaikawe o tēnei uiui māna ngā pātai nei e tuku:

One of the outcomes of the research project is to identify resources within the Māori language (including functions, features, form, patterns, and structures) that facilitate mathematical communication through a uniquely Māori cultural lens. Piata Allen will conduct the interview and ask the following questions:

1. Ka taea e koe te tautohu i ētahi o ngā rauemi o te reo Māori ka whakatauiria i te Māoritanga o ngā whakawhitinga whakaaro? He tauira o te reo ā-waha, o te reo ā-tinana hoki pea.

Can you identify resources within the Māori language that exemplify uniquely Māori ways of communicating? These examples could be both verbal and non-verbal.

2. Mā te aha ka ahurei ēnei tauira ki te reo Māori?

What makes these examples unique to the Māori language?

3. Tautohua ētahi o ngā rauemi o te reo pāngarau ka huawaere i te tirohanga Māori i ngā whakawhitinga whakaaro pāngarau?




4. Can you give some examples of resources within the pāngarau language that facilitate mathematical communication through a Māori cultural lens?



Appendix 3: Te Uiui Aromatawai | Diagnostic Interview

Ngā Pātai Rautaki


Te Tāpiri me te Tango (Ngā Pātai)

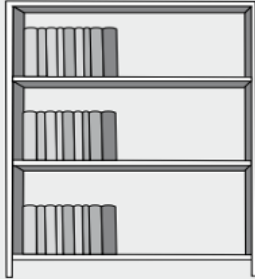

Ngā mahi a te kaiako me ngā pātai	He tauira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>1. Whakaputua ētahi taonga (pēnei i te porotiti) ki mua i te tamaiti.</p> <p>Ko te tatau pānga tahi i ētahi taonga e 8.</p>	Tangohia (Homai/Tikina/Tatauria) kia waru ngā porotiti.	Ki te kore e taea te tatau pānga tahi, kei te Kaupae 0 o Te Mahere Tau.	Ko te Pātai 1 te wāhi tūmata i ngā Pātai Mātauranga.
<p>2. Hoatu kia 4 ngā porotiti ki tētahi ringaringa o te tamaiti, kia 3 ki tētahi. Katia ngā ringaringa kia kore ai e kitea ngā porotiti.</p> <p>4 + 3 (mā te tatau ā-hinengaro)</p>	E whā ngā porotiti ki tēnei ringaringa, e toru ki tēnei. E hia katoa ngā porotiti?	Ki te kore e taea, kei te Kaupae 1 o Te Mahere Tau.	Ko te Pātai 1 te wāhi tūmata i ngā Pātai Mātauranga.
		<p>Mēnā ka tika te whiriwhiri mā te tatau i ngā porotiti, kei te Kaupae 2 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 1a</p> <p>Ka tika te whiriwhiri, mā te tatau ā-hinengaro i ngā porotiti.</p>	<p>Ko te Pātai 1 te wāhi tūmata i ngā Pātai Mātauranga.</p> <p>Haere tonu ki te Pātai Rautaki 3.</p>
<p>3. Uhia kia 8 ngā porotiti ki raro i tētahi kāri, kia 5 ki raro i tētahi atu kāri. Whakaaturia ngā porotiti ki te tamaiti, ka uhi anō ai.</p> <p>8 + 5</p>	E waru ngā porotiti kei raro i tēnei kāri, e rima kei raro i tēnei kāri. E hia katoa ngā porotiti?	Ki te kore e taea, kei te Kaupae 3 o Te Mahere Tau.	Ko te Pātai 1 te wāhi tūmata i ngā Pātai Mātauranga.
		<p>Taumata Whanaketanga: 1a</p> <p>Ka tika, mā te tatau ake i te 8.</p>	Haere tonu ki te Pātai Rautaki 4.
<p>4. Uhia kia 9 ngā porotiti ki raro i tētahi kāri, kia 8 ki raro i tētahi atu kāri. Whakaaturia ngā porotiti ki te tamaiti, ka uhi anō ai.</p> <p>9 + 8</p>	E iwa ngā porotiti kei raro i tēnei kāri, e waru kei raro i tēnei kāri. E hia katoa ngā porotiti?	Ki te kore e taea, mēnā kei te tika rānei mā te tatau ake, kei te Kaupae 4 o Te Mahere Tau.	Ko te Pātai 2 te wāhi tūmata i ngā Pātai Mātauranga.
		<p>Taumata Whanaketanga: 1e</p> <p>Ka whakamahia he rautaki wāwāhi tau mō te Pātai 3, te Pātai 4 rānei.</p>	Haere tonu ki te Pātai Rautaki 5.

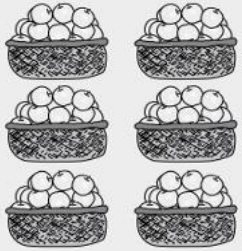

Ngā mahi a te kaiako me ngā pātai	He taurira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>5. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>37 – 9</p>	<p>E 37 ngā pukapuka kei roto i te pouaka a Whaea Wini. E 9 ka tangohia atu. E hia ngā pukapuka kei roto tonu i te pouaka?</p> 	<p>Mēnā i whakamahia he rautaki wāwāhi tau mō te Pātai 3, te Pātai 4 rānei, engari i tatau whakamuri mō te Pātai 5 nei, kei te Kaupae 5 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 2a</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 2 pea te wāhi tīmata i ngā Pātai Mātauranga.</p>
		<p>Ka whakamahia he rautaki wāwāhi tau⁶ hei whiriwhiri i te Pātai 5 nei.</p>	<p>Haere tonu ki te Pātai 6.</p>
<p>6. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>53 – 26</p>	<p>E 53 ngā māpere a Roimata. E 26 i hoatu ki tōna hoa. E hia ngā māpere e toe mai ana ki a Roimata?</p> 	<p>Ki te kore e taea, kei te Kaupae 5 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 2a, 2e rānei</p> <p>Mēnā ka whāia te hātepe ōkawa hei whiriwhiri i tēnei pātai, tirohia ngā kōrero kei te whārangi 9.</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 3 pea te wāhi tīmata i ngā Pātai Mātauranga.</p>
		<p>Ka whakamahia he rautaki wāwāhi tau⁶ hei whiriwhiri i tēnei pātai.</p>	<p>Haere tonu ki te Pātai 7.</p>
<p>7. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>394 + 79</p>	<p>E \$394 te pūtea a Roimata. Ka tāpiria te \$79 atu anō. E hia katoa tana pūtea ināianei?</p> 	<p>Ki te kore e taea, kei te Kaupae 5 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 2e</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 3 pea te wāhi tīmata i ngā Pātai Mātauranga.</p>
		<p>Mēnā ka whāia te hātepe ōkawa mō tēnei pātai, engari kāore e taea ētahi rautaki wāwāhi tau e rua te whakamārama, kei te Kaupae 6.</p> <p>Taumata Whanaketanga: 3a</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 5 pea te wāhi tīmata i ngā Pātai Mātauranga.</p>
		<p>E rua, nui ake rānei ngā rautaki wāwāhi tau hei whiriwhiri i tēnei pātai me te pātai o mua.</p>	<p>Haere tonu ki te Pātai 8.</p>



Ngā mahi a te kaiako me ngā pātai	He tauira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>8. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>$5.3 - 2.89$</p>	<p>He mahere huarahi tēnei. E rima ira toru manomita (5.3km) mai i Kaitaia ki Awanui. E rua ira waru iwa manomita (2.89km) mai i Kaitaia ki te kāinga o Mereana. E hia manomita (km) mai i te kāinga o Mereana ki Awanui?</p> 	<p>Ki te kore e taea, kei te Kaupae 6 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 3a, 3e rānei</p> <p>E rua, nui ake rānei ngā rautaki wāwāhi tau hei whiriwhiri i tēnei pātai me ngā pātai o mua.</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 14 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <p>Haere tonu ki te Pātai 9.</p>
<p>9. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>$2 - (\frac{3}{4} + \frac{7}{8})$</p>	<p>E rua rita (2L) te rōrahi wai ārani kei roto i te ipu. E $\frac{3}{4}$ rita ka ringia atu ki tētahi kapu, e $\frac{7}{8}$ rita ka ringia atu ki tētahi atu kapu. E hia rita wai ārani e toe ana i te ipu?</p> 	<p>Ki te kore e taea, kei te Kaupae 6 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 3a, 3e rānei</p> <p>Mēnā ka whakamahia he rautaki wāwāhi tau hei whiriwhiri i te Pātai 8 me te Pātai 9, kei te Kaupae 7 o Te Mahere Tau mō te tāpiritanga me te tangohanga.</p>	<p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 14 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <p>Haere tonu ki te Pātai Rautaki 10 mō te whakarea me te whakawehe.</p> <p>Ko te Pātai 14 pea te wāhi tīmata i ngā Pātai Mātauranga.</p>

Te Whakarea me te Whakawehe (Ngā Pātai)

Ngā mahi a te kaiako me ngā pātai	He tauira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>10. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>Uhia te pikitia o ngā rārangi tūru ki te kāri, engari kia kitea tonutia te kapa runga me te pou mauī o ngā tūru.</p> <p>5×8</p> <p>$15 + 5 = 3$ (e 3 atu anō ngā rārangi tūru)</p> <p>$8 + 3 = 11$ (11 katoa ināianei ngā rārangi tūru)</p>	<p>E 8 ngā rārangi tūru, e 5 ngā tūru ki ia rārangi. E hia katoa ngā tūru?</p> <p>15 anō ngā tūru ka whakatūria ki ētahi anō rārangi. E hia katoa ngā rārangi tūru ināianei?</p> 	<p>Mēnā ka whakamahia he rautaki tatau hei whiriwhiri i te wāhanga tuatahi o te pātai (5×8), kua e tukuna te wāhanga tuarua o te pātai. Kei tētahi o ngā Kaupae 2, 3, 4.</p> <p>Taumata Whanaketanga: 1c</p> <hr/> <p>Ka whakamahia he rautaki wāwāhi tau hei whiriwhiri i te 5×8, me te $15 + 5$, he meka mōhio rānei ēnei.</p>	<p>Haere toni ki te Pātai 17 mō te pānga riterite.</p> <p>Ko te Pātai 2 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <hr/> <p>Haere tonu ki te Pātai 11.</p>

Ngā mahi a te kaiako me ngā pātai	He tauira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>11. Whakaaturia te rapanga mō tēnei pātai. Uhia te wāhanga tuarua o te pātai, ka pānui ai i te wāhanga tuatahi.*</p> <p>$20 \times 3 = 60$</p> <p>Whakaaturia te wāhanga tuarua o te pātai, ka pānui ai.</p> <p>$18 \times 3 = 60 - 6 = 54$</p>	<p>E 3 ngā tūranga pukapuka, e 20 ngā pukapuka kei ia tūranga. E hia katoa ngā pukapuka?</p>  <p>Ka tika, e 60 ngā pukapuka katoa. Mēnā 18 ngā pukapuka kei ia tūranga, e hia katoa ngā pukapuka?</p>	<p>Ki te kore e taea, kei te Kaupae 5 o Te Mahere Tau, kei raro iho rānei.</p> <p>Taumata Whanaketanga: 2a</p> <hr/> <p>Ka whakamahia te otinga o te 20×3 hei whiriwhiri i te 18×3.</p>	<p>Haere tonu ki te Pātai 17 mō te pānga riterite.</p> <p>Ko te Pātai 2 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <hr/> <p>Haere tonu ki te Pātai 12.</p>
<p>12. Whakaaturia te rapanga mō tēnei pātai. Uhia te wāhanga tuarua o te pātai, ka pānui ai i te wāhanga tuatahi.*</p> <p>8×5</p> <p>Whakaaturia te wāhanga tuarua o te pātai, ka pānui ai.</p> <p>16×5</p>	<p>E 8 ngā pene kei te pākete kotahi. E hia katoa ngā pene kei ngā pākete e 5?</p>  <p>Mēnā 16 ngā pene kei te pākete kotahi, e hia katoa ngā pene kei ngā pākete e 5?</p>	<p>Ki te kore e taea, kei te Kaupae 5 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 2e</p> <hr/> <p>Ka whakamahia te otinga o te 8×5 hei whiriwhiri i te 16×5 (arā, mā te whakarearua i te 40).</p>	<p>Haere tonu ki te Pātai 17 mō te pānga riterite.</p> <p>Ko te Pātai 3 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <hr/> <p>Haere tonu ki te Pātai 13.</p>

Ngā mahi a te kaiako me ngā pātai	He tauria kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>13. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>24×6</p>	<p>E 6 ngā rourou, e 24 ngā āporo kei ia rourou. E hia katoa ngā āporo?</p> 	<p>Ki te kore e taea, kei te Kaupae 5, 6 rānei o Te Mahere Tau. Taumata Whanaketanga: 2e, 3a rānei</p> <p>Ka whakamahia he rautaki wāwāhi tau. Hei tauria:</p> <p>Wāwāhi Uara Tū: $24 \times 6 = 20 \times 6 + 4 \times 6$ $= 120 + 24 = 144$</p> <p>Tau Māmā: $25 \times 6 = 150$, nō reira, $24 \times 6 = 150 - 6 = 144$</p> <p>Whakaaro Pānga Riterite: $24 \times 6 = 12 \times 12 = 144$</p>	<p>Haere tonu ki te Pātai 17 mō te pānga riterite. Ko te Pātai 3 pea te wāhi tūmata i ngā Pātai Mātauranga.</p> <p>Haere tonu ki te Pātai 14.</p>
<p>14. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> <p>$72 \div 4$</p>	<p>Tokowhā ngā tāngata i haere ki te pikitia. E \$72 te katoa o te utu mō ā rātou tikiti. E hia te utu mō te tikiti kotahi?</p> 	<p>Ki te kore e taea, kei te Kaupae 6 o Te Mahere Tau. Taumata Whanaketanga: 3a, 3e rānei</p> <p>Ka whakamahia he rautaki wāwāhi tau. Hei tauria:</p> <p>Wāwāhi Uara Tū: $72 \div 4 = 40 \div 4 + 32 \div 4$ $= 10 + 8 = 18$</p> <p>Tau Māmā: $80 \div 4 = 20$, nō reira, $72 \div 4 = 20 - (8 \div 4) = 18$</p> <p>Huri Kōaro hei Whakareatanga: $4 \times \square = 72 \rightarrow 4 \times 10 = 40, 4 \times 8 = 32$, nō reira, $4 \times 18 = 72$</p> <p>Whakaaro Pānga Riterite: $9 \times 8 = 72$, nō reira, $18 \times 4 = 72$, nō reira, $72 \div 4 = 18$</p>	<p>Haere tonu ki te Pātai 17 mō te pānga riterite. Ko te Pātai 5 pea te wāhi tūmata i ngā Pātai Mātauranga.</p> <p>Haere tonu ki te Pātai 15.</p>

Ngā mahi a te kaiako me ngā pātai	He tauira kōrero mā te kaiako	Hei ārahi i te aromatawai	Ki tēhea pātai ināianei
<p>15. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> $2.4 \div 0.15$ $0.15 \times \square = 2.4$	<p>E rua ira whā manokaramu (2.4kg) te puehu parāoa a Māmā Merehēni hei mahi parāoa. Kore ira tahi rima manokaramu (0.15kg) te puehu hei mahi i te parāoa kotahi. E hia ngā parāoa ka taea e Māmā Merehēni te mahi?</p> 	<p>Ki te kore e taea, kei te Kaupae 7 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 4a</p> <p>Ka whakamahia he rautaki wāwāhi tau. Hei tauira:</p> <p>Whakarearua:</p> $0.15 \times 2 = 0.3$ $0.15 \times 4 = 0.6$ $0.15 \times 8 = 1.2$ $0.15 \times 16 = 2.4$ <p>Whakaaro Pānga Riterite:</p> $0.15 \times 2 = 0.3, \text{ ā, } 0.3 \times 8 = 2.4,$ $\text{nō reira, } 0.15 \times 16 = 2.4$	<p>Haere tonu ki te Pātai 17 mō te pānga riterite.</p> <p>Ko te Pātai 5 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <hr/> <p>Haere tonu ki te Pātai 16.</p>
<p>16. Whakaaturia te rapanga mō tēnei pātai, ka pānui ai.*</p> $22 \div 8$	<p>E 22 rita te rōrahi o te wai ārani. Ka wehea ki ētahi ipu e 8, kia ōrite te rōrahi ki ia ipu. E hia rita ki ia ipu?</p> 	<p>Ki te kore e taea, kei te Kaupae 7 o Te Mahere Tau.</p> <p>Taumata Whanaketanga: 4a, 4e rānei</p> <p>Mēnā ka whakamahia he rautaki wāwāhi tau, kei te Kaupae 8 o Te Mahere Tau. Hei tauira:</p> <p>Wāwāhi Uara Tū:</p> $22 \div 8 = 16 \div 8 + 6 \div 8$ $= 2 + 0.75 = 2.75$ <p>Whakawehe Whai Toenga:</p> $22 \div 8 = 2 \text{ (toenga 6)}$ $= 2 \frac{6}{8}$ $= 2 \frac{3}{4}$ $= 2.75$	<p>Haere tonu ki te Pātai 17 mō te pānga riterite.</p> <p>Ko te Pātai 5 pea te wāhi tīmata i ngā Pātai Mātauranga.</p> <hr/> <p>Haere tonu ki te Pātai 17 mō te pānga riterite.</p>

Appendix 4: Lesson Sequence 2

1	<p>Rich tasks/Intro to spatial reasoning student inquiry project</p> <p>Uruuru ao/uruuru whenua spatial narratives, mapping these on paper and on Google Maps, for example, Poutini:</p> <p>Te Wheke o Muturangi</p> <p>Grace, W. (n.d.). <i>Kupe and the giant Wheke: Māori myths and legends and contemporary stories</i>. https://eng.mataurangamaori.tki.org.nz/Support-materials/Te-Reo-Maori/Maori-Myths-Legends-and-Contemporary-Stories/Kupe-and-the-Giant-Wheke. Accessed December 2nd, 2022.</p> <p>Meihana, P. (n.d.) <i>The heritage trail of Kupe</i>. Malborough District Council. https://storymaps.arcgis.com/stories/18486bf9e060404f8f5b88121951a819</p> <p>Poutini</p> <p>Maipi, J. (2022). <i>Pūrākau, episode 4</i>. [Animated video] Cinco Cine. https://www.maoritelevision.com/shows/purakau/S01E004/purakau-episode-4</p> <p>Toitū te Whenua LINZ (2018). Poutini: A guardian Taniwha. https://www.linz.govt.nz/regulatory/place-names/about-new-zealand-geographic-board/nzgb-place-name-maps-and-publications/he-korero-p%C5%ABr%C4%81kau-mo-ng%C4%81-taunahanahatanga-ng%C4%81-t%C5%ABpuna/poutini-guardian-taniwha</p> <p>Google Maps. (n.d). The legend of Poutini https://www.google.com/maps/d/viewer?mid=1dp3zDUVlzUMDHqu-0okux5_UJ58Wqp3&ll=-40.85224426375098%2C172.07662799999997&z=6</p>
2	<p>Navigation concepts, environmental and standardised directional terms.</p> <p>The Natural Navigator. (2021) <i>How to navigate using Plants</i>. https://www.naturalnavigator.com/find-your-way-using/plants/</p>
3	<p>Students begin creating their own uruuru ao and start to identify significant location markers.</p>
4	<p>Students map their own uruuru ao both on paper and digitally</p> <p>Dalton, M. (2019). How to drop a pin on Google Maps. https://www.youtube.com/watch?v=InORWWNWAEg</p> <p>Gauging Gadgets. (2019). Create a public list on Google Maps. How to share Google Maps list. https://www.youtube.com/watch?v=xktpUAO1iUw</p>

Tohutoro | References

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