Making Sense of International Students' Experiences when Transitioning to Undergraduate Mathematics:

An Exploration of Mathematical Identities

Kim Locke

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Abstract

International students form a distinct cohort in first-year mathematics courses at many universities across the world. For these students, the transition to university mathematics may demand a significantly higher level of adaptation than would typically be required of students transitioning to university in their home countries. A research-based understanding of their experiences would illuminate challenges and enable host universities to provide effective support for international students as they transition from school mathematics into a university context. This study aims to better understand international student experiences of the transition by focusing on their mathematical identity constructions. Using a critical realist approach to identity, the study illuminates tensions that can arise when cultural understandings of being a mathematics learner interact with the institutional structures of first-year mathematics in a foreign country.

The study was undertaken in two phases. In the first phase, seven international students, who were at an advanced stage of their university studies in New Zealand, were interviewed about their learning experiences during first-year mathematics. The second phase involved three different international students who were enrolled in their first year of university mathematics. These three were observed as they participated in the collaborative tutorials that formed part of their first-year mathematics course. For this phase, richness and depth were attained by using data from repeated interviews, observations, and mathematical working.

Findings of the study show how the mathematical identities afforded international students by university structures might be threatened when cultural differences restrict access to learning resources. They further show how cultural resources provided by home country experiences can resource resilient mathematical identities that accommodate these tensions. The findings also offer evidence that mathematical identities constructed by international students can shift towards local norms, which can help to resolve tensions in collaborative contexts. I argue that these findings have implications for both teachers of international students and mathematics departments at host universities. I contend that teachers should be mindful that cultural backgrounds may restrict access to some learning resources, and I suggest resources that may be helpful to international students in their classes. I further call on mathematics departments to recognise the difficulties encountered by some international students and to implement structures that might address these.

Acknowledgements

My pathway from school teaching to research began with a sign from God – a clear response to a prayer for change. I knew then that doors would open but had no idea where they might lead. A PhD was not on my agenda and yet that was the rollercoaster on which I found myself. I am eternally grateful for the ride - the extreme peaks and troughs along the track have allowed me to view the world, and how I fit into it, from a whole range of new angles.

My wonderful supervisors were clearly part of God's masterplan. I am so thankful to both Dr Igor' Kontorovich and Dr Lisa Darragh for accompanying me on the ride. Their collective wisdom guided my progress, offering freedom to explore yet always holding me to account. Their searching questions and calls for clarity pushed me to think in new ways and to revisit my reasoning and explanations time and again. Their diverse styles of supervision complemented one another, and I count myself incredibly fortunate to have benefitted from their generous and genuine care.

Starting a PhD in one's later life might smack of poor career planning or impulsiveness. Yet impulsive decisions, poor timing, and a shared love of challenge are all part of the glue that has bound me to my amazing husband, Peter Locke, for near 40 years and counting. Thank you, Pete, for your patience with me on the peaks and your steady support through the troughs. Thanks for being the responsible one who worked while I played and kept the home fires burning while I flitted from conference to conference. Our agreement that I could do a PhD if you could get a motorbike has, I think, worked out better for me than for you. Hope the ankle fracture continues to heal well!

I am thankful to the University of Auckland for awarding me a doctoral scholarship and making my PhD possible. Thanks also to staff and fellow students in the Mathematics Department who have supported me in so many ways. My most enormous thank you goes to the participants who engaged so openly with me in my research. Without their willingness to share from their hearts this thesis would not have happened. I learned so much more from the participants than is written in this thesis and, with Peggy's permission, I share some of this now. In a catch-up coffee with Peggy some six months after our first meeting I asked how she was coping with her university studies. I half expected that she would be disappointed with herself given the high and seemingly unachievable goals she had shared with me in earlier interviews. I treasure her response:

> My goals change from one day to the next, so meeting them is not the most important thing. What is important is knowing that today I am better than I was yesterday. And I believe that I am.

As I acknowledge Peggy's wisdom I know that, thanks to those I have mentioned above and many more that I haven't, I too am better today than I was yesterday.

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Chapter 1: Introduction

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Foreign students boost to economy

By JO-MARIE BROWN and REBECCA WALSH

Auckland's international students are more valuable than realised, a study showing they contribute $198 million a year to the local economy.

Research paid for by the Asia 2000 Foundation on how much international students spend while living in New Zealand, shows that the city's 11,595 students pay $198 million in fees, accommodation, food, transport, books, appliances and other goods and services while studying.

(Brown & Walsh, 2020)

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Home / New Zealand / Politics

National wants new international students back in NZ - but with new work restrictions

International students who can be screened, tested twice, and who can pay for a 14-day quarantine should be allowed back in to New Zealand for the second half of this year, the National Party says.

But in an effort to keep jobs for Kiwis, new international students would no longer be able to work in New Zealand while they studied.

The Government is looking at opening up international student education - worth $5 billion a year in GDP terms - as soon as can be safely done.

(Cheng, 2020)

With headlines like these (and others that screamed “redundancies” and “job cuts” at universities) I, as a New Zealand educator, felt ashamed. Ashamed that international students were reduced, in the eyes of the public, to a commodity. Ashamed that those who chose to stay in this country, rather than disrupt their studies by returning home during a pandemic, might have to read these articles.

I found a glimmer of hope near the end of one of the newspaper pieces.

> Victoria University Vice-chancellor Grant Guilford said the value of international students was much more than the revenue to education institutes and their spending in the wider economy.

(Cheng, 2020)

It seems that some in academia might hold a different view from that touted by the media. Those who work daily with international students might be better placed to recognise them as the brave young (and not so young) folk that they are. People, rather than dollars, who leave the familiarity of home to pursue a dream, often at great personal cost. And I believe that those of us who teach, or have taught, international students should recognise our responsibility to support them, as well as we are able, in the mutually beneficial relationship of international education.

1.1 Background to this study

My research arises from a desire to understand more clearly how mathematics teachers and lecturers might support international students. UNESCO Institute for Statistics defines internationally mobile students as:

> individuals who have physically crossed an international border between two countries with the objective to participate in educational activities in the country of destination, where the country of destination of a given student is different from their country of origin (UNESCO Institute for Statistics, n.d.-b).

When using the term *international student* in this study, I am referring to these internationally mobile students from the UNESCO definition. I also refer to *local students* in this study. In contrast to international students, I incorporate in this term all students who consider the country in which they study to be their permanent home and have the right to live there indefinitely. In New Zealand, an international student is sometimes referred to as a “foreign fee-paying student.” These students, from other countries, are granted a temporary visa for

the express purpose of completing a programme of study at a New Zealand educational institution. In New Zealand, and in many other countries, they pay substantially higher tuition fees than their local peers in the same programmes, which is why educational arrangements of this nature are often dubbed "export education."

Despite the COVID pandemic, student mobility has continued its upward trend, with UNESCO statistics showing that there were over six million international students around the globe in 2021 (UNESCO Institute for Statistics, n.d.-a). New Zealand is a relatively small player in this market when compared with countries such as US, UK, and Australia. While down from pre-COVID levels, international students still make up some 10% of tertiary students in New Zealand (New Zealand Ministry of Education, n.d.), thus forming an important cohort within our universities. With mathematics acting as gatekeeper to prestigious careers (Wagner, 2021) and the common misperception that it is both language and culture neutral (McGee & Martin, 2011; Wagner, 2021; L. N. Wood et al., 2007), it is not unexpected that international students should be well represented in first-year mathematics courses, both in New Zealand and abroad.

Yet there is little research that provides insights into the experiences of international students studying mathematics at host institutions. For international students, the transition from school to university mathematics may demand significant adaptation. Curricular and other cultural differences between mathematical learning contexts in home and host countries (Kaiser & Blömeke, 2013; Leung, 2001; Yang et al., 2020) raise the likelihood that international student experiences of first-year mathematics may differ from those of local students. Given that the secondary to tertiary mathematical transition (hereafter STT) is well known as a time of disruption (Clark & Lovric, 2008) this gap in the literature has significant implications. Our responsibility to support international students in their transition to university mathematics demands that we understand their experiences, and this research study takes a small step towards a vast goal.

1.2 Personal Motivation

I have never been an international student. But I have moved countries. Twice. Both times with a family in tow. Despite both our moves being from one English speaking country to another, I noted signs of strain and tension each time my children had to establish themselves in a new and unfamiliar educational system. Yet each time they rose to the challenge and were, I believe, stronger for the experience.

But how must it be to move to a country where the language and culture are far from one's own? Where classroom norms are unfamiliar, prior knowledge mismatched, and where communicational challenges beset learning and personal relationships? This is the experience of many international students, and it is one that has long piqued my interest. My former role as mathematics teacher and dean of international students at a New Zealand secondary school brought me into close contact with school-aged international students. I saw many arrive, unsure of themselves and nervous in their new setting. Most left again, months or years later, with newfound confidence and independence. But the journey was seldom easy. It often involved confusion, feelings of otherness, misunderstanding others, and being misunderstood. It was a journey that called for large reserves of resilience.

A year of teaching first-year mathematics at a New Zealand university brought me into contact with international students in a different context. Conversations with a few international students in my lectures reminded me of how little quantified assessment results can reveal about the stories of students behind them. Yet in the large, impersonal context of a first-year mathematics course, assessment results were all that I knew of most students. Grades told me little about the efforts, the anxieties, the joys, the concerns that went into learning mathematics in a foreign, English-mediated context. I knew, from my experiences with international students at school, that even mathematics looked and felt different from what many were used to in their home countries. Yet how this played out for them in a university context I had little idea.

The focus of this thesis is thus the result of my underlying desire to better understand the experiences of international students in first-year mathematics. As a New Zealander in mathematics education, I feel some responsibility to ensure that those international students who choose to study at our universities have an experience that justifies the commitment they make. Without clearer understanding of what these experiences entail, effective support for international students in their transition to university mathematics cannot be clearly determined.

1.3 Research Goals and Aim

My research explores the experiences of international students in STT by focusing on relationships between the structures of first-year mathematics in a foreign country and those of their prior experiences in their home countries. My goals are:

- To illuminate tensions that arise when the structures of first-year mathematics interact with the social structures of prior experiences,
- To investigate how international students navigate these tensions.

To achieve these goals, I use the lens of mathematical identity. Research contends that mathematical identity is a useful tool for exploring learners' relationships with mathematics (Darragh & Radovic, 2020; Sfard, 2019). Identity both impacts on, and is shaped by, what learners say and do in mathematical interactions (Sfard, 2019). In this research I aim to explore the mathematical identity constructions of international students when the interaction of university structures (such as a university grading system or expected practices within a first-year mathematics tutorial) and their prior experiences create situations of tension in STT.

1.4 Outline of this Thesis

My thesis includes five published research outputs, each of which contributes to the three sets of findings of the thesis. In accordance with University of Auckland policies, these research outputs have been fully integrated into the thesis so that it reads as a coherent whole. To achieve coherence and avoid duplication, the research outputs have not been included in entire published form. Only the findings and discussion sections have been extracted from each output and blended into Chapters 4 and 5 of the thesis. Full details of these extracted portions and the pages of the thesis on which they feature can be found in the co-authorship forms. Here I offer an overview of the organisational structure of the rest of the thesis.

Chapter 2 – Literature Review

Chapter 2 offers a detailed review of literature in which my research is situated. I consider three bodies of research both from within, and outside of, mathematics education. I start the chapter by reviewing literature focused on STT. This offers insights on the disruption that students must accommodate when transitioning between school and university mathematics. I follow this with literature pertaining to country-specific differences in the teaching and learning of mathematics. This helps to shed light on how cultural differences might play a part in the transitional experiences of international students. In the final part of the review, I consider literature from higher education that offers insights on how transitioning from the educational context of one country to another might impact on international student experiences. At the end of the literature review, based on the gaps and opportunities identified, I state my research questions.

Chapter 3 – Methodology

The third chapter outlines the ontological and epistemological stance of my research before considering the theoretical framework. I discuss ways in which mathematical identity has been conceptualised by others, and evaluate how various theories of identity might contribute to my own research goals. I then outline the theoretical framework that I adopt. After outlining the theoretical framework, I discuss the procedural aspects of designing and implementing my two-phase research study and introduce the participants in each phase. I finally discuss the data collection and provide a detailed explanation of my analysis process. In this explanation I give examples of how I recognised the five constructs of social roles, social identities offered, social identities constructed, personal identity formation, and agency. I end the chapter with a discussion of quality and ethical considerations.

Chapter 4 – Storied Identities in First-Year Mathematics

Chapter 4 presents findings from the first phase of my research. These findings show how seven international student participants constructed mathematical identities when relating their experiences of first-year mathematics in interview settings. The chapter starts by focusing on Aadam, Irash and Jon (all student names are pseudonyms) who all tell stories of struggle in first-year mathematics. This section describes a common process, which I have called a manoeuvre, used by these students to construct storied mathematical identities. The identities constructed enabled engagement in first-year mathematics for two of the students and disengagement for one. The next part of the chapter focuses on Sunny, Mingyu, Liang, and May who all tell stories of success in first-year mathematics. It shows how their prior experiences resourced resilient mathematical identities that enabled these students to overcome the challenges of learning mathematics in English. Within this chapter, I discuss how insights from the first phase of the research study informed the design of the next phase.

Chapter 5 – Interactional Identities in First-Year Mathematics

This chapter presents findings from a fine-grained study of three international students from China who took part in the second phase of the research. The findings show how the participants constructed interactional identities during collaborative first-year mathematics tutorials. The first section focuses on Peggy as she interacts with her tutor during regular, bi-weekly mathematics tutorials. It shows how she acts agentively to transform her mathematical identity and considers how this relates to her mathematical output from these interactions. I then consider Peggy's mathematical identity in peer interactions. I show how

the mathematical identity constructed by Peggy restricts access to some of the learning resources available to her peers in collaborative tutorials. The final part of this chapter focuses on Yifei and Zixin showing how they construct mathematical identities in peer interactions during collaborative first-year tutorials.

Chapter 6 – Discussion and Implications

In this chapter I start by discussing the power of the theoretical framework for exploring situations of tension in mathematics learning contexts. I then synthesise the outputs from Chapters 4 and 5 to provide holistic answers for my research questions and illuminate:

- university structures that might threaten mathematical identities and create tensions for international students,
- resources drawn on by international students when constructing mathematical identities that mitigate tensions,
- how mathematical identities might change in ways that reduce the tensions experienced.

I finally call attention to the implications of these findings for mathematics teachers and their institutions, as well as for future research.

Chapter 7 – Conclusion

In the final chapter of my thesis I outline some limitations of this research study and offer my closing thoughts.

Chapter 2: Literature Review

To understand the experiences of international students as they transition from secondary school to university mathematics, one needs to consider three different aspects of this transition. Firstly, one must understand how the discipline of mathematics changes between school and university. It is also necessary to consider how cultural differences might impact on the mathematical learning of international students, as well as how they experience the change from home to host country educational systems. As a background to this study, I bring together literature from these three different areas of research. In section 2.1 I review literature on the secondary to tertiary transition in mathematics (STT) and the challenges it might bring for transitioning students. I follow this in section 2.2 with a review of literature within mathematics education that relates to cultural differences between mathematics teaching and learning in different countries. In section 2.3 I finally consider literature outside of mathematics education that focuses on the wider academic experiences of international students when transitioning to host universities. I close the chapter by synthesising intersections and gaps in the reviewed literature to situate my research study (section 2.4) and I recap my research questions in section 2.5.

2.1 The Secondary School to Tertiary Mathematics Transition (STT)

A rite of passage, traditionally understood in the context of anthropology, marks a change in status as a person leaves one social group and enters another. Clark and Lovric (2008) suggest that the crisis of finishing school and integrating into university mathematics can be viewed as a necessary disruption, and that support for students be aimed at helping them move from one place within the mathematical community to their new place in that community. They liken this disruption to a rite of passage, which is marked by three distinct phases. The first phase is *separation* which, in the context of STT, may just be a separation from familiar figures within the secondary school learning community, or it may also involve a separation from family, friends, or caregivers when students attend universities far from home. The *liminal* phase is the time during which the changes necessary for a successful transition are being achieved, such as orientation activities offered by the university. The final *incorporation* phase is when the student of mathematics re-joins the mathematical community equipped to occupy their new space. Clark and Lovric consider the entire first year of mathematics learning to be the incorporation phase and promote the idea that some

discomfort is a necessary part of the passage. Some of the factors contributing to this discomfort are outlined next.

2.1.1 Challenges experienced in STT

Reviews of literature on STT have summarised the commonly adopted perspectives of research in this space. In her frequently cited paper, Gueudet (2008) groups these into individual, social, and institutional perspectives. She categorises literature that investigates students' failure to achieve a desired standard of knowledge, or their ability to access different representations of mathematical concepts, as referring to individual ways of thinking. Social perspectives investigate the issue of formal mathematical language and proof, while institutional perspectives address inconsistencies between teaching style and assessment in school and university contexts.

Di Martino et al. (2023a) note that the focus of STT research published since Gueudet's (2008) study has changed from being largely cognitive to being more holistic. Both sociocultural and affective issues are better represented in recent STT literature. Di Martino et al. (2023a) highlight four themes in STT literature published after 2008. These relate to the gap between school and university mathematics, the affordances of technology in STT, factors related to success in the transition, and failure in STT. However, they suggest that more literature exploring affective factors in STT is needed to grow understanding of this complex issue. They also note that literature arises mainly from Europe, and that countries in Asia and Africa are underrepresented in STT literature.

The "gap" to be bridged

Researchers who focus on the gap between secondary school and university mathematics consider it in different ways. A gap suggests something that must be bridged. Yet what exactly must be spanned differs according to the study and may reflect a focus of the country from which the research originates. A German based study by Biehler and Kempen (2013), for example, addresses it as a gap in expected student thinking. In a bridging course for pre-service teachers, they asked students to prove a mathematical statement using both a generic proof (cf. Leron & Zaslavsky, 2009) and a formal proof. They found that few understood the idea of transferring commonalities from generic examples into formal proof. Even those who grasped this idea struggled with the formal language, symbols, and definitions of variables. A different approach to the gap is taken by Seldon (2012) who scrutinises literature from many countries to determine how proof at tertiary level is different from proof at school level. In

doing so she looks at a gap in what is considered an acceptable justification at school, and how this differs at university. Thomas and Klymchuk (2012) focus on the gap between instructional practices at New Zealand schools and universities, as well as differences in their assessment practices. They used surveys, interviews, and observations to collect data from teachers, lecturers, and students in New Zealand educational institutions. They found that instruction at school emphasised practices such as modelling, skills practice, collaboration, and factual knowledge. Tertiary institutions, on the other hand, valued conceptual thinking, rigour, and problem solving. Differences in assessment were also noted, with university lecturers having greater control over assessments than teachers. They noted a tendency amongst schoolteachers to teach only what was needed for achieving the curricular standards. All the different aspects considered in these studies contribute to what is commonly termed the gap between school and university mathematics. It hardly seems surprising that students might encounter difficulties as they traverse such a multifaceted divide. Yet these studies commonly approach STT from the researchers' perspective, where success relates purely to achievement. Their cognitive focus does not always recognise the role that social and affective factors play in mathematical learning. Sfard (2014) speaks of school mathematics and university mathematics as almost distinct disciplines (or discourses in her terms), where students must come to know mathematics that was familiar at school in an entirely new way. The process of relinquishing familiar ways and adopting new ways of acting suggests that social and affective approaches to research on STT could bring many new insights.

Pre-requisites for success

Studies that consider how cognitive, social, and affective factors relate to academic success usually involve quantitative methods (Di Martino et al., 2023a). Rach and Heinze (2017) conducted a large-scale study of first-semester students at a German university. They suggest that different learner characteristics are needed for school mathematics and the scientific discipline of mathematics studied at university. As a result, those students who can flexibly adapt their learning strategies are better equipped to succeed at university, while school-related knowledge, interest, or self-concept are not useful predictors of performance-related success at university. Rach and Ufer (2020) reanalysed data from former studies when attempting to predict a level of prior mathematical knowledge that might lead to academic success in analysis courses at two German universities. They found that prior knowledge was less of a pre-requisite for performative success than well-connected conceptual knowledge. They further suggest that formal symbolic representations were not a necessity for success in

these courses. Another study focused on academic success was that of Barnett et al. (2014), who used questionnaires to collect data from over 10 000 students across 134 colleges in the US. They found that student effort at US high schools did not necessarily translate into success in college calculus performance. This pointed to the notion of productive effort, which had a positive association with examination grades, and ineffective effort, which had either no association or a negative association with grades. One finding from this study was the relative success of Asian American students, whose higher school grades, but similar study patterns, led to higher performance in college calculus than their counterparts. The authors propose that a commonly held belief amongst these students, that effort was the main driver of academic success, supported their productive effort.

The findings of these last three studies recognise transitioning students as a non-uniform cohort. Despite involving large samples, they try to explain how different characteristics and attributes of learners might lead to different transitional experiences. However, they all consider success only in terms of academic achievement, or performance in assessment. Other ways of viewing success, such as attaining deeper levels of conceptual knowledge, were not considered in these large scale studies.

The studies of failure in STT are mostly qualitative, and they contribute to understanding of student dropout from university mathematics (Di Martino et al., 2023a). Di Martino and Gregorio (2019) investigated dropout amongst first-year students enrolled in a prestigious mathematics programme at an Italian university. The 127 students who took part in the research were considered “excellent in mathematics” in secondary school. Yet the authors found that, when the inevitable difficulties arose, some students could identify controllable factors and change the way they viewed success, whereas those who dropped out were unable to do this. Di Martino et al. (2023a) emphasise that a change in an individual’s view of failure or success infers a change in their vision of mathematics.

Thus, affective and sociocultural factors are closely related to mathematical learning and highlight a growing awareness of the complexity of STT. By considering the mathematical identities of international students in STT, my research focuses on how social, affective, and cognitive aspects of learning mathematics all contribute to their transitional learning experiences.

2.1.2 Mathematical Identities in STT

Conflicting identities

Some studies, such as that of Hernandez-Martinez (2016), use mathematical identity as a lens to explore student experiences in STT. In this study, narrative analysis was used to understand why two previously engaged students dropped out of university engineering degrees at UK universities. Hernandez-Martinez found that inflexible university practices, such as fast-paced lectures with little opportunity for interaction, and the perception that it was acceptable for students to perform mathematical processes without full understanding, did not align with the students' imagined future identities as professional engineers. The students believed that dropping out of the engineering programme was the most viable way forward. Solomon (2007) similarly proposes that when mathematics is presented as a non-negotiable finished product, as may be the case in some first-year programmes, it can generate compliance and procedural knowledge. This lack of agency is not necessarily a problem for students who align themselves with immediate communities of practice, such as the mathematics undergraduate community, where correct answers, following rules, and working at speed are valued. Yet for those who align with the wider mathematical community, it can be disempowering. Students who seek deeper, conceptual understanding, rather than merely attaining correct answers, might find themselves doubting their ability.

Jooganah and Williams (2016) suggest that the identity of "mathematician" at UK schools differs from that of universities, where students must know mathematics in a different way. They propose that conflicts between how mathematics is learned at school and university, or even across different university mathematics courses, can create conflict in student identities. If these conflicts between ways of learning mathematics at school and university in the same country can lead to conflicting identities for students, it seems likely that this might be exacerbated for international students who transition between the education systems of different countries.

Motivation and agency

Themes of motivation and agency emerge strongly from literature on mathematical learner identities in STT. Hernandez-Martinez et al. (2011) suggest that student perspectives of mathematical transition are often more positive than might be expected. These authors conducted a research study that focused on students who would traditionally be considered "at-risk" of failure (due to school leaving grades or socio-economic backgrounds) as they

transitioned from compulsory to non-compulsory mathematics. They found that, while students recognised the challenges that they faced, they described these obstacles as opportunities to develop new identities as learners. Black et al. (2010) suggest that activities that generate new motives can simultaneously offer new understandings of self which both evolve from, and drive, engagement, even when facing challenge. These authors term the altered self-understandings that arise from new motives as "leading identities." For example, transitioning from school mathematics to university mathematics might produce the new motivation of doing mathematics to become an engineer. In this context, becoming an engineer is the identity that leads students towards a self-imposed goal.

Mathematical identities that support resilience can enable students to cope with the challenges of STT. Hernandez-Martinez and Williams (2013) describe resilience as a process of interaction between a social context and the agency of the individual. They describe resilient students as "able to bring to bear particular reserves of capital that resonate with the new field [educational context]" (p.54). They suggest that students who have faced adversity and been able to reflect on, and break, cultural models gain educational capital that helps them negotiate the challenges of transition. This idea is strengthened by research from a British university, which suggests that first-year students who have had little previous opportunity to negotiate sustained struggle might become disenfranchised with mathematics (Ward-Penny et al., 2018).

Learning communities

Research also shows that communal learning can play an important role in the transition process. Di Martino and Gregorio (2019) surveyed students who had enrolled in a Bachelor of Mathematics at a prestigious university in Italy. They report that two thirds of those who had successfully passed their first year credited sharing their mathematical difficulties with others and attending study groups, as major factors that had contributed to their success. These actions helped them overcome initial feelings of shame and realise that the difficulties they were experiencing were not uncommon. Solomon et al. (2010) also consider the significance of undergraduate learning communities in adjusting to university mathematics. They contest the individualism of mathematics, suggesting that some students exhibit a preference for communal learning experiences. They report that informality, student agency, lack of time pressure, and psychological security is critical to success of collaborative learning groups. Their study found that an intervention by the university to limit overcrowding disrupted the system and eroded collaborative practices. These findings raise

questions about whether the collaborative learning environments commonly implemented in western universities (Kaiser & Blömeke, 2013; Oates et al., 2005) will be similarly effective. These institutionally established settings, in which students might be obliged to participate, lack the informality, student agency, and time independence deemed critical by Solomon et al. (2010).

2.1.3 Summary of STT literature

Literature reviewed in this section agrees that the process of transitioning between school and university mathematics programs is a complex endeavour with cognitive, affective, and sociocultural challenges. Although research in the broader field of mathematics education was initially characterised by cognitive approaches, the 1990's saw a switch in focus towards social theories that viewed thinking and reasoning as a product of social activity (Lerman, 2000). STT research prior to 2008, however, remained largely focused on the cognitive challenges facing students, and used data gathered through observations, questionnaires, or examination of student work. The generalised conclusions from these studies assumed a uniform cohort of students and there were few attempts made to explain how the unique understandings and circumstances of individual students might offer different experiences or perspectives on the challenges raised. The researcher's perspective, in most instances, saw the transition as problematic, and measured success solely in terms of academic achievement. Research since 2008 shows greater awareness of affective and sociocultural influences on transitional experiences. Studies of mathematical identity in STT speak of how motivation, the sense of agency, and collaborative opportunities available can impact on the transition experience for students. Yet cultural and country-specific aspects of STT are seldom addressed in this literature. How the influential issues summarised above might play out when students transition to university mathematics in a foreign country remains unexplored in this body of literature.

2.2 Learning Mathematics in Different Cultural Contexts

In this section I review literature concerned with learning mathematics in different cultural contexts. The literature reviewed in this section largely refutes a belief that mathematics is culturally neutral (Leung, 2017; Wagner, 2021). While the learners in these studies have not been identified as international students (whose specific reason for sojourning abroad is to study), this possibility is not precluded. By focusing on cultural aspects of learning

mathematics, the literature in this section illuminates how international students might experience contrasts when transitioning from home country education to host universities.

I start by reviewing literature pertaining to cultural factors that shape instructional practices and learner approaches to mathematics in different parts of the world (section 2.2.1). Given that China is the largest sender of international students (UNESCO Institute for Statistics, n.d.-a), studies offering insights on teaching and learning practices in Chinese classrooms are of particular relevance to my research. In section 2.2.2 I then focus specifically on how language influences learner experiences of mathematics. I focus on the language factor because international students undertake their early mathematics learning in their home language before switching to their host country's language at a more advanced stage of mathematics learning. This switch in the language of instruction might disrupt the mathematics learning trajectory of international students. In section 2.2.3 I summarise the literature reviewed in this section.

2.2.1 Mathematics Teaching and Learning Around the World

Variations in instructional practices

There are many studies that highlight differences in the instructional practices between school mathematics classrooms in different parts of the world. Cai and Howson (2012) describe the global mathematics curriculum reform that took place around the turn of the century and led to an emphasis on student-centred or inquiry-based learning in many countries. This reform did not initially influence Chinese instructional practices, which instead were shaped around basic mathematical concepts and skills (Yang et al., 2020). However, these authors describe how, in a later round of reforms, constructivist theories were also embraced by Chinese authorities to underpin their mathematics curriculum. Yet research shows that, despite this underpinning, mathematics instruction in China differs from many other global contexts (Kaiser & Blömeke, 2013; Yang et al., 2020). The influence of Confucian culture on classrooms in China places the teacher as an authority and expert (Leung, 2001; Zhou et al., 2022). This contrasts with the more student-centred approaches advocated in some other parts of the world such as Australia, Germany, and UK (Cai & Howson, 2012; Kaiser & Blömeke, 2013). Lillyman and Bennett (2014) suggest that respect for the wisdom of teachers may prevent Chinese students from questioning their teachers, even when studying in countries where this is accepted.

Classroom dialogue is often promoted as an effective strategy to enhance mathematical learning, yet some forms of student talk are more valued than others in different cultural contexts. In New Zealand, for example, the stated aims of the refreshed school curriculum sees, amongst other visionary aims, "Students participate as they take part in discussions with their peers about their mathematical and statistical thinking and the thinking of others" (New Zealand Ministry of Education, 2023). Xu and Clarke (2019) suggest that, although research from Europe and US privilege learning through talking with others, other forms of classroom communication are embraced in other parts of the world. They propose that choral responses generate active involvement in learning and are valued in some East Asian classrooms, where both silence and choral responses form part of the conventional classroom discourse. Zhou et al. (2022) investigated what was considered to be "good teaching" in China. They showed that Chinese school teachers seek to balance content-oriented instruction and student-centred learning. However, the examination driven culture of Chinese high schools, while nurturing broad and deep content, increases teachers' reliance on direct, whole class teaching and memorisation.

The growing commitment to groupwork in university mathematics is evident in many countries. For example, a recent study documents how the teacher of a topology class in New Zealand first asked students to attempt a proof in pairs, before inviting a student to present the pair's proof to the class and critiquing their work for the benefit of the class (Kontorovich & Greenwood, 2023). In the US, Wawro et al. (2012) describe an instructional sequence used in an introductory linear algebra course that alternates groupwork with whole class discussions. There is also a move to collaborative ways of working in tutorial settings. For example, a New Zealand based study by Oates et al. (2005) describes a departmental approach to tutorials which encourages first-year students to work collaboratively as they undertake tasks that consolidate content taught in lectures. These approaches to teaching and learning embrace the idea of student-focused activities often promoted by literature on effective teaching practice. Yet the move to groupwork might not always be equally beneficial to all students. Hwang et al. (2022) consider how students experienced groupwork in multilingual mathematics classrooms at a Midwestern university in the US. In this study Hwang et al. showed how groupwork disadvantaged students whose home language was different from others in the group. One student told how her difficulties expressing mathematical ideas in English made her feel that she was wasting others' time. This inclined

her to remain silent rather than actively contributing to group discussions. Such findings demonstrate how groupwork might restrict learning for some students.

Cultural beliefs about learning

While literature reviewed in the preceding paragraphs shows how instructional practices might vary across countries, cultural beliefs also play a part in how students experience mathematics learning. Mok (2020) suggests that a pervading view amongst Chinese parents is that mathematics is a skill that can be learned with sufficient practice and appropriate support. This view encourages positive mathematical engagement. While the insistence on hard work may become a burden for students from East Asia, Leung (2001) suggests that they strive to achieve a deeper level of pleasure that can be derived through diligence. This deeper sense of pleasure or enjoyment experienced by Chinese students is supported by Frenzel et al. (2007) in their quantitative study comparing 8th grade Chinese and German mathematics students. However, they also showed that Chinese students experienced greater levels of anxiety than students learning mathematics in Germany. They attributed these feelings to student achievement and parental expectations, offering that family approval is prioritised more highly in China than in countries like Germany. In collectivist cultures such as China, the importance of community is highly emphasised, and unity and selflessness are valued. Individualist cultures like Germany, on the other hand, value independence and place their emphasis on the rights and concerns of each person. According to Imada and Ellsworth (2011), students from individualist contexts experience different emotions in response to success and failure than those from collectivist contexts. They propose that, in the US, students are encouraged to feel pride in their accomplishments, whereas Chinese society sees accomplishment as the product of relationships with supportive others. Mok (2020) proposes that it is cultural factors, rather than superior education systems, that enable Chinese students to perform at a high level in mathematics. She substantiates this by pointing to the performance of Chinese immigrant students in New Zealand and Australia whose performance in PISA 2009 resembled that of their counterparts in Shanghai. Bowden et al. (2015) noted a tendency to "deep learning" and memorisation both in students from Confucian heritage countries like China, as well as other countries in Asia. They propose that, in cultures where teachers are viewed as esteemed members of the community, the students' trust in, and deep respect for, their teachers might inspire learning of this form. They further suggest that in cultures connected to Buddhism and Hinduism, which do not have a holy text and therefore rely on oral transmission of knowledge, memorisation is highly

valued as a steppingstone to deeper knowledge. This might contextualise the tendency to deep learning approaches and memorisation techniques that they noted in students from these and Confucian heritage backgrounds. Together these studies suggest that cultural beliefs and practices might shape the learning experiences of local and international students in very different ways.

Stereotypical identities

Country specific differences in instructional practices and beliefs about learning can lead to international students being recognised in stereotypical ways. Chalmers and Volet (1997) identify five misconceptions that were commonly reported in research emanating from Western countries in the late 1980's and early 1990's. These misconceptions led to international students from Southeast Asian countries being stereotyped as rote learners who were passive in class and unwilling to mix with their local peers. Later research countered these views, showing instead that students from Confucian heritage countries were active and engaged learners who used memorisation only as a step towards understanding (e.g., Biggs, 1994; Chalmers & Volet, 1997; Mathias et al., 2013). Yet stereotypical views of Chinese international students as rote learners still persist in some Western settings (J. Xu, 2022), holding the potential to influence the identities offered to these students and how they are recognised in university mathematics classrooms. International students, however, might not see themselves in the same way. A study of international students enrolled in a foundation programme at a UK university revealed their dislike of rote learning (Mathias et al., 2013). These students claimed that memorisation alone did not offer flexibility to respond appropriately in different situations. Instead, they saw memorisation as being part of a longer process through which they came to understanding, a process that applied even in mathematics where memorisation might be required until difficulties could be resolved.

Another persistent narrative about students from Asia is that they are inherently "good at mathematics." Shah (2019) and Trytten et al. (2012) argue that this characterisation, which is particularly common in the US where it is nurtured by the Model Minority Stereotype (MMS), is detrimental to students of Asian heritage. In the US, the MMS promotes Asian Americans as being economically and academically successful, and is often used to dismiss the effects of discriminatory practices against other minority groups. Shah (2019) suggests that labelling Asian students as being "good at mathematics" is both erroneous and dehumanising. It positions them as racial subjects. Trytten et al. (2012), in their mixed methods study of 159 racial minority students at a US engineering college, garnered student

voice on the MMS characterisation of Asian and Asian American people. Students of Asian heritage tended to accept, as part of their identity, the characterisation of them as hard working. However, they saw success as being due to work ethic rather than race, and the perception of Asian students possessing a particular aptitude for mathematics was not widely accepted by these students. For some students in this study, stereotyping led to fear that they would not live up to the MMS standards.

2.2.2 The Influence of Language on Mathematics Learner Experiences

One of the reasons commonly given by international students for choosing to study abroad is to improve their proficiency in the host language, which can be viewed as a form of immersion education. As Barwell (2020) notes, "The teaching and learning of mathematics depends fundamentally on language" (p. 441). Thus, international students, who learn mathematics in a non-native language, might experience both challenges and advantages that are different from those who learn in their home language.

Language structure and learning mathematics

Whorf's (1956) linguistic relativity hypothesis suggests that the structure of a language can affect the thought processes of those who speak the language. According to Edmonds-Wathen et al. (2016), since the development of mathematics is intricately linked to the development of language through which it operates, specific language features will determine what can easily be done in mathematics. Leung (2017) argues that, because language mediates between basic cultural structures such as skills and tools, and higher elements of culture such as ideals, beliefs, and ways of thinking, it plays a critical role in how mathematics is learned. According to Leung, there are many features of languages spoken in China (and on which Korean and Japanese are also based) that advantage the learning of mathematics when compared with English. Galligan (2001) also suggests that the linguistic attributes of Chinese languages provide cognitive advantage in mathematics, where a single word can offer access to an entire concept image, and syntax is critical to meaning. According to this author, the context dependent nature of the Chinese language may also activate different processing strategies from the English language, which is more word dependent.

The mathematics register refers to the specific kind of language used in mathematics (Halliday, 1978). Sometimes a word in everyday English may have a different meaning in mathematics. An example of how differences in the mathematics register of different

languages can lead to different understandings for students of mathematics is given by Kim et al. (2012). These authors explain that, in English, the colloquial base word “infinite” is adopted into the mathematics register and new mathematical meanings are built onto this. In Korean, the colloquial word for infinite gets replaced at school for a more formal mathematical word. This creates a disconnect between informal and formal Korean discourse on infinity. Consequently, English speakers in this study could speak confidently about infinity at lower levels of complexity but had difficulty engaging with higher level meanings. Korean students, on the other hand, could engage at higher levels, but their discourse mimicked what was taught and they could not draw from colloquial speech patterns. Durand-Guerrier et al. (2016) point to notational differences between countries, such as half open intervals which might be expressed as [a, b), [a, b[, or [a, b> depending on local conventions. They also highlight notational ambiguities that require context for interpretation, such as whether (a, b) represents an open interval or an ordered pair. Galligan (2001) suggests that, while learning mathematics in a native language might allow for the most efficient conceptual processing, there may be differences in how mathematics is processed by different language groups.

Disadvantaged by language

Active marketing of New Zealand as a study destination means that a high proportion of learners, both in secondary school and in tertiary programs, study in a language that is not their native tongue. Research undertaken in tertiary and senior secondary mathematics settings in New Zealand has explored the extent to which learners from other language groups might be disadvantaged by learning mathematics in English. Barton and Neville-Barton (2003) report that the impact for these learners in their first year of university mathematics was a reduction in achievement of around 10% due to language. Similar disadvantages were apparent, despite using different methods of assessment, in studies undertaken with Chinese students in senior secondary mathematics classes at two New Zealand schools (Neville-Barton & Barton, 2005). The difference in achievement was noticeably wider in questions with complex phrases, syntax, or vocabulary. As an example, a question offering information that there was “6 times as much milk as syrup” was correctly interpreted by 79% of students when presented in their home language of Mandarin Chinese. Yet the same question offered in English to the same group of students at a different time was correctly answered by only 35% of students. The authors suggest that this significant difference might be attributed to the complexity of expressing proportional relationships in

English relative to Chinese. Other schools in the same study reported difficulties with prepositions in questions like “add the answer to 60”, or “divide your answer into 100” and with creating diagrams from verbal descriptions.

Another source of disadvantage for mathematics learners from other language groups in Neville-Barton and Barton’s (2005) study was that of gaps in the mathematical vocabulary from earlier years. Students in their final years of school mathematics were not conversant with English words such as “fraction,” “decimal,” “perpendicular,” and many others used regularly in earlier-stage mathematics classrooms. Schinke (2016) attests to a similar issue in her research on European immigrants studying mathematics at university in New Zealand. Students in this study found it difficult to communicate mathematically after arrival, despite considering themselves to be proficient speakers of English. They reported that missing terminology from high school mathematics made it much harder to understand new concepts presented in lectures. Students often had to wait until after class before they could explore the terminology and make connections with prior studies. Barton et al. (2005) further report that undergraduates from other language groups struggled with questions pertaining to logic structures, regardless of their mathematical abilities.

Students from other language groups might develop strategies to cope with the challenges of learning in a non-native language. European students in Schinke’s (2016) study used texts and lecture recordings to review concepts not clearly understood. New arrivals overcame vocabulary challenges through extensive reading and additional practice. They reported this to be effective, but time consuming. Neville-Barton and Barton (2005) reported that some students who struggled with the language of instruction would disengage during teaching, relying on texts for later assistance. This sometimes resulted in knowledge of procedures with little understanding of contexts in which they applied. Strategies for interpreting questions in English included decoding each known word and guessing those that were unfamiliar. If keywords were unknown students were often unable to continue. Similarly, if questions were too confusing, students would often give up. According to Barton et al. (2005) strong prior knowledge and effective coping strategies enabled many learners from other language groups to cope with the procedural nature of first-year mathematics, but may not adequately prepare them for more advanced mathematical studies.

Bilingualism is sometimes seen as offering the resources of two languages to mathematical learning or providing multiple modes for thinking. Cummins (1981) demonstrated how differences in the level of bilingualism influenced academic attainment in specific ways. He

showed how higher order thinking skills in a second language took five to seven years to develop. Barton (2008) suggests that practicing mathematics in as many languages as possible will result in mathematical advantage. According to Schinke (2016) European undergraduates in her study expressed a wish to make greater use of their home language within their new English medium environment, yet found it less confusing to use only English resources than to switch between resources in different languages. These students also reported that some concepts had been taught slightly differently in their own country, which resulted in a different view that was unhelpful in their new environment. When studying alongside peers from the same country, however, they might use first language understandings to discuss some concepts. Schinke suggests that students who further their studies only in a single language do not benefit from the alternative "mode" of thinking that their home language might offer. Research from across the world (including UK, US, Australia, South Korea, and Europe) on learning mathematics in a second language context shows how students often make use of a home language to interpret problems and for some mathematical thinking (Barwell, 2020).

While disadvantage due to language was measured to be 10% for diverse language learners during their first year at university (Barton & Neville-Barton, 2003), Barton et al. (2005) suggest that this disadvantage increases in more advanced mathematical studies. These authors advise that the change from repetitive and predictable examples often offered in first year lectures, to one-off explanations and examples based on logical trains of reasoning so typical of more advanced courses, can create a stumbling block for diverse language learners. Many students, however, are unaware of their disadvantage relative to home language learners (Barton et al., 2005; Neville-Barton & Barton, 2005). This disadvantage can result in international students who enter university with strong mathematical backgrounds making limited progress in their first year at university. The masking effect of hard work and repetitive processes might preclude international students from seeking and receiving the support that would lay a foundation for the new pedagogies more common in subsequent years.

Overcoming language difficulties

Following lectures or classroom explanations can be a challenge for some learners from other language groups, who often must resort to time consuming strategies to access mathematical content. Barton et al. (2005) advise that students learning in a non-native language often struggle more than is realised, but coping strategies and good prior knowledge often hide the

effect of language issues in process driven first-year mathematics courses. Wood et al. (2007) suggest that university teachers can improve accessibility to lecture content by appealing to more than one sense at the same time. Spoken words and lecture notes together provide auditory and visual learning stimuli that support one another. This can be further strengthened by directing student attention to different representations alongside verbal cues. Students in Schinke's (2016) study advise how visual representations helped them communicate mathematically with lecturers and peers, as they could point to features of diagrams and manipulate visual representations to gain understanding.

Enabling other language learners to access home language support in the early stages of their transition to a new language environment may benefit understanding. Schinke (2016) reports how concepts previously learned in a native language get gradually translated and added to new knowledge developing through English based instruction. In instances where concepts previously learned at home were relearned in the new environment, understanding did not deepen, but English terminology was more easily connected to the concept. Barwell (2020) recommends that teachers of students from other language groups should encourage them to switch between languages as a way of promoting mathematical learning. Barton et al. (2005) supports this idea when proposing that universities offer a pre-semester "English for mathematics" program, home language tutorials, and encouragement for students to talk during lectures.

2.2.3 Summary of Literature on Cultural Aspects of Mathematics

Although mathematics is often deemed to be independent of culture, literature in this section highlights connections between the learning of mathematics and the cultural context in which this happens. Classroom practices and beliefs about mathematics vary widely from country to country. Differences in these beliefs and practices suggest that different forms of engagement with mathematics learning are valued in different cultural contexts. These differences can lead to stereotyping of students from other cultures.

Language structure and vocabulary have also been shown to influence mathematical thinking of learners from those language groups. There is evidence of disadvantage to mathematics learners when the language of instruction changes, and new understandings must develop in a different language altogether. The literature in this section clearly illuminates the potential for significant disruption to mathematical learning when international students move from home to host country.

Yet literature in this section does not provide a fully-fledged, holistic explanation of how the language, cultural, and pedagogical differences of classrooms across the globe might play out when international students engage with the process of learning mathematics at a host university. Understanding these aspects of international students' experiences in STT opens doors to fine-grained studies that will foreground the voices of international students. Studies of this nature could more fully address how cultural differences might impact on the engagement of international students when moving to a new and foreign educational setting, or how they might shape international students' perceptions of themselves as mathematics learners.

2.3 International Student Experiences in Tertiary Education

In contrast to Section 2.2 which focused on cultural practices and beliefs in mathematics classrooms around the world, this section draws on literature from outside of mathematics education. It focuses on the teaching and learning of international students in wider academic contexts. The number of international students studying at host institutions around the globe has risen steadily since the turn of the century, exceeding six million in 2021 (UNESCO Institute for Statistics, n.d.-a). With this growth has come a significant increase in the research on international students, most of which has occurred in the period since 2006 (Abdullah et al., 2014; Jing et al., 2020). Jing et al. (2020) undertook a large, quantitative, scientometric review of 3685 journal articles published since the turn of the twentieth century to map knowledge of the field. Using high frequency keywords, they identified five clusters, one of which was cross-cultural adjustment. Literature in this cluster concerns the process of interacting with and adapting to a foreign environment and provides some background for my research. Abdullah et al. (2014), who conducted a qualitative review of the literature on international students, report that only 11.4% of articles in their review took a teaching- or learning-focused approach, while Agostinelli's (2021) systematic review of literature homed in on articles focusing specifically on the teaching or learning of international students in "Western" universities. These reviews guided my search for literature that could background my study. Of note is that all these reviews, with the possible exception of Jing et al. (2020), focus entirely on research published in English language journals. Since many international students study in English speaking host nations, one might expect much of the research on international students to be published in English language journals. However, countries such as China and Russia also run international student programmes. Research emanating from

these countries has not been included in the literature reviewed, most of which emanates from US, Australia, Canada, and the UK (Andrade, 2006; Jing et al., 2020).

One of Agostinelli's (2021) criticisms of research on the teaching and learning of international students is that there are few studies that differentiate between the experiences of undergraduate and postgraduate students, different programme majors, or different linguistic groups. This raises questions as to why more homogeneous groupings are not identified when exploring the teaching and learning of international students. A closer examination of the literature highlighted by Agostinelli's systematic review shows that some of the studies are classroom based and may include students from different backgrounds who are drawn together by a common class. However, more than half of the articles identified by Agostinelli's review approached teaching and learning of international students from the perspective of teachers, who speak about their experiences of teaching international students in general terms. While classrooms and teachers provide convenient data sources for studies of international students, the preponderance of studies relying on these sources of data may point to barriers between researchers and more homogeneous participant groups.

When considering literature on international students, my interest in their experiences of transitioning to host universities directs my focus towards literature on adjustment and acculturation, including language acquisition, in academic settings. I have limited my review to studies involving international students at undergraduate or pre-university (foundation) levels as far as possible, although participants in one of these studies included a mix of undergraduate and postgraduate students. I have also included research from a range of countries including New Zealand. Most of the research studies in this wider body of literature on international student experiences do not discern between students in different disciplines of study. They will thus include, but are not limited to, students of mathematics. I found only one study in this body of literature that focused entirely on international students studying mathematics, and a further two that focused on international students in STEM or Engineering.

Literature on adjustment and acculturation of international students offers a vast range of methodologies, contexts, viewpoints, and research focuses. Ramsay et al. (1999) view academic adjustment as the process of achieving a fit between students and their academic context. Part of the process of academic adjustment involves overcoming obstacles, which give rise to affective, cognitive, and behavioural responses. It is thus affected by emotions, moods and feelings (Ramsay et al., 1999). Acculturation might be viewed as a process

through which individuals both accept, and try to incorporate, the norms and values of a host culture into their own norms and practices (B. S. K. Kim & Omizo, 2005). Thus acculturation includes not only adjustment, the affective part, but also adaptation, the part in which new cultural rules and norms are learned (Rajapaksa & Dundes, 2003). Rothe et al. (2010) propose that acculturation is a complex process and is not the inevitable outcome of being in extended and direct contact with members of a host culture. To navigate this disparate body of knowledge I start, in section 2.3.1, by considering literature in which international student experiences are compared with those of local students. In section 2.3.2 I then look at literature that offers perspectives from the teachers of international students before turning to student insights on their experiences (section 2.3.3). In section 2.3.4 I finally present literature on the concept of international student identities before summarising this body of literature in section 2.3.5.

2.3.1 Comparisons between International and Local Student Experiences

Studies that compare international and local student experiences focus on aspects such as the relationships between adjustment to university life and students' social networks (Hechanova-Alampay et al., 2002; Rajapaksa & Dundes, 2003), academic adjustment and learning processes (Ramburuth & McCormick, 2001; Ramsay et al., 1999; Zhao et al., 2005), and academic achievement (Iannelli & Huang, 2014; Loong & College, 2012). I expand on these studies in the paragraphs that follow. However, it is interesting to note that social aspects of adjustment feature strongly in the findings of research from the US (Hechanova-Alampay et al., 2002; Rajapaksa & Dundes, 2003; Zhao et al., 2005), while the two Australian studies (Ramburuth & McCormick, 2001; Ramsay et al., 1999) emphasise the impact of other aspects of learning on adjustment. The studies on achievement of international students originate from Malaysia (Loong & College, 2012) and the UK (Iannelli & Huang, 2014). Six of the seven studies in this section used quantitative data for their comparisons, and all of them identify differences in the experiences of local and international students. I chose to review these studies as they offer insights into how local and international student experiences can differ in wider academic contexts. This provides helpful background when considering the experiences of international students in mathematics learning contexts.

Academic adjustment

In the US, comparisons of international students with American students who relocated to universities away from home indicate that international students might adjust better than their

local counterparts. A quantitative study undertaken by Hechanova-Alampay et al. (2002) used questionnaires to investigate adjustment and strain during the first six months at university, where strain measured problems such as anxiety or depression. This study, involving 188 American and 106 international students, showed that international students faced greater adjustment difficulties during their transition to university than American students. However, the higher strain measurements for internationals subsided more quickly than their local counterparts, reaching lower levels than local students after six months. Rajapaksa and Dundes (2003), who compared the experiences of 182 international students and 100 American students, found that international students who reported satisfaction with their social networks also showed high levels of adjustment. Local students did not show the same pattern. In this study, the authors recognise that the construct of social network needs further exploration. While noting that it does not correspond to the number of close friends, they give little consideration to the fact that the concept of a social network can vary widely. Some students might consider teachers, peers, or others who offer academic support to be part of their social network, whereas for others this might not be the case. It seems possible that cultural differences in how social networks are interpreted might play a role in the differences noted between local and international students. Hechanova-Alampay et al. (2002) also report social support as being a significant influencer of adjustment. They found that patterns of adjustment differed when international students interacted with American students and that higher levels of interaction with host nationals helped international students to adjust better to American universities. The authors do not, however, consider how the factors that enable higher levels of interaction with local students, might themselves contribute to adjustment. I would argue that international students with higher levels of English proficiency, for example, might find it easier to interact with local students. It thus seems feasible that their proficiency in the host language may impact on their sense of contentedness or fit with the local environment. Despite these concerns, the two US based studies mentioned indicate that international students appear to adjust more quickly to their new university environment than local students, and that social support plays an important role in this.

Engagement with learning

Engagement with learning is identified as another area of difference between local and international students during their first year at university. Ramburuth and McCormick (2001) compared the learning approaches of 78 international students and 110 local students at an

Australian university. The international students came from various countries in Asia and attended a pre-university English language institute at the university. The local students were in their first month of study in mainstream courses. These authors found that the local students tended to balance hard work and failure avoidance, whereas international students aimed for high achievement through organised resources, study skills, and self-management. The international students in this study also showed a much stronger preference for group learning and a lower preference for auditory learning resources than the Australian students. In the only qualitative study reviewed in this section, Ramsay et al. (1999) interviewed 8 international and 12 local students at another Australian university at the end of their first year in a commerce program of study. The international students perceived the Learning Assistance Centre to be their preferred form of academic support, whereas local students found greater benefit from peer support within study groups. Both local and international students noted some lectures as being negative experiences that led to feelings of frustration. However, these difficulties were comparatively more severe for international students, who attributed their difficulties to the speed of lecturers' speech, lecturers' use of complicated words, and to their own lack of proficiency in English. Zhao et al. (2005) also noted differences in learning processes between international students and American students. In their study, international students in their first year of university showed higher levels of student-faculty interaction and higher levels of engagement with academic challenge than local students. These three studies (across two countries) thus all note differences between the engagement of local and international students during their first year of tertiary study. While each study considers a slightly different aspect of student engagement and arrives at a somewhat different set of findings, all point towards high levels of effort expended by international students in their determination to achieve.

Academic achievement

Some comparative studies also highlight differences between the academic achievement of international students and local students at certain universities. These studies provide additional background for my own research, but the highly situated nature of the findings cannot be ignored. Studies in the UK and other parts of Europe have reported mixed findings on the relative achievements of international and local students showing, at times, that internationals outperform local students and, at other times, vice versa (Loong & College, 2012). Iannelli and Huang (2014) found that Chinese students gaining first degrees from the

Russell Group universities[1] in the UK displayed lower levels of achievement than home country students. They speculate that this is possibly due (amongst other reasons) to lower value placed on achievement by Chinese graduates since foreign qualifications are difficult to translate in China and are thus not very meaningful to potential employers. Given that variables analysed in this study were drawn from the limited statistics residing in an agency-owned database rather than researcher designed questionnaires, they did not include personal characteristics beyond gender, age, and domicile (from which personal characteristics were inferred). The study might thus provide a useful snapshot of trends in achievement at UK universities. Yet I believe caution should be exercised beyond this point and suggest reasons given for reported observations are speculative. Loong and College (2012), who investigated achievement in pre-university mathematics in Malaysia, also found that international students performed at a lower level than local students. The 76 international student participants in Loong and College's study came from 26 different countries, with almost one third of participants being from Indonesia, Sri Lanka, and Bangladesh. Despite lower performance measurements, they displayed good self-testing and time management strategies, and had higher anxiety levels than local counterparts. Other studies such as Ramburuth and McCormick's (2001) comparative study of international and Australian university students took a different approach. Rather than comparing the achievement of locals and international students, the study considered their *motivation* to achieve. Using two widely-adopted instruments with Likert type scales, the researchers found that international students were more motivated to aim for high achievement than Australian students. The variability of findings in studies related to achievement indicates the highly situated nature of these studies and their dependence on a vast array of influencing factors. These findings are interesting to those intimately connected to the environments in which the studies took place. However, their relevance to other situations must be judged according to contextual similarities or differences.

2.3.2 Teacher Perceptions of International Students

Some studies explore teacher perspectives of international students. These studies suggest that university teachers do not see it as their responsibility to support the linguistic needs of international students (Agostinelli, 2021).

[1] An association of 24 public research universities in the UK with headquarters in Cambridge

The "problem" of teaching internationals

Jin and Schneider (2019) reported mixed views amongst teaching staff at a US university, finding significant relationships between background characteristics of teachers and their views on teaching international students, but little relationship between their discipline and views. While many trends noted by these researchers did not lend themselves to easy categorisation, the authors argue that faculty with backgrounds most similar to those of international students are better placed to understand and empathise with them. They point out a tension experienced by teachers who participated in their study. Many teachers welcomed the presence of international students, appreciating their diverse perspectives, and the strong academic and multilingual skills that they brought to the classroom. However, teachers often felt that specific characteristics of international students, such as language proficiency and other cultural differences, created "problems" for teaching. Another study conducted by Heringer (2018) interviewed ten teachers across six faculties (in both sciences and social sciences) at a Canadian university. This study also found that teachers appreciated the diversity introduced to their classes by international students. Yet the study showed that teachers' intentions remained fixed on moulding students to fit "a Canadian way of knowing" (Heringer, 2018, p. 7). While these studies were each conducted in the unique setting of a single mid-size university, one in the US and one in Canada, their findings raise a flag - it seems that teachers steeped in the teaching and learning styles of their own countries may find it difficult to fully embrace the two-way relationship that underlies internationalisation of education. True academic hospitality involves both sharing and receiving, embracing the idea that engaging with diversity could lead to new self-understanding (Ploner, 2018). Yet the teachers in these two studies did not appear to question their pedagogical views or entertain the possibility that their teaching approach might disadvantage some students in a multi-cultural classroom.

Rejection of professional development

To support both teachers and the international students that they teach, host universities might consider offering professional development on culturally relevant teaching approaches. Yet Andrade (2010) reports that the teaching staff at a religiously affiliated university in the US, where almost half the undergraduate student body were internationals, did not desire this training. While sympathetic to the students' language difficulties, and seeing themselves as a part of the solution, they perceived themselves to be using helpful teaching strategies for international students when teaching disciplinary content. The findings of Haan et al. (2017)

in their survey of teaching staff at another US university showed that, while some were interested in developing their skills for teaching international students, they expressed concerns about the time and knowledge that professional development would require of them. Others responding to this survey did not believe that any adjustment to instruction was appropriate and should not be necessary. University teachers in a New Zealand based study by Skyrme and McGee (2016) recognised both value and challenge in teaching international students. Yet many in this study felt ill-equipped to respond to the challenges. The authors suggest that discussions need to take place at many levels, including both departmental and institutional levels, to help staff and international students work productively together.

All these studies show the complexity of accounting for the needs of international students when teaching. There are many overlaps in the findings despite the research coming from three different countries (US, Canada, and New Zealand). The difficulties recognised by teachers in these studies, for the most part, related to teaching and assessing students of lower language proficiency. However, there is less evidence that teachers recognise how different cultural understandings might play out in the classroom. What also stands out are the reservations, for differing reasons, that many teachers expressed about participating in professional development. A common cry by researchers, however, is that universities enrolling international students have an ethical responsibility to ensure that international students receive the support that they need (e.g., Agostinelli, 2021; Andrade, 2010; Jin & Schneider, 2019).

2.3.3 International Student Insights on Their Experiences

Qualitative research studies offer insights from international students that are less visible in the comparative studies and studies of teacher perspectives reviewed earlier in this section. Literature reviewed in section 2.3.3 arises from the US, the UK, Canada, Russia, Australia, and New Zealand. None of the studies focus specifically on mathematics but, many participants are enrolled in programs that include courses in mathematics. I noted four themes evident throughout this literature, being those of academic relationships, language, expectations, and (more recently) studying abroad during the COVID pandemic.

Academic relationships

Academic relationships, which include relationships between international students and their classmates, can be challenging for international students. Chinese students at a US university report that it is more difficult for them to make friends in mathematics classes as structured

group learning is less common in mathematics than in other subjects (Heng, 2019). These students found first-year mathematics easier in their new context than they did in China, which Heng attributes to "more rigorous training" at home and the "more universal language" of mathematics. However, second-year mathematics created greater challenges as it became more abstract and complex than what they were used to. International students reported that they seldom formed informal study groups for mathematics, with a participant sharing her opinion that it was a subject that "requires you to understand on your own" (Heng, 2019, p. 617). Wu et al. (2015) similarly report that international students in their US based study initially felt isolated from their classmates, having adopted a passive role towards developing friendships at the start. Students in this study reported that they were usually the last to be invited to join a group discussion, or they would wait for classmates to come and speak to them as they did not know how to approach others in their classes. With the passing of time, they discovered strategies that enabled them to join classroom discussions. Asian international students in Campbell and Li's (2008) New Zealand based study echoed this sentiment as they spoke of the challenges of developing relationships with their Kiwi classmates. During the COVID crisis, relational difficulties were intensified for Asian international students across the globe, and many faced microaggressions and discrimination in their host institutions (Bilecen, 2020; Hari et al., 2023; Zhai & Du, 2020). Despite the challenges, students in Campbell and Li's (2008) study expressed an affinity for group discussions as part of their learning. However, they felt a strong dislike for group-based assessment where all students in the group receive the same grade regardless of their effort or contribution. Simpson's (2017) exploration of groupwork experiences of Chinese students at a UK university showed that some found groupwork challenging and rewarding, whereas for others it was baffling and led to feelings of exclusion when pushed beyond their relational or linguistic competence. In this study, language played an important part in how prepared students felt for collaborative tasks.

Together, the studies mentioned show the complexity of relationships in the classroom and how creating opportunities for students from different cultures to work together is not a simple matter. International students face many challenges in group settings. While language might be the most visible one, other cultural conventions, such as when one might interject or interrupt others in the group, may produce issues that are less overt. This makes it difficult to infer how these findings from wider academic settings might play out in a mathematics classroom where the language of mathematics might further compound, or perhaps simplify,

efforts to work collaboratively. These aspects will have a direct impact on the learning opportunities available to international students.

International students also speak frequently of their academic relationships with university teachers. Campbell and Li (2008) report that students in their study enjoyed the non-hierarchical relationships between students and university teachers in New Zealand, yet some struggled to engage with their teachers. These students still maintained the deep respect for the role of a teacher inherent to their home cultures. This prevented them from approaching teachers with their concerns. Some students in this study expressed views that New Zealand teachers were not 'strict' enough. The choice to study (or not) was left to the students and some felt that this failed to impart successful study techniques. They compared Chinese teachers to parents who pushed students to move forward. Students in the US study undertaken by Wu et al. (2015) reported similar experiences of teachers who were kind and approachable. However, the fear of being misunderstood and the communicational practices expected of them in the new environment (like speaking without first raising your hand) created barriers for these students. They expressed a desire to be recognised as international students and accommodated differently to local students. International students enrolled in STEM programs at a Canadian university felt positive about many aspects of their learning, but suggested that time be set aside in lectures specifically for students to ask questions (Smith et al., 2020). What is evident from these findings is that international students often have to learn how to negotiate new forms of relationship with teachers when studying at host universities. Despite recognising how student-teacher relationships at their host university may differ from those to which they are accustomed, adapting to the new expectations appears not to be an easy matter. Both communicational difficulties and deeply held cultural differences may provide obstacles to accessing help and support from teachers.

Relationships outside of class that support learning are very important to international students. Students in Heng's (2019) study of Chinese students at a US university worked hard and deliberately to develop out of class friendships during their first year of study, exhibiting agency over their circumstances. Attending shared interest clubs or international student gatherings helped them to connect with other international students during their first year. Connections to local peers seemed to develop more slowly and were more evident amongst second-year students. Montgomery and McDowell (2009) similarly found that international students form well connected social groups who support each other in both general and learning needs. Their study, which involved international students who had completed a year

of study in the UK, revealed that friends were purposefully selected. A focus on reciprocity and functionality was evident. The student group, which in the instance of this study was mediated through a chess club, enriched learning experiences for international students and displayed characteristics of a community of practice (Lave & Wenger, 1991). Members of the community supported one another academically, and "oldtimers" passed on their experience to "newcomers" in the community. Montgomery and McDowell's study was methodologically unique in that it included an extensive shadowing programme. The aim of the shadowing was "to see the university context from a student-led perspective" (p. 458) and, when combined with interviews, offered another view on student experiences.

Language

Language proficiency has a strong influence on the ease with which international students adjust to their new learning environment, and literature shares many helpful insights from students on issues experienced in an English mediated environment. The ten students in Wu et al.'s (2015) US based study had learned English for several years in their home country, yet still reported language as a significant hindrance to academic adjustment. Their difficulties arose from the speed with which native speakers spoke English and their pronunciations. These difficulties caused embarrassment for students who had to ask others to repeat themselves or slow down and created barriers during groupwork when international students struggled to follow the discussion. Campbell and Li (2008) also confirm language to be one of the biggest barriers to learning for international students from Asia studying in New Zealand. Students in their study not only experienced difficulties communicating with lecturers and other students, but they also struggled at times to understand lectures, follow instructions, and interpret assessment requirements. Five of the 22 participants in the study reported that, even after a year of study in New Zealand, they understood only around 60% of instructions and examination questions. These students all spent considerable time working to improve their English language skills and prepare for upcoming lectures so that they could keep up with the rest of the class. The students in this study reported that both the high level of effort required to overcome language difficulties, and unsatisfactory learning outcomes often led to high levels of stress. STEM students in Canada suggested that clearer articulation by university teachers, prior sharing of teaching materials, and adding terminology explanations would be helpful for understanding (Smith et al., 2020).

Expectations

The expectations that international students have of themselves, and those placed on them by their families and communities are also frequently mentioned in literature on international student experiences. According to Wu et al. (2015) the significant financial burden of study abroad carries with it the responsibility of attaining externally expected outcomes. These outcomes often include full proficiency in English and the shortest possible timeframe to completion of the study program. Campbell and Li (2008) report similar pressures imposed not only by parents but by the students themselves, who maintained the high expectations common within their home cultures. The rule that "practice makes perfect" appears to be widely adopted by international students, and Ramsay et al. (1999) suggest that the response of international students in their Australian study was to put in more effort as they attempted to overcome the obstacles that stood in their way.

Pandemic related issues

Recent studies of international student experiences during the COVID pandemic have heightened awareness of the challenges that these students face, as well as their strength and resilience. Unwelcoming attitudes from Australian authorities and exclusion from financial relief packages left many international students at an Australian university feeling shamed and reluctant to access support structures offered by their host institution (Gallagher et al., 2020). International students in Canada were also exempt from government relief programmes resulting in severe anxiety (Hari et al., 2023). This left these students vulnerable in many ways, including impaired academic performance. The pressures highlighted in preceding paragraphs can significantly impact on the experiences of international students when studying at host universities. Bilecen (2020) calls on universities to be active defenders of international student rights in the emotional and financial commitment that they make to study abroad.

2.3.4 The Changing Identities of International Students

Some of the literature on international students shows how their identities change as they settle into their new environment. A Chinese student in Heng's (2019) US based study explained that it was only in his second year of study that he felt a sense of "belonging." Montgomery and McDowell (2009) describe how international students at a UK university induct newcomers into their community, providing advice to help new students negotiate the difficulties they themselves have overcome. Sablina et al. (2018), who conducted a study of

Chinese undergraduate students undertaking university studies in Russia, noted how they changed from a unified group conforming to circumstance to autonomous individuals whose choices led to diverse experiences. Prior to arriving in Russia, the students viewed their educational path as one prescribed by third parties and circumstance. The experience of learning the Russian language and the value of obtaining a high-quality education in a specific major opened doors to a wide range of future possibilities and led to greater awareness of the choices available to these students. Campbell and Li (2008) report students perceiving themselves as initially dependent and later as autonomous people, again showing how their sense of self had been changed by the experience of studying in New Zealand. Tran and Gomes (2017) note how emerging research on international student identities shows these to be cosmopolitan, rather than linked to a home nation, and based on transient circumstances and aspirations for global mobility. All these studies evidence how the experience of studying abroad can change the way that international students perceive themselves.

2.3.5 Summary of International Student Literature

Research on international student experiences suggests that these may differ from local student experiences, particularly in the early stages of university study. Measures related to adjustment, achievement, and learning strategies have all shown differences from those of local students in a range of different contexts. The studies in this section show how teachers of international students often express appreciation for the diversity that these students bring to the classroom yet fail to consider how their teaching approaches might disadvantage some international students. Some teachers even consider language and cultural differences to be a problem when teaching international students. Yet the studies also show how many teachers are reluctant to attend professional development that could prepare them to teach international students, citing time pressure and other concerns as issues. Research with international student participants illuminates the influential role played by language and other cultural differences when international students interact with their teachers and peers at host universities. The studies mentioned show how these differences might create barriers for international students in groupwork settings or when seeking help from teachers. Other studies show how international students and their families often subscribe to expectations of high achievement and full language proficiency within the shortest space of time. These, and other pressures when studying in a foreign country, can have a significant impact on

international student experiences. The literature shows how these experiences might shift international student identities in various ways.

Yet the research in this section is largely based in the wider academic setting and seldom addresses the experiences of international students in the specialised domain of mathematics. The abstract nature of mathematics, which has its own unique vocabulary and language, and its own logic structure may offer a very different experience for international students than, say, the language rich social sciences, or the creative practices of the arts. Collaborative learning practices promoted in mathematics also require different forms of interaction than would be the case in many other subjects. Examining international student experiences in wider academic settings might offer some useful insights, yet it cannot be taken for granted that the same findings will be evident in mathematics learning contexts.

2.4 Intersections and Gaps in Literature

The secondary to tertiary mathematical transition, being a mathematics learner in different cultural contexts, and the experience of being an international student are the three bodies of literature which background my research. When viewed together they offer a context in which the transitional experiences of international students in first-year mathematics are located. For linguistically diverse students, the challenges of new terminology and foreign grammatical structures in the language of instruction and learning are likely to be augmented by the formality of the new mathematics encountered at university. Access to the abstract concepts of scientific mathematics requires strong conceptual links between different representations. Yet for students schooled in foreign education systems and languages, one might expect these links to be different than for local students. While adjusting to new levels of academic autonomy may challenge both international and local students, the simultaneous adjustment to unfamiliar customs, relationships, and instructional practices in the learning environment might add further discomfort for international students. Adapting to new social expectations can lead to shifts in how international students perceive themselves as learners, and higher workloads as they strive to overcome the obstacles that they face. Bringing these three bodies of literature together suggests possible differences in how international students and local students experience the transition to university mathematics. University structures at different levels, such as programmes of study, assessment systems, course curricula, language of teaching and learning, and pedagogical practices (to name but a few) have been shown to influence mathematics learning experiences. How these structures influence the

transition to university mathematics for students schooled in foreign educational contexts has not been investigated. It exposes a gap not addressed by existing literature.

In each of the three bodies of literature reviewed, I presented some studies focused on learner identity. Studies of mathematical identity in STT showed how students' transitional experiences might be influenced by factors such as motivation, agency, and collaborative opportunities. Cultural differences in how mathematics is taught and learned in classrooms across the world led to research (from a Western perspective) that at one point characterised students from China as "rote learners." More recently, the US Model Minority Stereotype has been shown to reinforce a view that all Asian students are good at mathematics. And the literature reviewed on international students in wider tertiary settings speaks of how the experience of being an international student can lead to shifts in identity. The findings of these studies suggest that identity could provide a useful tool for exploring the STT experiences of international students.

The growth in international education, and the popularity of programmes such as mathematics, computer science, and engineering amongst international students, makes it imperative that host universities develop understanding of how to support international students in their mathematical learning. Understanding how these students might successfully transition from one educational context to another may also be of benefit to wider STT knowledge. The experiences of this under-researched cohort of students in STT is thus worthy of investigation, and studies of social and affective factors impacting their STT experiences are needed. Mathematical identity provides a lens through which to view how social, affective, and cognitive factors can impact on the experiences of mathematics learners. I thus situate my research within a gap highlighted in the literature as I explore how mathematical identity is constructed by international students during their first year of university mathematics.

2.5 Research Questions

Here I state the three research questions that underpin this research study:

RQ1. How do the *structures of university mathematics* shape the mathematical identities of international students in their first year of studies?

RQ2. How do *prior experiences* of international students in their home countries shape mathematical identities in their first year of university mathematics?

RQ3. How are the mathematical identities of international students transformed by the continued interaction of *university structures* and *prior experiences* during collaborative first-year mathematics tutorials?

Chapter 3: Methodology

Methodology concerns the methods for creating and making sense of data, as well as the reasons for choosing these methods. I begin this chapter by outlining my philosophical perspective (3.1) and my theoretical framework (3.2) before describing the methodological processes adopted (3.3 to 3.7), quality criteria (3.8) and ethical considerations (3.9).

3.1 Philosophical Perspective

Lincoln et al. (2011) describe five paradigms of qualitative research, differing in their ontological, epistemological and methodological bases. The approach taken in this research study stands outside of the paradigms described by Lincoln et al., subscribing instead to the realist ontology, subjectivist epistemology, and dialectical methodologies of *critical realism*. While the word "critical" might lead one to believe that critical realism falls within the critical paradigm, critical theorists seek to address issues of social justice by confronting situations of social oppression, struggle, or power relationships (Kivunja & Kuyini, 2017; Lincoln et al., 2011). Critical realism focuses instead on structure and agency rather than following the ideological approach of critical theories. Yet it shares with them an aim to improve the world and thus has emancipatory implications (Gorski, 2013; Stutchbury, 2022).

Critical realism is a philosophical position that emanates from the work of Roy Bhaskar and has been further developed by a pool of scholars (Archer et al., 1998). In an essay that summarises the progression of Bhaskar's ideas since the 1980's, Gorski (2013) notes the emphasis placed on ontology in Bhaskar's original and most basic form of critical realism. He highlights differences between the realist ontology promoted by Bhaskar and those of the more commonly adopted philosophies. Whereas positivists do not distinguish social and natural entities as being ontologically different, those from a constructionist philosophy either separate the natural from the linguistically constructed social domain, or view both domains as being constructed by language. For Bhaskar, reality is a stratified, open system comprising three ontological domains: *the real*, *the actual*, and *the empirical*. The domain of the real includes all structures, both natural and social, that exist in the world. Yet not all these mechanisms or structures are activated in any particular space or time. Only those structures that are active in a specific context form part of the actual level of reality in that space and time. Further, not all active structures can be observed. Only some of the mechanisms that have been activated will be evident in the empirical layer (Gorski, 2013). Considering how this view of reality might be applied to social science research, it is helpful to take a top-

down view. The empirical layer reveals how people interpret, observe or experience the middle layer of active structures, which are emerged by causal (or generative) mechanisms embedded in structures at the deepest level (Park & Peter, 2022).

Consistently, in this research study, the social structures of first-year mathematics become part of the active reality and contribute to the social identities constructed by participants in the study. Prior social structures are not active except in how they are remembered or reflected upon by participants. It is these embodiments of prior structures that move into the actual reality, rather than the structures themselves, contributing to personal identity. Social identity can be observed in the empirical reality, being formed through its interaction with personal identity and human agency. This layering of reality offers a perspective through which to understand how university structures might impact on the lived and felt experiences of transitioning international students.

Marks and O'Mahoney (2014) suggest that critical realist research:

> can ask 'traditionally' constructionist questions about the discursive effects on identity at the micro-level, and research these in a similar manner—but can locate these findings within a wider framework that retroduces information about both social structures and the self and the relations between them (p. 77)

An important consideration, according to Marks and O'Mahoney (2014), is that the relativist epistemology of critical realism means that the methods do not uncover reality. They merely move us towards an understanding of the processes that might enable and constrain identity production. The goal is to pursue knowledge of structures at the deeper layer of reality that produce experiences (Park & Peter, 2022). However Gorski (2013) describes the critical realist view of knowledge as comprising "(provisional and fallible) descriptions of structures and powers" (p. 669) rather than statements about events. Although conclusions cannot be viewed as true or false, recontextualising phenomena in different ways allows comparative judgements about validity to be made (Danermark et al., 2002). Stuchbury (2022) cautions that critical realism requires that researchers be "critical" of the theories used and explanations proposed when exercising judgemental rationality.

Retroduction is a key part of the critical realist approach as it seeks to move towards understanding of foundational structures and the powers they exercise (Danermark et al., 2002). Danermark et al. stress that it is not in any way a logically valid inference, but rather a "thought operation" that seeks to clarify the basic pre-requisites for observed relationships,

actions, reasoning, or knowledge. They note that transfactual argumentation of this nature (using arguments that go beyond the empirical) has been criticised by those who view reality as being limited to only that which can be observed. According to Marks and O'Mahoney (2014) any critical realist study of identity should both illuminate the social structures that influence identity, and suggest properties of the individual that produce or alter these structures. A retroductive approach to this task would ask what the world would have to be like for the findings to be possible. Marks and O'Mahoney suggest that answering this question could be done in a variety of ways, including approaches that use coding, draw on convincing theories, or compare findings against an array of explanations to find a "best fit" for the data. Danermark et al. (2002) suggest counterfactual thinking, social and thought experiments, studying extreme cases, and comparing different cases as techniques to understand the causal effects of underlying mechanisms.

Explanations (such as those given above) of the retroductive process in critical realist literature appear somewhat loose. Yet they form an important part of the critical realist philosophy. A decision to adopt a critical realist approach to identity research requires engagement with some form of thinking beyond the empirical, and the suggestion of individual participant properties deemed to have contributed to their personalisation of social roles. In the context of my own research, it offers a way to form tentative connections between a new and unfamiliar social structure and the subjective experiences of transitioning international students. This requires a multi-level analysis with data collected from a range of different sources (e.g., interviews, observations, mathematical working etc.). Marks and O'Mahoney (2014) note that analysing data from different sources is not an attempt at crystallisation or triangulation, which are validation techniques in other paradigms. Multi-level analysis remains open to the idea of converging or diverging causal explanations from distinct data sources.

Although Bhaskar's work on critical realism continued into dialectic and meta-theories, the basic critical realism, as previously described, offered a view of reality that resonated with me. By adopting a critical realist perspective, my research could acknowledge the unknowable. Emotions and actions could exist in the same reality, residing in different layers. And explanations of how mechanisms existing in deep reality (such as the social structures of prior learning experiences) might influence actions in the empirical layer could be pursued. The critical aspects of the approach sat well with my goals of illuminating tensions between

structures in the various layers and understanding agentive steps that international students might take to mitigate them.

In all interpretive research, the researcher is a central instrument in the research process (Stewart, 2010). In critical realist research, the researcher is understood to be an intentional agent (Olsen, 2007) who becomes part of the actual reality when conducting interviews or undertaking observations in a classroom. How I structure a research interview, or my presence as an observer amongst those in the actual reality of a participant, will influence other structures and their powers. The interview questions I ask, the placement of a video camera, power relationships between myself and the participant, etc., will all contribute to what emerges in the empirical domain.

The findings that I present in this thesis also surface to the level of the empirical. They emanate from my own interpretation, observation, or experience of the events that took place, and become visible to the reader through this text. The interpretations I present are formed by my unique passage through time and space. My background as teacher, South African, mother, and immigrant, will have coloured my interpretations in specific ways. Stewart (2010) suggests that researchers must consider not only what they know and how they know it, but also their response to the topic under study itself (in this case the experiences of international students transitioning to university mathematics). My response stems from many informal conversations with international students entering the New Zealand education system for the first time. I noticed how different cultural understandings could lead to unanticipated conflicts for these students in their new learning context. Misunderstandings of this nature elicited my empathetic response and intellectual curiosity, leading to this research which seeks to give voice to a rarely heard cohort in first-year mathematics.

3.2 Theoretical Concepts and Perspectives

In this section I present and justify the theoretical framework that has guided this research. A theoretical framework offers the process through which empirical material from a research study will be reduced to meaningful data (Williamson, 2018c). It also frames the research in ways that focus attention on certain aspects, while limiting our view of others (Spangler & Williams, 2019). Frameworks must thus be chosen carefully to ensure that the processed data meets the intended purpose. The goal of this research study is to understand how international students experience the transition to university mathematics, illuminating tensions that arise and how students respond to these tensions. To achieve this, I wish to explore the interaction

between the structures of first-year mathematics and the personal experiences of international students within that structure.

Mathematical identity offers a focus that can simultaneously bring both the social and the personal into vision. It is a construct that brings together the affective, cognitive, and social factors at play when learning mathematics (Sfard, 2019). In its broadest sense, mathematical identity might be viewed as a "socially produced way of being, enacted and recognized in relation to learning mathematics" (Darragh & Radovic, 2020, p. 1). By exploring instances of tension between social and personal aspects of identity I hope to gain insight on how personal perspectives about learning mathematics, formed by structures within different social systems, might play out in a new social context. To achieve this, a framework that illuminates these relationships is required.

I begin by presenting (in section 3.2.1) some theories of identity that are grounded in different areas of research such as anthropology, psychology, and sociology. I start from this broader view, as these theories are foundational to many studies of mathematical identity. In section 3.2.2, I then turn my focus to the field of mathematics education, showing how it has both taken up these ideas and produced some of its own. In sections 3.2.3 and 3.2.4, I offer more detailed descriptions of the theories that I draw on to frame this research study. The theories described in section 3.2.3 are those through which I understand *how* and *why* mathematical identities might come into being. Positioning theory, described in section 3.2.4, offers the means through which to recognise *characteristics* of identities being constructed. I close the chapter with a discussion of the advantages and limitations of this framework.

3.2.1 Identity in Different Fields

While the concept of identity has gained in popularity amongst mathematics education researchers this century, it finds its roots in other areas of research (Darragh, 2016). Harré and van Langenhove (1999) suggest that, traditionally, psychologists concerned themselves with one type of identity (a social identity), whereas philosophers were more concerned with a different form of identity (personal identity). Historical tensions to reconcile two seemingly incompatible versions of identity are described by Holland et al. (1998) who explain how the debate eventually resulted in a major shift in perception on the relationship between culture and self. They suggest that identity became commonly accepted as being a social practice constructed through discourse in multiple contexts.

Identity as a social practice

Two well-known social theories of identity are Lave and Wenger's (1991) communities of practice and Holland et al.'s (1998) figured worlds. Lave and Wenger consider identity as being constructed through participation in a community of practice. A community of practice is one which offers a social identity defined by a shared interest. Members develop a repertoire of shared resources for their practice and participate in ways that construct their identities. The experts share information with the novices, guiding them along an inward trajectory towards full engagement in the community.

A different view of identity is put forward by Holland et al. (1998). They consider identities to be self-understandings that are acted out by individuals to let others know who they are in socially produced or culturally constructed activities termed figured worlds. Figured worlds are social contexts where people are grouped according to commonly accepted beliefs and rules, and individuals must figure out how to play their part. Identities develop through continued participation in the positions offered by a figured world. But Holland et al. (1998) also speak of another aspect of identity that they refer to as positional. Positional identities are formed with reference to "one's position relative to socially identified others, one's sense of social place, and entitlement" (p. 127). Positional identities intersect with figured identities as individuals author their identities with the resources available to them.

Yet other researchers critique Holland et al.'s (1998) theory of figured worlds. A situation giving rise to such criticism might be seen in a case offered by Holland et al., which involved an individual called Kondo conducting fieldwork by immersing herself in the daily lives of a community. Kondo's narrative explained how she, at one point, caught sight of her reflection while out shopping. She was shocked to realise that she had adopted the dress, posture, and habits of the gendered identity assigned her within the community. She told how the identity that she had constructed did not match how she saw herself. Clammer et al. (2004) caution that cases such as this one, where positional identities and figurative identities are in conflict, raise ontological concerns. During her fieldwork, the discourses of a figured world determined Kondo's actions and interactions. Within a post-structuralist ontology, an observer of Kondo (prior to hearing of her revelation) could not know, so must deny, the existence of a non-discursive reality. Yet Kondo's subsequent narrative showed how even prior to her shopping trip her reality differed from that of the taken for granted discourse.

Marks and O'Mahoney (2014) argue that an ontology that restricts reality to only that which can be known discursively weakens the potential for researchers to explain *how* identity is formed. They claim that, because research from a constructivist viewpoint must take interviews at face value, it cannot judge between the validity of different discourses. Consequently, even if an identity acted out by an individual in one context does not support an identity narrated by the same individual in an interview, both must be accepted as equally valid. Marks and O'Mahoney claim that a reality constituted only by discourse removes the possibility for subjects to be either untruthful or incorrect about their own identities, as in the case of Kondo. They further argue that a radical discursive approach cannot explain how discourses generated by organisations can be resisted. If power determines which discourse prevails, then the resources available for building identities are restricted, and identities other than these would not be recognised. The dismissal of a non-discursive reality (or knowledge of this) thus limits the capacity for constructivist accounts of identity to engage in emancipatory ways (Marks & O'Mahoney, 2014).

These criticisms of constructivist perspectives on identity appear as an important consideration in research focused on international students. The collective discourses of the host country may be unfamiliar, or only partially familiar, to many of these students, while the familiar discourses of home are unlikely to be recognised by others. How students, who do not fully understand the prevailing discourse of the host institution, might construct identities from available resources appears unclear. To avoid this issue and understand how international students might construct their mathematical identities in a foreign educational context, a different perspective might be helpful. One that can acknowledge the existence of, and take steps towards understanding, any factors at play that are not evident in the prevailing discourse.

A critical realist view of identity

The ontological difficulties described above may be avoided when adopting a critical realist view of identity. Perceiving reality as layered, and recognising the existence of mechanisms beneath the empirical level, provides a perspective which allows that which is personally embraced to differ from that which is performed. In keeping with Archer (2002), Marks and O'Mahoney (2014) assume a distinction between personal and social identities. They view agency as being an emergent property of humans, generated through reflexivity, that has power to challenge social structures. These authors suggest that the emergent and stratified ontology of critical realism might address what they perceive to be weaknesses in the theories

that dominate identity research. They consider critical realism as able to provide middle ground between social constructivism (in which an incorrect or untruthful identity is logically impossible) and social identity theory (where the individual is solely determined by group membership).

3.2.2 Identity in Mathematics Education Research

Having outlined some theories of identity rooted in, and widely adopted by, areas of research such as anthropology, psychology, and sociology, it would now be prudent to consider how identity has been approached in mathematics education research. There is little doubt that figured worlds (Holland et al., 1998) and communities of practice (Lave & Wenger, 1991; Wenger, 1998) have featured strongly amongst the theories framing studies of mathematical learner identities (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019). However other ways of theorising identity, such as the narrative identities of Sfard and Prusak (2005) and discursive identities of Gee (2000), also feature prominently in mathematics education research (Darragh, 2016). In a more recent review, Graven and Heyd-Metzuyanim (2019) extend the list of frameworks evident in mathematical identity research to include, amongst others, Archer's (2002) social realist theory of identity.

In the paragraphs that follow I provide examples of empirical research on mathematical identity that is framed by one or more of these theories, showing how all have contributed to the field. In the light of these examples, I further discuss the potential of the exemplified theories to achieve the goals of my own research study. To achieve my goals I required a framework that could illuminate tensions in the relationship between university structures and the prior experiences of international students, showing how these shaped their construction of mathematical identities. I also sought to explore mathematical identities both at a macro level through reflective interviews, and at a more fine-grained level through observations of students as they participated in the learning of mathematics. My framework thus needed to accommodate both of these contexts.

Empirical research framing identity as belonging to a community of practice

A study that adopts a communities of practice (Lave & Wenger, 1991; Wenger, 1998) approach to mathematical identity is that of Boaler et al (2000). Their large scale, qualitative study involved students aged from 14 to 18 years and spanned twelve different schools across UK and USA. Findings showed that the vision of mathematical success presented by most of these schools was not one with which students identified. The authors termed this a "failure

to belong" within the mathematical learner community. Even those students who qualified for further mathematical studies often chose not to continue with mathematics, showing that this choice did not relate to their academic achievement. Solomon's (2007) study of twelve undergraduate students at a UK university showed that many experienced a similar failure to belong within their community of practice. All of the students in this study were enrolled in a basic first-year mathematics course offered by their university, while half of them also took an additional course for students intending to major in mathematics. Students in this study described the activities of first-year mathematics as requiring them to blindly following mathematical rules without full understanding. For some students, this mechanical process left them feeling vulnerable and disenfranchised, whereas for others rule following was not an issue. Solomon argued that for the students who became disenfranchised, legitimate peripheral participation (Lave & Wenger, 1991), or the early stages of the process through which newcomers to university mathematics were mentored towards full engagement, was not a functional identity. These students, who desired a more participatory role, did not perceive themselves as able to succeed at undergraduate mathematics.

Both of the mentioned studies consider mathematics learners at school or university to be a community of practice in which students transitioning from one level to another are initiated into the practice by those more experienced. As showed in the literature review (section 2.3.3) it is certainly not uncommon for international students to form communities in which those students with longer acculturation periods mentor the novice into the wider cultural context of university life (Montgomery & McDowell, 2009). However, within the confines of a lecture theatre or a tutorial classroom of first-year mathematics (where much of my data would come from), a different type of relationship exists. In these contexts, there tends to be one expert (the lecturer or tutor) and many novices. The fact that my observational data would be generated in this formal and asymmetric setting makes the concept of identity in communities of practice less useful to me.

Empirical research framed by figured identities

Many studies in mathematics education research use the concept of figured worlds (Holland et al., 1998) when considering identities. Voigt et al. (2021) showed how the figured worlds of three different undergraduate calculus classrooms in the US nurtured different mathematical identities. The socially constructed narrative around the calculus course for students of the life sciences affirmed their lower mathematical value than students in other courses. Physics calculus nurtured identities of capable and collaborative students, while the

standard calculus course promoted an identity of doing mathematics without understanding. The identities of students in the standard calculus course are not dissimilar to those described in the findings of Solomon (2007). In a different study, Solomon et al. (2010) considered the figured worlds of mathematics support centres at two UK universities. They report how these centres enabled a shift in tutor-student relationships and the development of group-learning strategies. Yet when overcrowding forced changes in who could access the centre, the previously established figured world was disrupted.

Understanding the figured world of first-year mathematics requires tacit understandings of the student body frequenting this world. Given that some of my data would come from retrospective interviews, the figured worlds in which these participants' first-year experiences were formed would be unknown to me and could not be taken for granted. Understandings of what it means to be a "good student," for example, could be substantially different across the many different contexts in which participants interviewed had studied. This raised a red flag warning that figurative identities might be difficult to recognise in some of the data I could collect.

Although Cobb et al. (2009) also drew on the idea of figured worlds, they offered a view of identity that allowed for the sort of "misrepresentations" or disconnects between the social and the personal raised in section 3.2.1. Their study involved two middle school mathematics classrooms, which they viewed as figured worlds. Prevailing narratives within each classroom determined the expectations or obligations with which students could identify. Yet in these figured worlds, mere compliance with the expectations, or even resistance to them, were also optional responses. Cobb et al.'s framework allowed for a compliant identity, where an individual "fits in" by portraying a socially valued identity despite not understanding themselves in this way. This mirrors the case of Kondo, previously illuminated in the work of Holland et al. (1998), and similar ontological questions as those raised by Clammer et al. (2004) might be asked. In this framework, Cobb et al. (2009) perceived mathematical identity as being related to learners' alignment with, compliance with, or resistance to the obligations of their mathematics classroom.

Ontological issues aside, the view of identity proposed by Cobb et al. (2009) offered a framework which might be useful for understanding international student identities in situations where their figured worlds could be observed. In Cobb et al.'s study the researchers used observations and field notes to document figured identities in two classrooms prior to interviewing students. However, the fragmented nature of the university mathematics

learning environment in which my research would take place differs markedly from the school classroom for which their framework was developed. In middle schools, mathematics learning tends to take place in relatively small and intimate mathematics classes which can be easily observed. At university students move between large impersonal lectures, smaller collaborative tutorials, and self-initiated learning (both alone and with peers) as they engage in mathematics learning. Each of these contexts places a different set of expectations and obligations on students, which would need to be documented for this framework to be used in my study. Given that identity construction is intimately tied to time and context (Darragh & Radovic, 2020), I envisaged the need for a tool more adaptable to the patchwork of situations in which mathematics learning takes place at university, and more attuned to how identifying activity might change over time and between contexts.

Empirical research viewing identity as a narrative

Many identity studies in mathematics education adopt a narrative view of mathematical identity, drawing on Sfard and Prusak's (2005) work. Here the narrative does not reflect some internal identity, but the narrative itself is construed as the identity. Identity narratives are those that are reifying, endorsable and significant. How words are used in the narrative is important, as reifying narratives use verbs in ways that turn actions into properties or use adverbs that stress the repetitiveness of actions. For a narrative to be endorsable, the storyteller must believe that it reflects a true state, while significant narratives are those that, if changed, would affect the storyteller's feelings about the one they identify. To operationalise the idea of significance in identifying narratives, Heyd-Metzuyanim and Sfard (2012) added the idea of an emotional hue. Changes in emotional hue capture moment by moment flows of emotional expression. Any identifying narrative also has a subject, an author, and a recipient (Sfard & Prusak, 2005). This gives rise to multiple identities for any one person, depending on who is telling the story to whom. Sfard and Prusak make a further differentiation between actual identities (or stories about how things actually are) and designated identities (or stories about expectations of what one is to become), suggesting that designated identities offer a link between the social and personal aspects of identity. Sfard and Prusak suggest that it is through mathematics learning that the gap between one's actual and designated identities can be closed. Identities thus "play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure" (p. 19). Heyd-Metzuyanim (2015) drew on these ideas to show how the narratives endorsed by authority figures led to ritual participation (or rule following) and subsequent

mathematical failure of a once high achieving 9th grader. Clashes between her actual identity and a discourse motivated by grades resulted in anxiety when mathematical difficulties were encountered. Another study set in South Africa drew on these theories to show how instructional practices shaped the identities of two 3rd grade learners (Heyd-Metzuyanim & Graven, 2016). This study showed how the student who employed ritualised practices when learning mathematics, and was eager to listen, participate and follow instructions, was viewed by her teacher as a good student. Yet she required fingers or concrete materials to perform calculations. The second student showed far more flexible routines yet was viewed by the same teacher as having limited numerical skill on account of his inadequate social behaviour.

The commognitive framework for identity (Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005) offers tools for studying identity in a wide range of mathematical settings, including interviews and observations. Yet the appropriateness of using a framework so heavily dependent on word use and language is questionable when participants are not necessarily fluent in the language in which they are being interviewed or the language in which their mathematics learning is mediated. In section 2.2 I presented literature evidencing the critical roles played by both language structure and language proficiency on the mathematical learning experiences of students. Consideration of these issues raises cautionary flags for a framework heavily dependent on use of language when researching international student experiences. For some participants in my research study, language limitations might restrict not only what stories could be told in interviews, but also the way in which these could be narrated.

Empirical research using positional identities

Positional identities (Holland et al., 1998) are commonly encountered in studies of mathematical identity. They are relational, arising from dynamic relationships between individuals as they interact with one another. Positional identities tend to focus on one's social position, or status, relative to another. Authors who draw from more than one framework often use positional identities alongside other theoretical frameworks. For example, Hernandez-Martinez et al. (2011) use Cultural Historical Activity Theory (CHAT) as their main analytical framework. However, they also draw on positional identities when analysing data generated mainly through student interviews. They showed how students transitioning from school to university mathematics constructed new identities when positioned in different ways by their new institutions.

While many studies of mathematical identity look at the more stable traits of identity, others focus on the moment by moment shifts in positioning in mathematics classrooms. Wood (2013) termed the positions taken in these moments, when individuals interact momentarily within the constraints of their social space, to be micro-identities. Both Wood and Esmonde (2009) show how micro-identities influence student opportunities to learn in classroom situations. Wood's (2013) study focused on the self-positioning acts of a student who switched between the micro-identities of mathematical student, mathematical explainer, and menial worker. When recognised as mathematically capable by others, the student engaged with their arguments, or articulated arguments for his own solutions, enacting positions of mathematical student or mathematical explainer. However, after repeated attempts by another student in his group to position him as incapable, the student finally capitulated by following the orders given by his peer to complete the task. In accepting the offered position, he self-positioned as menial worker. In another example that demonstrates the relational character of positional identities, Esmonde (2009) considered how the micro-identities that teachers offered to students in cooperative groupwork could promote equity. She describes equity as the fair distribution of opportunities to learn or opportunities to participate. Esmonde suggests that positioning marginalised students as competent and necessary to the group facilitates equitable learning opportunities within the group. For English language learners in mathematics classes, Esmonde further proposes that using multiple languages in groupwork might be one strategy through which these students might be positioned as competent and necessary. Another study drawing on positioning theory (Harré & Van Langenhove, 1999) to explore mathematical identities was that of Gardee and Brodie (2023). These authors analysed the micro-identities offered and constructed by secondary-aged students at a school in South Africa. They considered the frequency of positioning utterances made by students in peer interactions, categorising them as being of higher, equal, or lower status. Gardee and Brodie also analysed how students positioned themselves in relation to others during interviews. By comparing the narrated macro-identities with observed micro-identities, they showed how the micro-identities thickened to form macro-identities across lessons.

These studies illustrate how positional identities have been used in both interview and observational settings in mathematics education. Some of the studies in the preceding paragraphs showed how the micro-identities offered and constructed in mathematics classrooms influenced learning opportunities for students in these spaces. Given my aim to explore how the structures of first-year mathematics might influence identity constructions of

international students in both interview and observational settings, I saw positional identities as offering a versatile analytical tool.

Empirical research based on emergent identities

Archer (2002) put forward a theory of identity based on a realist perspective (see section 3.2.1). This theory, which will be more fully described in section 3.2.3, views identity as emerging when human agency mediates between social structures and reflexive considerations. Gardee and Brodie's (2023) conceptualisation of mathematical identity as emergent from a structured relationship between distinct social and personal identities, and agency, builds on these ideas. They focus on emotions as a key aspect of personal identities, offering insights into what participants valued when learning mathematics. By interpreting emotional hues (Heyd-Metzuyanim & Sfard, 2012), they explored the emotional investment of participants in positioning acts observed during interactive mathematics learning. This helped to understand not only how, but also why, participants constructed the micro- and macro-identities seen in mathematical peer interactions.

Another researcher in mathematics education whose work uses emergent identities, based on a realist approach to identity, is Westaway (2019). Drawing on the work of Archer (2000) Westaway considers reflexivity in the emergence and expression of teacher identities. She noted a contradiction between historical and new systemic teacher roles created after a significant curriculum change in South Africa. Teachers' experiences as learners and trainee teachers fitted better with historical roles than with new roles, which then influenced their teacher identities. The new roles, which required new actions, disrupted identities demanding that reflexivity (or the internal conversations people have about themselves and their situations) mediate. The teacher participant in Westaway's study agentively rejected the new teacher role offered by the changed education system. Westaway and Graven (2019) also used a critical realist approach in a different study exploring the identities of four teachers in South Africa. The teachers in this study had been learners at school under the old South African curriculum and for some, their initial teacher training also fell within this era. These prior experiences, embodied by the teachers, created tensions when they encountered the new systemic roles expected of them and they chose to hold onto beliefs of the past.

There are similarities between Westaway and Graven's (2019) study and my own. Both focus on the identities constructed when roles or positions from prior experiences conflict with those in a new social context. My aim is to focus on tensions that might arise for international

students when their individual experiences of learning mathematics in foreign educational settings interact with the social structures of first year mathematics. Westaway and Graven demonstrate that a critical realist approach to identity is well suited to this purpose.

The considerations previously outlined when reviewing empirical research framed by a range of identity theories led me to the theoretical framework for this research study. In the next section (3.2.3) I outline the theoretical framework that I adopted.

3.2.3 Mathematical Identity as an Emergent Social Phenomenon

Archer (2002) offers a theory to suggest how social and subjective influences might coalesce into identities. She promotes the distinction between the *concept* of self (social) and the *sense* of self (subjective) as being necessary for organised social relations to exist. Archer claims that without this distinction social expectations could not be appropriated as there would be no way of internalising obligations. She demonstrates this with an example from Zuni culture who believe that an ancestor lives again in the body of one who bears his name. Individual Zuni are given two names, one for summer and one for winter, and, according to Archer, correct appropriation of names across seasons could not happen without a continuity of consciousness. The Zuni needs to sense that his two names apply to the same self, despite understanding himself to be one ancestor in summer and another in winter. This somewhat extreme example demonstrates the need for a continuous sense of self in a specific social context. Yet Archer proposes that a continuous sense of self is needed in any organised society. Individuals must recognise not only the existence of social expectations, but they also need a sufficient sense of self to feel that certain of these expectations apply to them.

The Zuni example bears similarities to the situation faced by some international students who take on English names when they move to English speaking countries. Their English names become part of their new concepts of self which conform to a whole new set of social norms. Yet adopting a new name and new social norms does not break the continuity of self across home and host country contexts. Archer (2002) argues that it is the continuous sense of self that enables personal identities to emerge from an individual's interactions with the natural, practical, and social orders of the world. These interactions prompt emotional commentaries at the levels of physical well-being (natural), performative competence (practical), and self-worth (social). Emotional commentaries in the natural order arise as individuals anticipate how something in the natural environment might affect physical well-being. While examples of this in a modern educational context are likely to be rare, it is not impossible that fear

could arise in anticipation of pain on encountering a schoolyard bully. Emotional commentaries of the practical order relate to accomplishment or performance, and the emotions that this may nurture. In a classroom for example, engaging with a specific mathematical problem, or carrying out a certain action, might offer a sense of satisfaction. The standards to be met and the emotions that arise are a matter of the relationship between the individual and the task. On the other hand, emotional commentaries of the social order relate to self-worth and are concerned with society's normative evaluations. Someone who has invested self-worth in academic performance, for example, might feel a sense of inferiority on receiving examination grades that are below average. Yet this may not be a concern for another. Which normative evaluations matter enough to raise emotional responses depends on our own definitions of self-worth. Building on these ideas, Archer views personal identity as being "a matter of what we care about in the world" (p. 15). It emerges as we strive to strike a liveable balance that may prioritise one of the natural, practical, or social orders but does not neglect the others. She further views social identity as an expression of what we care about in social roles that are appropriate for doing this. In other words, we don't simply animate a role offered by the social context but personify the role that we adopt in unique ways. Archer outlines a structured process through which this occurs. She proposes that nascent (or emerging) personal identity effectively tests the offered role, giving feedback to nascent social identity at three levels (natural, practical, and social), before choosing the terms and conditions of investment in the role. Through this structured process, identities become defined.

Inspired by the theories proposed by Archer (2002) and Marks and O'Mahoney (2014), I have chosen to conceptualise mathematical identity as the personification of a social role made available by a mathematics learning context. I align with the definition of mathematical identity put forward by Gardee and Brodie (2023) as being "a social phenomenon, which emerges from a structured relationship between personal identity, social identity and agency" (p. 446) in the context of learning mathematics. For international students transitioning to university mathematics, I see mathematical identity as a navigated stance between how they believe they should be seen as first-year mathematics students and the things important to them as individuals. By choosing to view identity in this way I am able to separately consider the influence of:

- Prior experiences that have been reflected on or remembered and then brought into the social context of first-year mathematics as part of the continuous sense of self,

- Social roles available in the active social structure of first-year mathematics,
- Agentive actions that reproduce or transform identities.

A schematic diagram of how I see these aspects of identity working together in the context of my research is given in Figure 3.1.

Figure 3.1

Mathematical identity as an emergent social phenomenon

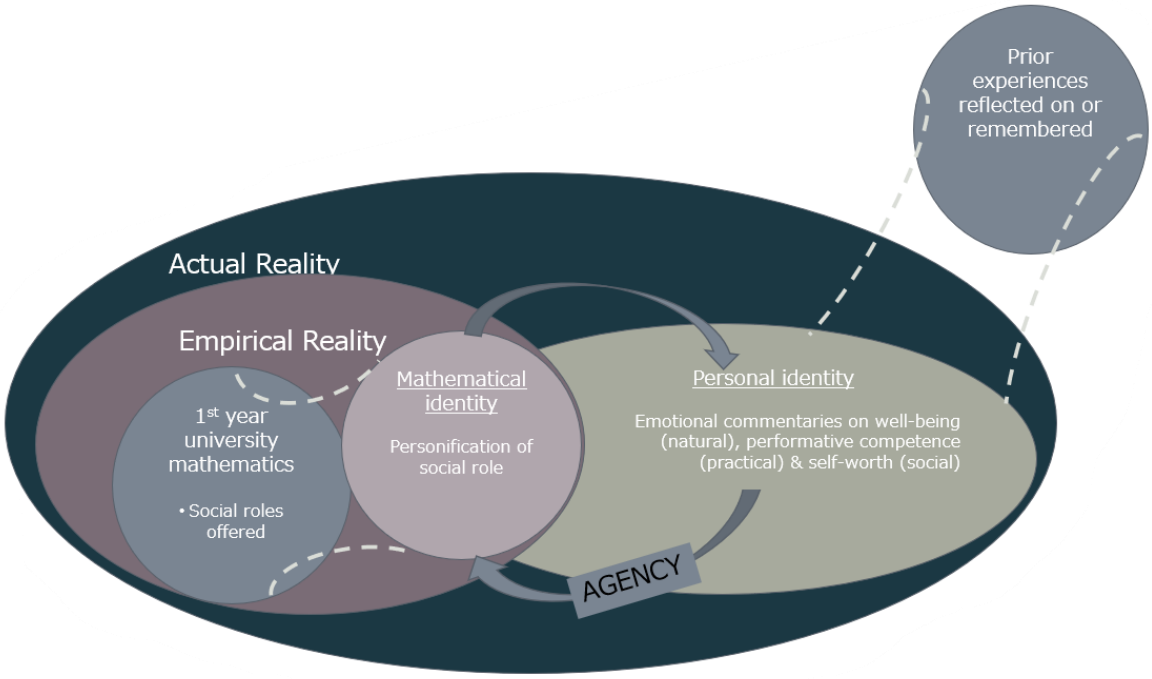


Figure 3.1 offers a diagrammatic representation of mathematical identity formed by a structured relationship between personal identity, social identity, and agency. The empirical layer offers insights on social roles in first-year mathematics, with the dashed lines leading to the social identities emergent from these. Social identities emerge in the empirical layer as individuals interpret, observe, or experience the social roles of activated structures. Personal identity is contained in the next layer of active structures and is emerged (dashed lines) by causal mechanisms embedded in deeper structures. Causal mechanisms are the myriad systems of structures, material and non-material, which have causal effect on the world. The social structures of prior educational experiences are one of the causal mechanism that reside at this deepest level. The arrows depict the dialectical relationship between nascent personal and social identities, through which the social role is personalised, and mathematical identity is formed.

Critical realism, as described in section 3.1, provides a meta-theory within which explorations of identity can take place. Marks and O'Mahoney (2014) propose that identity researchers will still need to draw on mid-level constructs to frame their studies.

3.2.4 Constructing Mathematical Identities Through Acts of Positioning

The previously described theory of emergent identities must be teamed with appropriate methodological tools to conduct a study of identity. Marks and O'Mahoney (2014) propose that:

> Whilst critical realists might use the same categories of methods as any other research position, they will seek to move towards understanding the processes that enable and constrain identity construction and discursive activity (p. 77).

Thus traditional qualitative methods are used to investigate data at the empirical level, although causal mechanisms at the deep level require inference and retroductive processes (Price & Martin, 2018; Stutchbury, 2022).

Positioning theory (Harré & Van Langenhove, 1999) is a research tool often used by identity researchers in conjunction with concepts such as figured worlds (Darragh, 2016). This theory introduces the notion of positioning as a way to understand how individuals locate themselves and others as coherent participants in conversations. The first part of Harré and Van Langenhove's positioning triangle involves clusters of behaviour types, or *positions*, that relate to one another in specific ways. Unlike roles which are fixed, positions are open to change and offer rights (what is owed to the position holder by others) and duties (what the position holder owes to others). In any interaction, rights and duties limit the actions available to individuals. For example, in a group of mathematics learners, we may note one person offering mathematical advice and demonstrating mathematical concepts on a whiteboard. In performing these actions the individual positions themself as a "teacher." By their actions, they simultaneously position those to whom the advice is offered or the concepts demonstrated as "students." In turn, those positioned as students might communicate their acceptance of this positioning act by engaging with the explanations and demonstrations. Alternatively, they could communicate their rejection of this act of positioning by ignoring the assistance offered and turning to other sources of help.

The second part of Harré and van Langenhove's (1999) positioning triangle is the *storyline.* This is a set of narrative conventions, through which the positioning unfolds and can be interpreted. In the previous example, the student/teacher storyline might be viewed as a

stereotype in which the narrative conventions are culturally resourced. In this instance the storyline may be understood in different ways by those in the interaction. One student might expect that a teacher demonstrates exactly how to solve a mathematical problem, while another might see the teacher's duty as being to provide guiding principles only. In a similar vein, the efforts of one student to point out another's error may be viewed by one as supportive and the other as condescending. In this instance one student may bring a storyline of peer collaborators, while the other may bring the storyline of smart student/weak student to the interaction. Storylines are usually taken for granted rather than articulated and must be discerned through the actions of the one telling the story.

The final piece of Harré and van Langenhove's (1999) positioning triangle is the *illocutionary force*, or the intention of the communication. A student whose intention is to position themselves as smarter than another might point out an error in the other's working through a supercilious announcement of the incorrectness of a particular step. Whereas one who intends to position themselves as an equal collaborator might gently question the misstep by pondering aloud why their working diverges at that step. When learners are positioned by others in an interaction, they exercise agency by choosing whether or not to appropriate positions that are offered to them. This in turn helps them understand more about themselves and to evaluate how they fit into the community (Gardee & Brodie, 2023).

The examples given above describe how positioning might play out in a classroom situation. Yet positioning happens in any social interaction including that of an interview. During interview, interviewees position themselves for the interviewer through their stories. Harré and van Langenhove (1999) describe three positioning practices that offer an expression of self in autobiographical accounts. These are achieved through claiming agency or expressing responsibility for an action, by indexing statements with one's own point of view, or by describing or evaluating a past event as contributing to one's biography. For example, an interviewee might claim to have invited a peer to contribute to a mathematical discussion. In saying this they perform an act of positioning by declaring themselves responsible for an action.

Positioning theory offers insight into how actions are embedded in social contexts, and how the rights and duties of some positions may not be accessible to all in the interaction. In the context of this research study, positioning theory offers a tool that can be used at the empirical level of reality to describe identities constructed. Further, by seeking instances in which participants reject the social identities offered them in favour of others, it can

illuminate moments of tension between the structures of first-year mathematics and subjective experiences of international students. Positioning theory is thus a useful tool to understand what mathematical identities are constructed in an interview or classroom setting (the empirical level of reality). To understand why or how these identities might form, retroductive processes must be used to explore generative mechanisms (such as prior experiences of learning mathematics) at a deeper level.

3.2.5 Advantages and Limitations of an Emergent View of Mathematical Identity

A lack of coherence regarding the conceptualisation and operationalisation of identity in mathematics education research has long been noted (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019; Radovic et al., 2018; Sfard, 2019). The critical realist framing of this research thus sits on the edge of what is already a confusing accumulation of ideas. To communicate effectively across a paradigmatic divide brings many challenges which may limit the reach of this research.

The complexity of how mathematical identity is conceptualised in this study might also be seen as a limitation. My approach to mathematical identity as a social phenomenon, formed through the interaction of social identity, personal identity, and agency, and recognised through the positioning acts of an individual, requires an overlaying of theories. One theory is needed to describe what can be seen and another to understand why this might be so. This complexity may draw criticism. Yet it also has certain advantages. It neither restricts identity to something that is long term and stable, nor to something instantaneous. Rather it allows for a continuously changing identity to emerge over time. It also skirts the ontological inconsistencies that might otherwise arise when exploring links between individual and social structures.

Thus, identity perceived in a structured and emerging way provides a best effort attempt to understand things that cannot be known. The lived experiences of participants in my research can never really be known by others. How their prior experiences contribute to their mathematical identities, or why they construct the mathematical identities that were observed, may be unknown even to themselves. While I might suggest that aspects of prior experience, or mathematical understanding, contribute to the identities constructed at different times in first-year mathematics, this is merely my rational judgement of what might underlie these identities.

3.3 Designing the Research Study

When undertaking a research study, the project must be designed in a way that ensures the research goals are met and the research questions can be answered (Maher & Wilkinson, 2019). In this section I outline the decisions that I made concerning the research setting, the design of the study, and the implementation of the project.

3.3.1 Research Setting

My first step in the design process was to identify the context in which I would conduct my study of international student experiences in the transition from secondary school to university mathematics. I chose a large research university in New Zealand which invests significant resources into the recruitment and support of international students. International students are well represented in both undergraduate and graduate programmes at this university and make up around 20% of the student base. The university requires all international students to demonstrate proficiency in English language, either through scores attained in recognised English language tests (such as IELTS or TOEFL) or previous academic study completed in English. To meet this requirement, some international students enrol at foundation colleges in New Zealand prior to undertaking studies at the university. This serves as a means of improving their English and meeting the required language standard for enrolment. Others meet entrance requirements by other means, such as attending local schools for a period and attaining New Zealand university entrance (UE) through the national education system.

The wide range of first-year mathematics courses offered by the university was another influential factor in my choice. It offers first-year mathematics courses tailored for students with different requirements, thus serving the needs of international students with diverse reasons for learning mathematics. Students intending to major in mathematics must complete three courses that set them up for advanced study of mathematics. In this thesis I refer to these as Maths 101 courses, despite each having a different focus (linear algebra, calculus, and computational mathematics respectively). Students enrolled in alternative programmes of study such as chemistry, computer science, or economics are likely to choose from four service courses in mathematics that offer skills and contexts relevant to their programme. I refer to these as Maths 100 courses. Maths 100 courses might include some topics in basic linear algebra, calculus, or computational mathematics, but demand a lower level of formality and rigour than Maths 101 courses.

First-year mathematics courses at the university also offered a combination of impersonal and more intimate learning contexts. This suited my research aim by providing different situations in which international student learning experiences might be explored. All the large first-year mathematics courses deliver content through lecture styled teaching, usually offering one-hour long lectures thrice weekly. Students are also scheduled to attend regular, weekly tutorials for one hour whose purpose is to consolidate concepts, and practice skills, related to recent lecture content. Tutorials offer a small group setting with up to thirty students on the roll, although attendance can vary significantly from week to week. Each course has a unique style of tutorial with some offering group problem solving tasks on vertical whiteboards, and others opting for worksheet styled questions that students complete at their own pace. Desks in tutorial rooms are usually organised in groups, and there are often whiteboards on all the surrounding walls. Despite differences in tutorial style, the intention across all courses is that students confer with one another, discuss their approaches and solutions, and work together to develop their knowledge. Their efforts are supported by a tutor who is usually a postgraduate student in mathematics, and who circulates to help students when they encounter difficulties.

My doctoral programme began in the latter half of 2020, some seven months prior to the novel coronavirus outbreak being characterised as a pandemic. The COVID-19 pandemic thus also forms an important part of the research context with implications for the design of the study. New Zealand's borders were closed to international visitors from March 2020 and did not fully open again until the latter half of 2022. By the time I was ready to start collecting data for the first phase of my study, student mobility throughout the world had already been interrupted and many international students had returned to their home countries. While some of the international students who were already living and studying in New Zealand before the borders closed remained in New Zealand, many returned to their home countries and continued their studies online. This change in how international students could participate in first-year mathematics had implications for the pace and design of my research. Many of my target group were only accessible through online channels, and opportunities for the social interactions in which I was interested were limited. Those international students who stayed in New Zealand could continue to learn on campus as and when New Zealand's COVID protection framework permitted. This meant that there were periods in which all teaching and learning moved online, while at other times students residing in New Zealand could attend learning activities on campus. However, the situation

could (and did) change frequently, often with little warning. This again impacted on my research activities and these uncertainties had to be incorporated in my research processes.

3.3.2 Two-phase Research Design

I chose to conduct my research in two distinct phases. In the first phase I would conduct a single, semi-structured interview with international students who had already completed a first-year mathematics course at the participating university. During this stage I sought to understand the STT by exploring the narrated experiences of advanced students who had had time to reflect on their first-year mathematical learning experiences. I would analyse stories told by these participants in interview by noting their acts of positioning and the storylines underlying these. Through exploring positioning acts resourced by both university structures and home country experiences I would seek to understand how social and personal identities interacted in the storied construction of mathematical identities.

In the next phase I would conduct a fine-grained study of international student participants during their first year of university mathematics. The second phase would include both interviews, observations, and scrutiny of students' written mathematical tasks. These multiple perspectives would be used as an aid to interpret participants' acts of positioning in collaborative settings. I would seek to identify storylines resourced by university structures and those resourced by prior experiences to understand how social and personal identities interacted and mathematical identities were constructed in collaborative settings.

The two-phase approach was a considered methodological choice (Taylor et al., 2016), although the anticipated scope of these phases was later adjusted to accommodate both the restrictions imposed by the pandemic and other, unexpected, recruitment difficulties. Wood (2013) distinguishes between macro- and micro-identities, emphasising the part both play in establishing links between identity and mathematical learning. According to Wood, most identity research takes place at a macro level, often based on students' self-reports. Yet she suggests that the time lag between the moment of learning and the moment of recounting identity restricts the ability to see how learning and identity interact in these moments. My intention in conducting a two-phase study was to explore international student identities at these two different levels. The storied mathematical identities constructed by international students who had already advanced beyond their first year of mathematics at the participating university would offer a perspective filtered by time and reflection. This would illuminate how university structures might shape mathematical identities of international students at a

macro-level. In the next phase I planned to explore the identities of international students as constructed through their actions in first-year mathematics learning contexts. This would offer a micro-level study in which interactions between the social structures of local learning contexts (such as tutorial classrooms or lecture theatres) and individual experiences could be explored in real time. The focus of this phase would be to illuminate tensions in the day-to-day learning activities of international student participants, and to understand actions taken by these students to mitigate them.

A further benefit of a two-phase approach is that insights gleaned from phase 1 could guide the second, finer-grained phase of the study. Ideally, the identity stories from participants who, with the benefit of time, had reflected on their experiences might provide focus areas for the next phase to explore at a deeper level. Additionally, the macro-level study appeared easier to implement and would provide real data at an earlier stage that would help with theoretical decision making. It also offered protection against pandemic related disruptions, in that phase 1 could be undertaken entirely via Zoom, if necessary, and would ensure that data collection could begin even under lockdown conditions.

3.4 Implementing the Research

3.4.1 Phase 1 Implementation

In this phase I sought to collect data through interviews with international students who had completed a first-year mathematics course at the university. The timeline for implementing the first phase of my data collection is shown in Table 3.1. In the paragraphs that follow I elaborate on the research processes taking place during this time.

Table 3.1

Timeframes and data collection activities for phase 1

Date	Activity
Mar 2021	Ethics approval received
Apr – Jul 2021	Recruitment and interviews of Aadam & Irash
Jul 2021	Ethics amendment approved
Aug – Sep 2021	Recruitment and interviews of Mingyu, Liang, May, Jon & Sunny

Recruitment of phase 1 participants

I initially recruited participants for phase 1 via email invitations sent to departmental mailing lists by an administrator in the mathematics department. The mailing lists were created for departmental use and were organised according to academic criteria such as postgraduate status, or enrolment in a mathematics course. They thus included both students who fit my inclusion criteria and those who did not. The email specified the purpose of the study and the inclusion criteria, and invited those who were interested to contact me by email for further information.

The email invitation was initially sent only to postgraduate students and was repeated after a two-week period. However, a lack of response necessitated a wider reach. The same email invitation was sent on two occasions each to various undergraduate mailing lists, this time with some (though very limited) success. Eide and Allen (2005) report similar challenges in recruiting participants for their transcultural study through traditional channels. For these researchers, connecting with a known and trusted member of their target population was critical to their recruitment process. I discussed my own recruitment problems with a postgraduate member of the university mathematics department who was well-connected amongst international students. His word-of-mouth advertising supplemented my email efforts and finally, after six months, I had seven participants for this phase.

Although participants had completed one or more of the first-year courses offered by the university, they had little else in common and came from different countries, schooling systems, pathways into university, and first-year mathematics courses. I accepted all seven volunteers who fitted the selection criteria as participants. I viewed the diversity in their backgrounds as a strength rather than a weakness, given that one of the purposes of this first phase of the research was to inform my approach for the in-depth phase to follow. The recruitment difficulties highlighted how important personal connections were to my target group. This discovery is supported by McClean and Campbell (2003) who suggest that different methods of recruitment have higher success rates in different ethnic communities. They found interpersonal contacts to be crucial in recruiting participants from South Asian communities. I noted this as an issue to be addressed when recruiting for the next phase of my study.

Conducting phase 1 interviews

Due to COVID-19 pandemic conditions, I interviewed six of the phase 1 participants via Zoom and one in person. Interviews were semi-structured and ranged from 15 to 53 minutes in length. The longest was the in-person interview with a participant of high English proficiency. With one exception, there appeared to be a positive relationship between participants' language proficiency and the length of their interviews. I began each interview by asking participants about their pathways into university mathematics. After this, the questions I asked explored five aspects of their first-year mathematics learning experiences, encouraging participants to speak about:

- the differences between prior and university mathematics experiences,
- how being international shaped learning experiences,
- their thoughts on success and challenges to success,
- connecting with others in the mathematics community,
- their engagement with learning resources.

I anticipated that data from the first two lines of questioning might offer insights on how home country experiences contributed to personal identities. Other questions related to the new learning context, and I anticipated that these might facilitate understanding of tensions experienced by participants and their actions to mitigate these. Interviews were video recorded, and I later transcribed these for analysis.

3.4.2 Phase 2 Implementation

In phase 2 I sought to collect data through a series of interviews and observations. I planned to conduct an initial semi-structured interview with each participant at the start of the semester. This would be followed by two tutorial observations and clarifying interviews. I would then do a final interview with each participant at the end of the semester. The timeline for implementing the second phase of my data collection is shown in Table 3.2, followed by an elaboration of the processes taking place during the various stages.

Table 3.2

Timeframes and data collection activities for phase 2

Date	Activity
Dec 2021	Ethics approval received
Summer semester (Jan – Feb 2022)	
Week 2	Recruitment and 1st interview of Peggy
Week 3	1st tutorial observation & clarifying interview (Peggy)
	Recruitment and 1st interviews (Yifei & Zixin)
Week 4	1st tutorial observations & clarifying interviews (Yifei & Zixin)
Week 5	2nd tutorial observations & clarifying interviews (Peggy, Yifei & Zixin)
Feb 2022	Final interviews of Peggy, Yifei and Zixin

COVID context of phase 2

The summer semester spans only six weeks (rather than being a more traditional 12-week semester) and offers a condensed, fast-paced learning experience. Students attend daily two-hour lectures, and two collaborative tutorial sessions every week. Typically, students enrol in a single course during the summer semester but are permitted to take a second course should they choose. The reduced time frame of the a-typical semester placed some pressure on the recruitment and data collection process. However, I could not ignore the opportunity that it presented, in very uncertain times, to collect data from students who were present on campus and able to participate in collaborative learning activities.

Recruitment of phase 2 participants

To address the need for personal connections in the recruitment phase (McLean & Campbell, 2003), I sought permission from course coordinators to speak with their tutors. I was invited to address tutors in two of the Maths 100 courses at their pre-semester briefings. In these meetings I explained my research project and requirements for international student participants. I invited tutors to contact me by email should they be willing to help with the research as auxiliary participants. Three tutors volunteered their assistance and advertised the research to their tutorial groups. This was done via a whole class announcement made by the tutor in the second bi-weekly tutorial. While this did not provide much time for tutors to establish trust relationships with students in their tutorial groups, their presence and position

within tutorial groups provided a personal connection to my target population. I provided tutors with details of the project, flyers, and a script for the announcement. After describing the project, tutors invited students who were interested in the research opportunity to pick up a printed flyer from a pile in the classroom, and to contact me by email for further information. Through this tutor to student advertising three international students, who were currently enrolled in a first-year mathematics course, and present on campus, volunteered for the second phase of the research. They participated in a series of interviews and two tutorial observations.

Initial and final phase 2 interviews

I conducted the first one-to-one, semi-structured interviews in person during the second or third week of the semester. The interviews took place as early in the semester as possible to capture stories told by participants about themselves and mathematics at an early stage of their university mathematics course. Interviews ranged from 16 through to 36 minutes in length with the shortest interviews in this phase being those of the participant with the highest level of English proficiency and the longest period of study in the New Zealand education system. In the first interview, the questions I asked guided participants to speak about their prior mathematical learning contexts; how being an international student influenced mathematical learning experiences; their actions when participating in tutorials, lectures, or other mathematical learning contexts; and their thoughts on success in mathematics. I also asked them to describe themselves as mathematics learners (see Appendix B). The final interviews took place via Zoom due to COVID restrictions and were only conducted once the semester had ended and participants had received their final grades. The intention of this timing was to capture storied identities after the learning experience. During the final interview I asked similar questions to those of the initial interview.

Observations and clarifying interviews in phase 2

Between the first and final interviews, I conducted video recorded observations of participants as they interacted with peers and their tutor in one-hour long, collaborative tutorials. I also conducted a clarifying interview of between ten and 20 minutes after each observation. The first tutorial observations took place as soon as possible after the initial interview. This gave students and tutor a short period to become accustomed to working together. The next observation took place as late in the second half of the semester as possible, providing a gap for any changes in interactional patterns to become more evident.

The short time frames of the summer semester (only six weeks in total) and the uncertainty of pandemic restrictions meant that a significant gap was, in fact, difficult to achieve and there were only two or three tutorials separating the first and last observations.

Two participants coincidentally attended the same tutorial session for their final observation as the threat of a new COVID wave resulted in rescheduling of the original tutorial timetable. I took the opportunity to seat both at the same table (together with a local peer), gathering data simultaneously for both participants. All other observations included a single participant working with peers who had consented to be part of the data collection process. Each observation was followed within a day or two by a clarifying interview during which participant actions during the tutorial were discussed. The intention of the clarifying interviews was to offer a space for participants to comment on any actions from the observation that had drawn my attention. For example, I may have noticed a participant return unexpectedly to a previously completed question or drop suddenly out of a peer discussion. The clarifying interviews provided opportunities for participants to explain some of these actions.

During tutorial observations, I used two video cameras. One focused on the wider group with the main participant central to the recording, and the other focused on the workspace of the participant to capture mathematical working. Due to the noisy classroom environment, I also placed an audio recorder on the participant's table for supplementary recording. During the tutorial, I used my phone camera to record mathematical working written on surrounding whiteboards as part of tutor explanations. I also recorded the final worked solutions produced by participants during the observation.

3.4.3 Transcribing the Data

Before any analysis could begin, recordings of interviews and observations had to be transcribed. I undertook the transcriptions myself after each of the interviews or observations. As transcribing requires careful listening and watching, and often produces insights along the way, it is a first step in data analysis (Bailey, 2008; Taylor et al., 2016). Bailey suggests that many decisions must be made when choosing how to represent audio and video data in written form, including the level of detail needed for a particular project. Because I did not plan to analyse linguistic aspects of the data, and because participants paused often as they sought to express themselves in English, I did not pay close attention to pause lengths unless I sensed these to be significant to the context.

In many instances I also chose not to transcribe utterances exactly as spoken. This decision arose largely due to difficulties sometimes experienced by participants when expressing themselves in a language that was not their home language. Efforts to express a thought were sometimes revised more than once as the participant discarded initially chosen words and replaced these with others that better expressed their intent. Sentences were sometimes abandoned part way through, in favour of another attempt to say what was intended. In instances like this, I transcribed only the outcome of these attempts, removing repetitions and false starts. For example, one participant, when speaking about his pathway into the New Zealand university where he studied first-year mathematics, uttered the words, "But I .. you know like uh .. 2019, this year I have, I got offer." I transcribed this as "But 2019 I got offer [from university]", leaving out the false starts, filler words, and overlaps that would have cluttered the text. In doing so I attempted to strike a balance between readability and accuracy (Bailey, 2008).

I also often had two video recordings, an audio record, and one or more photographs all related to the same episode in the observation. For the transcripts, I chose one of the video recordings as the base for the transcript and transcribed the spoken words of all participants appearing in the video (main participant as well as peers and tutor). I used other recordings to assist when audio from the main recording was indistinct. To the verbal transcription I added descriptions of non-verbal communications such as gestures or voice tone where I deemed these may be useful to the interpretation. When synchronised with video evidence of participants' written mathematical progress at that moment, and actions of peers or tutors in the vicinity, a detailed record of each moment was created. I used all these details when analysing participant acts of positioning in interactions. The final transcripts are necessarily a reduction of a huge and complex data set (Bailey, 2008), but include a level of detail that I perceived to be appropriate for the aim of my research.

3.5 Introducing the Participants

3.5.1 Phase 1 Participants

The seven participants in phase 1 of the study were advanced students who had completed a first-year mathematics course at the participating university. They came from four different countries and were enrolled in a variety of programmes at the university. I introduce them in Table 3.3.

Table 3.3

Phase 1 participants and their background details

Phase 1 participant (pseudonym)	Background
Aadam	Aadam, an international student from Malaysia, was in his final year of a BSc majoring in mathematics. He had attended school in Malaysia and studied under the Malaysian national curriculum until the age of 14 years. At this point he changed from the local curriculum to the Cambridge international assessment system[2] to complete his schooling in Malaysia. He enrolled in an undergraduate mathematics programme (Maths 101) in New Zealand directly after this.
Irash	Irash was born in Sri Lanka, but moved to Singapore when he was four years old. He undertook most of his schooling in Singapore under the Cambridge international assessment system, before moving to New Zealand where he completed his final A-Level year. He then enrolled in a conjoint programme at university with mathematics and commerce as majors. He studied Maths 101 as part of the former major. After his first semester he restructured his programme, dropping mathematics and switching to BCom.
Mingyu	Mingyu attended a top-level high school in China before studying pharmacy for one year at a Chinese university. She then enrolled to study media, film, and television at university in New Zealand. Although mathematics was not a mandatory subject for her programme, Mingyu elected to study three first-year courses (two from Maths 100 & one from Maths 101) over three different semesters while an undergraduate student.
Liang	Liang completed school (Grade 12) in China and then moved to New Zealand where he studied at a local school for one year. He met the requirements for University Entrance (UE) through his Level 3 National Certificate of Educational Achievement (NCEA), the most advanced of three school leaving certificates granted by the New Zealand Qualifications Authority. The pandemic forced his return to China where he worked for six months before enrolling with a New Zealand university as an online student. He completed a first-year mathematics service course (Maths 100) during his first semester and was enrolled in another first-year mathematics course when interviewed during his second semester.
May	May completed her schooling in Vietnam before undertaking a year of foundation studies at a college in New Zealand. She then attended

[2] Cambridge offers the internationally recognised IGCSE (International General Certificate of Secondary Education) and O-Level (Ordinary Level) qualifications for upper secondary school students (age 14+). These qualifications are equivalent although there are curricular differences. Their advanced programme (age 16+) can lead to an A-Level (Advanced Level) qualification. (https://www.cambridgeinternational.org/programmes-and-qualifications/)

	university in New Zealand where she studied ecology and statistics. During her second year at university, she enrolled in a university mathematics course (Maths 100) for the first time, as this was a requirement for further studies in statistics. May was in her final undergraduate year at the time of interview.
Jon	At the time of interview, Jon was nearing the end of his first year of undergraduate studies in computer science at a New Zealand university. He had attained an O-Level qualification under the Cambridge international assessment system while at school in Malaysia, before signing up to an American Degree Transfer Programme (ADTP). This entailed two years of study at a Malaysian university followed by a further two years at a university in the US. Unfortunately, the programme was interrupted by the pandemic, and Jon then enrolled with the New Zealand university where he studied Maths 100 as an online student.
Sunny	Sunny finished school in China before coming to New Zealand. She completed a nine-month foundation course to gain entry to university, where she enrolled to do a BSc with mathematics (Maths 101) and computer science as major subjects. At the time of her interview, Sunny was doing her honours year specialising in computer science.

3.5.2 Phase 2 Participants

The three participants who volunteered to take part in the second phase of my research were all Chinese citizens. All were enrolled in a first-year mathematics course, but there were significant differences in their prior mathematical learning experiences leading up to this point. I introduce Peggy, Yifei and Zixin, the phase 2 participants, in Table 3.4

Table 3.4

Phase 2 participants and their background details

Phase 2 participant (pseudonym)	Background
Peggy	Peggy was halfway through her second year of a BSc (majoring in computer science) at the time of her participation. She had completed high school in China before studying for a year at a foundation college for international students in New Zealand. During her first year at university, Peggy did not study mathematics. She enrolled in a university mathematics course (Maths 100) for the first time at the start of her second year and, during the summer semester, was in the latter portion of her first year of mathematics.
Yifei	Yifei was beginning her second year at university at the time she participated in this research. She had attended a middle school in China,

	before moving to New Zealand for her last three years of school. In New Zealand, she studied under the Cambridge international assessment system attaining IGCSE and A-Level qualifications. A year into her bachelor's degree she changed from a health science programme, choosing instead to major in pharmacology and statistics. This required that she enrol for the first time in a university mathematics course (Maths 100).
Zixin	Zixin was well acquainted with education in New Zealand prior to participating in this research. Having undertaken his early years of schooling in China, he had attended school in New Zealand for five and a half years. He obtained his Level 3 NCEA gaining University Entrance. When volunteering to participate in this research, he had just begun his first year at university and was enrolled in an engineering programme (Maths 100).

3.6 Data Collection Methods

Data for my research came from video recorded interviews and observations. I also added the mathematical working produced by participants during tutorial observations to the data set. My purpose in collecting data from these different sources was to provide richness and depth through which the experiences of the individual could be explored and interpreted. This would enable a multi-layered analysis of participant identities (Marks & O'Mahoney, 2014).

3.6.1 Collecting Data through Interviews

All participants took part in at least one semi-structured interview. The purpose of an interview is to "gather descriptions of the life-world of the interviewee" (Kvale, 1983, p. 174). These are then used to make sense of phenomena of interest. In the context of my research study, interviews are thus an appropriate way to gather participant descriptions of their learning experiences in first-year mathematics. These descriptions offer the context through which mathematical identities can be interpreted. Kvale further suggests that qualitative research interviews must focus on certain themes rather than containing exact questions. Although I had a list of questions, I wanted the conversation to flow as freely as possible, and to keep the interview focused but not directed (Kvale, 1983; Taylor et al., 2016). This meant that I did not ask all participants exactly the same questions. Some spoke to an area of my interest without being directly asked, while others offered information on something I had not anticipated, and I explored openings of this nature through further questioning (Williamson, 2018b). For some, conversation came easily, and they had no difficulty offering wide ranging thoughts and experiences. Others found it more difficult to

express themselves in a conversation conducted in English. These interviews were generally shorter as the conversation flowed less freely.

Given that the focus of my research study is on international students, encountering communication difficulties was not unexpected. Marschan-Piekk and Reis (2004) emphasise that conducting interviews in a language that is not the first language of participants exacerbates the potential for communicational issues. Not only might misunderstandings arise, but they suggest that interview bias and response bias are also a possibility. My experience in phase 1 of conducting interviews in English with participants from other language backgrounds confirmed this. When interviewing I found that I sometimes had to simplify my questions or repeat them using different words for some of the participants. This may have influenced how they answered, as my spontaneous rewording sometimes changed the essence of a question or narrowed it towards the answer that I perhaps anticipated. An extreme example of this can be seen when my question "How connected do you feel with others in the mathematics community?" was met with confusion by one participant. Subsequent iterations finally resulted in the question "Do you feel like you are part of the university, or do you feel like you are outside because you are overseas - that it's all a bit difficult?" a somewhat leading question.

My interpretation of participants' answers may also not always have been as they intended. To check my understanding, I would regularly echo back to them what I believed I had heard, providing opportunity for clarification or correction. At times further elaboration was forthcoming, but I believe there were instances when participants probably settled for what they felt able to say in English when put on the spot, rather than what they may have wished to say. While misunderstandings are bound to have arisen, my intention was not to create anxiety, but rather to foster a positive experience for participants by seeking "to understand as well as possible [their] experiences of a subject matter" Kvale (1983, p. 178). In this instance, the subject matter that I sought to understand was their STT experience.

To minimise communicational misunderstandings, I sent interview transcripts to participants for review. Given the challenges of verbal communication in a language that is not one's first, the transcripts enabled participants to translate written text and use technology to generate more accurate answers should they wish to. I asked that they suggest revisions if their given answers did not represent their intended meanings. Only one participant requested a minor change to a transcript due to a misrepresentation of their intended meaning. Another participant asked that I not reveal details of a personal incident related during interview.

However, he confirmed that the incident could be alluded to in reports, a request which I honoured in all analysis and reporting. As suggested by Kvale (1983), I do not view misunderstandings that are likely to have occurred as errors but accept them as part of the interview process.

The second phase of the research included a series of at least four interviews with each participant. This helped to develop rapport with my research participants and they may have felt more comfortable sharing information with me as trust developed (Grinyer & Thomas, 2012). The series of interviews also helped to reduce the potential for miscommunication in that it afforded regular interactions with participants, and thus repeated opportunities to revisit questions asked in earlier interviews. This structured relationship with participants provided multiple opportunities for participants to clarify their meanings and intentions. Regular interactions with participants facilitated better access to underlying storylines than was the case in the single interview technique of phase 1.

3.6.2 Collecting Data through Observations

I also used observations as a tool to gather data in phase 2. Rather than relying on field notes, I used video recordings and camera shots. During these observations I collected artefacts in the form of mathematical working of participants (Creswell, 2014). Williamson (2018a) suggests that observations are an appropriate tool for studying people in their natural environments. Whereas participants in an interview may be "unaware of important nuances or unwilling to disclose certain information" (p. 406), observations make it possible for the researcher to capture this data. For my research with international students, observations offered an important data collection tool in which the issue of language played out in an entirely different way from interviews.

However, observations also have some downsides. While I wanted to observe participants' natural encounters with the structures of first-year mathematics at their host university, observing natural encounters is an unattainable goal, as observation itself is not part of the natural structure. My presence (or even just the presence of a camera) in a tutorial imposes another structure, that of a research observation, which may change how people act and what they say. An obvious example of this was the self-conscious response to my presence of a peer at Peggy's table who remarked to no one in particular, "Senior maths lecturer's looking at me like 'what's she doing?'" (MS1 Obs1 16:23). Angrosino and Rosenberg (2011), while

recognising that unobtrusive, objective observation is idealistic, suggest that it is still a good data collection technique that is "as unobtrusive as possible" (p. 467).

My data collection tools included both observations and interviews. This is a common research technique that allows researchers to consider events from different sources (Williamson, 2018a). Yet it also raised my awareness of how my presence as observer and interviewer influenced the data I collected. Stewart (2010) noted how two studies undertaken four years apart revealed unexpected shifts in findings. He attributed the differences to his own influence on the research. Differences in the way he had asked questions or approached the analysis shifted his research findings in unexpected directions. In my study, I saw how my presence in a participant's world changed how she described herself. Having initially described herself as interacting with peers in tutorials (MS1 Int 1 21:48), she later described her hesitance to speak with peers (MS1 Obs2_int 15:53). These contrasting descriptions were offered in interviews that spanned two tutorial observations. I believe it was my presence in the tutorials observing her actions, and the questions that I asked, that changed how this participant perceived herself in interactions with peers.

3.7 The analytic process

From an early stage in the recruitment period, I engaged in an informal process of analysis using data collected from Aadam and (later) Irash, the earliest participants to sign up. Taylor et al. (2016) caution that data should not be forced into another's framework, emphasising the importance of extending and revising ideas in the early stages of analysis. The delays enforced on me by the slow pace of recruitment offered an ideal opportunity to trial different ideas and analytical approaches to understand their effectiveness in achieving my aims. Lessons learned through this extended period of reading and testing of different theories culminated in the analytical framework detailed in section 3.2, and the operationalisation of concepts that will be described in section 3.7.2.

3.7.1 Early Analytical Process

Analysis began at transcription of my first recorded interview and continued throughout the data collection phase and beyond through a dynamic and flexible process (Taylor et al., 2016). Taylor et al. suggest that analysis is a gradual process that combines insight, intuition, and familiarity with the data. I began by reading and re-reading the finished transcripts, writing notes to myself about thoughts and ideas that struck me (Creswell, 2014). At the same

time, I continued to read literature on mathematical identity, searching for an analytical framework that I could adapt for my own purposes.

Following Taylor et al.'s (2016) approach to data analysis, I began coding themes and creating typologies even with the data I had available to me at the time. I wrote analytical memos documenting my developing ideas. I developed visual representations such tables or grids that showed intersecting ideas, and I finally transferred my transcripts to Excel spreadsheets where I had the flexibility to sort and code in different ways with relative ease. As new participants came on board, I transcribed their data and added another sheet to my Excel file where I could code, revise, compare, and develop new ideas. I continued to write analytical memos, refining my ideas, and constantly searched for patterns within and across transcripts. By returning to my data regularly and trying to view it in different ways, I eventually began to create concepts and propositions. I then returned to the literature to search for studies related to my developing findings. Through many iterations of this process I gradually moved towards clarity in how I would operationalise concepts for interpretation (Taylor et al., 2016).

Having settled on a conceptual framework, participants' acts of positioning were deductively coded according to whether they stemmed from the structures of first-year mathematics, or from prior experiences. I undertook the initial coding myself, and validation of codes was achieved through regular discussions with my supervisors. During these discussions I presented extracts from the data together with my coding choices. At times my choices were challenged and required revisiting or justification before consensus was attained.

3.7.2 Operationalising the Constructs of my Research

While only published after I had undertaken a large part of my analysis, Gardee and Brodie's (2023) framework for analysing relationships between peer interactions and learner mathematical identities offered a helpful cross-check on how I had operationalised concepts such as social and personal identities. Being grounded in a critical realist perspective, their use of positioning theory and their definition of mathematical identity as being "a social phenomenon, which emerges from a structured relationship between *personal identity*, *social identity* and *agency*" (p. 446) fitted with my own conceptualisation of identity. In addition to operationalising each of the concepts referred to in this definition, I also needed to consider the concepts of *social roles* and *offered identities*. I operationalise these five concepts in the paragraphs that follow.

I view social roles as long term, stable categories offered by different structures within the hierarchies of a social system. Within a university we find professors, lecturers, and postgraduate students amongst many roles available in an academic structure. We also find students ranked according to a grading system or classified by their programme of study. The roles exist independently of the individuals in the system. Social roles commonly resource mathematical identities over a longer time frame, referred to by Wood (2013) as macro-identities. Participants in my research constructed their social identities in interviews by selecting and personifying, or inhabiting, social roles in ways that they valued. To construct social identities, they described alignment with some properties of a role, while distancing themselves from others.

Within the space of a tutorial, learners cycle frequently between positions made available to them in the tutorial classroom. Wood (2013) terms these micro-identities. I, in line with Gardee and Brodie (2023), refer to the positions made momentarily available to participants by others in the mathematics classroom as social identities offered. These are characterised by their rights and duties which offer different ways of working in the context of a tutorial. Although not formalised in the way that the more long-term social roles might be, they are determined by classroom norms and are made visible through the actions of those participating in the tutorial. By repeatedly aligning with, or distancing themselves from, the social identities available to them in a tutorial, participants in my study constructed their social identities in observations. I recognised four different social identities across all the tutorials that I observed, namely students working *independently* of peers or the tutor (SI), students *depending* on peers or the tutor for assistance (SD), students *collaborating* with peers (SC), and students *helping* peers by offering their knowledge (SH). Students in tutorials positioned themselves and others by performing actions associated with these four social identities at different moments throughout the tutorials.

Personal identities are formed when an emotional commentary is appraised simultaneously at each of three different levels – how it speaks to physical well-being, performative competence, and self-worth (Archer, 2002). To operationalise these in interviews I looked for stories in which the participant referred to feelings associated with learning mathematics. Operationalising personal identities in observations was somewhat more difficult. I considered that personal identity was being reproduced or transformed in two different situations. Firstly, if a participant's verbal and non-verbal acts in an observation showed mutuality with a personal identity described by the participant during interview, then I

viewed the observed acts as contributing to personal identity. However other situations, such as an apparent contrast between verbal and non-verbal acts, or rejection of an offered identity, also suggested to me that an emotional commentary was being responded to. I flagged both of these situations as being the visible evidence of occasions when personal identity was being reproduced or transformed during tutorials.

An agent is both partly formed by society, and partly transformative of society (Archer, 2002). In this study, agency refers to the idea that an individual, whose mathematical identity is partly determined by their social context, can transform this identity. Individuals might exercise agency in different ways and to differing degrees. Action taken to identify with a social role by personalising it in unique ways hints at agentive choices. For example, a student might narrate the identity of a capable student in an interview by telling how they complete tasks quickly or attain high grades. They exercise agency by choosing which of the socially determined rights or duties normally associated with capable performance they align with. Yet these choices do not suggest the same degree of transformative intent as one who describes actions not normally associated with a role afforded them. Choosing to speak of actions misaligned with the rights or duties of an afforded social role suggests a much higher level of transformative intent. To operationalise agency in interviews, I looked for instances when participants described their choice to act in deliberately transformative ways. I viewed stories of actions misaligned with the normative duties of their social role as evidence of agency. I also viewed stories of deliberate actions to overcome an obstacle and maintain a valued social role as evidence of agency. In observations I deemed participant actions to resist an offered identity as agentive.

In table 3.5 below, I outline the concepts of social roles, social identities (offered and constructed), personal identities, and agency. I also show how each of these were operationalised in interviews and in observations.

Table 3.5

Operationalising five concepts related to mathematical identity

Concept	What it is	How it is recognised in INTERVIEWS	How it is recognised in OBSERVATIONS
Social Roles (macro)	Categories that exist independently of the participant, and are offered by an activated social system (actual level)	Participant's utterances acknowledge existence of structured aspects within first-year mathematics that categorise them in a certain way.	

		Example Peggy: "Oh, most of the students .. is focused on the answer, answer the question and do the question, and ask the question to tutors if they don't know how to solve it." Interpretation Peggy acknowledges the normative actions of students in a tutorial.	
Social identity offered	Position offered momentarily to participant by others within the social system		Utterances by peer or tutor that position participant as SD, SI, SC, or SH. Example Nina to Peggy when she deemed her to be confused: "If it's a polynomial, I usually just divide that by the highest factor. Like if the highest is n squared then I divide everything by n squared." Interpretation By offering help, Nina positions Peggy as SD
Social identity constructed (emerges from a social role or offered identity)	How the participant personifies a role in the activated social system by cleaving to a subset of role properties that they deem worthy of reproduction, while transforming, downplaying or rejecting others	Participant utterances that describe their position with reference to a specific aspect of a university generated structure (example 1). OR Participant utterances that describe their actions in terms of the rights/duties of a social role (example 2). Example 1 Aadam: "when it comes to exam it is obviously I'm not doing well with it ... I am not great … I'm probably below average" Interpretation Aadam describes the relative positions of his grade and his peers' grades. Example 2	Participant actions (verbal or non-verbal) indicating momentary positioning of themselves as SD, SI, SC, or SH. Example Peggy: Yeah okay I got it I got it [takes tablet back, picks up pencil and hovers over work, inserts two equals signs, scrolls down slightly. Long pause without any visible written progress, followed by a "note to self" written on her script and the absence of a final solution.] Interpretation With her words Peggy positions herself as SI – a student who can continue independently.

		Peggy: "I will, at the beginning I will answer the question … I will solve for [my classmate] … I will ask the tutor or just close to me the students." Interpretation Peggy describes herself as fulfilling specific duties of a student in a tutorial (SI, ST, SD).	
Personal identity formed (emerges from continuous sense of self as a mathematics learner)	What learners care about in relation to learning mathematics – reflexive appraisal of emotional commentaries on different levels (physical well-being, performative competence, and self-worth)	Participant utterances referencing an emotional commentary about learning mathematics. Example Peggy: "But uh, I'm a little shy, I'm not sure they will discuss with me." Interpretation: Peggy references an emotional commentary related to anxiety in mathematical communications.	When participant actions (verbal or non-verbal) align with a personal identity described during interview (example 1). OR When actions suggest the reflexive appraisal of an emotional commentary e.g., apparent contrast between verbal and non-verbal acts, or rejection of offered identity (example 2). Example 1 Peggy's silence when peers discuss a question for which she has just written a correct answer. Interpretation Peggy's reluctance to contribute her knowledge to the discussion aligns with the personal identity as a shy student described in interview. Example 2 Peggy: (In response to Nina's explanation of how she approaches similar problems) "Yeah yeah, I'm also same." Interpretation Peggy rejects Nina's offer of SD and ends the conversation abruptly. This suggests that she has appraised an emotional commentary about the conversation (unwelcome).

Agency (the actions of an agent who is both partly formed by and partly transformative of society)	Actions taken by a participant to maintain a threatened social identity or transform an unwelcome social identity.	Participant utterances that describe their deliberate actions to overcome obstacles threatening an afforded social role (example 1). OR their choice to act in a certain way regardless of the constraints of a social role (example 2). <u>Example 1</u> Mingyu: "vocabulary is very difficult for me, but when I search it I can remember it" <u>Interpretation</u> Mingyu describes how she overcomes some language difficulties by looking up unfamiliar mathematical terms. <u>Example 2</u> Aadam: "I don't really practice that much compared to all my friends. But what I always do is understanding the logic behind it… put it in a sense, where it makes sense for you, for yourself" <u>Interpretation</u> Aadam chooses to learn in a way that enhances his understanding rather than repetitively practicing skills to enhance exam performance.	Participant actions (verbal or non-verbal) to resist a social identity offered, or to transform a social identity previously constructed. <u>Example</u> Nina (inviting Peggy to help her evaluate her answer): "…negative one to the power of negative one?" [laughs] Peggy: "Oh really." [laughs without looking up from work] <u>Interpretation</u> Peggy resists Nina's efforts to position her as SC by offering a non-committal response and remaining intensely focused on her own work (remains SI).

3.7.3 Artefacts collected during observations

The mathematical working completed by participants was photographed and added to the dataset at the end of each observed tutorial. During the analysis phase, both work in progress (captured in video recordings), and these artefacts were examined. I compared participant working with the published solutions to understand what progress had been made towards expected solutions at specific points throughout observations.

3.8 Quality Aspects of the Research Process

The quality of a research study must be judged in terms of its own paradigm which spans ontology, epistemology and methodology (Healy & Perry, 2000). These authors outline six quality criteria for research with a critical realist perspective.

Given the belief in a real, but not perfectly knowable reality, Healy and Perry (2000) suggest that the first two aspects concerning the quality of any study in this paradigm must relate to ontology. *Ontological appropriateness* considers whether the world being investigated is appropriate. Research in this paradigm should seek to investigate complex phenomena in the social sciences involving reflective people. C*ontingent validity* asks whether the study seeks to uncover broad generative mechanisms operating in that world. My research investigates students navigating the complex social situation of learning mathematics in a foreign university. I seek to explore how mathematical identities come into being by considering the prior experiences or cultural backgrounds of international students, and how these might shape mathematical identities in the new social context. I choose to focus on how or why certain identities are enacted, rather than merely describing what I see. Yet the social world is an open system where human actors have infinite choices at their disposal. In this context, revealing generative or causal mechanisms is not about direct cause and effect, but rather about movement towards understanding the processes that might enable or constrain a participant's enaction of identity in a specific context. In my study, for example, I consider what might have generated the changes noted in Peggy's interactions with the tutor, or how past experiences might have shaped the mathematical identities enacted by Aadam, Irash and Jon in their interviews.

The next quality aspect suggested by Healy and Perry (2000) relates to epistemology. Critical realism considers the perspective of a participant, not as reality itself, but rather as a "window to reality" (p. 123). Healy and Perry suggest that *triangulation*, in realism, concerns obtaining multiple perceptions of a single reality by triangulating multiple data sources, or the interpretations of several researchers. Marks and O'Mahoney (2014), on the other hand, dispute that triangulation can be achieved by analysing data from different sources. They suggest that distinct data sources may offer different information about a phenomenon, and that convergence is not a necessity. They offer an example of this from their own research in which they noted a gap between the workplace relations described by participants in an interview situation, and those identified through observations and analysis of emails. Marks and O'Mahoney claim that, although triangulation is used as a validation technique in other

paradigms, it is not appropriate in critical realist studies where multiple data sources are used to provide depth and richness. My study uses multiple data sources such as interviews, observations and written mathematical solutions. However, I do not view these as a form of triangulation, but rather a means to provide depth and context to my interpretations. For example, storylines from interviews provided context that helped me interpret participants' actions in observations. And an examination of artefacts from observations added insight when analysing transcripts of participant interactions with their tutor or peers.

Healy and Perry's (2000) last three quality criteria pertain to methodology. *Methodological trustworthiness* aligns to some extent with the qualities of consistency or dependability in constructivism. In an alternative approach to rigour in critical realist research, Ryan and Rutty (2019) speak instead about transparency and accessibility. Together these criteria resemble methodological trustworthiness which pertains to the extent to which the research can be audited. In my research, I aim for transparency by including descriptions of constructs and citing quotations from participants to demonstrate my approach to interpretation. *Analytic generalisation* concerns theory building and is somewhat akin to the quality of transferability in constructivism (Healy & Perry, 2000). With small participant studies in specific contexts, the feasibility of generalising to a wider population is limited. By disclosing the specific circumstances of my study (describing participant backgrounds and tutorial contexts in as much detail as possible) I enable readers to judge the relevance of my findings to circumstances that they investigate. Creswell (2014) suggests that thick descriptions of this nature enhance validity of the findings. The final quality criteria proposed by Healy and Perry (2000) is that of *construct validity* and refers to how well the research measures constructs. Ryan and Rutty (2019) suggest the criteria of accuracy, purposivity, and propriety. Taken together these are similar in nature to Healy and Perry's (2000) construct validity. While mathematical identity is an abstract concept that cannot be precisely measured, I propose that a form of construct validity is attained in my research, by using positioning theory to abstract and understand participant utterances and actions. Positioning theory goes beyond what is said in the moment and considers both the context and intention of utterances. While the potential for misunderstanding between interviewer and participants will always be exacerbated when one is communicating across different cultures in a language that is not a home language, the use of a theory that looks beyond mere words mitigates this as much as it is possible.

Aside from striving to meet theoretical, paradigm-related measures, I also took a more pragmatic approach to assuring the quality of my research by engaging with peer review processes. The five research outputs included in my thesis have all been peer reviewed by members of the international mathematics education research community prior to acceptance for presentation (or future presentation) at international conferences and publication in conference proceedings or journals. Feedback from the international community has corroborated and enhanced the quality of my research outputs.

3.9 Ethics

Much of social science research strives to say something about the conditions under which human beings can "flourish" (Archer et al., 2016). Any researcher who intends to contribute to the wider good of mankind must surely wrap ethical practices into every step of the research process. While the aspects described in the previous section concerning the quality of the research process are part of these practices, engaging ethically with participants is another important aspect. Ethical practices are strongly related to the *principle of respect* which infers a moral concern for the autonomy and privacy of participants recruited (Fisher & Anushko, 2012).

3.9.1 Informed Consent

Prospective participants in a research study should be informed about all aspects of the study that might influence their willingness to participate. This might include aspects such as the purpose of the research, the right to withdraw, adverse effects and prospective benefits, confidentiality, incentives for participation, and who to contact with questions (Fisher & Anushko, 2012). All participants in my research were given a participant information sheet (PIS) in which all this information was detailed. The PIS explained the aims of the study, what would be required of participants, and the role of the researcher. They were also advised that participation was voluntary and that they could withdraw at any time. Importantly, the PIS informed participants that their details would be removed from the data to maintain confidentiality but, because of the nature of observations, anonymity could not be guaranteed.

I emailed the PIS to those who expressed interest in participating after an initial conversation about the research. I invited them to come back to me with any questions after reading the information provided in the PIS. All participants signed a consent form, acknowledging their acceptance of the conditions outlined in the PIS, prior to their participation. This confirmed

their agreement to be video recorded in interviews and observations. They were given vouchers as reimbursement for their time and effort.

3.9.2 Confidentiality

The privacy of participants might be threatened by participating in a research study, so it is important to take all necessary steps to protect their confidentiality. Taylor et al. (2016) suggest that this can be done by:

> using pseudonyms, omitting identifying information in your data and written products, maintaining all data in a secure location, and destroying video or audio recordings after they are transcribed and analyzed (p. 40)

In my research I removed all identifying information at the time of transcription, using pseudonyms when communicating with my supervisors and in all public communications and reporting from this point onwards. I stored all electronic data, including digitised consent forms, on a University of Auckland managed device in a password protected account. As per the ethics approval received, I will keep all data for six years and will destroy it after that.

3.9.3 Perceived Conflict of Interest

Relationships between researchers and participants should be grounded in trust. Conflicting interests that might impair the objectivity of the research would breech this trust and compromise the research (Fisher & Anushko, 2012). Since the research was taking place in the context of a university, participants were advised that their participation (or non-participation) would have no bearing on their course grades or any other aspect of their enrolment at the university. Additionally, no participants were recruited from courses taught by either myself or my supervisors.

3.9.4 Approval for the Research

Ethical approval for the first phase of this study was granted by the University of Auckland Human Subjects Ethics Committee on 22nd March 2021 (Ref. UAHPEC22140). An amendment to broaden the inclusion criteria, publicise the research opportunity through different channels, and offer a monetary token of appreciation for participation was approved some four months later in July.

The second phase of the study received the approval of the same Ethics Committee in October 2021 (Ref. UAHPEC22975). An amendment to accommodate pandemic-related difficulties during data collection was approved on 7th December 2021.

PHASE 1 (IDENTITIES ON REFLECTION)

Chapter 4: Storied Identities in First-Year Mathematics

In this chapter I show how the seven participants in the first phase of my research study constructed mathematical identities in an interview setting. Extracts from two published research outputs have been incorporated in the chapter and have been referenced in footnotes to the relevant sections. In this chapter I show how social roles afforded these students by university structures informed the social identities that they constructed. I also show how personal identities, formed by reflecting on their experiences of learning mathematics, interacted with social identities. This interaction was mediated by agentive actions that transformed or reproduced mathematical identities.

In my research I view social roles as being resourced by university structures. They exist independently of the individuals within a social system and arise from benchmarks that categorise individuals in specific ways. For example, at the higher administrative levels, a university grading structure will categorise students according to their performance in formal assessments, while a degree structure categorises according to area and level of study. At department level, social roles such as tutor and student determine how participants in a tutorial are categorised, and what their roles entail. At a course level the structure of a mathematics course provides benchmarks against which students must constantly measure themselves – those who understand the content of a lecture and those still uncertain, those who can successfully complete a quiz on the first attempt and those who need three attempts, those who can complete assignments independently and those who need help. At course level, university structures might categorise students in ways that reflect their performance when doing mathematical tasks. Social roles at this level are thus performance oriented, effectively offering students a measure of their "competence" as mathematics learners.

In interview settings, individuals construct social identities by describing their position relative to others in structured aspects of a social role (e.g., grades that are higher/lower or solutions that are more/less efficient than their peers). Alternatively, they may describe how they meet, or fail to meet, duties connected with a particular social role, as they perceive it. The participants in phase 1 of my research study all recognised social roles related to performance. Some saw performance in terms of grades, others speed and efficiency, or their ability to complete tasks independently. While each may have perceived these roles somewhat differently, they all aspired to a role that they saw as requiring a certain level or

type of performance in course activities. I term the role that they desired that of *capable performer*, while clarifying that this role looked different for each participant. In these findings I evidence how phase 1 participants positioned themselves in a performance-oriented storyline. Sunny, Mingyu, Liang, and May spoke of mathematics as being relatively easy, showing how for these students, the role of capable performer was not difficult to attain. I refer to these as stories of success as participants successfully attained a role with which they felt satisfied. Aadam, Irash, and Jon, on the other hand, told stories of struggle, showing how the role of capable performer was less easily accessible to them.

As elaborated in section 3.2.3, personal identities are formed in response to an individual's reflexive appraisal of emotional commentaries on different levels (physical well-being, performative competence, and self-worth). They emerge from the continuous sense of self as a mathematics learner and can be thought of as what learners care about in relation to learning mathematics. In interviews, individuals might reference an emotional commentary, or their response to an emotional commentary, as they construct their identities. I view utterances resourced by reflexive accounts of learning mathematics as expressions of personal identity. The findings that follow also detail instances in which participants outwardly expressed feelings, memories of social systems from the past, or beliefs or values that shaped their actions when learning mathematics.

I start, in section 4.1, by outlining the mathematical identities constructed by each of the three students who told stories of struggle when interviewed about their experiences in first-year mathematics. In section 4.2 I outline the mathematical identities constructed by the four students who told stories of success. Each section ends with a synthesis of key findings and how they contribute to existing knowledge.

4.1 Stories of Struggle (Aadam, Irash, and Jon)

In this section I explore the identity constructions of Aadam, Irash, and Jon, three participants from phase 1 of my study. Aadam, Irash, and Jon all described themselves as having struggled with first-year mathematics. Their stories show that they recognised the performance-oriented storyline of first-year mathematics and the socially valued role of capable performer. Yet they found it difficult to meet the obligations of a performance-related role. In this section I show how each student described aspects of this social role with which they struggled, and how they each also described feelings or beliefs about learning mathematics that expressed their personal identities. In these stories of struggle, personal

identities contrasted with the social identities afforded these students and showed how they wanted to be recognised. Aadam and Irash described agentive actions that portrayed them as mathematics learners in ways that were not related to performance and that they valued more highly. Jon described prior accomplishments, portraying himself as a capable performer in a more familiar educational context, but disadvantaged by the unfamiliar obligations of this role in New Zealand.

The two-fold conceptualisation of identity illuminated a process employed by these three students, who were faced with social roles that they did not fully embrace, as they constructed mathematical identities in an interview situation. I refer to this process as a *manoeuvre* as it involves three parts working together to yield a complex mathematical identity as a result of their action. By elucidating the action of each part of the manoeuvre (in section 4.1.4), I again show how both past experiences and the institutional structures of first-year mathematics, informed their mathematical engagement. Although the same manoeuvre was employed by all three when constructing mathematical identities, Aadam and Irash constructed identities that enabled positive engagement with mathematics during their first year at university, while Jon's mathematical identity was one that led to disengagement from mathematics in the new social context.

Sections 4.1.1 to 4.1.3 show how social structures and prior experiences contributed to Aadam, Irash, and Jon's identity constructions in an interview situation. In section 4.1.4, I offer an overview of the manoeuvre and how it was employed by these three participants as they constructed mathematical identities within stories of struggle. Section 4.1.5 pulls together what has been learned by these stories of struggle and how they inform the next phase of the research study. Finally in section 4.1.6 I demonstrate the contribution to knowledge made by the stories of struggle, showing how they address the first two research questions.

4.1.1[3] Aadam – One with Deep Understanding but Low Average Performance

In this section I present an extract from Locke et al. (in print) describing how Aadam constructed his mathematical identity in interview. At the time of interview, Aadam was in

[3] Section 4.1.1 and parts of section 4.1.4 have been extracted from a conference paper. The Version of Record is available as:

Locke, K., Darragh, L., & Kontorovich, I. (in print). A mechanism used by international students when enacting identities in first-year mathematics. In *Proceeding of the 28th International Conference on Mathematical Views, 2022*. University of Oviedo, Spain: MAVI

the final year of his mathematics major. He had completed his schooling in Malaysia, initially studying the Malaysian national education system before transferring to the Cambridge international assessment system for his final years.

How university structures shaped social identity in first-year mathematics

In his interview, Aadam made the evaluative comment:

> I enjoy doing [mathematics], but when it comes to exam it is obviously I'm not doing well with it. So when, in terms of success, I say that I am not great. In terms of, if I put myself in the class, I'm more or less the average or probably low average.

In this act of positioning Aadam considered his examination grades relative to those of his peers, evidencing a recognition of the role offered him by the grading system. Rather than accepting the grade-based role exactly as offered, Aadam then positioned himself loosely within the constraints of this system, describing himself as "low average." With this remark, Aadam inhabited the role offered him by blurring any clearly demarcated grade boundaries and amalgamating it to some extent with surrounding roles. In so doing, he invited me, as the interviewer, to interpret his position according to my understanding of first-year structures. The social identity constructed by Aadam in this way complied with the social structure. Yet it did not align with positions adopted by Aadam at other times in his interview.

How individual experiences shaped personal identity in mathematics learning contexts

Aadam described how his cultural understanding of student-teacher relationships formed in Malaysia had prevented him from asking lecturers for help in his first year of university mathematics.

> Even though I keep telling myself that "you have to see the lecturer," ask if you not too sure. And then, when I wanted to do that, somehow I have this feeling of like, the feeling of scared I guess? I mean, as a first-year, you just, you do have a feeling of like a fear to ask your lecturers, in case they don't have time or something like that.

This extract shows Aadam's response to an emotional commentary raised by a university support structure. Aadam's New Zealand university expected students to seek help from lecturers when needed. Yet, despite understanding this action as both a right and a duty of his student role, it created tension for Aadam who found it difficult to meet these expectations.

Aadam also introduced another storyline from his prior learning experiences. It involved the two positions of learning mathematics for interest and understanding versus learning

mathematics to pass examinations. In this storyline, Aadam spoke of his school days in Malaysia when his motivation for learning mathematics changed.

> Yeah, and from then on, instead of me taking the local exams, because I realised there's too much in a sense, then at the age of 14 years old, I changed from local exams to Cambridge education. And then I can see there's a big gap between the local exams and the Cambridge education, where the understanding and how do we think to solve a problem. It's a really big difference of 'Oh, how we are doing this? I have to think through – is it correct or not?' Something like that.

He then developed this storyline on learning for interest and understanding by transferring it into the context of first-year mathematics.

> And then, I decide to take maths [at university] for that particular reason, because I believe maths is more like, there are some understatements [underlying statements] where there is more to it more than numbers. So that's why I was keen to take it.

In doing so, he positioned himself in first-year mathematics as an interested student who recognised mathematics as being deep and nuanced – a student for whom exploring these depths was the main purpose behind choosing to study mathematics at university. Regular utterances throughout the interview such as, "for me to see, to grasp, the idea of what these formulas are holding is a fair game for me," reinforced his position and communicated the value he placed on deep understanding rather than a grades-driven approach. The accumulation of positioning utterances of this nature evidenced a personal identity of a keenly interested student whose goal was to understand mathematics to increasing depths. Thus, the mathematical identity constructed by Aadam during the interview involved social and personal aspects that were not aligned towards the same goals.

Agentive actions to transform mathematical identity

When constructing his mathematical identity in interview, Aadam articulated his approach to learning mathematics. Despite receiving advice from those more experienced to "keep doing the practices," Aadam explained that "I don't really practice that much compared to all my friends. But what I always do is understanding the logic behind it." He also described how he actively engaged with course content through strategies that were both collaborative ("So it's good to have a friend where you debate [mathematics]") and individual ("then you have to imagine yourself in an existing world where there is something like this, and then you have to like put it in a sense, where it makes sense for you"). Aadam chose to tell of an approach that eschewed common practices geared towards examination success in favour of those that led

to deeper understanding. These communications of agentive action suggested some potential to enhance his performance-related social role. However, they also served to transform his mathematical identity to one more aligned with personal identity, in that they described the actions of a learner striving for deeper understanding through debate and visualisation.

The extracts show how the grading structure of university informed Aadam's identity, and how he drew on a storyline from individual experiences that offered different positions for success in mathematics. Through agentive action he sought to alter a social role afforded him by the structures of first-year mathematics and construct, in an interview situation, his mathematical identity.

4.1.2 Irash – An Independently Collective Process Person

Irash spent most of his school years in Singapore, before moving to New Zealand for his final school year. He studied under the Cambridge international assessment system throughout. When interviewed, he was an undergraduate student in a Commerce programme, having completed Maths 101 in his first semester.

How university structures shaped social identity in first-year mathematics

Irash spoke repeatedly of a persistent struggle with mathematics during his first year at university.

> But I found that I struggled with the maths at uni level. I scored an A in Cambridge A-levels, but when it was uni it really is another level, you know? Like, and that was especially algebra for me, I couldn't really figure it out.

In this statement Irash positioned himself as no longer categorised amongst the top performers in mathematics. He described being unable to make sense of linear algebra in his first year of university mathematics, suggesting this to be an obligation of capable performer that he was unable to meet. Irash also viewed efficiency of solutions as being an obligation of this role. He described some of his peers as "efficient types that just get to the answer quickly," whereas he categorised himself as being a "process person" whose answers were sometimes "like a round-about. You know instead of trying to make a U-turn straight, I have to go around to just come back." He also spoke of "really trying to speed up, so I can get the answer more quickly." With these utterances, Irash positioned himself as not fulfilling the obligations of speed and efficiency that he perceived to be important to the social role to which he aspired.

Through an accumulation of positioning acts, Irash identified as having limited success in accomplishing the obligations of the role he recognised as capable performer. He said that "even with the practicing, [algebra] caused me to struggle and that's why I decided to make the switch to commerce." By describing how his sincere efforts did not alleviate his difficulties, he positioned himself as precluded from a social role that he desired. Recognising his exclusion, he described how he had, on completion of his course, distanced himself from mathematics. He stated, "but now I'm in the Bachelor of Commerce side because I find that as the more interesting. More interesting than my math side which I used to major."

How individual experiences shaped personal identity in mathematics learning contexts

When speaking of his struggles with mathematics at university, Irash described a variety of actions undertaken to enhance his understanding and performance. One of these actions was to move to the front of the lecture theatre because he perceived those at the back to be less engaged.

> A lot of them [students seated at the back] try to keep up, but of course there are going to be some who, I think, probably because they find the subject boring, and sometimes I fall into the trap, but that's only sometimes, because many times I'm trying to listen and keep up.

With this reflexive statement, Irash portrayed himself as a student who was generally committed and engaged, despite having lost interest in the subject itself. I interpret his move to the front of the lecture theatre as an effort to distance himself from the disengagement that he perceived in others sitting at the back. He also described how "that uneasiness" that he felt when learning algebra in his first year at university had turned to feelings of "relief" after being helped by a friend. This offers further evidence of the conflicting feelings that his struggle generated when trying to meet the obligations he deemed necessary for capable performer.

Throughout his interview, Irash emphasised the importance of collaborative efforts, not only in first-year mathematics, but also beyond. He described how peers played a very significant part in his first-year experiences.

> For me that collaboration [in mathematics] is really useful because I believe in this thing about being independently collective, or the letters IC, as I prefer to call it. I haven't seen it mentioned anywhere and it's probably just self-invented [laughs]. So the basic definition is to take individual responsibility to work collectively with the people around you. This is because I believe you will also benefit if you learn to work together within and beyond yourself.

Irash explained that his main goal, when working with his peers, was to work together to benefit the group. He perceived that helping everyone to move forward together also benefited him as an individual. He elaborated at length on his very firm belief that everyone benefitted when individuals contributed, in whatever way they could, to support group upliftment. Irash told how these ideals concerning collaborative interactions were shaped by an incident from his childhood in his home country of Sri Lanka. In the story that he recounted, Irash found himself benefitting from a public-spirited act by an individual in his community. He later named the benefactor's attitude of offering whatever you have as an individual (regardless of how little or how much it might be) for collective good, as being "independently collective" or "IC." Irash's belief in an independently collective way of being in all situations influenced his interactions with peers in first-year mathematics, thus shaping his mathematical identity.

Agentive actions to transform mathematical identity

Irash described his independently collective approach to mathematical collaboration during tutorials.

> Sometimes I felt uncomfortable, but I just noticed others in my group. Some of them, they were just very quiet. So I had no choice but to explain to them. …But I also open up, allow them to share as well.

Here Irash described how, even when feeling vulnerable, he contributed to his group if he believed this would benefit the group in some way. In this instance his contribution may or may not have been the mathematics that he offered. However, he perceived that his action of sharing paved the way for others to do likewise and for the group to benefit from the contributions of all.

Irash also spoke of how meeting up with friends to discuss different approaches to mathematical problems was beneficial to everyone.

> Meeting in groups where we can, as I mentioned before, share different methods and how different methods can be leading to the same answer, all this is part of being IC. Because you get to learn something but everyone, including yourself, gets to solve the problem together as a whole. And this really benefits not only yourself, but also the other person as well.

Here Irash elaborated on what it meant to be independently collective, offering another example of actions in first-year mathematics that portrayed him in this way. In these, and other, utterances Irash agentively sought to transform a performance-oriented identity into

one that he valued more highly by describing actions whose main purpose was collective upliftment rather than personal achievement.

Thus, social structures and past experiences interacted in ways that enabled Irash to portray himself as a process driven mathematics learner who lived by his ideal of being "independently collective" even in mathematical interactions.

4.1.3 Jon – An Experienced Student Unaccustomed to Local Practices

Jon resided in Malaysia when he completed a first-year mathematics course through online studies with a New Zealand university. He had previously completed his schooling in Malaysia attaining his O-Level qualification (the "Ordinary Level" programme is normally studied by students between the ages of 14 and 16 years) under the Cambridge international assessment system. He had then enrolled in a tertiary programme which would have allowed him to complete the first two years of his degree at a Malaysian university before transferring to a US College for the final two years. The pandemic had interrupted his plans prior to this transfer.

How university structures shaped social identity in first-year mathematics

Jon described his struggle with mathematics during his first year with the New Zealand university.

> When I did maths in Asia it's very straightforward, like you do this certain question, you do this certain stuff and you get the answer, that's about it. [Mathematics in New Zealand] is challenging I guess you can see, because I'm not used to it yet … To me it feels pretty similar to what I've learned before, but the way that it's conveyed over here in New Zealand, like the way the teaching style .. it's completely different. Well, New Zealand teaches based on your understanding of certain things, so sometimes they can ask certain questions which can be worded really funny and I will get confused, basically.

Here Jon positioned himself as struggling to meet the performance requirements of Maths 100. His statement suggests that the obligations of a capable performer in New Zealand are different to those in Asia and were not yet familiar to him. He spoke further of his lack of familiarity with New Zealand teaching and learning practices. This shows how, while he was familiar with the mathematical content of his course, Jon perceived there to be a locally determined pedagogical and assessment style which he could not access. He described how, despite his knowledge of the mathematical content, his performance did not meet the

obligations of capable performer. He thus positioned himself as being unable to attain a role that he valued.

He gave further evidence of the peculiarity or disconnectedness of assessment questions from his own understanding of course materials.

> Most of time I just Google and stuff like that. I read the notes and try to understand what they're trying to say to me. But I mean most of the time it kind of works, but the exam always comes up with completely different questions that doesn't even match up.

Here Jon described how his actions of using online resources and reading course notes usually helped him make sense of content. Yet he saw his efforts as ineffective in terms of examination performance. He perceived examination questions at his New Zealand university to be unrelated to content learned, and thus an obstacle between him and the social role that he desired.

How individual experiences shaped personal identity in mathematics learning contexts

Jon's account of his pathway into the New Zealand university showed how the interaction of university structures and prior experiences gave rise to emotional commentaries. Jon's understanding of who he was as a mathematics learner contrasted with the social role afforded him by the structures of his New Zealand university. His social role, in New Zealand, was based on his enrolment status as a first-year student, and the storyline of success associated with this. However, during his interview, Jon explained that, despite being enrolled as a first-year student, he had already spent two years at a university in his home country.

> So, I study at this university called [university name] before. Yeah, I'm actually a university student. I mean I'm 21 years old this year so pretty old really… Oh I did quite a lot of math courses. I did Calculus 1, Calculus 2, Calculus 3, Pre-calculus, Discrete Maths, things like that.

While Jon had little choice but to comply with the first-year status afforded him by his New Zealand university, this extract shows that he did not embrace this role. He explained that this was not actually his first experience of university, and he portrayed himself as a mature and experienced student.

Jon described his pathway into Maths 100 at his host university in New Zealand.

> But my plan changed, so I'm no longer going to America, so I ended up going to New Zealand. But my credits didn't get transferred over, so basically, I'm restarting university all over again.

Jon offered an account of a change in circumstances brought about by the pandemic, and the refusal of his New Zealand university to recognise his prior studies. He repeated the same story a few seconds later, this time saying, "I no longer have my plans of going to the US," and "everything is gone to waste." With these statements it is evident that Jon felt a sense of loss about which he cared deeply. On two other occasions in his interview, Jon described his experience of studying first-year mathematics in New Zealand as "disappointing." He was unsure whether his disappointment related to "the online thing or [university name] style of teaching." While his situation as an online student contributed to his disappointment, Jon's feelings of disappointment also related to the barriers between him and the social role that he desired.

Agentive actions to transform mathematical identity

Jon drew on his prior knowledge when completing mathematical tasks in Maths 100 saying, "I use my own knowledge, my past experiences, to kind of help with my answers and all that in certain things." When reviewing lecture content, he claimed:

> I don't even open like the textbook or the coursebook, I usually just read the slides… I mean I understand the things on the slides and on the lecture sometimes. But like sometimes they ask out of the book questions.

Here Jon described actions that did not align with those normally available to first-year students, such as relying on the sufficiency of prior knowledge, rather than on new resources, to access content taught in lectures. By speaking of mathematical content as being familiar and by naming mathematics courses studied in his home country, Jon acted agentively to transform the identity of first-year student that was afforded him by university structures. His repeated allusion to inaccessible examination questions positions him as disadvantaged by his new social context. Jon's prior experiences of studying mathematics in Malaysia interacted with university structures in a way that enabled him to portray himself as an experienced mathematics student whose unfamiliarity with local practices prevented him from performing to his full potential in examinations.

4.1.4 Illuminating the Manoeuvre

Aadam, Irash and Jon all voiced a struggle with mathematics and employed a common manoeuvre as described in a paper by Locke et al. (in print) to construct mathematical identities in their interviews. The manoeuvre involved a non-linear process comprising three parts:

- Acknowledging a performance-related role offered them by university structures and using features of the role to construct a social identity that complied with this.
- Introducing a storyline that called on experiences prior to entering the new social system, developing this in the new social system of first-year mathematics, and positioning themselves in this alternative storyline.
- Taking agentive action to effect change from the proffered social role to a mathematical identity better aligned with their personal identity.

In this section I draw from the previous extracts of Aadam, Irash, and Jon to illuminate how these students employed each part of this manoeuvre.

Acknowledge and comply with social role

Aadam's evaluative comment about his own examination performance acknowledged the social role that this generated for him, and he described himself as "low average." Irash similarly recognised his struggle with inefficient and often procedure driven solutions, describing himself as a "process person." By also describing himself as a student who would be majoring in Commerce, he constructed a social identity that distanced him from future engagement with mathematics (Lave & Wenger, 1991). Jon too acknowledged and complied with his role as a first-year student in New Zealand but differentiated himself by way of age and mathematical background.

Introduce a new storyline and position in this

Aadam, Irash and Jon all related stories from past experiences in their home countries that introduced alternative positions to those afforded them by the performance-oriented storyline of first-year mathematics. They carried positions with which they identified from these storylines into the context of their mathematics studies in New Zealand. For Aadam the storyline arose from his experiences of switching to the Cambridge mathematics curriculum at school in Malaysia. It offered a position in which success in mathematics involved deeper understanding, rather than examination performance. For Irash the storyline had its genesis in

an everyday experience in his own country where he had benefited from the community-oriented actions of an individual. He carried into first-year mathematics a position whose goal was collective growth rather than individual achievement. Jon's storyline came from unwelcome, pandemic-related changes to study plans over which he had little control. He brought a similar position into his mathematics learning context, in which he had little power to control the circumstance that created tensions. Unlike Aadam and Irash, Jon's position did not provide new goals and his view of success in mathematics remained bound to academic performance.

Exercise agency to transform mathematical identity: Aadam told how he did not repetitively practice mathematical skills but preferred to debate mathematical problems with his peers and, when working independently, to visualise complicated mathematical concepts in ways he could understand. These actions were targeted more toward satisfying his interest, and deepening his understanding of underlying theories, than toward achieving high levels of performance in examinations. Irash explained how he drew his peers into group discussions by making himself vulnerable if necessary. At times he felt that he brought little more than leadership to the group but offered this as his contribution to group learning. These actions aligned with his philosophy of being "independently collective" and benefitted not only his own mathematical understanding but those of his group as well. Jon spoke of using Google, prior knowledge, and notes or slides from the recorded lectures to access the content of his mathematics course. However, he found himself faced with assessment questions that did not match with his understanding of this material. His actions to understand, together with his struggle to answer assessment questions, reproduced Jon's identity of having limited control over disadvantageous circumstances.

Summary and discussion

In section 4.1 I have described how three international students, who were faced with unwelcome social roles, constructed mathematical identities in an interview setting. I offered evidence of a manoeuvre rendered visible by the methodological choice to view social and personal identities as distinct. University structures afforded these students social roles that they did not embrace. When constructing their mathematical identities, they drew on past experiences to transform performance-related social identities into mathematical identities that more closely aligned with how they saw themselves. Experiences from their home countries offered storylines with alternative positions. During interview, they brought these storylines into the context of first-year mathematics. Their prior experiences thus coloured

and shaped their narrations of first-year university mathematics. The manoeuvre outlined in section 4.1.4 enabled them to bring valued positions from their past experiences into their new learning contexts to transform an unwelcome identity afforded by university structures.

4.1.5 Learning from Stories of Struggle

Exploring the identity stories of Aadam, Irash, and Jon showed how both social roles afforded by university structures and personal experiences of learning mathematics shaped the mathematical identities constructed by these participants. They all spoke of how they struggled to meet the perceived obligations of capable performer in first-year mathematics. This struggle created tensions for Aadam, Irash, and Jon. For Aadam the struggle related to graded examination performance. Irash spoke of difficulties understanding a new "level" of mathematics and performing this in an efficient manner. Jon attributed his difficulties to country-specific differences in pedagogical and assessment practices. They mitigated these tensions by agentively constructing identities that differed from the performance-oriented roles that university structures afforded them. These identities portrayed them in ways that they valued more highly than their afforded social role. In table 4.1 I summarise the identities constructed by these three students.

Table 4.1

Transforming social identities based on performance-oriented roles

	Aadam	Irash	Jon
Social Identity	Low average mathematics student.	Process-oriented person.	Mature first-year student.
Tensions	Struggle to meet performance obligations of capable performer.	Struggle to meet performance obligations of capable performer.	Loss of previously gained academic credits and struggle to meet performance obligations of capable performer.
Personal Identity	Someone who understands the logic behind deeply layered	Independently collective mathematics learner.	Experienced mathematics learner

	mathematical arguments.		in unfamiliar local context.
Agentive action to transform social identity	Describes success as deep understanding rather than in terms of grades.	Describes overarching goal as collective rather than individual benefit (IC).	Describes mathematics courses completed and wasted efforts.
Actions described that support personal identity	Doesn't practice much. Debates mathematical concepts with friends. Visualises problems in a way he understands.	Offers leadership to tutorial group. Shares different ways to arrive at same solution. Friends 'teach' others what they know.	Fruitless note reading to prepare for unmatching examination questions. Preparations quashed by "out of the book" questions.

These findings contributed to the second phase of my research study in two ways. Firstly, they showed that, in interview situations, the chosen theoretical framework effectively illuminated tensions that arose in the interaction of university structures and prior experiences. Secondly, they exemplified how agentive action might be employed by international students to mitigate tensions between social and personal identities. I elaborate on both points next.

Understanding the usefulness of the theoretical framework

The framing of this research study evolved over time into its current form. The cases of Aadam, Irash, and Jon confirmed its versatility in interview settings in which an individual's perception of themself as a mathematics learner differed from how they perceived themselves to be recognised by the institution. In this instance the distinction between social and personal identities allowed the influence of university structures to be separated from the embodied influence of prior experiences in the identity stories of participants. The cases of Aadam, Irash, and Jon confirmed that the separation of social and personal identities could illuminate how international students negotiated tensions between prior and current social systems when constructing mathematical identities in an interview setting.

However, I could not yet tell whether my framework would effectively illuminate tensions when social and personal identities were more closely aligned. For this I explored the identities constructed by Sunny, Mingyu, Liang, and May, the four students who told stories of success in their interviews. I was also unsure of whether the framework would illuminate the contribution of university structures and prior experiences to mathematical identities constructed in observational contexts. Storylines from prior experiences, while discernible in an interview setting, may be less so when participants were not authoring their own identities for an interviewer. This guided my methodological choice to include interviews as one of the data gathering tools in the next phase of the study. I realised that storylines gleaned from interviews might provide context through which observational data could be interpreted.

Exemplifying agentive action

An agent is both partly formed by society, and partly transformative of society (Archer, 2002), yet recognising agency is not necessarily straightforward. Aadam, Irash, and Jon storied agency in a way that contributed to how I operationalised it for the second phase of the study. Their stories led to my first operationalisation of agency in interviews as "participant utterances that describe their choice to act in a certain way regardless of the constraints of a social role." I later extended this to include "descriptions of deliberate actions to overcome obstacles threatening an afforded social role."

I chose this somewhat radical approach to operationalising agency as, in situations where actions align with the constraints of a social role, agency might be taken for granted. Simply personalising a social role might appear to imply choice rather than inevitability, yet I felt unconvinced that aligning (albeit in a unique way) with some aspects of a social role was sufficient to evidence transformative intent. Aadam, for example, described himself as a "low average" student. He chose this description from amongst those afforded him by the university grading structure. He might instead have chosen to describe himself as, say, a "passing" student, or to use an alphabetic grade that complied with the role that the grading structure afforded him. While his description of himself as a low average student involves a choice and might be construed as personalising a social role, it appears to hold little of the transformative intent that would typically be associated with agency. In my research I thus chose to attribute agency only to those actions (or stories of actions) that clearly showed disregard for the rights or duties of an offered social role, or deliberate efforts to overcome obstacles in the path of a valued social role.

While my operationalisation of agency in interviews did not immediately offer an approach to agency in observational settings, it provided a useful beginning. From this starting point I arrived at the description "participant actions (verbal or non-verbal) to resist a social identity offered, or to transform a social identity previously constructed" as a way of recognising agency in observations.

4.1.6 Contribution to knowledge

This section explored the storied identity constructions of international students for whom university structures did not afford valued social roles. It addressed research questions RQ1 and RQ2 by showing how both university structures and prior experiences shaped the mathematical identities constructed by Aadam, Irash, and Jon in an interview setting.

The main contributions to knowledge offered by this section are:

- International students who recognise performance-related social roles in first-year mathematics might construct mathematical identities by describing aspects of their own performance that falls short of a perceived obligation of capable performance. (RQ1)
- When struggling to attain a valued social identity, international students might employ a common process that draws on prior experiences to personalise mathematical identities in ways that they value. (RQ2)
- When struggling to attain a valued social identity, international students who can redefine success in non-performative ways might maintain higher levels of engagement in first-year mathematics than those whose view of success remains linked to performance.
- International students may feel unable to access help from the lecturers, or to access assessment questions in ways that enable them to demonstrate knowledge. (RQ1)

4.2 Stories of Success (Sunny, Mingyu, Liang, and May)

Sunny, Mingyu, Liang, and May all spoke of the relative ease with which they managed to meet performance related requirements of their courses. They thus identified as capable performers. In this section I show how each student constructed their social identity by describing how they met obligations that they perceived to be important to this social role. They also expressed feelings and beliefs about mathematics, evidencing what they cared about as mathematics learners, or their personal identities. Sunny, Mingyu, and Liang further described challenges that threatened their identities as capable performers and told of actions

that they had taken to overcome these challenges. In this way their mathematical identities were constructed in interview through a structured relationship between social identities, personal identities, and agency.

In the previous section (4.1) I illustrated a manoeuvre used by Aadam, Irash, and Jon who told stories of struggle when constructing their mathematical identities. For Sunny, Mingyu, Liang, and May, it seemed that close alignment between social and personal identities rendered an extreme manoeuvre of this nature unnecessary. These students embraced the social role afforded them by the performance-oriented storyline of first-year mathematics and did not seek to be recognised in a substantially different way. Their social identities did not require significant transformation. However, the absence of a visible manoeuvre does not mean that a common process was not employed by these students as they constructed mathematical identities in interview. Further research would be needed to understand whether a common process might be employed by international students who tell stories of success in STT.

In the next section (4.2.1) I present extracts from a published research output that show how Sunny constructed a resilient mathematical identity when faced with the challenges of learning mathematics in a new country. I then show how Mingyu, Liang, and May each constructed their mathematical identities in sections 4.2.2 to 4.2.4. In section 4.2.5, I draw these together to understand what has been learned about sources of tension, and actions to mitigate these, from the storied identities of Sunny, Mingyu, Liang, and May. Section 4.2.6 demonstrates how these stories of success address the first two research questions by showing how the structures of first-year mathematics, and their prior experiences, shaped the mathematical identities of international students.

4.2.1[4] Sunny – A Capable Performer Who is Accustomed to Challenge

At the time of interview, Sunny was enrolled in a postgraduate level honours programme specialising in computer science. She had completed her schooling in China before moving to New Zealand where she completed a foundation (or bridging) course in English, which

[4] Section 4.2.1 has been extracted from a conference paper. The Version of Record is available as:

Locke, K., Kontorovich, I., & Darragh, L. (2023). Secondary-tertiary transition of international students: One student's efforts to overcome the challenge of learning mathematics in English. In Ayalon, M., Koichu, B., Leikin, R., Rubel, L., & Tabach, M. (Eds.), *Proceeding of the 46th Conference of the International Group for the Psychology of Mathematics Education, 3* (pp 291-298). University of Haifa, Israel: PME. https://pme46.edu.haifa.ac.il/conference-schedule/conference-proceedings

included mathematics and other subjects. She then enrolled in a Bachelor of Science programme at the participating university, with majors in computer science and mathematics. Her first university mathematics course, Maths 101, covered standard topics in linear algebra and calculus.

How university structures shaped social identity in first-year mathematics

The analysis offered in Locke et al. (2023b) revealed that the storyline within which Sunny viewed success as a mathematics learner was resourced by the structures of her course. She explained how she evaluated her progress in first-year mathematics.

> I think for me is like just, when each assignment counts, if I can do all the questions by myself, or maybe I'm not, but that is my goal – I would like to do a lot of questions by myself. And also like, try to be as fast as possible, I mean just take shorter time as that I can easily remember, I have already remember all those things.

Her perception of success as being able to complete tasks both independently and quickly replicated the performance requirements of the mid-semester course test and the final examination.

Sunny went on to describe the challenges standing between herself and success of this nature. While she found that the content in her first year was "not that difficult," she believed that "the biggest problem would be just the problem of learning English, and that is the hardest thing." She explained that she could understand concepts and communicate using written mathematical symbols but had difficulty communicating her understanding verbally.

> I can understand those things, like the symbols. But if you asked me to explain it in English of how those symbols like you can explain to others, it will be a bit hard to do that part.

Understanding course content was not entirely without challenge either. For instance, she shared her struggle to grasp "the definition part, with the, like epsilon all those things." Sunny asked her course lecturer for help.

> I mean I asked them, and I still don't understand. They want to help, but you know I just don't understand. I think it's the language along with the concept. The concept is also hard.

In this reflection we see how language presented a barrier to understanding. In these, and other instances, Sunny constructed the mathematical identity of a student proficient in

meeting most mathematical requirements of her first-year course, but struggling to meet communication requirements in the English language. Cultural understandings of student-teacher relationships may also have contributed to Sunny's reluctance to persist in questioning her lecturer until concepts were clear.

How individual experiences shaped personal identity in mathematics learning contexts

Sunny also described feelings and beliefs that emerged from her individual experiences of learning mathematics. She explained that she had completed high school in China before introducing a storyline in which she described "the content here [in New Zealand] is not that difficult comparing to the content learning in China." By positioning herself within this storyline Sunny portrays herself as a first-year student with advanced prior knowledge and one accustomed to learning difficult mathematical concepts. She later described how "I quite enjoy learning math itself, but there's still some difficulties there. But I think it's okay to have some difficulties." This relayed her sense of enjoyment when learning first-year mathematics and strengthened her position as a student who was not only accustomed to challenge, but one who embraced it.

Agentive actions to overcome barriers to success

Sunny's difficulties with language threatened her social identity as a capable performer. But her personal identity as one accustomed to challenge shaped her actions in first-year mathematics. Her reflections show language difficulties manifesting in three aspects: understanding the course lecturers, communicating mathematics verbally, and collaborating with peers.

Sunny found lectures difficult to follow, saying that understanding was easier when verbal explanations were accompanied by symbolic notation. She explained how she would revise the lecture content "again and again by seeing, for me it's more about seeing the lecture notes, and also maybe listening to the recording to help me understand." She also sought external resources that offered visual explanations, such as an English-mediated YouTube channel with videos where "they explain things in some animations, which is better." These actions show how Sunny invested considerable time and effort to overcome the challenge of accessing lecture content delivered in English. Communicating her mathematical understanding to others also presented a challenge. She told how she spent significantly more time completing assignment questions requiring verbal explanations than those requiring calculations.

Collaborative sense-making is an important aspect of first-year mathematics at Sunny's university. Students work together in problem-based tutorials. The mathematics department also offers 'assistance rooms' which are specific spaces where students can gather informally to work on mathematics in the company of others and a tutor. Sunny shared that in both tutorial sessions and the mathematics assistance room, she would "tend to find people speaking Chinese and discuss together about those questions." So she took part in collaborative sense-making activities by seeking out Chinese speaking students with whom she could communicate fluently.

Discussion

Sunny's experience highlights some factors that contributed to her successful negotiation of STT and illuminates areas where additional support might have been helpful. The agentive actions described by Sunny evidence how her experience of productive struggle with mathematics in China supported practices that enabled her to overcome the new, language-based challenges of learning mathematics at a foreign, English-medium university (Di Martino et al., 2023b; Hernandez-Martinez & Williams, 2013). Her actions reproduced the identity of one who was both accustomed to, and embraced, challenge when learning mathematics, and reinforced her identity as a capable performer in the new sociocultural context. Other studies (e.g., Hernandez-Martinez & Williams, 2013) have shown how prior experiences can contribute to resilience in STT, but studies of international students in STT are scarce.

The idiosyncrasy of Sunny's circumstances must be acknowledged, and it must also be recognised that not all students have such well-established storylines to draw on, where positions support success in STT. However, Sunny's experience of language difficulties is certainly not unique. Sunny expressed a preference for "seeing" the lecture content by repeatedly re-reading the course notes after lectures. She also did not rely entirely on symbolic representations (L. N. Wood et al., 2007). Instead, she referenced a range of conceptual representations, seeking animations online and listening to lecture recordings.

Sunny's strategy to overcome language barriers when working with other students (Hwang et al., 2022) was to join groups where others spoke her own language, making mathematical communication easier and communal sense-making more productive. Yet communicating with university lecturers remained challenging on two fronts. Both the academic English encountered in these interactions and cultural understandings of the student-teacher

relationship (Lillyman & Bennett, 2014) raised barriers to understanding. These findings show how overcoming language barriers demands significant effort from international students, and host universities should be encouraged to take steps to support them.

4.2.2 Mingyu – A Capable Performer Who is Passionate About Mathematics

Mingyu completed her schooling in China and studied pharmacy at a Chinese university for one year. However, she did not enjoy her studies, finding her course difficult, so opted to enrol at a New Zealand university instead. In New Zealand she studied a programme unrelated to mathematics, but flexible enough to allow her to include mathematics courses if she chose.

How university structures shaped social identity in first-year mathematics

Mingyu described her performance in first-year mathematics.

> I think I'm doing alright because the course is not very difficult. And the teacher of the lecture is really clear, and the tutorial teacher is very good, so I can enjoy every lesson, so I can do everything.

In this statement, Mingyu expressed her opinion on the difficulty of the course. In doing so she positioned herself as a capable performer who was able to meet course requirements. She reiterated similar perceptions at other points in her interview. Although she did not articulate the performance criteria very clearly, she attributed her success in meeting them to her relationships with supportive instructors, as is common in Chinese society (Imada & Ellsworth, 2011).

However, Mingyu also spoke of how the change from learning mathematics in her home language to learning in English had been a challenge.

> Just some vocabulary is a little, vocabulary is very difficult for me. But when I search it I can remember it, it's just like the sign and some other things. In China we have a different one. And of course, in some inside, in here, we use 'C' and some other number to add the symbol of something. In China we use another word, we use 'A' or something, so that is different.

In this extract Mingyu told of how she not only had to learn a new mathematical vocabulary, but also new symbolic conventions in mathematics. Here she refers to "C" as typically representing the constant of integration in New Zealand, whereas in China an "A" would have been more conventional. While the mathematical requirements of her course offered little difficulty, Mingyu described some of the challenges she faced in meeting the communicational obligations of her course, thus threatening her social role.

Agentive actions to overcome barriers to success

Mingyu described actions to overcome these challenges such as searching unknown terminology and adapting formulae with which she was familiar. She also told how she would ask a Chinese tutor to explain questions to her at times, and how she would "often send email to the teacher because sometimes we can't understand the teacher talk in the lecture, so we have to talk to them after school." Here Mingyu describes a choice to present her questions in written rather than verbal form after class. These actions describe Mingyu's efforts to overcome language challenges and meet the duties of a capable performer.

How individual experiences shaped personal identity in mathematics learning contexts

Mingyu also spoke about her experiences of learning mathematics in China. She told how she had attended a "top level" school in China which offered "a high level of education." She compared learning mathematics in China to learning mathematics in New Zealand.

> In China, in middle school, maybe we study the set. And then in high school we will never study the set again, we study another things, we study the definition. And then in college we study much difficult things and we never touch the things in high school … and each period we study very difficult. In high school it's much difficult for a high school student.

Mingyu's description of mathematics in China shows that she holds memories of difficult, disconnected units of learning that, once covered, disappear from the landscape. She compares this to her mathematics learning in New Zealand.

> But here every course we study from the beginning and the end. And the depth of the things we study is deeper and deeper. So I prefer to, so the way in [New Zealand] we can study in a system, every course, I just like this.

Here Mingyu describes her perception of a more continuous curriculum that spirals down to deeper levels as courses progress. Her response to the emotional commentary that this comparison evokes favours her New Zealand based studies and shows positive feelings towards first-year mathematics. Her personal identity is further reflected in her statement, "But I really like maths and I'm a second-year student and every semester I choose maths." While mathematics was not a pre-requisite course for her programme of study, Mingyu emphasised her choice to study a mathematics course each semester. These utterances portray Mingyu's passion for mathematics on transitioning to university in New Zealand.

Mingyu's experiences of learning mathematics interact with social roles afforded by university structures in ways that enable her to portray herself as a capable performer with a passion for mathematics.

4.2.3 Liang – A Capable Performer Who Understands Why

After completing his schooling in China, Liang moved to New Zealand and studied at a local school for another year. Pandemic related travel restrictions forced Liang to return to China and complete his first semester of university mathematics as an online student.

How university structures shaped social identity in first-year mathematics

Liang also found it relatively easy to meet the performance requirements of his mathematics course.

> I think [Maths 100] actually is not so hard than to school, because you know international student in China we still learning the maths and in New Zealand you still study that… So I think [Maths 100] is okay for all students.

In expressing this opinion, Liang positioned himself as a capable performer who was able to meet the obligations of this role, suggesting that it was not much harder than school mathematics. He then elaborated on how he evaluated his eligibility for the role, saying "Oh yes I was pass [Maths 100]." This showed that he perceived attaining a passing grade to be an obligation of the role. He also positioned himself relative to his peers when telling a story about how they were unable to do some of the course quizzes, yet "I do know how to do that [quiz] because I was in China for study that. But their whole high school is in New Zealand so they can't understand how do that." In saying this, he positioned himself at a higher level of performance than his peers in this aspect of the course.

Yet Liang also found the communicational obligations of his course to be challenging.

> I think the most challenge is language. Because I mean in New Zealand the first year my English actually not good. Still now, still not good [laughs]. So when I study so many time, like local student they can understand what the teacher talking about. So in mine, I have to more time to understand what the teacher talking about. Maybe I can know the meanings, but I don't know how to speak or how to write.

In expressing an opinion about his language proficiency, Liang positioned himself as less capable than local students in meeting the communicational demands of his course. However, he indicated that he was able to reach an understanding of teacher communications, but this required a time commitment beyond that of his peers.

Agentive actions to overcome barriers to success

Liang described some of his actions to overcome language difficulties while attending school in New Zealand prior to the pandemic, saying, "Like after class I will find some local friends. We will try to speak and to understand that [mathematics learned in class]." During his first year of university mathematics when New Zealand's borders were closed to international students, Liang advised that the "China Learning Centre is better than in home because you have so many friends they can help you." He thus described informal, collaborative meetings with peers as a strategy to overcome language difficulties and meet the obligations of a capable performer.

How individual experiences shaped personal identity in mathematics learning contexts

Liang described positive feelings while learning mathematics in New Zealand saying, "I'm feel very happy." He believed that "the China maths is harder for New Zealand maths," and described his learning in China.

> Actually, it's very boring, specially in the maths because we mostly just try to focus your course. You don't have time to bring, like you can't understand why these function, how to that because they just tell me "you should do that." And you try more times for you can talks that, and remember that and use that.

Through this extract and other utterances, Liang introduced a storyline that portrayed mathematics in China as being taught at a faster pace and in a more procedural manner than in New Zealand. He positioned himself within this storyline, perceiving himself as a student who understood mathematical concepts more clearly than those who had schooled only in China.

> But mostly [Chinese] students do not understand why that. Like one plus one equals two. They just know that's equal two. But I knew that because I'm one year high school experience.

In this quote, Liang attributed his deeper understanding of first-year mathematics to having undertaken his final year of school in New Zealand where teachers have smaller classes and can "help you what do you want to do." Through these reflexive accounts, Liang told how his experiences of school mathematics in both the fast-paced, content-focused environment of China and the more student-focused environment of New Zealand had contributed to his personal identity. He inferred that these gave him both breadth and depth as a mathematics learner.

Liang's prior experiences interacted with the social structures of first-year mathematics in ways that enabled him to portray himself as a capable performer with a conceptual, rather than procedural, understanding of mathematics.

4.2.4 May – A Capable Performer Who Can be Energised by Dynamic Peers

May schooled in Vietnam before attending foundation studies for one year in New Zealand. She was required to complete a first-year mathematics course as a prerequisite for her programme of study at university.

How university structures shaped social identity in first-year mathematics

May's performance also afforded her a social role with which she was satisfied. She shared that she had studied hard to attain her university entrance qualification in Vietnam and still remembered a lot of her school mathematics. She later remarked that "the [mathematics] lecture content compared to what I learned in stats and biology, I feel it's much less." This meant that she did not have to expend too much effort on mathematics in her first year.

> I mean I feel like as long as I come to the lectures frequently because the lecturers include a lot of examples there. So I thought if I come to the lectures frequently and do my assignments I don't have to do extra work outside.

These opinions about mathematical content and descriptions of her actions positioned her as one who found it relatively easy to attain the role of capable performer. She substantiated her claim to this role saying, "I mean I have a fair [mathematics] grade, so it's fine." This shows how, for May, fair grades were one of the eligibility criteria for this role. When asked, she could not recall any challenges experienced in first-year mathematics that prevented her from meeting her performance obligations.

How individual experiences shaped personal identity in mathematics learning contexts

May described her feelings at the start of the year when required to undertake a diagnostic quiz to ascertain readiness for the course she had enrolled in.

> So I was in quite a panic about it and I spent the whole night trying to do a sheet full of differentiation and integration, and when I had done the quiz I realised I was kind of over reacting.

This statement shows how, after her initial panic, May's experience of completing the quiz led to a sense of complacency. This complacency characterised much of her story about first-year mathematics. She spoke of feeling "comfortable with number," being "confident" when

calculating, and learning new concepts with ease. However, when speaking of her interactions with a particular group of students during lectures she described a stronger response to an emotional commentary.

> So there are some students that sit at the front and they very, kind of like enthusiastic about like the lecture. And because I sit near them due to my eyesight, I can only see close, their energy kind of like transferred to me.

May speaks of having been energised by the enthusiasm of other students, rather than by the mathematics itself. She makes it clear that her position at the front of the lecture room was due to weak eyesight rather than free choice. She also claims an energy transfer from others rather than energy generated from within herself. When speaking about tutorials she again referenced an energy transfer from other students, believing that:

> those people who already passionate about the lecture in the group, can kind of like fuel their positive energy to other people and make them want to be include in, and so like participate, add on their opinion in the discussion or like group in the tutorial as well.

These statements reflect a personal identity of one who is not ordinarily inspired by mathematics but can be energised by others more passionate than herself.

May's personal experiences interacted with the social structures of first-year mathematics in ways that enabled her to portray herself as a capable performer who could be energised by others.

4.2.5 Learning from Stories of Success

The novel way of conceptualising mathematical identity proved useful for illuminating the effect of past experiences on more recent actions in a different social context. Exploring the identity stories of Sunny, Mingyu, Liang, and May showed the theoretical framework also to be useful in situations where participants embraced their social roles. It showed how both social roles afforded by university structures and personal experiences of learning mathematics shaped the mathematical identities constructed by these participants. In one way or another, they all portrayed themselves as capable performers, describing how they met obligations of this role that they perceived to be important. Yet they also offered many more insights into how their personal identities, partly formed by experiences of learning mathematics in other countries, interacted with the social structures of first-year mathematics. This highlighted some tensions in their experiences and the agency exercised to overcome

obstacles in the path of a valued mathematical identity. In table 4.2 I summarise the identities constructed by these four students.

Table 4.2

Mathematical identities based on the role of capable performer

	Sunny	Mingyu	Liang	May
Social Identity	Capable performer who completes tasks independently in a quick and efficient manner.	Capable performer who can do all that is required.	Capable performer who attained a passing grade.	Capable performer who has a fair grade.
Tensions	Understanding spoken lecture content and explaining mathematical concepts in English.	Understanding the lecturer.	Understanding spoken lecture content and explaining mathematical concepts in English.	None acknowledged.
Personal Identity	One who is accustomed to challenge and embraces difficulties.	One who is passionate about mathematics in New Zealand.	One who works with understanding.	One who is energised by dynamic peers.
Agentive actions to mitigate tensions	"See" lecture content using notes and YouTube videos, together with recordings.	Email the lecturer after class and look up unfamiliar vocabulary.	Spend time reviewing course notes and speak with local peers after class.	N/A

	Find Chinese speaking peers in the assistance room.

There were three threads that spanned the identity constructions of these participants, namely perceptions of success, language concerns, and teachers and peers as mathematical resources. These threads are elaborated in the paragraphs that follow. They offered potential areas on which to focus the second phase of the research.

Perceptions of success

Sunny, Mingyu, Liang, and May all positioned themselves as successfully attaining a social role that they cared for. Mingyu's "I can do everything," May's "I get a fair grade," and so on, show how they perceived themselves as successfully fulfilling their duties as capable performers. All suggested that school mathematics in their home countries (China and Vietnam) had prepared them to the extent that the cognitive challenge of university mathematics was not significant. Liang and May indicated that there was barely a gap between school mathematics and their first semester at university, and that the university course provided a useful review of work already learned. Sunny, the only one of the four taking mathematics as a major subject (Maths 101), found some aspects of her first mathematics course challenging, but her most frequently adopted position was one for whom the mathematics itself was not too hard. Repeated positioning of this nature constructed capable performance-oriented identities for these participants.

Language concerns

With the exception of May, all participants spoke explicitly about the challenges of learning mathematics in English. Both Liang and Sunny spoke of their struggle to understand lecture content and to express their mathematical knowledge in English. These language issues threatened their identities of capable mathematical performance. Yet Liang and Sunny both spoke of time-consuming actions undertaken to reach understanding of English-mediated lecture content. Mingyu also related how she communicated with lecturers by email after class to clarify her understanding. The descriptions of actions undertaken by Sunny, Mingyu, and Liang to overcome language challenges contributed to their identities as capable performers.

Teachers and peers as mathematical resources

Although there were few commonalities in the accounts of Mingyu, Liang, May, and Sunny when speaking of their interactions with others in the first-year mathematics community, it is evident that these interactions formed important resources for all of them. Liang referred to the support he received from both English-speaking peers, as well as peers from the China Learning Centre that he attended. Sunny sought to collaborate with Chinese speaking peers in the assistance room, while May was drawn into dynamic mathematical discussions with local peers during lectures and tutorials. The purpose of these interactions was to discuss mathematical concepts (in either English or Chinese) and to clarify understanding. Only Mingyu and Sunny spoke explicitly about their interactions with teachers. Mingyu relied on a Chinese speaking teacher to explain concepts that she did not fully understand. Sunny, on the other hand, found it difficult to seek help from her lecturer despite their willingness to help her. Descriptions of actions to clarify mathematical concepts by interacting with peers or lecturers further contributed to the mathematical identities constructed by Mingyu, Liang, May, and Sunny.

Sources of tension

Despite portraying mathematical identities of performative success, the storied identities of the three participants from China (Mingyu, Liang, and Sunny) highlighted two potential areas of tension as personal identities met the social structures of first-year mathematics in New Zealand. For international students from China, it is possible that:

1. Accessing lecture content and expressing mathematical knowledge in English might threaten identities of success.
2. Cultural understanding of student-teacher relationships may preclude international students from accessing help offered by teachers.

These insights informed the second phase of my research study. To explore both areas flagged as potential sites of tension, I made the methodological move, in phase 2, to focus on participant interactions with peers, and participant interactions with tutors, during collaborative tutorials.

4.2.6 Contribution to knowledge

Section 4.2 has explored the storied mathematical identities of international students who were afforded roles that they valued by the structures of first-year mathematics. It addressed

the first and second research questions, showing how both university structures and prior experiences shaped the storied identities of Sunny, Mingyu, Liang, and May.

The main contributions that this section offers to research are:

- Although international students might recognise social roles related to performance in first-year mathematics, they construct their identities as capable performers by cleaving to different aspects of performance. (RQ1)
- International students personalise their mathematical identities by offering reflexive accounts of themselves as mathematics learners. These accounts include comparative judgements between learning mathematics in host and home countries, and expressed beliefs or feelings related to learning mathematics. (RQ2)
- International students from China might find their social identities threatened by communicational difficulties that restrict their ability to verbally evidence knowledge, restrict their access to spoken or wordy course resources, and require significant time and effort to overcome. (RQ1)
- International students might experience difficulties when accessing help from lecturers. (RQ1)

PHASE 2 (IDENTITIES IN ACTION)

Chapter 5: Interactional Identities in First-Year Mathematics

The second phase of this research study explored the mathematical identity constructions of three international students from China (Peggy, Yifei, and Zixin) as they interacted with peers and the tutor in collaborative tutorials. While the previous phase addressed only two of the three research questions, it also guided the design and focus of this next phase of the study. This phase now addresses all three research questions. It shows not only how university structures and prior experiences shape mathematical identities, but also how mathematical identities might be transformed in STT through continued interaction between university structures and prior experiences. The chapter incorporates extracts from three research outputs. Each published output contributes to my research goals by illuminating tensions between social roles in the new social context of first-year mathematics and participants' reflexive understandings of themselves as mathematics learners. The full paper from which each extract is taken is referenced in a footnote to the relevant section.

Local discourses determine structures that exist in any social setting, such as a mathematics classroom. These discourses offer benchmarks against which social worth can be assessed (Holland et al., 1998) and shape the identities of individuals in that landscape. There are many studies that evidence how the instructional practices in school classrooms or other collaborative educational settings offer a collective view of what is valued, thus shaping students' mathematical identities in these spaces (Cobb et al., 2009; Heyd-Metzuyanim & Graven, 2016; Voigt et al., 2021). To be recognised as an effective mathematics learner, students must identify with an accepted set of norms or obligations in their classroom (Cobb et al., 2009). Students' actions in the classroom are thus shaped by their interpretation of these obligations. For international students, identifying with the norms of their new educational setting may require a significant cultural shift.

Although mathematics is commonly viewed as being language- and culture-neutral, this is not the case (Wagner, 2021). Research shows that foreign students learning mathematics in a non-native language are disadvantaged (Barton et al., 2005). One might expect this disadvantage to reduce over time. Yet Cummins (1981) claimed that it could take between five and seven years for proficiency in a non-native language to develop to a point where higher order thinking skills are supported. Prior to this, cognitive demands of the language present barriers to synthesis and analysis of ideas at higher interpretive levels.

International students must often adapt to additional cultural differences. In Chinese culture, active listening is highly valued, and children are taught that opinions should not be expressed unless the speaker has earned the right through authority, experience, or expertise to do so (Rublik, 2018). These values carry through to school classrooms where the teacher is typically viewed as authority and expert (Leung, 2001; Zhou et al., 2022). Values like this contrast with the student-centred approaches to learning that are valued in some other countries (Kaiser & Blömeke, 2013). Collaborative groupwork and mathematical communication are encouraged in first-year tutorials in New Zealand universities (Oates et al., 2005). Such approaches might create tensions for students unfamiliar with these practices.

During the summer semester in which I collected my data, students were expected to attend hour long, bi-weekly, tutorials where they worked on pre-assigned problems relating to recent lecture content. In these tutorials, students were encouraged to access course texts or other resources, and to discuss problems freely with one another, and the tutor, during tutorial sessions. Tutorials were run by senior mathematics students in the department who circulated through the tutorial room supporting individuals and groups as they worked. The context of these tutorials fosters particular discourses concerning student-tutor relationships through which both tutors and students make sense of their roles in the tutorial classroom (Kontorovich & Ovadiya, 2023). A culture of collaborative problem solving and mathematical communication between students is encouraged in these tutorials (Oates et al., 2005).

For some international students, practices of this nature may feel unfamiliar or even uncomfortable. In addition to being culturally foreign, students from other language groups might also find collaborative work daunting when conducted in a language in which they are not fluent. All of this suggests that international students in these tutorials may encounter a wide range of new pedagogical practices to which they must adapt. The norms and structures of their new social setting will shape their mathematical identities.

The first research output (section 5.1) shows how Peggy acted agentively to transform her mathematical identity when interacting with the tutor. The second (section 5.2.1) also focuses on Peggy, but this time explores her interactions with peers. It shows how cultural understandings and language difficulties restricted her access to learning resources in collaborative tutorials. The final research output in this chapter (section 5.2.2) considers how Yifei and Zixin, who had both attended schools in New Zealand for a period prior to

transitioning to university mathematics, constructed mathematical identities when interacting with peers.

Together these research outputs provide snapshots of mathematical identities constructed by international students who have been immersed in the education system of their host country for different lengths of time. The identities constructed show how international students might reconcile some of the tensions created for them by university structures. They also suggest how mathematical identity constructions might change over time as students become more familiar with local norms. The three outputs from this chapter are followed by a summary (section 5.3) in which I contrast the mathematical identities constructed by the three participants in phase 2.

5.1 Constructing Mathematical Identities in Student-Tutor Interactions

In this section I explore the experiences of Peggy, an international student from China, as she engaged with the tutor, Leigh, during collaborative tutorials in first-year mathematics. The research output presented in section 5.1.1 is followed, in section 5.1.2, by a demonstration of how it addresses the first and third research questions. The study of Peggy's interactions with the tutor show both how her mathematical identity was shaped by university structures, and how her mathematical identity was transformed by the continued interaction of university structures and prior experiences.

5.1.1[5] Peggy – Changes in Positioning During Student-Tutor Interactions

This section describing Peggy's construction of mathematical identity is extracted from Locke et al. (2023a). At the time of her participation in the study, Peggy, who had schooled in mainland China, had studied in New Zealand for two and a half years. She had completed a foundation course and her first year of a BSc programme before enrolling in Maths 100. Peggy was recruited to the research study at the start of her second semester in mathematics.

I observed Peggy's participation in two collaborative tutorials. The observations took place in the third and seventh (out of ten) tutorials of the course. For this research output I isolated all episodes in which Peggy interacted with her tutor, Leigh, and analysed her positioning acts in these episodes. Copies of the tutorial worksheet and Peggy's worked solutions from both the

[5] Section 5.1.1 has been extracted from a journal article. The Version of Record is available as:

Locke, K., Kontorovich, I., & Darragh, L. (2023). Transforming mathematical identity: Changes in one student's positioning during first-year mathematics tutorials. *International Journal of Mathematical Education in Science and Technology, 54*(9), 1785-1803. https://doi.org/10.1080/0020739X.2023.2259917

observed tutorials were also added to the dataset and I examined Peggy's written solutions from both tutorials to understand how well these resembled an expected solution for each problem.

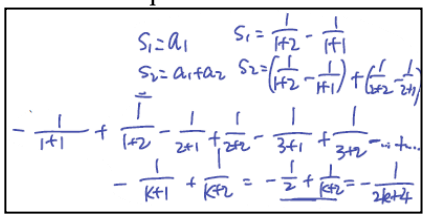
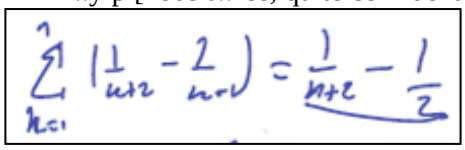
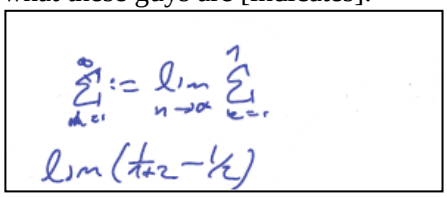
Findings

In the paragraphs that follow I explore Peggy's interactions with Leigh, noting how they changed over the course of the two observations. When considering interactions between Peggy and Leigh, I refer to them as *productive* if they enabled Peggy to progress her work towards an expected solution at the close of the interaction, and *unproductive* if they did not. I also show how Peggy's interactions with Leigh became more productive as her positioning evolved.

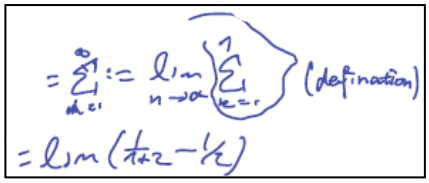
During the two observations, I noted seven instances when Leigh interacted directly with Peggy concerning tutorial questions. Three of these interactions lasted less than a minute each, while the other four interactions were more protracted spanning around twelve minutes in total. I focus on the four protracted episodes in the paragraphs that follow. By examining Peggy's positioning of herself and Leigh in these four episodes I show how she increasingly assimilated the rights embraced by her local peers into her own storyline about student-tutor relationships. I also scrutinize Peggy's written mathematical output from these interactions noting evidence of progress after the interaction with Leigh.

Episode 1: Peggy affirms understanding of tutor's explanation

The first protracted mathematical interaction observed between Peggy and Leigh occurred almost half an hour into her third tutorial. Just prior to this episode, Peggy had watched as Leigh explained to another student, while writing on a nearby whiteboard, how to find the sum of the series $\sum_{k=1}^{\infty}\left(\frac{1}{k+2}-\frac{1}{k+1}\right)$ by examining its partial sums. Having guided the student towards a partial sum, Leigh concluded his explanation with, "So, to get the sum, you just have to take the limit." Peggy continued to work independently on this question, simplifying her partial sum of 'k' terms a step beyond Leigh's answer, but not going as far as to find the infinite sum. When Leigh arrived at her side some moments later, she slid her working across to show him and the following conversation ensued:

1	Peggy	Um, is it negative two ‘k’ plus four .. for the sum of the series [indicates the worded question on the tutorial sheet with her pencil as she says it]?
	Peggy’s working	
2	Leigh	Hmm.. not quite. So, yeah, so that’s correct [indicating with thumb]. That is the partial sum. Then, remember [takes pencil and slides tablet towards himself to write], so I’ll not use the ‘k’. Instead I’ll write it like this, so [starts to write] the sum from ‘k’ equals one up to ‘n’ of one over ‘k’ plus two minus one over ‘k’ plus one [writes $\sum_{k=1}^{n}\left(\frac{1}{k+2}-\frac{1}{k+1}\right)$].
3	Peggy	[makes short verbal sounds of affirmation, nodding her head and watching Leigh’s working throughout]
4	Leigh	You’ve correctly pointed out that this is one over ‘n’ plus two minus one half. You got that there?
5	Peggy	Mmm..yip [nods twice, quite confidently, while peering at working]
	Leigh’s working	
6	Leigh	Okay. Then, by definition, this sum from ‘k’ equals one up to infinity is sort of by definition just the limit as ‘n’ goes to infinity of these partial sums as ‘k’ goes from one up to ‘n’. So to calculate that, all you have to do is take the limit of one over ‘n’ plus two minus one half. ‘Cos those are what these guys are [indicates].
	Leigh’s working continues	
7	Peggy	Aah [she has been nodding her head throughout Leigh’s explanation]
8	Leigh	Those, so when you get the full infinity sum one you just need to take the limit.
9	Peggy	[Sits back and looks up] Okay
10	Leigh	Is that clear?
11	Peggy	Okay, so I just need to uh .. write the limit of the ..
12	Leigh	So a series, by definition, is the limit of partial sums.
13	Peggy	Oh
14	Leigh	And these here up to ‘n’ are the partial sums [points to working]?
15	Peggy	Yeah
16	Leigh	So the whole series, which is to infinity, is the limit of those partial sums.
17	Peggy	Yeah
18	Leigh	Now so you have worked out what those partial sums are, one over ‘n’ plus two minus a half?
19	Peggy	Yep
20	Leigh	So then well the series is just the limit of that.
21	Peggy	Yeah okay I got it, I got it. [She takes back her tablet as Leigh leaves and hovers her pencil over the working before inserting two equals signs. After ten seconds of inactivity she circles the partial sum which Leigh had indicated and writes ‘(definition)’, but does not compute the required limit.

Peggy's additions to Leigh's working

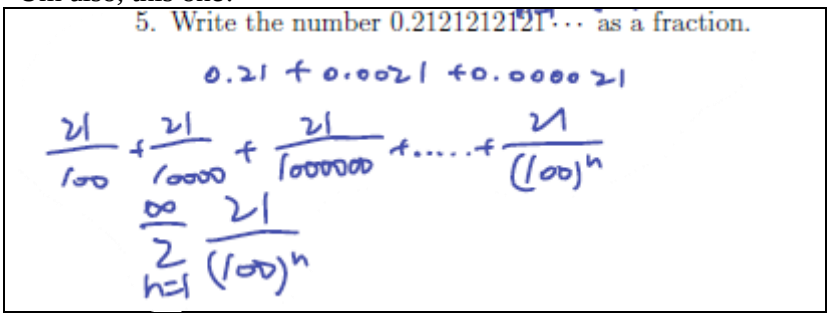


Through her short verbal affirmations (e.g., 'Yeah', 'Aah'), Peggy positioned herself as a student who found Leigh's explanation helpful and, by implication, she positioned Leigh as a helping tutor. Although Leigh attempted to confirm the effectiveness of his explanation by posing a question (line 10) and re-explaining (lines 12-20), he later accepted Peggy's position as having received the help that she needed (line 21). In this episode, Peggy's construction of student incorporated a very limited repertoire of rights. Having initially checked an answer with Leigh, Peggy affirmed her understanding of his explanation throughout the rest of the episode.

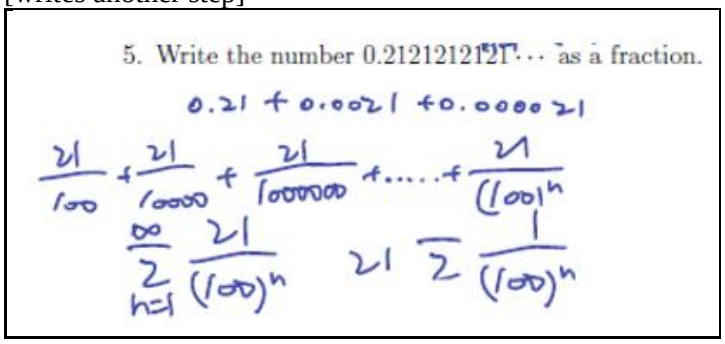
By considering Peggy's written output after Leigh had stepped away, I noted that Peggy had not advanced the question beyond where Leigh had left it. Her unspoken actions, such as the long pause, hovering pencil and insertion of a few symbols, together with the lack of final answer, suggested that the exchange may not have enabled Peggy to progress autonomously and was therefore unproductive. However other interpretations (such as Peggy, having gained clarity, felt no need to write a final solution) are also possible.

Episode 2: Peggy expresses uncertainty to elicit further help

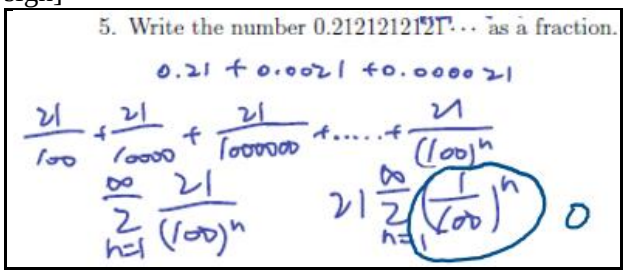
Some five minutes after the interaction presented in Episode 1, Leigh returned to Peggy's side where she checked her solution to another question that asked students to "Write the number 0.2121212121 ... as a fraction."

1	Peggy	Um also, this one?
	Peggy's working	
2	Leigh	Yes [enthusiastically] just calculate now.
3	Peggy	Oh, okay .. This is the final answer?
4	Leigh	No, not quite because remember you want it as a fraction so you want to actually calculate what this infinite sum is.
5	Peggy	Ohh..[moves pen hesitantly but doesn't write] hmm … [pen to mouth]

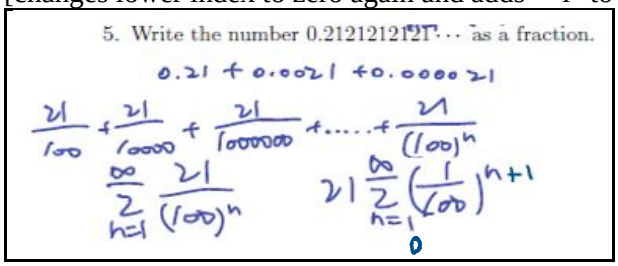
6	Leigh	So my first step would be to pull out that constant twenty one [points]
7	Peggy	[writes another step]



8	Leigh	Yep. And then that's when n equals one to infinity. You would want that sum, to use the geometric series, to be from 'n' equals zero.
9	Peggy	/zero [simultaneously with Leigh's last word]. So that's zero. Noo.. [hand to mouth, thinks then circles formula following summation sign]



10	Leigh	No, we want the initial 'n' to be zero not one [points]
11	Peggy	Oh okay [erases circle and changes lower index from one to zero]
12	Leigh	No no, you had it right the first time. It's supposed to be one, but we need to make it to be starting from zero.
13	Peggy	[Has erased former change while tutor was speaking] Ohh.. [changes lower index to zero again and adds '+1' to power]



14	Leigh	Not quite. I'll go over here [moves to whiteboard]. You have twenty one sum from 'n' equals one to infinity of one over a hundred to the power of 'n'. [writes on whiteboard $21\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^{n}$]
15	Peggy	Yip
16	Leigh	So this is the same as going twenty one the sum from 'n' equals one to infinity of one over one hundred times one over one hundred to the 'n' minus one, right? [writes $21\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)\left(\frac{1}{100}\right)^{n-1}$]
17	Peggy	Yip

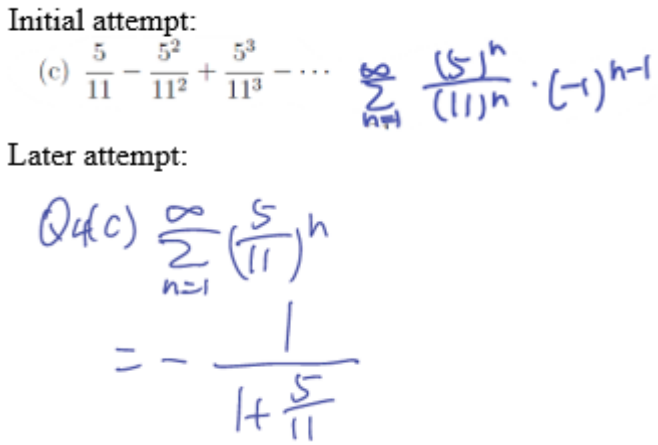
18	Leigh	So twenty one over one hundred times sum from ‘n’ equals one to infinity one over one hundred to the power of ‘n’ minus one. [writes $\frac{21}{100}\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^{n-1}$]
19	Peggy	Yip
20	Leigh	But when ‘n’ equals one this is zero [points].
21	Peggy	Yip
22	Leigh	Right? So this is the exact same thing as going twenty one over one hundred sum from ‘n’ equals zero to infinity one over one hundred to the ‘n’ [writes $\frac{21}{100}\sum_{n=0}^{\infty}\left(\frac{1}{100}\right)^{n}$]
23	Peggy	/’n’ minus one [pre-empts Leigh’s last word incorrectly]. Oh ‘n’ yeah.
24	Leigh	Right? ‘Cos I’m starting from zero now.
26	Peggy	Oh yeah okay.
27	Leigh	So then, on this one you can use your geometric series formula so it’s twenty one over one hundred times one over one minus one over one hundred .. right?
28	Peggy	[no audible response for 3 seconds]
29	Leigh	This is just a geometric series.
30	Peggy	[changes body posture slightly to left to see board better] Um..
31	Leigh	On that board [points to another whiteboard where formula for geometric series has been written] remember ‘r’ in this situation is one over one hundred … see?
32	Peggy	[After pause as she turns to look] Oh yeah
33	Leigh	Yeah, good, cool?
34	Peggy	Yip

In this interaction we see how Peggy’s position wavered between understanding and uncertainty while Leigh explained verbally in the initial part of the interaction (lines 1 to 13). Peggy, at various times, affirmed her understanding, pre-empted Leigh’s words and attempted to perform his verbal instructions. At other times she hesitated, expressed confusion, and erased what she had written. Leigh, noting her difficulties, moved to the board where he could write as he spoke. From lines 15 through 23, Peggy affirmed her understanding, but made an incorrect attempt to pre-empt his words (line 23). This led Leigh to position her as still in need of help, and he offered further clarification for why the exponent should be n rather than $n-1$ (line 24). Leigh’s response to Peggy’s errors invited her into a storyline in which expressions of confusion or uncertainty were an accepted means by which students could negotiate further help from tutors. Peggy had not, up to this point, embraced a right to request further clarification. However, in this instant, and for the first time, Peggy openly communicated her uncertainty through a long and deliberate pause (line 28). In doing so she accepted Leigh’s positioning. After further input from Leigh, Peggy once again declared herself as not needing further help, a position which Leigh accepted this time.

In this interaction we see how Peggy's construction of student in her mathematics tutorial changed by legitimizing the expression of uncertainty in student-tutor relationships. This opened a feedback loop that informed Leigh's further actions. Once Leigh had stepped away Peggy continued to work on her solution and her final script shows her use of the geometric series formula but does not offer a fully evaluated answer. In this instance Peggy's written output suggests that Leigh's explanation enabled her to progress autonomously towards a solution of the problem. However, towards the end of the tutorial, Peggy returned to a question previously "completed" and attempted to work this further. To do so required that she use the same strategy demonstrated by Leigh in lines 14 to 22 and manipulate the expression into a format commensurable with the use of the geometric series formula. Her output, shown in Figure 5.1, indicated recognition of the similarity of the situation but failed to consider the lower limit for the sum. I attribute Peggy's decision to progress her working on this question to her earlier interaction with Leigh but note also that it did not lead to an expected solution.

Figure 5.1

Peggy's initial and later attempts to complete question 4(c)

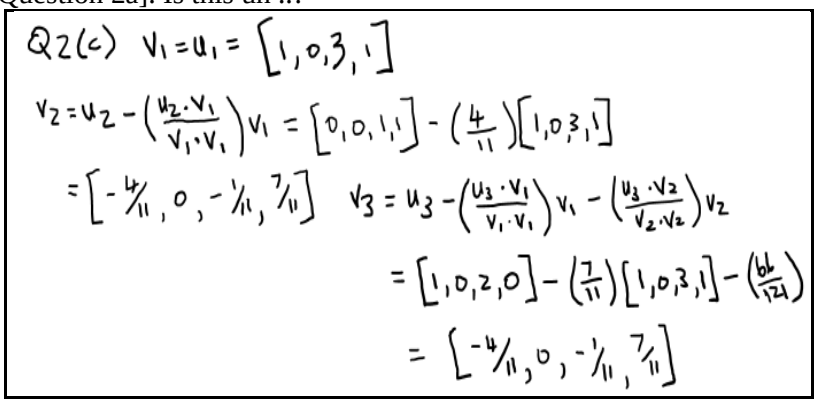


Episode 3: Peggy redirects tutor's explanation

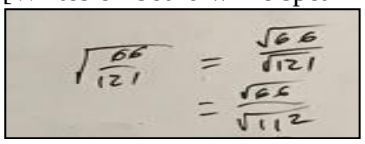
The next observation of Peggy took place during her seventh tutorial. The second question on the tutorial worksheet comprised two parts. The first part asked students to use the Gram-Schmidt method to find an orthogonal basis for the column space of a given matrix, while the second asked for an orthonormal basis for the same matrix. Many students found this

question challenging, and Leigh offered advice to groups of students using nearby whiteboards to aid his verbal explanations. After overhearing Leigh's advice to another student and modifying one of her vectors, Peggy confirmed her process with Leigh and thereafter paid little heed to the surrounding discussions. She referred instead to her coursebook and other online resources for assistance. Some twenty minutes later when Leigh arrived at her side, she queried her result with him.

1 Peggy Hi. For the .. the result is so complex [points to her last worked step on Question 2a]. Is this uh ..?



2 Leigh You have to plug that in the length, right? So remember [moves to board] you are taking the square root of sixty-six over one hundred and twenty-one, which is square root of sixty-six over square root of one hundred and twenty-one? Which is square root sixty-six over square root of eleven squared.
[Writes on board while speaking]



3 Peggy Yeah but, uh ..

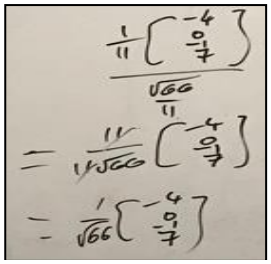
4 Leigh But Peggy, if I have square root 'a' squared, what does this equal [writes $\sqrt{a^2}$ on board]?

5 Peggy Ah, it's 'a', but ..

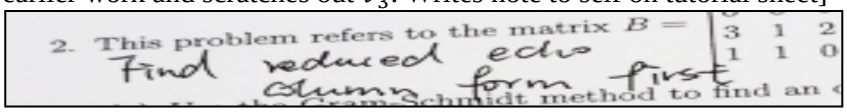
6 Leigh /Yeah, so this is root sixty-six over eleven [interrupts]

7 Peggy But it's uh..? [pointing to unrooted $\frac{66}{121}$ in her own working]. But it's not squared.

8 Leigh No no, root sixty-six is not squared. So just leave it like that. So now you've got one over eleven times minus four zero minus one seven, right? And you're dividing that by square root of sixty-six over eleven. And this equals eleven times eleven over root sixty-six times minus four zero minus one seven. Cancel it out. Over root six six. … That's all it is. That's all the answer is.



9	Peggy	But why I got different one here [laughs]?
10	Leigh	You haven't square rooted it. Remember for length you need to take the square root.
11	Peggy	Oh and then square it?
12	Leigh	[Leans over to her tablet and scrolls through coursebook] So you went past it. Oh, here you see, the length thing you take the square root and the length.
13	Peggy	Oh, but it's question two.
14	Leigh	Yeah yeah, so you finding the length of that vector right? But the length, you need to remember to take the square root for the length.
15	Peggy	Ah? .. But this is ..
16	Leigh	You found the basis in '2a', right?
17	Peggy	Yeah, '2a'. Ah no, I will calculate '2a'.
18	Leigh	I'm confused on what your question is Peggy?
19	Peggy	I will calculate the '2a', but I find the result is complex too.
20	Leigh	No, no that's correct.
21	Peggy	Oh, that's correct?
22	Leigh	Yes, that's the correct answer.
23	Peggy	Yeah, but for the 'v_3' ..
24	Leigh	'v_3'.. no no okay, so you need to first calculate a basis for Col A. So Gram Schmidt is just a method where you have a bunch of vectors. And it will turn them orthogonal. So you first have to find what the basis is.
25	Peggy	[Shakes head] Yeah but 'v_3' is a complex to ..
26	Leigh	Yeah, but you shouldn't need to use 'v_3'because 'v_3' won't be in the basis.
27	Peggy	Ohhh
28	Leigh	That's what I meant. So the first thing is actually to find the basis. That's the very first thing you do. Before you do any Gram Schmidt.
29	Peggy	Oh okay. Okay, thank you. [Returns to own work and row reduces the matrix. Arrives at two linearly independent vectors, then returns to earlier work and scratches out v_3. Writes note to self on tutorial sheet]



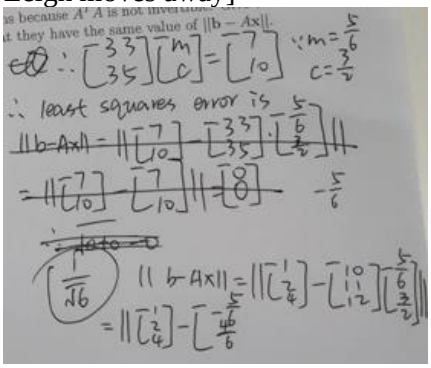
In this episode we see a significant change in Peggy's positioning of herself compared to Episodes 1 and 2. She opened the interaction by voicing doubt about her working, rather than merely presenting answers for confirmation. Her regularly uttered "but" expressed objection to the help dispensed by Leigh, and she practically challenged Leigh to find her error (line 9). After several tutorial classes, we see how Peggy's tutorial storyline has developed to incorporate a wider repertoire of rights. It offered her the right, as a student, to respectfully redirect the focus of a tutor's advice. In this interaction, Peggy's persistent objections

eventually turned the exchange towards a conclusion in which Peggy positioned herself as no longer needing help. Peggy's subsequent progression of the question to an expected written solution, as well as her "note to self" written alongside it, suggests that the interaction with Leigh in this episode was productive.

Episode 4: Peggy initiates request for help (and Leigh checks own understanding)

The final interaction observed between Peggy and Leigh took place as the seventh tutorial neared its end. It concerned the fourth question on the tutorial sheet which required students to calculate the least squares solution for an inconsistent system, and to give the least squares error for their system. Peggy looked up from her work towards where Leigh was working with another student and raised her hand high and deliberately to summon him as he finished. Leigh crossed the room in response to Peggy's signal.

1	Leigh	Yip?
2	Peggy	Hi. Is this the square error for b minus .. [briefly points to her answer for Q4]
3	Leigh	No, it's definitely not zero.
4	Peggy	Oh [scratches head, tucks hair behind ears].
5	Leigh	Definitely not zero [pensively].
6	Peggy	Um
7	Leigh	For which one is it?
8	Peggy	For question four.
9	Leigh	Question four. I believe the answer there should be, um, one over square root six
10	Peggy	[Writing] One over square root six?
11	Leigh	Yip. So .. those are correct [points to top line] which means just I think that the error is somewhere here [indicates].
12	Peggy	Ohh..
13	Leigh	[Still examining working] Oh no no, so you have basically five over six and then three over two [points to m and c from line above] right? It should be 'A' applied to those values back here. Not to this three three three five one [points to matrix in line above] it should be to the original matrix 'A' [points to matrix A in the tutorial question]
14	Peggy	Oh okay.
15	Leigh	And then your 'b' is not this one, it's that 'b' [points to vector b in the question]
16	Peggy	Oh yeah yeah. Okay, thank you. [Crosses out working and corrects as Leigh moves away]



In this episode we see Peggy initiating an interaction with Leigh for the first time. Her construction of student incorporated an active request for help, rather than waiting for help to be offered. Leigh's actions this time also incorporated a check to ensure that his advice was directed towards Peggy's difficulty (line 7). After a short exchange, Peggy positioned herself as no longer in need of help and Leigh, accepting her position, moved to another student. Peggy's written solution showed that she had corrected both the matrix and the vector to be used in her calculation, although this was followed by an incomplete effort to arrive at the answer offered by Leigh. As the tutorial was ending and some of the other students were beginning to pack up, I interpret Peggy's incomplete working in this instance as being due to haste rather than confusion. I justify this by noting that the step between her final full line of working and the answer offered by Leigh was merely a matter of arithmetic manipulation. I interpret Peggy's actions in this instance as having been enabled, through a productive interaction with Leigh, to progress toward an expected solution.

Bringing insight from beyond the discourse

In the tradition of the critical realist approach (Archer et al., 2016; Marks & O'Mahoney, 2014; Park & Peter, 2022), I sought to identify social structures and conditions that might shape observed phenomena (in this case, the developments in Peggy's interactions with Leigh). The retroductive inference process required that I bring insights from beyond the discourse between Peggy and Leigh into consideration as I sought to construct a 'best fit' account of what was observed. To shed further light on the findings, I returned to the dataset to examine more closely how Peggy's peers, who were local to New Zealand, interacted with Leigh. I considered the positioning of Nina, Jared, Luke, and Jing, the four students who shared a table with Peggy in either of the tutorials. I grouped their utterances, as they interacted with Leigh, according to what I perceived to be the intention of the utterance.

Coding the utterances of Peggy's four local peers as they interacted with Leigh led to ten rights. These are presented in Table 5.1.

Table 5.1

Rights embraced by local peers in student-tutor interactions

Right	Intention	Example
Initiate a request for tutor's help	Students asserted this right when they needed help with solving the assigned problems and the tutor was not at their table to give assistance.	[Turning to tutor who was standing a short distance away] "Just on question two 'b' I was thinking …"
Check an answer with tutor	This right was claimed by students who wished to confirm with the tutor whether an answer they had obtained was correct.	"This one [referring to Q1d], it doesn't converge?"
Affirm understanding of tutor's explanation	Students used this right to signal comprehension when the tutor explained a concept to them.	"Yeah I got that."
Express uncertainty	This right was asserted when students did not understand the tutor's explanation. It signaled a need for further help.	"Hmm, I don't remember that formula."
Redirect focus of tutor's explanation	Students claimed this right when they perceived that the tutor's explanation was not directed towards the issue with which they needed help. Its purpose was to refocus the explanation on their area of need.	"Yeah, right. I'm just doing the longer way of working out" [points to coursebook].
Reject help from tutor	This right was used in response to the tutor's offer of help if students felt confident that they could progress without help.	"No I think I'm alright."
Ask tutor to identify error in their working	Students claimed this right when they knew that their solution was incorrect, but needed help to identify where things had gone wrong.	"Maybe I wrote it all wrong. What have I done to this?"
Challenge tutor's explanation	Students used this right when they believed the tutor may have made an error in their explanation.	"Alright sorry, you said one over 'n' plus two and I was like 'what's happened to the half?' That's what I was asking."
Suggest next step	This right was used by students to signal to the tutor how they planned to proceed and confirm whether this approach was appropriate.	"So that's my basis for 'A,' the orthogonal one, and then I'm going to find the length."

Engage in non-mathematical talk with tutor	Non-mathematical communication with the tutor was used by students for the purpose of social bonding.	"How did you remember my name?"

I observed these ten rights being regularly claimed by local students over both observations. They illustrate some of the permissible social acts available to students during the observed tutorials.

Returning to the analysis

The final stage of my analysis was to reconsider the findings in the light of this knowledge. This process enabled me to identify changes in Peggy's positioning of herself when interacting with Leigh, and to consider mechanisms that may underlie these changes. By examining Peggy's four interactional episodes with Leigh, I identify which of the rights embraced by local students Peggy claimed in each episode. These are shown in Table 5.2.

Table 5.2

Rights claimed by Peggy in each episode

Right	Episode 1	Episode 2	Episode 3	Episode 4
Initiate a request for tutor's help	-	-	-	Yes
Check an answer with tutor	Yes	Yes	-	Yes
Affirm understanding of tutor's explanation	Yes	Yes	Yes	Yes
Express uncertainty	-	Yes	Yes	Yes
Redirect focus of tutor's explanation	-	-	Yes	-
Reject help from tutor	-	-	-	-
Ask tutor to identify error in their working	-	-	Yes	-
Challenge tutor's explanation	-	-	-	-
Suggest next step	-	-	-	-
Engage in non-mathematical talk with tutor	-	-	-	-

The table shows that the way Peggy positioned herself included a growing number of rights over time.

Discussion

The findings focus on four snapshots that illuminate how Peggy positioned herself in collaborative first-year mathematics tutorials. Her later constructions of the mathematics student identity incorporated more of the rights embraced by her local peers in their interactions with the tutor. It also appeared that Peggy's interactions with Leigh became more productive in later episodes. I propose that Peggy's positioning of herself towards the end of the semester facilitated a process through which Leigh was able to support her mathematical learning more effectively than in earlier episodes.

At some universities in New Zealand, collaborative tutorials for first-year mathematics students might be led by experienced students in the mathematics department (Kontorovich & Ovadiya, 2023; Oates et al., 2005). This was the case at the university where the study took place. My observation of Peggy's peers revealed aspects of the social structure in a tutorial of this nature, and I noted some of the rights attributed to students in this setting. To be recognised as an effective student in a tutorial would require alignment with the local storyline and the social roles it affords (Cobb et al., 2009; Holland et al., 1998).

Yet personal identity determines what is prioritized and is shaped by reflection on prior experiences and tacit understandings (Marks & O'Mahoney, 2014). What Peggy valued as a mathematics student was formed, in part, by her experiences of learning mathematics in China where teachers are typically viewed as authority and expert (Leung, 2001; Zhou et al., 2022). This is consistent with what I observed at the beginning of the semester. In the first episode Peggy chose to uphold Leigh as an expert tutor, affirming her understanding (and thus his skill as a tutor) throughout his explanation. Although expressions of uncertainty were commonly used by local students when interacting with the tutor, Peggy initially declined to exercise this student right. Viewed in terms of Archer's (2002) theory, Peggy's personal identity engaged with the active social structure and she chose to decline a right afforded her. This gave rise to an internal commentary regarding her well-being (natural level), competence (practical level), and self-worth (social level). Her efforts to balance these concerns in her ongoing interactions with Leigh resulted in further choices that personified her student role. I suggest that the changes observed in Peggy's positioning in later episodes evidence the dialectical relationship between personal and social identities as theorised by

Archer. I believe that it was Peggy's sense of having received (or not) the help she required that may have spurred her to adopt a wider range of student rights.

Yet I note that Leigh's positioning of Peggy may also have played a part in facilitating the changes in her student identity. Leigh's gentle, but persistent, efforts to help Peggy despite her mis-attempts to show understanding (in the second episode, for example) validated uncertainty as acceptable within the student-tutor storyline to which he subscribed. In Episode 4, his check that he had understood her issue correctly validated her earlier attempts to redirect the focus of his explanation. I believe that these negotiations were also instrumental in facilitating changes in how Peggy positioned herself.

5.1.2 Contribution to knowledge

This section explored changes in an international student's acts of positioning in time-sequenced interactions with a tutor. It addressed the first and third research questions by showing how university structures shaped Peggy's mathematical identity when interacting with Leigh, and how the continued interaction of university structures and prior experiences transformed Peggy's mathematical identity.

The contributions that this section makes to knowledge are:

- International students may struggle to access help from tutors, which can threaten competence and thus mathematical identities. (RQ1)
- The mathematical identities of international students might gradually shift towards local norms during ongoing, regular interactions with tutors. (RQ3)

5.2 Constructing Mathematical Identities in Peer Interactions

In this section I present two research outputs that show how Peggy (in section 5.2.1) and Yifei and Zixin (in section 5.2.2) construct mathematical identities in peer interactions. In section 5.2.3 I show how these research outputs contribute to knowledge and address the three research questions.

5.2.1 Peggy[6] – Barred From Collaborative Interactions

The focus of this research output is on Peggy's positioning relative to her peers in the first tutorial observation, and uses interview data to understand storylines. In examining her

[6] Section 5.2.1 has been extracted from a conference paper. The Version of Record is available as:

interview data, I looked for instances when Peggy spoke about the setting of a mathematics tutorial. I noted two storylines constructed by Peggy in interviews through which I interpreted her positioning of herself in tutorial observations. In one of these storylines Peggy revealed her understanding of classroom norms and positioned herself relative to her perception of a prototypical student. For ease of reference I named this position, afforded by an institutionally generated role, the *ideal student*. I viewed Peggy's positioning relative to her ideal student as contributing to her social identity. A second storyline constructed by Peggy in interviews related to her individual experiences of communicating with peers and the tutor in tutorials. In this storyline Peggy expressed aspects of her personal identity by voicing feelings, or responses to emotional commentaries, as she positioned herself relative to her idea of a *proficient English-speaker*. I finally considered the observation data, noting Peggy's actions to position herself when interacting with others in the tutorial classroom. One would not expect a storied identity constructed in interview to be replicated in the complex social setting of a tutorial classroom. However, I believe that juxtaposing Peggy's positioning acts in both settings is insightful.

Two storylines: Constructing mathematical identity in interviews

Examining the interaction of social and personal identities through the stories Peggy told in interviews, illuminates her perception of her rights and duties as a student in mathematics tutorials. In her first interview, Peggy shared a story in which she positioned herself as a student of a certain type:

Peggy:	Oh, most of the students .. is focused on the answer, answer the question and do the question, and ask the question to tutors if they don't know how to solve it.
KL:	So they mostly work by themselves?
Peggy:	Yeah and a few students will .. uh .. the brain is go here, go there, just uh .. how do you say that? A few students they not focused on the questions. They may thinking another thing, thinking something, I don't know. So for me, I will, at the beginning I will answer the question. After I answer all the question, my classmates will ask me something he don't know. Yeah, then I will solve for him.
KL:	Right, okay. And if there's something that you don't know?
Peggy:	I will ask the tutor, or just close to me – the students.

From this interview transcript the storyline pertaining to Peggy's perception of classroom norms is clarified. Peggy perceived the ideal student as being focused on answering questions independently and asking the tutor only when stuck. Her ideal student would also help other

Locke, K., Kontorovich, I., & Darragh, L. (2023). How mathematical identity informed collaborative learning opportunities for a first-year international student from China. In Drijvers, P., Csapodi, C., Palmér, H., Gosztonyi, K., & Kónya, E. (Eds.). *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 2443-2450). Alfréd Rényi Institute of Mathematics and ERME. http://erme.site/wp-content/uploads/2024/01/CERME13_proceedings_full.pdf

students when their work was done and could ask peers for help as well. It should be noted that Peggy's storyline, in which students work independently before exchanging help, differs somewhat from the university's storyline of tutorials being a setting for collaborative problem solving. While Peggy initially positioned herself as her ideal student, in later interviews other positioning is also evident. For example, on one occasion she admitted that to "ask the tutor maybe more helpful for me [than to ask peers]." Here she described herself as not necessarily seeking help from her peers. She also mentioned that "I'm not very frequently to ask the question to tutors." All of these positioning acts layer one upon another and contribute to Peggy's social identity as a not quite ideal student.

In a second storyline evident in her interview data, Peggy expressed feelings about communicating in English. Although other Chinese-speaking students attended the tutorial group, Peggy repeatedly reiterated her preference for speaking English when in the tutorial room "because it can practice my English." However, Peggy claimed, "I prefer speak English to discuss the mathematics with who speaks English. But uh, I'm a little shy. I'm not sure they will each discuss with me." This storyline, constructed by Peggy from her individual experiences, suggests that proficient English-speakers have a right to discuss mathematics together, but that Peggy may not enjoy this right. In her final interview, she positioned herself as a less than proficient speaker of English, explaining how she often preferred to work independently because "I know my English is not good, so I'm kind of afraid to speak English. So I'm rather to study by myself at the first."

In the following transcript one sees how Peggy's self-proclaimed limited proficiency in English informs, not only those situations in which she must express herself in English, but also those in which she must understand the discourse of others in the tutorial room:

Peggy	Sometimes, maybe because the accent, because they have accent or they speak too fast, so maybe sometimes I cannot hear correctly. But sometimes I can understand, yeah.
KL	Okay, so if there's a conversation happening over there, and you think, "Ah, that's what I'm working on," you can link into it?
Peggy	Yeah, but I think as long as one person speaking English clearly so I can understand very well yeah.

Peggy's positioning entailed limited rights to learn from the mathematical discussions of proficient English-speakers around her. However, accents and conversational speed sometimes excluded her from these learning opportunities.

Interpreting in-the-moment classroom positioning through the stories told in interview

During the first observation of Peggy in her tutorial, I noted that, for most of the time, her gaze focused largely on her tablet where she both wrote solutions to tutorial questions using an electronic pen, and could access an electronic version of the course book. When approached by the tutor, Leigh, to find out if she needed help, Peggy responded in a strongly patterned manner. She repeatedly offered one or more of her answers for confirmation, sometimes by merely pointing at her answer as she spoke the number of the question aloud. Incorrect answers exposed specific learning needs and enabled Leigh to offer help in these areas. By checking answers Peggy was able to communicate her problem (or lack of one) with limited recourse to language. I viewed this as an expression of personal identity. She simultaneously reproduced her social identity as an ideal student who first attempts questions and asks for help only when stuck.

In the second half of the tutorial, her peers, Nina and Jared, in separate instances, sought Peggy's input on specific questions. Peggy willingly offered what she was able, using a mix of words, symbols and gestures to explain her work. The following extract evidences the final moments of a seven minute interaction in which Peggy determinedly tries to help Nina:

Peggy	So you just, from question 'a' you can get a point, it's the minimum point value.
Nina	Mmmmm
Peggy	But
Nina	/[interrupts] I don't really know how to explain that with words.
Peggy	Ohhh
Nina	It doesn't make sense, maybe he [tutor] can help me.
Peggy	Yeah okay [sits back]

Despite Peggy's efforts, Nina finds her mathematical difficulty unresolved, and declares that she will turn instead to the tutor for help. The outcome of Peggy's attempt to help Jared was the same and he too turned to Leigh for assistance. Here one sees Peggy positioning as her ideal student by helping others once her own work neared completion. The actions of her peers, however, suggest that her efforts may have been hindered by communicational difficulties, and they turned to the tutor instead. Peggy readily accepted her ejection from the discussion in both cases, reproducing her personal identity as a less than proficient speaker of English who is excluded from mathematical discussions.

When Nina and Jared discussed tutorial questions with one another, Peggy did not appear to engage with them in any way. She remained focused on her tablet, even when they discussed a question on which she too was working. However artefacts from the tutorial evidenced Peggy's engagement with the early part of a protracted discussion between Nina and Jared,

but stopped well short of their solution. When questioned about this incomplete working on her tutorial sheet after the observation, Peggy responded that she "just heard from classmates and think about what they actually talk about." Peggy's limited engagement in the collaborative discussions of her peers, reproduced her storied identity as being somewhat excluded from collaborative learning. However the duties of Peggy's ideal student involved an exchange of help rather than collaborative sense making. Reproducing her ideal student may also have contributed to Peggy's reticence to participate in mathematical discussions of this sort.

A final extract shows Peggy's actions when offered help by her peers. The following interaction occurred within minutes of the start of the tutorial when Peggy checked her answer to the first question with her peers. The question asked students to find the limit to the sequence $\left\{\frac{n-1}{2n+1}\right\}_{n=1}^{\infty}$:

Peggy	So it's the limit to one, the first one?
Nina	The limit? Should be one over two.
Peggy	One over two? Oh.
Nina	Yeah, you divide it by ..
Jared	[interrupts] Because that's just 'a' 'n' to the 'k' and that's 'b' 'n' to the 'l', and they're both to one so 'a' over 'b.' [As Jared speaks, he tracks with his finger in course book, which he turns to show Peggy. Then he passes the book across to her and places it on table to her left. Peggy looks at book and Nina leans across from her seat to look as well]
Nina	What formula are you using? That doesn't make sense.
Jared	I just read the book.
Nina	Oh, there's something like that, okay. I don't remember that.
Peggy	Yeah [returns gaze to own work but doesn't write anything].
Nina	If it's a polynomial, I usually just divide that by the highest factor. [Peggy turns to look at Nina, concentrating on what she is saying. Then returns to look at own work.]
Nina	Like if the highest is 'n' squared then I divide everything by 'n' squared.
Peggy	Yeah yeah, I'm also same.

When a comparison of answers to the first question elicited a surprised response from Peggy, Nina and Jared positioned her as one in need of help. Nina explained her own approach, while Jared offered his course book showing Peggy the section on l'Hopital's rule. However Peggy, after listening for about thirty seconds, returned her focus to her own work saying, "Yeah, yeah, I'm also same," effectively refuting their positioning of her and closing down her peers' attempt to help. Peggy also rejected two subsequent offers of help from Nina later in the tutorial saying, "No, I'm okay." These efforts to reject the help of her English-speaking peers may be partly explained by Peggy's description, in interviews, of her fear of speaking English in mathematics tutorials. Peggy's mathematical identity as ideal student was

overshadowed, in tutorials, by anxiety about communication. This restricted her ability to learn from English-speaking peers in a tutorial context.

However, holding Peggy's repeated rejections of help from her peers against her storied ideal student also illuminates some tacit understandings. Firstly, in her storied construction of identity, Peggy acknowledged her right to ask nearby peers for help when needed. In the tutorial setting, however, Peggy did not ask for help. While her visible uncertainty sparked offers of help from her peers, Peggy had not expressly verbalised such a request. A second item of note, when comparing classroom and storied identities, is the timing of these offers of help. While Peggy's storied construction of student did not specify when she could request help from her peers, an interpretation compatible with the timing of when she could help others, is that peers help one another only when all questions have been attempted alone. Again Peggy's ideal student may differ from her peers' in terms of giving and receiving help. Her repeated rejection of help from peers may be Peggy's attempts to reposition herself as her ideal student.

Discussion

The conceptualisation of mathematical identity as having distinct social and personal aspects, together with the methodological move to juxtapose Peggy's positioning in interviews and observations, afforded a rich understanding of her interpretation of classroom norms. This approach provided evidence of how students from different cultural backgrounds might be barred from collaborative work in university mathematics classrooms. The significance of this finding becomes clear when one considers the large number of international students in tertiary mathematics programmes (Institute of International Education Project Atlas, n.d.). The findings support, and move beyond, Hwang et al.'s (2022) study of students in multilingual mathematics classrooms by showing how not only language, but also cultural understandings, might lead to inequitable learning opportunities in collaborative settings. While Peggy's case relates to a specific combination of circumstances, the findings may be relevant to other situations. For international students, differences in language and classroom norms (L. Xu & Clarke, 2019) raise many opportunities for inequity. By showcasing an example of inequitable resource distribution in collaborative work, I illuminate some of the challenges faced by international students in these settings.

5.2.2[7] Yifei and Zixin – Alignment With Local Storylines

While many international students transition directly from their home country schooling system to university in their host country (often via a foundation or bridging college), this is not always the case. Indeed, it is not uncommon for international students to undertake their final years of secondary school within the education system of their host country. For students who have already experienced a period of socialisation in the schooling system of their host country, one might expect learner identities to align with those of local students, at least to some extent. This research output explores the STT experiences of Yifei and Zixin, two international students from China who attended school in New Zealand for some years prior to transitioning to a New Zealand university. The aim is to explore how international students with substantial socialisation periods in their host country schooling system experience collaborative classroom settings in first-year mathematics.

Yifei and Zixin, the participants in the focus of this research output, had completed three and five years, respectively, in the New Zealand schooling system. Yifei's school offered the Cambridge International curriculum (CIE, 2016), and she completed her Cambridge A-Level qualification. Some of the mathematical content covered in this curriculum overlapped with that of her first-year university course, and Yifei was already familiar with vector operations, vector equations of lines, and scalar products before transitioning to university. Zixin's school followed the New Zealand national curriculum, which is focused on calculus topics rather than linear algebra, and where vectors are not discussed.

Although Zixin and Yifei took part in a series of interviews which provided some background for interpretation, the focus of this research output is their participation in two collaborative tutorials. In one of the tutorials, I observed as Yifei worked at a table alongside two local students, Kathy and Natalie. In the next observation, Yifei and Zixin shared a table with local student, Kathy.

I analysed the students' positioning acts as they interacted with their peers. I define the acts as being *collaborative* when students jointly constructed solutions to problems by building knowledge together, and *cooperative* when one student helped another from a position of

[7] Section 5.2.2 is extracted from a paper entitled "Collaborative learning experiences of international students in 1st year mathematics" by Locke, K., Kontorovich, I., and Darragh, L. The paper has been peer reviewed and accepted for presentation at the 2024 ICME15 conference in Sydney, Australia.

knowledge. Through these acts of positioning Yifei and Zixin constructed their identities as mathematics learners in action.

Zixin's positioning in peer interactions

In the second tutorial observation, Zixin, Yifei, and Kathy were seated at a table together and worked on questions about matrices. From an early stage, Kathy and Zixin could be heard checking their understanding, asking questions, expressing uncertainty, or suggesting procedures as they worked through the questions together. This pattern of interaction, exemplified in the following extract, occurred at regular intervals throughout the tutorial.

[1] Zixin:	Question four. Where's the 'I' [identity matrix] come from?
[2] Kathy:	So I think it would be this here [points to something in her course book and turns it to face Zixin]. Um, I've lost it [Zixin stretches across and indicates]. Yeah, and on this page here, but I am confused with it. I don't understand it.
[3] Zixin:	Yeah I can't get it. Page?
[4] Kathy:	Hundred and nine. And then there's more working on one ten.
[5] Zixin:	Yeah .. oh 'I' is the .. 'I' should be the [writes before pushing his page towards Kathy].
[6] Kathy:	Okay, so we need to row reduce it.
[7] Zixin:	So 'I' will be .. yeah yeah it should be that .. 'I' times four [writes and shows Kathy as he speaks].

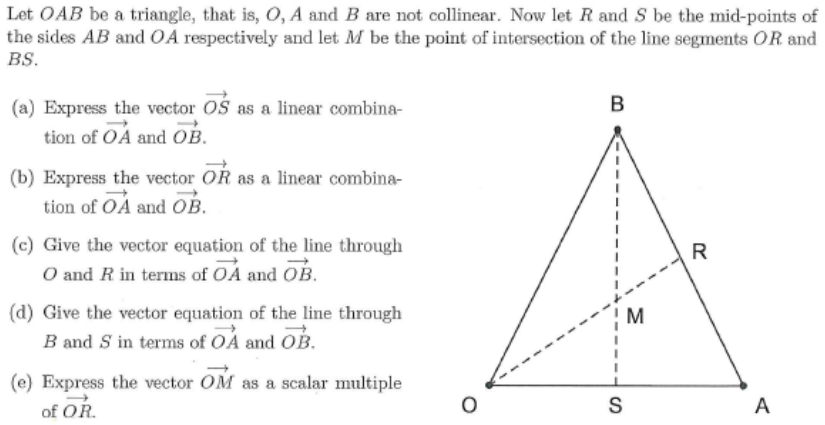
This interaction typified the way that Zixin and Kathy worked together to produce their solutions for the tutorial questions. They regularly positioned themselves, and one another, as equal collaborators during the tutorial. In these interactions, neither held the key to a solution, but by sharing what they knew, or came to understand, they jointly progressed towards an outcome. A similar pattern was evident in interactions between Natalie and Kathy (both local students) in the first tutorial observation. Across both tutorials I noted eleven episodes in which peers at the table engaged in collaborative interactions like this. This offers sufficient evidence to propose that Zixin's construction of student rights and duties aligned with those of the local students with whom he interacted in tutorials.

Yifei's positioning in peer interactions

During the two tutorials, Yifei was eager to help her peers and to contribute to discussions around the table. Yet I noted only one instance in which she contributed to a discussion without having previously completed the question independently. In the extract that follows I show how Yifei worked with her peers to solve the problem shown in Figure 5.2.

Figure 5.2

Arithmetic vector problem from first tutorial observation



Let OAB be a triangle, that is, O, A and B are not collinear. Now let R and S be the mid-points of the sides AB and OA respectively and let M be the point of intersection of the line segments OR and BS.

(a) Express the vector $\overrightarrow{OS}$ as a linear combination of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(b) Express the vector $\overrightarrow{OR}$ as a linear combination of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(c) Give the vector equation of the line through O and R in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(d) Give the vector equation of the line through B and S in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(e) Express the vector $\overrightarrow{OM}$ as a scalar multiple of $\overrightarrow{OR}$.

Yifei had finished part (a) of the question independently before joining the discussion. On hearing her peers express their confusion Yifei checked which question they were working on.

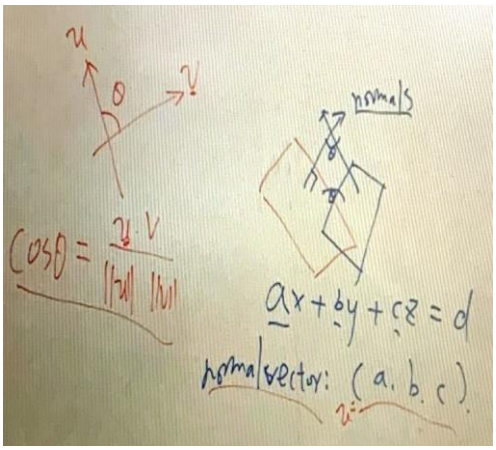
[21] Yifei: Five? … Five 'a'?
[22] Kathy: Oh just all of it. I was confused on all of it. And I was thinking it was values. And so 'OB' was like a value, but it's not, it's just the direction that they're like making you like walk to.
[23] Yifei: Like the first one, is it 'OS' is equal to one half 'OA'? [Kathy nods] So that's the five 'a''s answer? Is that the answer?
[24] Kathy: Because it's just asking/
[25] Yifei: [interrupts]/Ah. So 'OR' is 'OB' plus [looks more closely at sheet, eyes flick up to board, back to sheet, then starts to write, crosses out, tries again]
[26] Natalie: Right okay those are the same thing. And then 'OR' is the same as that.
[27] Kathy: These .. so we don't know what .. say this is a map, and we've only explored 'OB' and then we go back on 'OA'.
[28] Natalie: So we don't know what 'BR' is.
[29] Kathy: Yeah, so we don't know what 'BR' is.
[30] Yifei: But we know what 'BR' is [excited tone]. 'BR' is like one half 'AB'.
[31] Kathy: But we don't know that yet.
[32] Yifei: Oh 'AB' is like this [holds paper out to Kathy, pointing out vectors as she speaks] so I think it's first 'AB' is 'OB' minus 'OA'. So 'BR' is like there .. half of this.

This episode took place towards the end of the tutorial. After confirming that they were all working on the same question, Yifei offered her answer to the first part for confirmation (line 23). Then, despite not having attempted subsequent parts of the question independently, she participated fully in the group effort to construct a solution to part (b). This interaction, where peers helped one another by pooling their understanding to arrive at a solution, aligned with a collaborative student identity. In this interaction she positioned herself as an equal collaborator.

In all other instances, Yifei refrained from discussions if she had not yet completed the question independently. The following extract presents a situation in which Yifei rejected the position of equal collaborator when it was offered. Here I focus on an interaction between Kathy, Natalie, and Yifei, also from the first tutorial observation. Question 3(c) asked students to find the angle between planes $(x, y, z) = (1,2,1) + t_1(1,0,1) + t_2(0,1,1), t \in R$ and $2x - y + z = 5$. Yifei had not yet attempted this question. Instead, she had chosen to begin working from question 4 which, as she explained in a clarifying interview afterwards, was the easiest for her. Question 4 was very similar to 3(c). The main difference between the two was the form of the given planes, which were both in general form in Yifei's question. The discussion that follows took place once the tutor had left their table after explaining to the group that the technique needed for question 3(c) was to find the normal vectors of the planes. The tutor used the board to write up the scalar product equation needed to find the angle between normal vectors as shown in Figure 5.3. Natalie, nodding her head, took out her phone to photograph the diagrams.

Figure 5.3

Tutor's sketch of normal vectors to planes and formula to calculate the angle between them



[31] Natalie: [to Yifei] Do you understand what he [the tutor] did there, on the board?
[32] Yifei: I think it's like that [turns her sheet to show the working she had already completed for question 4]? This is what I did. I don't know how it is right. I must use that equation [motions at board]?
[33] Natalie: Oh yeah, you did …
[34] Yifei: Is it right? 'u' times 'v'. And you calculate the length of 'u' and 'v' and you use that formula?
[35] Natalie: Yeah. Oh… you get 45 degrees. Hmm interesting … Very smart.
[36] Kathy: [after all have been working independently for about two minutes] Do we need to try and put this [indicates first plane] into the same thing as this [indicates second plane]? Because this is one thing and this is another thing, so do we need to put them [the planes] into like the same .. like .. form, almost?

[37] Natalie: Yes, I think you do need to find the general equation.
[38] Yifei: Is it question 4? […]
[39] Kathy: Three 'c'. You have to find like the vectors.
[40] Natalie: How did you find the vectors? Yifei, how did you find the vectors?
[41] Yifei: Oh .. vectors ..oh I haven't done question 3.
[The discussion continued for another minute but Yifei's efforts to explain her working did not progress their solution. Yifei eventually left the discussion]

In this extract Natalie and Yifei respectfully negotiated their positions, with Natalie initially assuming a duty to help Yifei (line 31). Yifei positioned Natalie as the authority by offering her own working to question 4 for scrutiny (line 34). This prompted Natalie to declare her, or her work, "very smart" (line 35), a declaration of higher status. In line 40 Natalie positioned Yifei as one who might help them resolve question 3(c), but Yifei rejected this position saying she had not completed that question (line 41). In the extended discussion that followed Natalie and Kathy continued to seek Yifei's input. They again positioned her as key to resolving their difficulty, since she had managed to find normal vectors to planes in another question. For a while she complied, before rejecting their positioning of her once again, and exiting the discussion. Yifei, in this instance, did not take up the collaborative duties of the student role that was offered her.

In the first episode Yifei accepted the collaborative position offered by her peers, yet in the second she rejected it. This points to a potential difference between Yifei's perception of her student duties and those of her local peers. In both situations, her peers invited her into a shared activity of group sense making. In one instance Yifei met the duties as she understood them, and in another she did not. I note that the question's diagrammatic representation in the first episode, and her prior experience with vector operations at school, may have contributed to Yifei's sense of competence. This may have given confidence that the question was within her grasp and that she was capable of solving independently. The question in the second extract, where Yifei rejected a collaborative position, used only symbolic representations and involved planes. Her school curriculum did not include planes and this question may have felt less familiar. I argue that, for Yifei, mathematical interaction with peers in tutorials could only happen if she deemed herself able to advance independently. This suggests that her notion of the duties of a student involved cooperation from a position of expertise, rather than collaboration.

Yifei's positioning in a different tutorial, when attempting to solve questions related to matrices, supports this argument. In this instance, Yifei sat at a table alongside Kathy and Zixin, who initially worked at a slightly faster pace through the tutorial questions than what

she did. On occasions Yifei left out questions, skipping ahead to work either on the question that her peers were discussing or, in the final instance, on the question that followed it. In each instance, Yifei did not join the discussion until she herself had made significant written progress. She then contributed to the discussion by offering answers for confirmation or suggesting pages in the course book that she had found helpful. These actions support my proposal that Yifei regularly positioned herself as a cooperative, rather than a collaborative, peer.

I also noted a tension for Yifei of working in this cooperative manner when the content was unfamiliar and peer discussions tended to be collaborative. The discussion often moved on before Yifei had made sufficient progress on her own that she could contribute cooperatively. I interpreted her final move to skip ahead of where her peers were working as an agentive attempt to resolve this tension. By completing the question before her peers reached it, Yifei would be able to contribute cooperatively. Of course, other explanations are also possible. She may, for example, have considered this question to be more accessible than the one on which her peers were working. However, the speed with which she progressed the question that had been skipped when she returned to it later suggests otherwise. I argue that her decision to move ahead of her peers was an agentive move that would enable her to work cooperatively with her peers when they reached this question.

Summary and discussion

This research output explored the positioning acts of two international students who had attended high school in their host country prior to transitioning to university mathematics. I argue that Zixin, who spent five years at school in New Zealand before transitioning to university, constructed a collaborative mathematical identity. In the observed tutorials, he positioned himself as an equal collaborator in peer interactions, contributing to the joint solution of problems. Yifei, on the other hand, constructed a cooperative mathematical identity. After three years at a New Zealand school, she generally positioned herself as contributing from a position of knowledge when interacting with peers in first-year tutorials. One possible explanation for this difference in mathematical identities might be differing levels of proficiency in academic English. With less than five years of immersion in an English medium learning environment, Yifei's language may not have adequately developed to support the higher order thinking skills (Cummins, 1981) required for mathematical collaboration.

Yet language may not fully account for these differences. For Zixin, the entire experience of secondary school mathematics had been lived within the host country. His collaborative identity resembled those of local peers. Yifei, on the other hand, had moved from China to New Zealand part-way through her secondary school years. She brought her embodied experiences of learning in both of these social contexts into first-year mathematics. Archer's (2002) theory suggests that Yifei's personal identity emerges from these experiences. The internal commentaries that arose in response to her encounter with a collaborative student role in first-year mathematics offered feedback on natural, practical, and social concerns. In prioritizing some of these concerns over others, cultural beliefs about who holds rights to speak in interactions may have played a part. For example a common belief in Chinese society is that one must earn the right to speak and should not express an opinion without the authority, expertise, or experience to do so (Rublik, 2018). Understandings of this nature may have influenced her actions as she transformed collaborative aspects of the student role and personalised her mathematical identity.

The study has implications for teachers of international students in interactive contexts. The mathematical identities of international students who have had shorter socialisation periods in New Zealand mathematics classrooms may bar them from collaborative learning. Teacher sensitivity to both the level of academic language development and to differences in cultural understandings of international students is essential in groupwork situations. The study might also inform international students considering different pathways into university mathematics. It paves the way for future research on the impact of different socialisation periods and pathways into university mathematics for international students.

5.2.3 Contribution to knowledge

Section 5.2 explored the positioning acts of three international students with different periods of immersion in the New Zealand education system prior to enrolling in their first university mathematics course. It addresses all of the research questions by showing how Peggy, Yifei, and Zixin positioned themselves in interactions with their peers during collaborative tutorials.

The main contributions to knowledge made by this section are:

- Limited access to collaborative mathematical discussions in first-year tutorials might threaten the mathematical identities of some international students. (RQ1)

- International students might resolve tensions in collaborative interactions by enacting mathematical identities that were valued in prior (home country) learning contexts. (RQ2)
- The mathematical identities of international students with longer immersion periods in host country education systems might align more closely with local norms than those with shorter immersion periods. (RQ3)

5.3 Summary of Findings from Phase 2

Acculturation refers to the process by which the cultural norms and values of their host culture are accepted by individuals and incorporated into their own norms and practices (B. S. K. Kim & Omizo, 2005). The research outputs in this chapter explore the identities constructed by three international students from China, all at a different stage of acculturation, as they interact with peers and tutor in collaborative mathematics tutorials.

The first research output focuses on Peggy as she interacted with the tutor, Leigh, during four observed episodes. By comparing Peggy's positioning in successive episodes, Peggy was seen to agentively transform her mathematical identity by incorporating more of the rights embraced by local students into her own social identity. Peggy's actions to transform her mathematical identity appeared to facilitate interactions in which Leigh was able to support her more effectively.

The next research output again focused on Peggy, showing how her repeated rejections of help from peers stemmed, in part, from apprehension about discussing mathematical concepts in English. However, differences in cultural understandings may also have played a part in Peggy's actions. It appeared that Peggy's identity as an 'ideal student' valued independent work prior to a cooperative exchange of help between peers once tasks had been completed. Peggy sought to mitigate tensions between her personal identity and the social role she was offered by rejecting the help of her peers when offered too early.

In the final research output the mathematical identities of Yifei and Zixin were explored as they interacted with peers in a tutorial setting. Zixin, who had spent five years in a New Zealand school prior to entering university, constructed a collaborative identity not unlike those of local students. Yifei, who had undertaken her early high school years in China before moving to New Zealand to complete her final three years at a local school, constructed a different mathematical identity. While embracing interactions with her peers, Yifei

constructed an identity that was cooperative and helped others from a position of knowledge, rather than one that was collaborative and developed knowledge jointly with her peers.

The findings from these three research outputs show how experiences of schooling in China shaped the mathematical identities of the three international students, and how they acted agentively to mitigate occasional tensions between personal identities and social identities. There is a noticeable contrast between the identities that Peggy, Yifei, and Zixin construct when interacting with their peers that I suggest can be partially attributed to differing periods of time spent in mathematics classrooms in New Zealand. Peggy, the most recent arrival to New Zealand mathematics classrooms, perceived a duty to work independently before cooperatively helping peers when her work was complete. Yifei, whose high school years were split between China and New Zealand, constructed the identity of an interactive student who readily helped her peers when her mathematical understanding was sufficient to do so. Zixin, whose mathematical education in New Zealand spanned the longest period of the three, constructed a collaborative mathematical identity much like those of his local peers. These contrasting identities support the notion that the negotiation between social roles and personal identities is ongoing (Archer, 2002), and might gradually transform one's perception of what it means to be a mathematics learner in the local context.

Chapter 6: Discussion and Implications

My research aimed to explore how international students construct mathematical identities when the interaction of university structures and prior experiences create situations of tension in STT. My goals were to illuminate tensions that might occur when university structures interact with the prior experiences of international students, and to understand how international students accommodate these tensions. The three research questions detailed in section 2.5 pertain to (RQ1) how university structures shape mathematical identities of international students in STT, (RQ2) how prior experiences shape their mathematical identities, and (RQ3) how the continued interaction of university structures and prior experiences transforms the mathematical identities of international students in collaborative contexts.

The outputs from phase 1 and phase 2 of my research were presented in four separate sections of this thesis (sections 4.1, 4.2, 5.1, and 5.2). These individual contributions all come together in ways that offer more holistic answers to my three research questions. A synthesis of the research outputs suggests that mathematical identities afforded international students by university structures might be threatened by cultural differences that restrict access to learning resources. They further show how cultural resources provided by home country experiences can enable resilient mathematical identities that accommodate tensions. A final thread drawn from the cluster of outputs reveals that mathematical identities can shift towards local norms, helping to resolve tensions in collaborative contexts. These findings will be discussed in more detail in section 6.2. In the next section (6.1), however, I first draw on the research outputs to discuss the power of a critical realist approach to mathematical identity when exploring situations of tension in mathematics learning contexts. I finally discuss the implications of the findings in section 6.3.

6.1 On Distinguishing Between Social and Personal Identities

The decision to view mathematical identity as being formed by a dialectical relationship between distinct social and personal identities (Archer, 2002) is not common in mathematics education. Gardee and Brodie (2023) used this approach to investigate how and why learners constructed mathematical identities in secondary school classrooms. In their investigation, they viewed mathematical identity as "a social phenomenon, which emerges from a structured relationship between personal identity, social identity, and agency" (p. 446). I have taken their novel approach into a different mathematical context by considering how

mathematical identities are constructed in STT. I propose that it was this view of mathematical identity as emergent from distinct personal and social identities that allowed me to see how both individual experiences and university structures shaped the mathematical identities constructed by international students in my study.

In section 4.1 I showed how Aadam, Irash, and Jon, who were faced with social roles they did not embrace, constructed mathematical identities in an interview. I saw how they used a common manoeuvre that recognised a role offered them by the university. They constructed from this a mathematical identity that complied with the role they had been offered. They then drew on storylines from prior experiences that offered positions more aligned to how they saw themselves in mathematics learning situations and acted agentively to transform their social identity by personalising it in ways that they valued. In each of these cases the social role alone did not define the student. Personal identity, evident in the agentive actions of these students to transform their storied identities, offered a strong countering effect. The manoeuvre employed by Aadam, Irash, and Jon which saw mathematical identity as being both partly formed by, and partly transforming, their social roles can be understood through the critical realist framing of identity (Archer, 2002; Marks & O'Mahoney, 2014). In this instance it revealed contrasting goals between social and personal identities for both Aadam and Irash. This offered an insightful perspective on why Aadam and Irash maintained high levels of engagement despite the social roles afforded them, whereas Jon was less able to do so.

The power of a two-entity conceptualisation of mathematical identity facilitated other research outputs as well. Section 4.2 showed how, when Sunny's valued social identity was threatened by communicational issues, her personal identity as one accustomed to challenge provided a resource that enabled her to embrace and overcome the challenges she faced. We also saw how Peggy, who was excluded from the collaborative student role promoted in first-year tutorials, drew on personal identity to position herself in ways that were valued in social systems from prior experiences (section 5.2). The critical realist framing of social and personal identities as distinct (Archer, 2002; Marks & O'Mahoney, 2014) again facilitated understanding of Sunny and Peggy's actions in these moments of tension.

Section 5.1 showed how Peggy's mathematical identity shifted towards local norms through successive interactions with Leigh, the tutor. This transformation can also be understood through Archer's (2002) explanation of how nascent personal identity and nascent social identity engage in a dialectical relationship. Through this relationship, we understand that

Peggy chose elements of the social role that she saw being enacted by her peers and determined, based on feedback from personal identity, whether they were worthy of reproduction. Iterations of this process led Peggy to ascertain how she would invest in the role. In Peggy's case successive interactions with Leigh evidenced different stages of the dialectical relationship between her social and personal identities.

Viewing social and personal identities as distinct, but connected through agency, facilitated understanding of how international students constructed identities in moments of tension created by university structures. I propose that conceptualising mathematical identity as two distinct but interacting entities provides a useful tool for exploring situations of tension that arise for mathematics learners in many different learning spaces.

6.2 Answering the Research Questions

In this section I address my research questions by arguing that the mathematical identities afforded international students by university structures might be threatened by cultural backgrounds that restrict access to the resources of their mathematics courses. On the other hand, home country experiences can also help to mitigate these tensions by providing a resource for resilient mathematical identities. I finally argue that the mathematical identities of international students can shift towards local norms, resolving tensions experienced in interactive mathematics learning contexts.

6.2.1 Answering RQ1: How do university structures shape the mathematical identities of international students in their first year of university mathematics?

The findings of this study show how the mathematical identities of international students might be impacted by restricted access to course resources. Sections 4.1, 4.2, 5.1, and 5.2 provided evidence that international students, particularly those from China, faced difficulties accessing spoken lecture content, assessment questions, help from lecturers or tutors, and collaborative peer learning during their first year of university mathematics. To maintain, or attain, a valued social identity actions to overcome these obstacles were required.

Threat to identity posed by English-mediated lectures and assessment questions

Despite their stories of success, we saw in section 4.2 that Sunny, Mingyu, and Liang attributed their biggest struggle to learning mathematics in English, rather than to the mathematics itself. Both Sunny and Liang spoke of how they understood the mathematical concepts of their courses but found it extremely difficult to demonstrate their knowledge to

others in spoken and written English. This had the potential to change their social standing when performance depended on worded, rather than symbolic, mathematizing. Understanding lectures delivered in English also presented challenges for all three of these students and required their time-consuming actions to access lecture content and maintain their performance-related identities. In section 4.1, on the other hand, we saw how Jon found himself locked out of a social identity that he valued through being unable to interpret the requirements of assessment questions in New Zealand. This prevented him from demonstrating the mathematical understanding that would have afforded him a more valued identity.

The issue of accessing lectures and assessment questions in English is perhaps not unexpected as language is reported to be a significant hindrance across all learning areas for many international students (Campbell & Li, 2008; Ramburuth & McCormick, 2001; Ramsay et al., 1999; Wu et al., 2015). In mathematics, difficulties with teachers' speed of speech and pronunciation, gaps in earlier mathematical vocabulary, and differences in how some concepts may have been taught in other countries, can all make it hard for international students to access new concepts presented in lectures (Barton & Neville-Barton, 2003; Neville-Barton & Barton, 2005; Schinke, 2016; L. N. Wood et al., 2007; Wu et al., 2015). These issues of access have long been highlighted in literature, yet these studies do not consider the impact that this may have on learners' mathematical identities. Gutiérrez (2013) suggests that, in mathematics education, those who are "successful" at mathematics are often conferred with a higher status than those less successful. My study thus shows how restricted access to English-mediated lectures and assessment questions has a direct bearing on the mathematical identities of international students, threatening lower status identities if these communicational obstacles cannot be overcome.

How accessing help from teachers might compromise identity

Accessing help offered by lecturers and tutors may compromise the mathematical identities of international students. Lecturers and tutors are an important source of support for students in first-year mathematics courses. University support structures might include "office hours," or regular time periods during which students can approach lecturers for help. Universities might also offer regular tutorial sessions during which first-year students meet with tutors in their tutorial groups. Yet international students are not always able to access this help in the ways envisaged. To benefit from help offered by lecturers or tutors, transformation of international students' mathematical identities may be needed.

In section 4.1 Aadam related how his cultural background prevented him from making use of office hours to seek help from his lecturer. In section 4.2 we again saw how, despite a lecturer's willingness to help Sunny, she did not attain mathematical clarity through the one-on-one interaction. In both these instances, the international students were unable to access support through this structure. Section 5.1 also showed how Peggy's early interactions with her tutor, Leigh, did not appear to provide the support she needed to progress her mathematical working in a tutorial. Yet a shift in her identity in later tutorials facilitated interactions that were more productive. These findings suggest that accessing help from teachers may require transformation in the mathematical identities of international students.

Literature from higher education tells us that cultural understandings of how students and teachers relate to one another can vary significantly across global contexts. It is thus well known that cultural differences can raise issues for international students like Aadam, Sunny, and Peggy who might feel uncomfortable, or unfamiliar, with communicational practices acceptable at their host universities (Campbell & Li, 2008; Leung, 2001; Lillyman & Bennett, 2014; Wu et al., 2015; Zhao et al., 2005; Zhou et al., 2022). These studies do not, however, consider the issue of student-teacher relationships either in the context of mathematics learning, nor in terms of learner identities. My study goes beyond a repeated statement of their difficulties by showing how mismatched understandings of student and teacher roles can result in ineffective support from teachers. In Sunny's account her access to this source of help remained inaccessible and she found other ways to overcome her mathematical difficulty. Peggy, on the other hand, appears to have responded by agentively shifting her identity in ways that enabled access to help from her tutor. Peggy's case demonstrates how restricted access to a university support structure might generate changes in the mathematical identities of international students.

For international students, the complexity of accessing help from teachers should not be understated. While differences in interactional patterns between students and teachers are often attributed to differences in cultural understandings, the findings of my study show that transformation of mathematical identities may be required before this resource can be accessed.

How collaborative peer learning can threaten valued identities

While learning mathematics through collaborative practices is valued at some New Zealand universities (Kontorovich & Greenwood, 2023; Oates et al., 2005), groupwork in language

diverse classrooms is not straightforward. For international students, language difficulties can restrict access to collaborative practices, threatening their mathematical identities in these contexts.

In section 5.2 I demonstrated how both Peggy and Yifei found themselves unable to fully engage with the collaborative learning practices of local peers in their tutorial groups. Despite her desire to discuss mathematics with her English-speaking peers, Peggy perceived herself to be a less than proficient speaker of English who was somewhat excluded from peer discussions. Yet Peggy's reticence to join peer discussions went beyond language. Her cultural experiences of interactive learning practices tended towards cooperative exchanges of help amongst peers once work had been completed independently. The collaborative practices of her peers in first-year mathematics thus created tensions for Peggy, whose valued mathematical identity embraced interactions of a different form. Yifei, too, welcomed interactions with her peers, contributing eagerly to the mathematical talk at her table whenever she felt able. However, her mathematical identity embraced cooperative practices, where contributions to group discussions were made from a position of knowledge. This identity precluded collaborative sense making with peers, creating tensions for Yifei when peers worked in this way.

Hwang et al. (2022) noted how students whose home language was different from that spoken by other group members had negative experiences in mathematical groupwork. Simpson (2017) also noted that groupwork, while providing welcome opportunities for some international students, pushed others beyond their linguistic competence. My study supports these findings, showing that language demands of collaborative work can create tensions for some international students. Both Peggy and Yifei, while embracing the idea of mathematical discussions in tutorials, found themselves somewhat locked out of collaborative learning. However, my findings go beyond the language focus of such studies (Hwang et al., 2022; Simpson, 2017) to show how differences in student perceptions of their rights and duties in peer interactions might further contribute to tensions in mathematical groupwork for international students.

6.2.2 Answering RQ2: How do prior experiences shape the mathematical identities of international students in their first year of university mathematics?

Chapters 4 and 5 show how home country experiences can act as a resource for resilient mathematical identities, as international students negotiate STT. This can help to mitigate

some of the tensions described in the previous section. In section 4.2 we saw how Sunny called attention to the difficult mathematics encountered in China and proclaimed her belief that difficulties were "okay." Her prior experiences of overcoming difficulties contributed to a resilient mathematical identity in her first year of university mathematics. In section 4.1, Aadam and Irash drew on prior experiences from their home countries to redefine success in non-performative ways. For Aadam, success in mathematics related to deep understanding, whereas for Irash, the goal was to benefit his learning group. Despite their struggles in first-year mathematics, Aadam and Irash's home country experiences contributed to resilient mathematical identities and full engagement in mathematical learning during their first year at university. In section 5.2 I showed how understandings formed through home country learning experiences contributed to Peggy's mathematical identity when language proficiency barred her from the interactive practices that she desired. By recalling social structures from her past she constructed an identity that that offered meaningful ways for her to engage with her learning despite the communicational challenges that she faced.

Literature suggests that international students record higher levels of engagement with challenge (Zhao et al., 2005) and higher motivation to achieve (Ramburuth & McCormick, 2001) than local students. However, comparative studies make no claim as to why this may be the case. My research offers an explanation for the engagement of the international student participants, even when facing challenges, in terms of mathematical identity. My findings suggest that home country experiences offered a resource for mathematical identities in times of tension. To understand this one might turn to Hernandez-Martinez and Williams' (2013) claim that resilient students bring "particular reserves of capital that resonate with the new field" (p. 54). I infer from this that home country experiences offered cultural resources that resonated in some way with the new learning context and enabled resilient identities that contributed to student engagement. The findings thus offer a possible explanation for the higher engagement and motivation levels of international students recorded by Zhao (2005) and Ramburuth and McCormick (2001).

My research also shows how certain types of cultural resource were brought into play by the international student participants from China as they constructed resilient identities in their first year of university mathematics. Literature suggests that students of Asian heritage commonly subscribe to the idea that effort is a major contributor to success (Barnett et al., 2014; Campbell & Li, 2008; Mok, 2020; Ramsay et al., 1999), and that diligence brings a deep level of pleasure (Frenzel et al., 2007; Leung, 2017). These perceptions offer cultural

resources upon which international students from China might draw during first-year mathematics. In my study, Chinese participants such as Sunny, Liang, and Mingyu, described their time-consuming efforts to overcome difficulties. In doing so they evidenced an agentive leveraging of appropriate cultural resources related to diligence. Literature further shows how instructional practices of mathematics classrooms in China might differ from those of New Zealand (Kaiser & Blömeke, 2013; Kontorovich & Greenwood, 2023; Oates et al., 2005; Yang et al., 2020). When restricted from joining peer discussions, Peggy agentively deployed cultural resources valued in Chinese mathematics classrooms that related to silence (L. Xu & Clarke, 2019) and independence (Heng, 2019) to construct her mathematical identity in New Zealand. My study thus exemplifies some forms of cultural resource that contributed to their resilient mathematical identities in STT.

While my research focuses on how prior experiences contribute to mathematical identities in STT, beliefs about mathematics formed in home countries can also be linked to future identities, or what students hope to become in the future. There are many studies of mathematical identity in STT that show future identities as being instrumental in influencing mathematical engagement. Some show how new identities that develop in STT might lead to new motives and positive engagement for some students (Black et al., 2010; Hernandez-Martinez et al., 2011). Other studies show how changes that fail to align with an imagined future identity can lead to disengagement (Hernandez-Martinez, 2016; Jooganah & Williams, 2016). In my study, Irash offered an example of one whose motive to learn mathematics pertained to becoming someone who would bring future economic benefit to his country through entrepreneurial activities. Yet this so-called future identity was formed in past experiences. Being "independently collective" arose from a storyline birthed in his past and envisaged in his future. Despite Irash's struggle with university mathematics, he remained faithful to this storyline which offered "a way of being" in all situations, mathematical or not. While Black et al. (2010) show how leading identities can influence mathematical engagement, these findings show how future identities, invoked by storylines from prior experiences, can also impact on mathematical engagement.

6.2.3 Answering RQ3: How are the mathematical identities of international students transformed by the continuous interaction of university structures and prior experiences during collaborative first-year mathematics tutorials?

My findings show how mathematical identities of international students can shift towards local norms. This happened through the ongoing interaction of university structures and

personal identities resourced by prior experiences. The findings further showed that closer alignment with local norms might help to resolve tensions for international students in collaborative contexts. Section 5.1 showed how Peggy incorporated more of the rights of local students into her later interactions with the tutor. I suggest that the changing nature of Peggy's mathematical identity in successive episodes was influenced by the social norms observed in the tutorial classroom, social structures from Peggy's past (now internalised in her personal identity), and her requirements for help when attempting to solve tutorial questions. Peggy offers an example of how a dialectical relationship between social and personal identity mediated by agency (Archer, 2002) can lead to transformation of mathematical identity. By understanding identity in this way, we see how Peggy engaged with selected aspects of a social role, which in turn generated emotional commentaries of the natural, practical, and social orders for Peggy. Her ensuing actions sought to balance feelings of competence and feelings of worth in a way that, for her, was acceptable. The process was repeated in subsequent interactions with Leigh, transforming her mathematical identity in ways that were evidenced in section 5.1.

In section 5.2 we saw how Peggy, Yifei, and Zixin, all of whom had been immersed in the New Zealand education system for differing periods of time, embraced different interactional practices when working with peers in tutorial groups. Zixin, who had completed the entire five years of his high school education in New Zealand enacted a mathematical identity most closely aligned with his local peers. Yifei, with three years in the New Zealand school system, showed closer alignment with local norms than Peggy, who had transitioned from school in China to university in New Zealand via a one-year bridging programme.

Clark and Lovric (2008) describe STT in terms of a rite of passage through which students move from one place in the mathematical community to another. They further propose that the entire first year of mathematics learning is part of the incorporation phase in this rite of passage, during which students construct identities aligned with their new social status. Based on my findings, I suggest that for international students (who must traverse the global mathematical community) the incorporation phase could be much longer. Studies of international student experiences when transitioning to host universities record factors affecting their adjustment (Hechanova-Alampay et al., 2002; Rajapaksa & Dundes, 2003; Zhao et al., 2005), which is part of the acculturation process (Rajapaksa & Dundes, 2003). Wu et al. (2015) report language to be a significant hindrance to academic adjustment by preventing students from joining classroom discussions until they discover strategies that

enable this. The interactional mathematical identities evidenced by Peggy, Yifei, and Zixin indicate different stages of acculturation, suggesting that time spent in New Zealand mathematics classrooms may play a part in this process. Considering STT as a rite of passage, the findings of section 5.2 suggest that, for some international students, the incorporation phase might extend beyond their first year of university mathematics. My research shows that aligning with collaborative learning practices is not a straightforward matter for some international students and may require protracted periods in the educational systems of their host country.

Together the findings of sections 5.1 and 5.2 suggest that the mathematical identities of international students might shift towards norms promoted by new social structures. As personal identities (emergent from structures of the past) and social identities (resourced by university structures) interact in ongoing cycles, mathematical identities might transform in ways that mitigate the tensions experienced by international students.

6.3 Implications

The findings of my study have implications in practice and research. Institutions that enrol international students have an ethical obligation to these students, both to understand their needs and to ensure that these needs are met (Agostinelli, 2021; Evans et al., 2017; Heringer, 2018; Jin & Schneider, 2019). My research shows how international students may have inequitable access to some of the resources of first-year mathematics. This has implications for teachers of international students, for mathematics departments at host universities, and for future research.

6.3.1 Implications for Teachers of International Students

Teachers are often unaware of the international students in their classes, and even less aware of the role they could play in enhancing the learning experiences of these students. My research showed how difficulties experienced by Sunny, Mingyu, and Liang (section 4.2) when learning mathematics in English led these students to employ time consuming strategies as they sought to overcome the obstacle of language. While these language difficulties are unavoidable for some international students, it is important that teachers of international students are aware of the obstacles that they face. Teachers can support international students by providing resources that enable them to prepare for lectures ahead of time, as well as by providing resources that students can access in their own time after lectures. Helpful preparatory materials might include subject specific vocabulary and prior reading to make

lecture time more productive (Evans et al., 2017). Wood et al. (2007) suggest that lectures themselves should include verbal, written, diagrammatic, and symbolic representations that appeal to more than one sense at a time, as these build important concept images essential to learning. The verbal accompaniment to written, diagrammatic and symbolic representations helps students to switch between these representations. However, these authors caution that teachers of international students should avoid metaphors or humour that may have little meaning within a different culture. Another suggestion to make lecture time more productive for international students is to include lecture-interrupting processes that encourage small group interrogation and reconstruction of content during lectures (Skyrme & McGee, 2016). To enable international students to review mathematical content in their own time outside of lectures, recordings and online posts or resources could also be provided (Huong et al., 2017). This range of resources may help international students develop robust mathematical identities and feel more confident to ask questions either in person or through online channels.

It is also important for teachers of international students to understand that the tacit understandings of the student role held by international students may differ from those of local storylines. When storylines differ teachers may find it difficult to provide meaningful help. Such difficulties were evident in the case of Sunny (section 4.2) and also, initially, with Peggy (section 5.1) whose positioning in student-teacher interactions prevented dialogue that may have facilitated understanding. A teacher sensitive to these differences might seek to develop strategies that can effectively probe understanding throughout explanations and rely less on students affirming their own understanding. Such strategies may reveal difficulties not yet fully resolved and might also validate a negotiated approach to understanding as an acceptable classroom practice. Relationships that foster trust and connection will not only enhance learning experiences for international students (Huong et al., 2017), but might further facilitate alignment of student-teacher storylines to enable effective support.

Peggy and Yifei's experiences (section 5.2) showed how groupwork in multicultural classrooms can be a source of tension for international students. While some international students may relish the opportunities that groupwork provides for social interaction and English practice, this is not the case for all. Others may find the relational and linguistic challenges they face in groupwork to be beyond their current level of competence (Simpson, 2017). Teachers need to be aware of group dynamics and to intervene as necessary to support international students. This might include reallocating students to groups where at least one

other speaks their language (Hwang et al., 2022), or offering more of their own time to students barred from collaborative discussions. Esmonde (2009) suggests broadening the notions of competence in ways that separate understanding from being good at explaining. This might allow students who understand, but struggle to explain, to contribute more confidently to group discussions. Such actions from teachers sensitive to the issues faced might help to boost mathematical identities for international students.

6.3.2 Implications for Host Universities and Mathematics Departments

For many international students, their time-consuming efforts, together with advanced prior knowledge from learning in their home countries, may mask language difficulties and make their struggle invisible to institutions. Yet their disadvantage was noted by both Barton et al. (2005) and Ramburuth and McCormick (2001) some twenty years ago. Barton et al. (2005) proposed a range of interventions for implementation by mathematics departments wishing to address the language difficulties experienced by international students. These included implementing first language tutorials for students from other countries, offering "English for mathematics" courses, and providing staff development. Andrade (2010) also suggests that departments should do more to support language development of international students. She suggests that they might offer buddy schemes for students on the same course or provide international students with peer mentors. In a similar vein to Barton et al.'s (2005) suggestion of an English for mathematics course, Andrade (2010) suggests linking courses with language support, and making use of technology to enhance subject specific vocabulary. Finally, a popular call amongst studies concerned with the learning experiences of international students is for universities to provide professional development that trains teachers in linguistically supportive practices (Haan et al., 2017; Jin & Schneider, 2019; Skyrme & McGee, 2016).

6.3.3 Implications for Future Research

Many universities use groupwork in their mathematics programmes (e.g., Kontorovich & Greenwood, 2023; Wawro et al., 2012). Although groupwork is often recommended as a means to improve student understanding and achievement, it does not always lead to equitable learning opportunities (Hwang et al., 2022). These authors suggest that power dynamics amongst peers and their teachers might lead to marginalisation of some students. This concern is particularly relevant in culturally diverse classrooms where the language proficiency or cultural competence of some international students may not be sufficiently

developed to cope with the demands of group activities (Simpson, 2017). While authors such as Hwang et al. (2022) offer some insights on how students in collaborative groups manage cultural differences, there is a need for further research in this area.

My research also showed how mathematical identities can shift towards local norms. The findings suggest that alignment with local identities may depend, to some extent, on the length of time spent in the host culture. However, factors other than time might also influence the acculturation of international students in mathematics courses at host universities. This study paves the way for future research to understand factors that might encourage agentive changes in identity that lead to more effective interactions with peers and teachers.

The critical realist approach to identity used in this study has been helpful for understanding how home country experiences influenced the mathematical identities of international students in a new social context. The distinction between personal and social identities might be similarly helpful in other situations where one seeks to understand how prior experiences influence identities in a new context.

Finally, my research has foregrounded the educational experiences of international students while paying less attention to the mathematical knowledge developed as part of these experiences. Monitoring changes in mathematics when students transition from one country to another might be an area of interest for further studies.

Chapter 7: Conclusion

Participants in this research study belong to a cohort of students in first-year mathematics who, despite their economic and cultural importance in many countries, remains relatively invisible in mathematics education research. While there exists a handful of studies that do focus on international students in either STEM or mathematics (Hwang et al., 2022; Loong & College, 2012; Sparks et al., 2019), very few of these appear within the domain specific literature of mathematics education. By exploring the experiences of international students transitioning to university mathematics, I have widened the space for international students in mathematics education research.

My research contributes to knowledge on affective and sociocultural issues in STT by showing how, for international students, mathematical identity might be shaped by cultural understandings and language proficiencies that restrict access to learning resources. According to Di Martino et al. (2023b), alleviating issues of equality and inclusion for students in STT rests partly on a better understanding of the impact of sociocultural factors. I thus contribute to this end by illuminating how social and cultural factors can shape international students' experiences when transitioning to university mathematics in a foreign educational context. Di Martino et al. (2023b) also highlight the need for research on students' transitional experiences to consider country-specific educational contexts. I address this need by considering the mathematical learning experiences of international students who transition across country borders to attend university in a host country. My focus on a New Zealand university also helps to diversify the US and Europe-centred body of STT literature.

7.1 Limitations

In a perfect world I would have interviewed and observed a cohort of international students, newly arrived from a range of sending countries. I would have interviewed them in their own language and observed them via inconspicuous cameras that could record every movement, sound, expression, and mathematical output for playback in 3D Sensurround. Sadly, these ideals were far from my reach and the gap between what I could practically do, and what I would like to have done, remain as limitations to my study.

Unanticipated and unavoidable difficulties arose with the onset of the COVID-19 pandemic. Participants meeting eligibility requirements for my study were in short supply, and opportunities for groupwork were limited and unpredictable. This resulted in a widening of eligibility criteria in the first phase of the study, and a reduced data collection period in the

second phase. Without the constraints imposed by the pandemic, I would have sought to recruit a more homogeneous group of phase 1 participants, so that patterns and contrasts might be more evident. Ideally, I would also have chosen a more typical 12-week semester in which to collect data for the second phase of the research, providing greater separation between the two observations. Given the chance, I would have repeated the process in another semester, increasing the number of phase 2 participants.

Although most participants understood English very well, it was not the first language of any participant in this study. It is thus highly likely that what participants would like to have said, were they speaking their own language, and what they felt able to say in English, may have differed substantially. It may also have influenced who was willing to participate in the study, standing as an obstacle to those less confident in their spoken English. While I sought to reduce the limitations of language proficiency by collecting observational data and conducting multiple interviews, first language interviews would have been preferable.

Just as the relative positions of interviewer and interviewee determines what data can be collected in interview, so the presence of recording equipment and an observer in tutorials disrupts the "natural" setting and is likely to impact the actions of participants. Data collected under such circumstances cannot be unaffected. The recordings also cannot capture the infinite detail of the setting, and much of what took place in the tutorial room would have played no part in the analysis.

7.2 Final Words

I began this research with the naive hope that I might right a perceived wrong. That I might change what I saw as a "taken for granted" attitude towards international students in university mathematics departments. That I might raise appreciation for the difficulties they face, even in mathematics, where high grades might mask underlying challenges. I went so far as to hope that I might stir mathematics departments into actions that could address some of these difficulties and help international students to thrive in their first year of university mathematics. I end my research with the uncomfortable feeling that right and wrong are a whole lot less clear than when I started, and there is a long road yet to travel.

While I still hope that my three-and-a-half-year journey will make a small difference to something I believe to be important, I know without doubt it has made a large difference to who I am now.

The minutes that passed between completing the sentence above and beginning the one I now write is perhaps evidence of just one of these changes. Just writing the words “who I am now” has my mind skittering in all directions. How I see myself? Or how others see me? Or how I hope I portray myself to others? Or all of these? Do I now write what I believe to be true? Or do I say what I think you should hear? Who am I as a researcher? Who am I as Kim? My identity has changed, and how I now think about identity is a whole lot different than before. Identity has become a tool I now wield, like it or not.

I have come to understand that undertaking a PhD is more about learning new skills than achieving an outcome. I might yet right that wrong. I might yet awaken appreciation and stir up change for international students in mathematics. Perhaps after three-and-a-half years I am ready to begin.

To support international students effectively in their first year of university mathematics requires not only an understanding of their needs, but also a willingness from teachers and mathematics departments to address these. As mathematics educators we all have a duty of care to our international students, who have invested more than most (both emotionally and financially) in their education. If this research is to make even the small difference I hope for, it marks the start of the real journey rather than the end.

--------THE BEGINNING-------

Appendix A: Phase 1 Interview Discussion Topics

Tell me about your background – where you went to school, what you're doing now?

Thinking back to when you began your first semester of mathematics studies:

How did you find your first semester of maths at university?

What expectations did you have of yourself as a first-year maths student?

How did you gauge your progress during your first semester of mathematics?

Do you have any memorable experiences of your first semester of mathematics?

Is it different for international students in their first semester of maths than it is for local students?

What were the greatest challenges in your first semester?

Where did you find support in your first semester?

What did success look like for you in your first semester of maths?

How connected did you feel to the community of mathematics students and staff at university during your first semester?

Is there anything else I should be asking you?

What three words would you use to describe your experience of learning mathematics in your first semester at university?

Appendix B: Phase 2 Interview Discussion Topics

Initial Interview

Why did you come to study in New Zealand?

Tell me about your background – where you went to school, what you're doing now?

How has maths changed now that you are at university?

How do you see yourself as a maths learner?

How would other people describe you as a maths learner?

How does being an international student influence your experience of learning maths?

What would I see if I came to a maths lecture with you?

What would I see if I came to a tutorial with you?

How do you engage with maths outside of class times?

Describe a successful mathematics student.

What advice would you give to a first-year maths student who wants to thrive?

Clarifying Interviews after Observations

Tell me how you felt about the work in today's tutorial?

While you were working, I noticed ……..Can you tell me about it?

(Examining artifacts together) Can you tell me about this question?

How confident are you about this section of coursework?

Do you have any other comments about today's tutorial?

Final Interview

How did maths go this semester?

What can you tell me about your experience in Maths 100?

Do you feel any different about maths now than you did at school?

How do you see yourself as a maths learner?

Were there any ways in which your background as an international student influenced your experience in maths this semester?

Do teachers or classmates treat New Zealand students differently than international students?

How did the tutorials run – how were students expected to work in tutorials?

What do you view as cheating in maths?

How do you feel about your success in maths this semester?

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