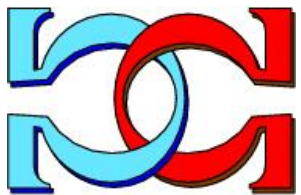
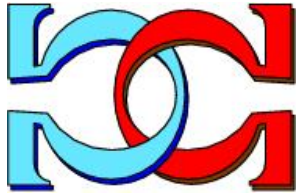
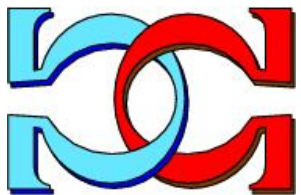


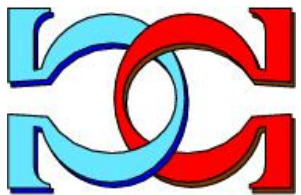
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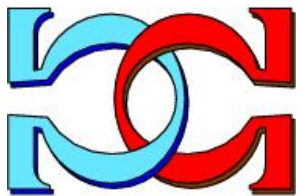
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the Infinity**



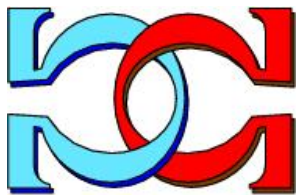
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CDMTCS-569  
March 2023



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# ChatGPT, Randomness and the Infinity

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August 25, 2023

## Abstract

ChatGPT, an application first released in November 2022 by Abacus.AI, has become very popular instantaneously and still dominates AI discussions in media and technical circles.

In this paper, we briefly discuss the merits and shortcomings of ChatGPT, the role of randomness in its capacity for diversification and a “dialogue” about infinity. We end with a few questions.

In November 2022, the start-up company Abacus.AI<sup>1</sup> has released the first version of ChatGPT, a free chatbot that can answer all sorts of questions with unprecedented “eloquence”.

ChatGPT, which has become very popular instantaneously<sup>2</sup>, is an improvement of GPT-3, an AI model that generates text based on patterns fed from massive Internet databases. ChatGPT stands out because it can answer questions in natural languages, a new interface everyone can use. Asking questions in natural language is not new: Siri, Apple’s virtual assistant, uses voice queries, gesture-based control, focus-tracking and a natural language to answer questions, make recommendations, and perform actions.

In early February 2023, Google and Baidu unveiled their chatbots called Bard and Ernie, which will compete with ChatGPT.

ChatGPT has been initially used as an alternative to Google because it can provide descriptions, answers and even solutions to complex questions. What does the program do? According to ATRIA Innovation<sup>3</sup> ChatGPT

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<sup>1</sup>Abacus.AI is the world’s first AI-assisted data science and end-to-end MLOps platform that enables real-time machine and deep learning at scale for common enterprise use cases. You can bring your own models or use our neural network and generative AI techniques to create highly-accurate models and operationalize them in production. cf. <https://abacus.ai>.

<sup>2</sup>Being free and sometimes producing fun errors helped.

<sup>3</sup><https://www.atriainnovation.com/en/how-does-chat-gpt-work/>, January 2023.

can:

1. generate coherent and well-written texts in various styles, topics and languages,
2. generate solutions and answers to various questions,
3. be used to generate attractive posts and messages for social networks,
4. generate reports, e-mails and other content for productivity applications,
5. analyse large data sets and extract valuable information.

How good is it? How does it do it? And why does it work?

Let's start with the first question. S. Altman, OpenAI, co-founder and CEO, believes that "Soon you will be able to have helpful assistants that talk to you, answer questions, and give advice. Later you can have something that goes off and does tasks for you. Eventually, you can have something that goes off and discovers new knowledge for you."<sup>4</sup>

For Stanford University Professor Christopher Potts, ChatGPT is "extremely impressive"<sup>5</sup>. Many others agree, but not all.

For Mike Pearl, "The ChatGPT chatbot from OpenAI is amazing, creative, and totally wrong. Need ideas? Great! Need facts? Stay away!"<sup>6</sup> For Professor G. N. Smith from Pomona College, "GPT-3's struggles with reality".<sup>7</sup> E. Musk, the startup co-founder, left the board and distanced himself from the company. The company itself acknowledged that at this stage<sup>8</sup> ChatGPT can give wrong answers and present misinformation as fact, writing "plausible-sounding but incorrect or nonsensical answers".

To give a short answer to the second question, we refer to a documented article by S. Wolfram [9] which "looks inside" ChatGPT:

My purpose here is to give a rough outline of what's going on inside ChatGPT-and then to explore why it is that it can do so well in producing what we might consider to be meaningful text. I should say at the outset that I'm going to focus on the big

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<sup>4</sup><https://tinyurl.com/28ypucjc>.

<sup>5</sup><https://tinyurl.com/8725fe6u>.

<sup>6</sup><https://mashable.com/article/chatgpt-amazing-wrong>.

<sup>7</sup>See <http://economics-files.pomona.edu/GarySmith/audiofiles/OnPoint.mp3>.

<sup>8</sup>December 2022.

picture of what’s going on – and while I’ll mention some engineering details, I won’t get deeply into them. (And the essence of what I’ll say applies just as well to other current “large language models [LLMs] as to ChatGPT.)”

ChatGPT’s primary objective is to generate a “plausible follow-up” to any text it receives. But what is “plausible”? Synonyms are “acceptable”, “adequate”, “passable”. In this context, a “plausible follow-up” could be a sequence of words that aligns with what a human familiar with many relevant sources would expect. According to [10], given the input,

The best thing about AI is its ability to

ChatGPT will produce a ranked list of words that might follow, each with some probability: “learn” (4.5%), “predict” (3.5%), “make” (3.2%), “understand” (3.1%), “do” (2.9%).

Simplifying, this is what ChatGPT does all the time: ask over and over again

given the text so far, what should the next part of a word (or even word) be?

and based on the answers (and probabilities), decide the adding (part of the) word. Quoting again [9]

One might think it should be the “highest-ranked” word (i.e. the one to which the highest “probability“ was assigned). But this is where a bit of voodoo begins to creep in. Because for some reason – that maybe one day we’ll have a scientific-style understanding of – if we always pick the highest-ranked word, we typically get a very flat essay, that never seems to “show any creativity” (and even sometimes repeats word for word). But if sometimes (at random) we pick lower-ranked words, we get a “more interesting” essay.

The key word is *randomness*: if we use the exact random prompt multiple times, we will get different essays each time! There is no underlying theory: empirically, this is the way to produce diversity.

If this seems surprising, think about “generative music”. In 1957 Lejaren Hiller and Leonard Isaacson, at the University of Illinois at Urbana-Champaign, used the ILLIAC computer to “compose” music.<sup>9</sup> First, they

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<sup>9</sup><https://historyofinformation.com/detail.php?id=1307>.

used simple rules: a melody with only notes within an octave, harmonies that tended towards the major and the minor with no dissonance, and a few more parameters. The University’s student orchestra members performed these compositions, which sounded like Bach and Penderecki. Still, they didn’t sound “original”. Eventually, the ingredient which helped was the random insertions of dissonances.<sup>10</sup>

Generalising the context, AI has undoubtedly made life easier. Implementing AI has also raised several concerns and questions. Here are some of them: Is the power of AI limitless? Will a form of AI surpass human intelligence soon?<sup>11</sup> Is AI understandable? Is AI ethical? Is AI accountable? Is AI free of bias and discrimination? Is AI safe and private?

In what follows, we will discuss only the first question: Is the power of AI limitless? If by limitless, we mean “AI can solve any problem”, then the reader would rightly suspect the answer is “no”. Why? AI ultimately consists of complex algorithms, whose limits have been proved by computability and complexity theories [8, 1, 2]. In a nutshell:

*most problems cannot be solved by algorithms.*

AI relies on algorithms, hence can “solve” only problems which cannot fall into the class covered by the undecidability theorems. The human brain can write and maintain massively complex software but cannot solve undecidable problems too.

There is an avalanche of articles regarding ChatGPT’s positive features – which include assisting education by making reference lists, generating “first drafts”, debugging, and tutoring – and adverse side-effects, like being used in exam cheating. As G. Smith noted,<sup>12</sup> the undesirable use of ChatGPT in exam cheating has a positive educational consequence:

the best response by educators is to teach and test critical thinking skills. These are what students need and they cannot be reliably faked by large language models (LLMs).

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<sup>10</sup>For details see <https://www.musicradar.com/how-to/ordered-chaos-a-guide-to-randomisation-probability-and-generative-music>.

<sup>11</sup>“Any intellect that vastly outperforms the best human brains in practically every field, including scientific creativity, general wisdom, and social skills”, is according to philosopher Nick Bostrom, a *superintelligence*.

<sup>12</sup><https://mindmatters.ai/2023/03/text-generators-education-and-critical-thinking-an-update/>.

How does ChatGPT perform in mathematics? The article [7] investigates

mathematical capabilities of ChatGPT by testing it on publicly available datasets, as well as hand-crafted ones, and measuring its performance against other models trained on a mathematical corpus, such as Minerva. We also test whether ChatGPT can be a useful assistant to professional mathematicians by emulating various use cases that come up in the daily professional activities of mathematicians (question answering, theorem searching).

The study concludes

that contrary to many positive reports in the media (a potential case of selection bias), ChatGPT’s mathematical abilities are significantly below those of an average mathematics graduate student. Our results show that ChatGPT often understands the question but fails to provide correct solutions. Hence, if your goal is to use it to pass a university exam, you would be better off copying from your average peer!

We next present a simple test of ChatGPT’s ability to deal with the infinite. The test is inspired by a conversation of S. Marcus<sup>13</sup> with a young girl:<sup>14</sup>

Antonia, a 10-year-old Romanian-Spanish girl from Tarragona, Spain, had a decisive influence in choosing the topic of the volume. Earlier than us, she called Professor Marcus *El infinito*, under the circumstances pointing to his manner of approaching kids. In his opinion, a child should rediscover Zeno’s paradoxes via a Socrates-type dialogue of the following kind: S.M.: “Look, Antonia, you have some usual bread, and you eat half of it today. How much does it remain for tomorrow?” A.: “Half a bread, of course.” S.M.: “Okay, but tomorrow you eat half of what you have. Does some bread remain for after tomorrow?” A.: “Yes, sir, some bread still remains.” S.M.: “Good, but after tomorrow you again eat half of your bread. Is it true that some bread remains?” A. (already doubtful): “Yes, but”... After about five iterations, the two parts are always separated: A., or any other

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<sup>13</sup>The author’s professor, supervisor, mentor, collaborator and friend for more than four decades.

<sup>14</sup>See the Preface of the book [5].

“victim”, knowing for sure that (s)he can eat a normal bread in a day or so, hence any continuation of the dialogue is senseless (because it’s breadless), S.M. looking desperately for the bright light of infinity in the others’ eyes: eating every day half of the piece of bread you have got means securing bread for infinitely many days. In many cases, both sides are disappointed . . . In at least one case, S.M. got a surname: *El Infinito*.

Below is the dialogue with ChatGPT:

Hello ChatGPT!

Hello! How can I help you today?

You have a loaf of bread, and Antonia eats today half of it. How much does it remain for tomorrow?

If Antonia eats half of the loaf of bread today, then half of the loaf remains for tomorrow.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

If Antonia eats half of what remains from the previous day, then the remaining bread will be halved again. If the original loaf of bread is divided in half, and then half of the remaining bread is eaten, then one quarter of the original loaf will remain.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

If Antonia eats half of what remains from the previous day, then the remaining bread will be halved again. If one quarter of the original loaf remains and half of that is eaten, then 1/8th of the original loaf will remain.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

Yes, if Antonia continues to eat half of what remains each day, some bread will remain for the following day. The amount of bread remaining will decrease by half each day, but it will never reach zero as long as Antonia only eats half of what remains each day.

After clicking “regenerate response” we got:

Hello ChatGPT!

Hello! How can I help you today?

You have a loaf of bread, and Antonia eats today half of it. How much does it remain for tomorrow?

If Antonia eats half of what remains from the previous day, then the remaining bread will be halved again. If  $1/8$ th of the original loaf remains and half of that is eaten, then  $1/16$ th of the original loaf will remain.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

If Antonia eats half of what remains from the previous day, then the remaining bread will be halved again. If  $1/16$ th of the original loaf remains and half of that is eaten, then  $1/32$ th of the original loaf will remain. And so on, following this pattern, the amount of bread will be halved every day.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

If Antonia eats half of what remains from the previous day, then the remaining bread will be halved again. If  $1/32$ th of the original loaf remains and half of that is eaten, then  $1/64$ th of the original loaf will remain. The pattern will continue as long as the bread is not completely finished.

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

If Antonia continues to eat half of what remains from the previous day, the amount of bread will continue to be halved each day. However, it will eventually run out, and there will be no bread left for future days.

The ChatGPT answers in the initial and regenerated dialogues are different in many respects, but the striking points are the conclusions:

1. In the first case:  
Yes, if Antonia continues to eat half of what remains each day, some bread will remain for the following day. The amount of bread remaining will decrease by half each day, but it will never reach zero as long as Antonia only eats half of what remains each day.



2. In the second case:

If Antonia continues to eat half of what remains from the previous day, the amount of bread will continue to be halved each day. However, it will eventually run out, and there will be no bread left for future days.

The last answer in the first dialogue

Yes, if Antonia continues to eat half of what remains each day, some bread will remain for the following day. The amount of bread remaining will decrease by half each day, but it will never reach zero as long as Antonia only eats half of what remains each day.

seems impressive because instead of responding “exactly” to the question

OK, the next day, Antonia eats half of what remains. Does some bread remain for the following day?

which refers to the *particular case 1/8th*, it offers a *generalisation* and, based on it, the correct answer. Could it be that ChatGPT was showing a form of mathematical “thinking”? ChatGPT does not rely on logical or mathematical inferences; hence these answers seem like an emergent<sup>15</sup> skill. The first conclusion is correct and includes a justification, if not a proof.<sup>16</sup>

After “regenerate”, ChatGPT seems to make it again a more general correct inference

The pattern will continue as long as the bread is not completely finished.

However, unexpectedly, it ends with a wrong answer.

The first conclusion is correct, and the second is wrong. Why? One can speculate that in the first case, ChatGPT was following a “mathematical” approach, while it offered a more “practical” answer in the second case. A more plausible explanation is that ChatGPT dialogues have been processed independently, despite using the command “regenerate”: quite surprisingly, ChatGPT processed the regenerated sequence of questions as a new dialogue and made no attempt to make any connection between the *same questions asked*. Consequently, it didn’t realise the contradictory final answers and

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<sup>15</sup>See also [6].

<sup>16</sup>Does this answer suggest the possibility of humanity running out of bread?

didn't "see" that the first answer was correct while the second was wrong. This "behaviour" seems to be characteristic of (current?) AI: the inability to make logical connections, hence recognising "true" vs. "false" answers and also to "learn from mistakes".<sup>17</sup>

Finally, I used the cloud-based AI-enabled typing assistant Grammarly<sup>18</sup> to check ChatGPT's answers. Grammarly found many issues. For example, Grammarly suggested changing "Yes, if Antonia continues to eat half of what remains each day, some bread will remain for the following day. The amount of bread remaining will decrease by half each day, but it will never reach zero as long as Antonia only eats half of what remains each day." to "Yes, if Antonia continues to eat half of what remains each day, some bread will stay for the following day. The remaining bread will decrease by half daily but never reach zero if Antonia only eats half of what remains daily.". Grammarly recommends various good styles and favours more compact texts; in contrast, ChatGPT often writes fluffy, non-compact sentences as a spandrel of its stochastic parroting, not quality writing. Of course, Grammarly and ChatGPT have different goals, but still, ChatGPT's answers should show a reasonable proficiency in English.<sup>19</sup>

Following B. Carpenter's suggestion<sup>20</sup> I repeated both dialogues with the addition of the first question-hint:

Can you explain Zeno's paradox?

The answer was:

Certainly! Zeno's paradox is a philosophical puzzle that was first proposed by the ancient Greek philosopher Zeno of Elea. The paradox is based on the idea that motion is an illusion, and it raises questions about the nature of space, time, and infinity.

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<sup>17</sup> "Regenerate response" instructs ChatGPT to answer the question from other angles or ways. I was not able to find an explicit "definition" of this command: In an email to the author from 24-Mar-23, A. Whithy suggested that regenerate is "intended by design not to look at the previous iteration of the answer, but instead, replace that answer with a new one as if the earlier had not happened". This "intention" suggests that answers are generated at random and can contradict themselves: ChatGPT makes no effort to guarantee minimal coherence and correctness, so it cannot be trusted.

<sup>18</sup> "Grammarly's team of computational linguists and deep learning engineers designs cutting-edge algorithms that learn the rules and hidden patterns of good writing by analyzing millions of sentences from research corpora." See <https://www.grammarly.com/blog/how-does-grammarly-work/>.

<sup>19</sup> ChatGPT can engage in conversations in at least 95 languages, with various degrees of competence.

<sup>20</sup> Email to Calude, 13-Mar-23.

Zeno's paradox is often presented in the form of a race between a tortoise and a faster runner. The runner, who is much faster than the tortoise, gives the tortoise a head start. The tortoise travels a certain distance, say 100 meters. Then, the runner sets off and tries to catch up with the tortoise. But by the time the runner reaches the point where the tortoise started, the tortoise has moved a little farther ahead. By the time the runner reaches the new point, the tortoise has moved a little farther again, and so on.

Zeno argued that, because there is an infinite number of points between any two points, the runner would have to cover an infinite number of points in order to catch up with the tortoise. And since it is impossible to complete an infinite task, the runner could never actually catch up with the tortoise.

Zeno's paradox has been interpreted in many ways over the centuries, and it has been used to support a variety of philosophical positions. Some have argued that the paradox demonstrates the impossibility of motion, while others have used it to argue for the existence of a higher reality beyond the physical world. However, most modern philosophers and mathematicians agree that the paradox is based on a flawed understanding of infinity and the nature of space and time.

The remaining answers have been similar to the ones obtained initially, even when I used the new GPT-4. This shows that (sometimes?) ChatGPT can't see the connection between topics, even in case some clear hints are given.

Under the title *Introducing ChatGPT* the opening of the Open.AI website<sup>21</sup> includes the paragraph:

We've trained a model called ChatGPT which interacts in a conversational way. The dialogue format makes it possible for ChatGPT to answer followup questions, admit its mistakes, challenge incorrect premises, and reject inappropriate requests.

The example discussed above does not confirm the claims above.

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<sup>21</sup><https://openai.com/blog/chatgpt>, checked on 12 March 2023.

Giuseppe Longo<sup>22</sup> is, first and foremost, a mathematician with important results in mathematical logic, computability, and complexity. His theoretical work has been expanded to foundations of physics, biology, cognitive sciences, epistemology and the digital world. So, he is eminently qualified to comment on the recent development of AI, its power, limits and future.

His recent lecture *A critique of digital reason: successes and limits of some powerful techniques in AI* delivered at the University of San Francisco<sup>23</sup> discusses the following questions relevant to this article:

1. How to relate mathematical constructions of optimal paths in complex networks of interactions to actual human action?
2. How does randomness,<sup>24</sup> used to produce unpredictable machine events, related to human creativity? Hypes follow hypes, often with major effects on the stock market and a little more.

It concluded with

A better understanding of the limits of these great technologies may help to produce better interfaces. A couple of examples of successful interface construction, human/machine, will be mentioned, beyond myth.

ChatGPT and similar AI applications capture a “statistical likelihood relationship” in impressive but imperfect ways; they raise many questions, including:

1. How far can they go?
2. Could they master syntactical questions correctly?
3. What about semantical ones?
4. Could they self-correct?

Without a doubt, ChatGPT has a disruptive effect. Emerging Generative AI has the potential to empower people at work, save lives, and possibly

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<sup>22</sup>[https://en.wikipedia.org/wiki/Giuseppe\\_Longo](https://en.wikipedia.org/wiki/Giuseppe_Longo).

<sup>23</sup><https://philosophy.sfsu.edu/event/giuseppe-longo-critique-digital-reason-successes-and-limits-some-powerful-techniques-ai>, 13 April 2023.

<sup>24</sup>Randomness was essential in our joint works [3, 4].

reduce some of the worlds worst inequities. At the same time, this trend raises hard questions about ethics, privacy, the legal system, education, and much more. It is equally important to remember that there are always challenges and difficulties in any given period and that humanity has always found ways to overcome them and progress. Individually it is essential to stay informed and engaged in these efforts. One such effort was the open letter “Pause Giant A.I. Experiments”,<sup>25</sup> calling for a temporary moratorium on training models larger than G.P.T.-4. The letter had 27,572 signatories (as of 2 May 2023), including the author of this article.

## Acknowledgement

I thank B. Bravi, B. Carpenter, M. Marcus, J. Sifakis, A. Whithy and S. Yang for many discussions and comments on this article which improved the presentation.

## References

- [1] C. Calude. *Theories of Computational Complexity*. North-Holland, Amsterdam, 1988.
- [2] C. Calude. *Information and Randomness—An Algorithmic Perspective*. Springer, Berlin, 2002 (2nd ed.).
- [3] C. S. Calude and G. Longo. Classical, quantum and biological randomness as relative unpredictability. *Nat. Comput.*, 15(2):263–278, 2016.
- [4] C. S. Calude and G. Longo. The deluge of spurious correlations in big data. *Foundations of Science*, 22(3):595–612, 2017.
- [5] C. S. Calude and G. Păun, editors. *Finite Versus Infinite. Contributions to an Eternal Dilemma*. Spriger-Verlag, London, 2000.
- [6] C. S. Calude and K. Svozil. Spurious, emergent laws in number worlds. *Philosophies*, 4(2):17, 2019.
- [7] S. Frieder, L. Pinchetti, R.-R. Griffiths, T. Salvatori, T. Lukasiewicz, P. C. Petersen, A. Chevalier, and J. Berner. *Mathematical Capabilities of ChatGPT*, 2023.

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<sup>25</sup><https://futureoflife.org/open-letter/pause-giant-ai-experiments/>.

- [8] P. Odifreddi. *Classical Recursion Theory, Vol. 1*. North-Holland, Amsterdam, 1989.
- [9] S. Wolfram. What Is ChatGPT Doing ... and Why Does It Work?, [writings.stephenwolfram.com/2023/02/what-is-chatgpt-doing-and-why-does-it-work/](https://writings.stephenwolfram.com/2023/02/what-is-chatgpt-doing-and-why-does-it-work/), March 2023.
- [10] S. Wolfram. *A New Kind of Science*. Wolfram Media, Inc., Champaign, IL, 2002.