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Cite as: AIP Conference Proceedings **2651**, 050011 (2023); https://doi.org/10.1063/5.0105416
Published Online: 27 March 2023

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# Sliding Mode Control Based on High-Order Extended State Observer for Flexible Joint Robot Under Time-varying Disturbance

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**Abstract.** In this paper, a high order extended state observer (HOESO) based a sliding mode control (SMC) is proposed for a flexible joint robot (FJR) system in the presence of time varying external disturbance. A composite controller is integrated the merits of both HOESO and SMC to enhance the tracking performance of FJR system under the time varying and fast lumped disturbance. First, the HOESO estimator is constructed based on only one measured state to precisely estimate unknown system states and lumped disturbance with its high order derivatives in the FJR system. Second, the SMC scheme is designed based on such accurate estimations to govern the nominal FJR system by well compensating the estimation errors in the states and the lumped disturbance. To verify the tracking trajectory performance, several simulations have been conducted on the simulated FJR plant model. In addition, a comparative study is carried out between the proposed method and the full state feedback linearization control (FLC) with first order ESO (ESO1).

## 1- INTRODUCTION

Flexible-joint manipulators have received a considerable attention from the control engineers due to their significance in the modern manipulators. In contrast to rigid manipulators, robotic manipulators with joint flexibility can offer various advantages, including lower cost, smaller actuators, lightweight, larger work volume, better transportability and maneuverability, higher operational speed and power efficiency [1, 2]. Industrial robots are equipped with transamination power system such as bearings, belts, gears, and hydraulic lines in order to transmit the low torque from the motor to the link with high speed, thus achieving specific tasks [1]. However, the deformation of such harmonic devices results in the flexibility in the joints that is the main reason of the oscillations generated in the end effector of robots [3].

Empirical evidences indicate that the joint flexibility should be considered in both dynamic modeling and control design if high tracking performance is required. To represent the joint flexibility in the dynamic modeling, the link of robot is connected to the motor via a torsional spring which represents the aforementioned-above harmonic drives components. Interestingly, the joint flexibility consideration will reduce the potential system uncertainties, which should be compensated and rejected in any control design scheme. However, the introduction of joint flexibility to manipulators results in oscillations and natural frequencies which limit a trajectory motion accuracy of the tip's link and increase a setting time and overshoot of trajectory response [4]. Furthermore, this will increase the order of related system to double compared to rigid manipulators and make the number of inputs less than degree of freedom to be controlled [5]. Thus, this complicates the equation of motions of the system and make the control design difficult task [1].

In order to tackle the above-aforementioned problems, many control schemes have been proposed and implemented to the FJR manipulators [6-23]. Most interestingly, SMC and disturbance observer based control methods [1, 22, 23] are well-known for their strong robustness to the system uncertainties and external disturbances in the FJRs.

In this paper, we consider the problem of robust trajectory tracking control of the FJR manipulator subjected to unknown time varying disturbances. Considering the fact that the FJR system is affected by fast time varying disturbances and state-dependent uncertainties which may be in the form of constant, ramp and parabolic disturbances. For such a situation, High order ESO estimator is first constructed for estimating the unmeasurable states and time

*8th Engineering and 2nd International Conference for College of Engineering – University of Baghdad*
AIP Conf. Proc. 2651, 050011-1–050011-9; https://doi.org/10.1063/5.0105416
Published by AIP Publishing. 978-0-7354-4296-2/$30.00

varying disturbances with its high order derivatives in the FJR plant model. The reason for using HOESO here is that the estimation of high order derivatives of disturbance helps to improve the estimation accuracy. Based on such estimations, a full order SMC (FOSMC) is designed based on its full order sliding manifold in order to well achieve robustness by compensating the estimation errors in the states and disturbances. Finally, the simulation results on the FJR system are conducted to verify the benefits of proposed control which is compared with the existing feedback linearization control FLC with the ESO in the literature. Not that the third order ESO (ESO3) estimator used in our controller is designed based on the assumption of third derivative of disturbance is bounded rather than the boundedness of the first derivative of disturbance as in the first order ESO [1].

The rest of the paper is organized as follows. Section 2 describes the dynamic modeling of the FJR system. In Section 3, the HOESO estimator design is introduced. Section 4 presents details on full order SMC controller based on HOESO. In Section 5, simulation results are provided to verify the efficacy of the proposed method. Finally, the paper is concluded in Section 6.

**TABLE 1:** Nominal physical values of the FJR system.

| Parameters | Meaning | Value |
|---|---|---|
| $R_m$ | Armature Resistance (Ω) | 2.6 |
| $K_m$ | Motor Back-EMF Constant (V. s/rad) | 0.00767 |
| $K_t$ | Motor Torque Constant (N. m/A) | 0.00767 |
| $J_{arm}$ | Total Arm Inertia (kg. $m^2$) | 0.0019 |
| $J_{eq}$ | Equivalent Motor Inertia (kg. $m^2$) | 0.0021 |
| $K_g$ | High Gear Ratio | 14:5 |
| $K_s$ | Joint Stiffness (N.m/rad) | 1.2485 |
| $B_{eq}$ | Equivalent Viscous Damping (N.m.s/rad) | 0.004 |
| $\eta_g$ | Gearbox Efficiency | 0.9 |
| $\eta_m$ | Motor Efficiency | 0.69 |

## 2- DYNAMIC MODEL OF FJR MANIPULATOR

The dynamical modelling of the FJR represented in the single-input single output (SISO) canonical form is given by [5]:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= x_4(t) \\ \dot{x}_4(t) &= f(x) + b(x)(u + d(t)) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where

$$f(x) = a_3\ddot{x}_1 + a_1\ddot{x}_1 + \frac{a_3}{K_s J_{arm}}\dot{x}_1,$$

with

$$a_1 = -\frac{K_s(J_{eq} + J_{arm})}{J_{eq}J_{arm}}; a_2 = \frac{K_s}{J_{eq}}$$

$$a_3 = \frac{\eta_m\eta_g K_t K_m K_g^2 + B_{eq}R_m}{J_{eq}R_m}, b = -\frac{\eta_m\eta_g K_t K_g K_s}{J_{arm}J_{eq}R_m}$$

in which all system parameters with their physical meanings are listed in Table 1. In the FJR system dynamics (1), $x_1(t)$ is a measurable link position, which is the output the FJR system. In addition, the state vector $x = x_1(t)$, $x_2(t) = \dot{x}_1$, $x_3(t) = \ddot{x}_1$, $x_4(t) = \dddot{x}_1{}^T$ includes the link angular position, angular velocity, angular acceleration, and angular jerk, respectively, $u(t)$ denotes the control input, $d(t)$ is the matched time-varying external disturbance, and $f(x,t)$ and $b(x,t)$ are unknown uncertain functions, which can be generally represented as

$$
\begin{aligned}
f(x,t) &= f_0(x,t) + \triangle f(x,t) + O_1(x) \\
b(x,t) &= b_0(x,t) + \triangle b(x,t) + O_2(x).
\end{aligned} \tag{2}
$$

As illustrated in the above functions, $f(x,t)$ has a nominal linear known function $f_0(x,t)$, an unknown state-dependent uncertainty $\triangle f(x,t)$ and a state-dependent nonlinearity $O_1(x)$, and also $b(x,t)$ has the similar aforementioned functions.

In the system dynamics (1), all of such impacts of system uncertainties, nonlinearities and external disturbances can be lumped with the nominal system dynamics $f_0(x,t)$ in the one variable $\xi$ as

$$\xi(x,u,d) = f_0(x) + \triangle f(x,t) + O(x,u) + \triangle b(x)u + b(x)d(t) \tag{3}$$

with

$$O(x,u) = O_1(x) + O_2(x)u;\ \triangle b(x) = b(x) - b_0(x).$$

For simplicity, the system dynamics in (1) with the lumped disturbance $\xi(x,u,d)$ can be rewritten as

$$
\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= \xi(x,u,d) + b_0 u \\
y(t) &= x_1(t)
\end{aligned} \tag{4}
$$

with

$$b_0 = -\frac{\eta_m \eta_g K_t K_g K_{s0}}{J_{arm0} J_{eq0} R_m}$$

where $b_0(x,t)$ is the nominal input constant of $b(x)$ and the values of the nominal system parameters are listed in Table 1. Accordingly, the dynamical model (4) represents the SISO system with fourth order, which is subjected by the unknown time-varying disturbance $\xi$.

Based on the plant model (4), the control objective is to design a robust FOSMC such that the system output, i.e, the link motion trajectory $x_1(t)$ is enforced to track a desired reference as quick and accurate as possible.

## 3- HOESO ESTIMATOR DESIGN

A HOESO estimator is here designed for estimating virtual system states and time-varying lumped disturbance with its high-order derivatives. In practical applications, most mechanical systems subjected to unknown time-varying disturbances which may be in the form of constant, ramp and parabolic disturbances. For unknown time-varying generalized disturbance $\xi(x,u,d)$, it can be modeled by the following time polynomial form

$$\xi = \sum_{j=0}^{m-1} \gamma_j t^j + r(t) \tag{5}$$

where $\gamma_j > 0,\ (j = 0, \cdots, m-1)$ are unknown constants, and $r(t)$ stands for an unknown residual term. Accordingly, it is assumed that there exists the first m time derivatives of $\xi$ at least and the m-th time derivatives of $\xi$ is bounded by $\left|\xi^{(m)}\right| = \left|r^{(m)}\right| \leq \mathcal{D}$ where $\mathcal{D}$ is a positive constant, i.e, $lim_{t\to\infty}\ \xi^m = 0$. Based on such assumptions with extending $\xi^j,\ j = 1, \cdots, m$ as a set of auxiliary states in the system dynamics (4), we define

$$x_5 = \xi, x_6 = \dot{\xi}, \cdots, x_{m+4} = \overset{(m)}{\xi}. \tag{6}$$

Let us to introduce a new state vector $\chi = [x_1, \ldots, x_{m+4}]$, the FJR system dynamics (4) can be extended by m extra states variables as in the following state-space model:

$$
\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_4(t) \\
\dot{x}_4(t) &= \xi + b_0 u \\
\dot{x}_5(t) &= \dot{\xi} \\
&\vdots \\
\dot{x}_{m+4}(t) &= \overset{(m)}{\xi}\ .
\end{aligned} \tag{7}
$$

In order to estimate the FJR system states and the lumped disturbance with its high order derivatives, the HOESO for the extended model of FJR system (7) is constructed as follows

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \kappa_1(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \kappa_2(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_3 &= \hat{x}_4 + \kappa_3(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_4 &= \hat{\xi}_1 + \kappa_4(x_1 - \hat{x}_1) + b_0 u \\
\dot{\hat{\xi}}_j &= \hat{\xi}_{j+1} + \kappa_5(x_1 - \hat{x}_1)\ \ (j = 1, \cdots, m-1) \\
&\vdots \\
\dot{\hat{\xi}}_m &= \kappa_{m+4}(x_1 - \hat{x}_1)
\end{aligned} \tag{8}
$$

where $\hat{x}_i\ (i = 1, \cdots, 4)$ are the state estimations of $x_i\ (i = 1, \cdots, 4)$, $\hat{\xi}_i\ (i = 1, \cdots, m)$ are the disturbance estimations of $\xi^{(j-1)}\ (j = 1, \cdots, m)$ and $\kappa_k > 0,\ (k = 1, \ldots, m+4)$ is the observer gains of HOESO.

For analyzing the stability of the HOESO dynamics (8), the estimation errors are defined as:

$$
\begin{aligned}
\tilde{e}_i &= x_i - \hat{x}_i\ (i = 1,2,3,4), \\
\tilde{e}_{j+4} &= x_{j+4} - \hat{\xi}_i\ (j = 1, \cdots, m)
\end{aligned} \tag{9}
$$

By combining the system dynamics (7) and the time derivative of (9), the observer error dynamics is obtained as:

$$
\begin{aligned}
\dot{\tilde{e}}_1 &= \tilde{e}_2 - \kappa_1 \tilde{e}_1 \\
\dot{\tilde{e}}_2 &= \tilde{e}_3 - \kappa_2 \tilde{e}_1 \\
\dot{\tilde{e}}_3 &= \tilde{e}_4 - \kappa_3 \tilde{e}_1 \\
\dot{\tilde{e}}_4 &= \tilde{e}_5 - \kappa_4 \tilde{e}_1 \\
\dot{\tilde{e}}_{j+4} &= \tilde{e}_{n+j} - \kappa_{j+4} \tilde{e}_1\ \ (j = 1, \ldots, m-1) \\
&\vdots \\
\dot{\tilde{e}}_{n+m} &= \overset{(m)}{\xi} - \kappa_{m+4}.
\end{aligned} \tag{10}
$$

The closed loop of HOESO dynamics (10) can be determined as follows

$$
\overset{(m+4)}{\tilde{e}_1} + \kappa_1 \overset{(m+3)}{\tilde{e}_1} + \cdots + \kappa_{m+3}\dot{\tilde{e}}_1 + \kappa_{m+4}\tilde{e}_1 = \overset{(m)}{\xi} \tag{11}
$$

Obviously, under the assumption of boundedness of the $n^{th}$ rate of change of the time-varying lumped disturbance $\xi^m$, the coefficients of HOESO are selected such that the following characteristics polynomial form in the complex variable s

$$
s^{(m+4)} + \kappa_1 s^{(m+3)} + \cdots + \kappa_{m+3} s + \kappa_{m+4} = 0 \tag{12}
$$

is Hurwitz stable and its roots are assigned in the left half of the complex plane. Accordingly, the trajectories of observer estimation errors $\tilde{e}_k\ (k = 1, \cdots, m+4)$ will converge to zero when time goes infinity, i.e., $\tilde{e}_1 = \dot{\tilde{e}}_1 = \cdots = \tilde{e}^{(m+3)} = 0$.

## 4- CONTROLLER DESIGN OF FOSMC

In this section, a FOSMC is designed to improve a tracking trajectory response of the FJR system in the presence of unknown time-varying matched disturbance. The tracking error trajectories are presented as

$$\begin{aligned} e_1(t) &= x_1(t) - x_d(t) \\ e_2(t) &= \dot{x}_1(t) - \dot{x}_d(t) = x_2(t) - \dot{x}_d(t) \\ e_3(t) &= \dot{x}_2(t) - \ddot{x}_d(t) = x_3(t) - \ddot{x}_d(t) \\ e_4(t) &= \dot{x}_3(t) - \dddot{x}_d(t) = x_4(t) - \dddot{x}_d(t) \end{aligned} \tag{13}$$

where $x_d(t)$ represents the time-varying input reference, which is differentiable up fourth-order. Accordingly, the error system dynamics of (4) is obtained as

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= e_3(t) \\ \dot{e}_3(t) &= e_4(t) \\ \dot{e}_4(t) &= \xi(x,u,d) + b_0 u - \ddddot{x}_d(t). \end{aligned} \tag{14}$$

A full-order sliding mode surface for the error dynamics (14) can be defined in the following form:

$$\sigma(t) = b_0 u + \hat{\xi}_1 - \ddddot{x}_d + \textstyle\sum_{i=2}^{4} \lambda_i \hat{e}_i + \lambda_1 e_1 \tag{15}$$

where $\lambda_i > 0, (i = 1, \cdots, 4)$ are constants, which guarantees that the following polynomial form

$$P_c(s) = s^4 + \lambda_4 s^3 + \lambda_3 s^2 + \lambda_2 s + \lambda_1 \tag{16}$$

is Hurwitz stable. For the high-order uncertain FJR system dynamics (14), the proposed output feedback of full-order SMC (FOSMC) is given as

$$u = -\frac{1}{b_0}\left(u_{eq} + u_n\right) \tag{17}$$

$$u_{eq} = \hat{\xi}_1 - \ddddot{x}_d + \textstyle\sum_{i=2}^{4} \lambda_i \hat{e}_i + \lambda_1 e_1 \tag{18}$$

$$\dot{u}_n = \eta \ sign(\sigma) \tag{19}$$

where $\hat{e}_2 = \hat{x}_2 - \dot{x}_d,\ \hat{e}_3 = \hat{x}_3 - \ddot{x}_d,\ \hat{e}_4 = \hat{x}_4 - \dddot{x}_d$ and the switching gain $\eta > 0$ ensures that the system output $y(t) = x_1$ will asymptotically converge to its reference trajectory $x_d$.

## 5- SIMULATION RESULTS

In this section, some simulations are conducted to show the merits of the proposed HOESO based FOSMC method. To compare the efficiency of the proposed scheme, the feedback linearization based full state feedback control (FLC) with ESO1 [1] is also applied for the FJR system in which the ESO1 for the system (4) is designed as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \beta_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + \beta_2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 &= \hat{x}_4 + \beta_3(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_4 &= \hat{\xi}_1 + \beta_4(x_1 - \hat{x}_1) + b_0 u \\ \dot{\hat{\xi}}_1 &= \beta_5(x_1 - \hat{x}_1) \end{aligned} \tag{20}$$

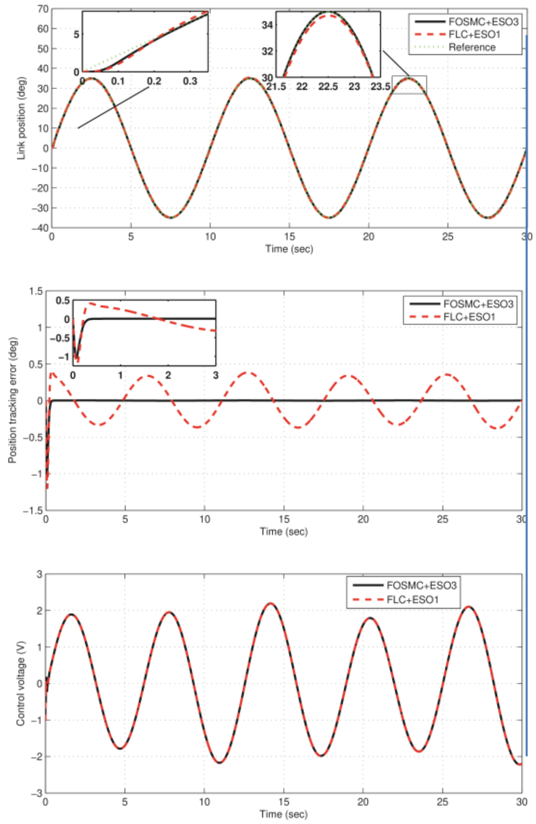


**FIGURE 1.** Response Curves of FJR system for two controllers with the external disturbance (top: link position trajectory; middle:link position tracking error; bottom: control input voltage)

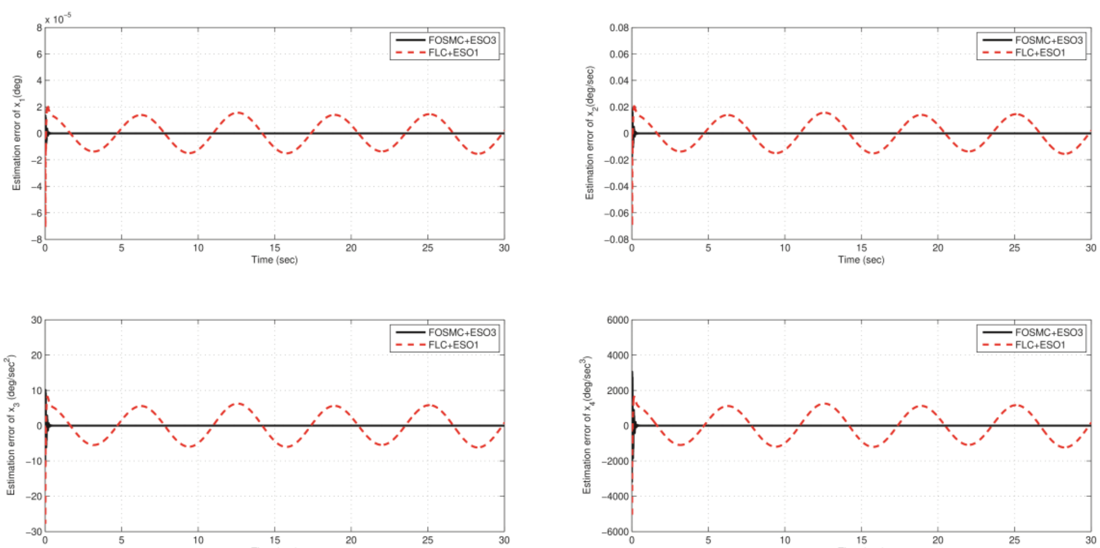


**FIGURE 2:** State estimation errors for FJR system.

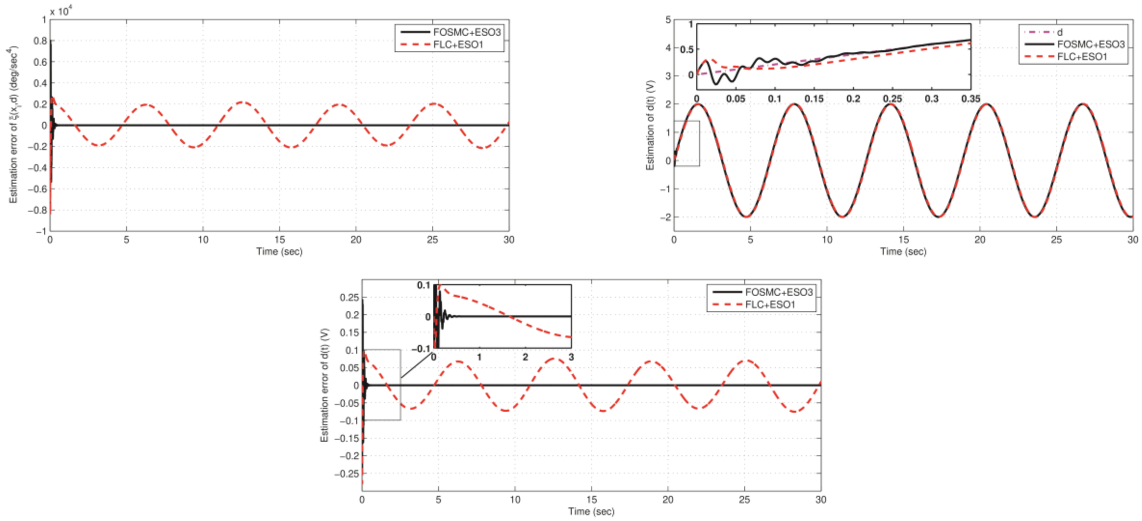

**FIGURE 3:** Disturbance observation performance for FJR system.

where $\beta_i > 0,\ (i = 1, \cdots, 5)$ such that the roots of the following polynomial

$$P_{ESO1}(s) = s^5 + \sum_{i=1}^{k=5} \beta_i s^{(k-i)} = 0. \tag{21}$$

are placed in the left hand side of the complex plane. The ESO dynamics (20) represents the first order ESO (ESO1), which is designed based on the assumption of boundedness of the first derivative of the lumped disturbance $\dot{\xi}$ for $m{=}1$. This assumption is true to be considered when the FJR system affects by either constant or slow disturbances. In this composite controller, the FLC law for the FJR system (4) is designed as

$$U_{FLC+ESO1} = -b_0^{-1}(\hat{\xi}_1 - \ddddot{x}_d + \sum_{i=2}^{4} c_i \hat{e}_i + c_1 e_1) \tag{22}$$

where $c_i > 0,\ (i = 1, \ldots, 4)$ are control gains, which are chosen such that the eigenvalues of the following characteristics function

$$P_{FLC}(s) = s^4 + c_4 s^3 + c_3 s^2 + c_2 s + c_1 \tag{23}$$

are placed in the left half of the complex plane. On the basis of HOESO presented in this paper, the boundedness of third derivatives of the time varying system lumped disturbances is taken into account to construct the third order ESO (ESO3) by selecting (m=3) as follows

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \kappa_1(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \kappa_2(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_3 &= \hat{x}_4 + \kappa_3(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_4 &= \hat{\xi}_1 + \kappa_4(x_1 - \hat{x}_1) + b_0 u \\
\dot{\hat{\xi}}_1 &= \hat{\xi}_2 + \kappa_5(x_1 - \hat{x}_1) \\
\dot{\hat{\xi}}_2 &= \hat{\xi}_3 + \kappa_6(x_1 - \hat{x}_1) \\
\dot{\hat{\xi}}_3 &= \kappa_7(x_1 - \hat{x}_1)
\end{aligned} \tag{24}$$

in which based on (12), the observer gains $\kappa_k > 0, (k = 1, \cdots, 7)$ in accordance to the following seventh-order polynomial

$$P_{ESO3}(s) = s^7 + \sum_{i=1}^{k=7} \kappa_i s^{(k-i)} = 0. \tag{25}$$

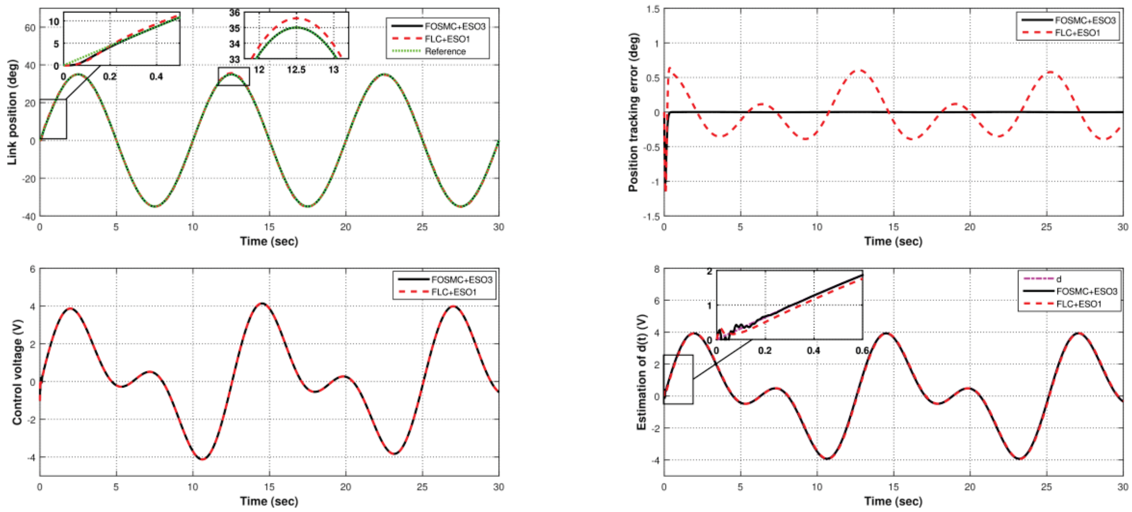


**FIGURE 4:** Response curves of FJR system for two controllers under the dual external disturbance.

Here, we would like to have a fair comparison, the observer and controller parameters in the proposed and comparative methods are equally tuned. In this particular consideration, the controller parameters in the proposed FOSMC with ESO3 (FOSMC+ESO3) in (17) and FLC+ESO1 in (22) are selected their polynomial equations (16) and (23), respectively. For simplicity of tuning process, the control coefficients in (16) and (23) are calculated as follows

$$P_{SMC}(s) = P_{FLC}(s) = (s + w_c)^4 \tag{26}$$

with

$$\begin{aligned}[c_1,\ c_2\ \ c_3\ \ c_4] &= \ [\lambda_1,\ \lambda_2\ \ \lambda_3\ \ \lambda_4] \\ &= \ [w_c^4,\ 4w_c^3,\ 6w_c^3,\ 4w_c]\end{aligned}$$

where $w_c = 30$ is the bandwidth of both controllers FOSMC and FLC schemes. To obtain the corresponding observer gains in ESO1 (20) and ESO3 (24), their polynomial characteristics (21) and (25) are simplified as in the two following functions, respectively as

$$P_{ESO1}(s) = (s + w_o)^5 \tag{27}$$

$$P_{ESO3}(s) = (s + w_o)^7 \tag{28}$$

where $w_o = 200$ is the bandwidth of both first order ESO1 and third order ESO3. Note that the time-varying lumped disturbance acting on the FJR system (4) contains the nominal system model $f_0(x)$ and the time-varying external perturbation $d(t) = 2sin(t)$.

Next, the performance of proposed controller in the existence of fast time-varying disturbances is evaluated. Note that we consider a sinusoidal reference in which its amplitude of $35^o$ and frequency of 0.1 Hz are set.

The simulation results under the two control schemes are demonstrated in Figures 1-3. To illustrate the simulation results more obviously, the zoomed figures of some profiles are given. From Figure 1, it can been clearly seen that the proposed control accomplishes better tracking performance in the transient and steady state error region in comparison with FLC+ESO1. In addition, the prediction accuracy for the system states and the lumped disturbance under the FOSMC with third order ESO (ESO3) is significantly better than the comparative controller FLC with ESO1 as shown in Figure 2 and Figure 3. Therefore, the better estimation quality nicely affects the tracking trajectory performance.

Figures 4 show response curves of FJR system for two controllers under the dual external disturbance that is defined as $d(t) = 2sin(t) + 2.5sin(0.5t)$. It can be seen from Figure 4 that the proposed control scheme is better than the comparative FLC scheme in terms of tracking, tracking error, and the disturbance estimation.

## 6- CONCLUSION

In this paper, a high order extended state observer based the SMC has been proposed for the FJR in the presence of external disturbance. The composite controller has integrated the merits of HOESO and SMC to enhance the tracking trajectory performance of the FJR under time varying and fast disturbance. First, the HOESO estimator is constructed based on only one measured state in order to precisely estimate unknown states and lumped disturbance with its high order derivatives in the FJR system. Second, the SMC controller is designed based on such accurate estimations to govern the nominal FJR system by well compensating the estimation errors in the states and lumped disturbance. To verify the tracking trajectory performance, several simulations have been conducted on the simulated FJR plant model. In addition the comparative study is done between the proposed method and the full state feedback linearization control with first order ESO. In the future work, the stability analysis of the closed loop FJR system under the proposed work and experimental validation will by conducted.

## REFERENCES


1. S. E. Talole, J. P. Kolhe, and S. B. Phadke, “Extended-state-observer-based control of flexible-joint system with experimental validation,” IEEE Trans. Industrial Electronics 57, 1411–1419 (2010).
2. R. Van Ham, T. G. Sugar, B. Vanderborght, K. W. Hollander, and D. Lefeber, “Compliant actuator designs,” IEEE Robotics & Automation Magazine 16 (2009).
3. K. Rsetam, Z. Cao, and Z. Man, “Hierarchical sliding mode control applied to a single-link flexible joint robot manipulator,” in 2016 International Conference on Advanced Mechatronic Systems (ICAMechS) (IEEE, 2016) pp. 476–481.
4. K. Rsetam, Z. Cao, and Z. Man, “Design of robust terminal sliding mode control for underactuated flexible joint robot,” IEEE Transactions on Systems, Man, and Cybernetics: Systems (to be published, doi:10.1109/TSMC.2021.3096835, 2021).
5. K. Rsetam, Z. Cao, and Z. Man, “Cascaded-extended-state-observer-based sliding-mode control for underactuated flexible joint robot,” IEEE Transactions on Industrial Electronics 67, 10822–10832 (2019).
6. L. Le-Tien and A. Albu-Schäffer, “Robust adaptive tracking control based on state feedback controller with integrator terms for elastic joint robots with uncertain parameters,” IEEE Trans. Control Syst. Technol. 26, 2259–2267 (Nov. 2018).
7. W. Sun, S.-F. Su, J. Xia, and V.-T. Nguyen, “Adaptive fuzzy tracking control of flexible-joint robots with full-state constraints,” IEEE Transactions on Systems, Man, and Cybernetics: Systems (2018).
8. Y. Yunda, C. Zhang, A. Narayan, J. Yang, S. Li, and H. Yu, “Generalized dynamic predictive control for non-parametric uncertain systems with application to series elastic actuators,” IEEE Transactions on Industrial Informatics (2018).
9. F. Petit, A. Daasch, and A. Albu-Schäffer, “Backstepping control of variable stiffness robots,” IEEE Trans. Control Syst. Technol. 23, 2195–2202 (Nov. 2015).
10. Y. Pan, H. Wang, X. Li, and H. Yu, “Adaptive command-filtered backstepping control of robot arms with compliant actuators,” IEEE Trans. Control Syst. Technol. 26, 1149–1156 (May 2018).
11. S. J. Yoo, J. B. Park, and Y. H. Choi, “Adaptive dynamic surface control of flexible-joint robots using self-recurrent wavelet neural networks,” IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 36, 1342–1355 (2006).
12. S. J. Yoo, J. B. Park, and Y. H. Choi, “Adaptive output feedback control of flexible-joint robots using neural networks: dynamic surface design approach,” IEEE Transactions on Neural Networks 19, 1712 (2008).
13. Z.-G. Liu and Y.-Q. Wu, “Modelling and adaptive tracking control for flexible joint robots with random noises,” International Journal of Control 87, 2499–2510 (2014).
14. Y. Li, S. Tong, and T. Li, “Adaptive fuzzy output feedback control for a single-link flexible robot manipulator driven dc motor via backstepping,” Nonlinear Analysis: Real World Applications 14, 483–494 (2013).
15. D. Ginoya, P. Shendge, and S. Phadke, “Disturbance observer based sliding mode control of nonlinear mismatched uncertain systems,” Communications in Nonlinear Science and Numerical Simulation 26, 98–107 (2015).
16. C. Ott, A. Albu-Schaffer, A. Kugi, and G. Hirzinger, “On the passivity-based impedance control of flexible joint robots,” IEEE Transactions on Robotics 24, 416–429 (2008).
17. J. De León-Morales, J. Alvarez-Leal, R. Castro-Linares, and J. Alvarez-Gallegos, “Control of a flexible joint robot manipulator via a non-linear control-observer scheme,” International Journal of Control 74, 290–302 (2001).
18. R. Marino and M. Spong, “Nonlinear control techniques for flexible joint manipulators: a single link case study,” in Proceedings. 1986 IEEE International Conference on Robotics and Automation, Vol. 3 (IEEE, 1986) pp. 1030–1036.
19. J. Kim, “Two-time scale control of flexible joint robots with an improved slow model,” IEEE Transactions on Industrial Electronics 65, 3317–3325 (2018).
20. Y. Pan, X. Li, and H. Yu, “Efficient pid tracking control of robotic manipulators driven by compliant actuators,” IEEE Trans. Control Syst. Technol. , 1–8 (2018).
21. J. Kim and E. A. Croft, “Full-state tracking control for flexible joint robots with singular perturbation techniques,” IEEE Trans. Control Syst. Technol. 27, 63 – 73 (Jan. 2019).
22. D. Ginoya, P. Shendge, and S. Phadke, “Sliding mode control for mismatched uncertain systems using an extended disturbance observer,” IEEE Transactions on Industrial Electronics 61, 1983–1992 (2014).
23. H.Wang, Y. Pan, S. Li, and H. Yu, “Robust sliding mode control for robots driven by compliant actuators,” IEEE Trans. Control Syst. Technol. (2018).