A Computational Model of the Ocular Lens

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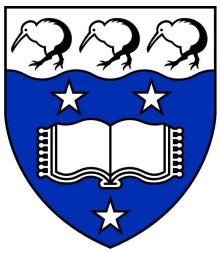

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Abstract

The aim of this project is to develop a computational model of the structure and function of the ocular lens, specifically the solute and fluid transport in the lens.

The modelling framework was based on finite volume methods. The intracellular and extracellular solute fluxes were modelled using the Nernst-Plank equation with an extra term to capture solute fluxes due to advection. The modelling framework included equations describing the flux through the Na^+/K^+ pumps and K^+ channels in the surface membrane, and Na^+ and Cl^- channels in the fibre cell membrane. The intracellular fluid flow between adjacent fibre cells was modelled by a homogenised transmembrane fluid flow equation and the intracellular fluid flow along the fibre cell was modelled as Poiseuille flow. The extracellular fluid flow was modelled as Couette flow with an extra term to capture electro-osmotic flow. The fluid flow through the fibre cell membrane and surface membrane was modelled as transmembrane fluid flow. The governing equations account for the structural properties of the lens, such as the tortuosity of the extracellular cleft, the intracellular and extracellular volume fractions, and the membrane density.

A one-dimensional model of the Na^+, K^+, Cl^- and fluid transport in the frog lens was developed. This model was based on the analytic model developed by Mathias (1985*b*). The results were consistent with the results from the analytic model and experimental data.

Two versions of the two-dimensional model were developed. In the first model, the parameters were spatially constant except for the distribution of the Na^+/K^+ pump currents at the lens surface and the fibre cell angles. The second model was the same, except the extracellular cleft width and fibre cell height was spatially varied to represent the sutures and the diffusion barrier. These models were solved and compared with each other and with experimental data.

Compared to the first, the second model predicted a significantly larger circulation of solutes and fluid between the pole and equator. It predicted a 12-20% increase in the penetration of Na^+, K^+ and fluid into the lens. The second model also predicted a 300-400% increase in Cl^- penetration and, unlike the first model, a Cl^- circulation between the poles and equator. This is significant since Cl^- is not an actively transported solute. These results highlight the strong structure-function relationship in the lens and the importance of an accurate spatial representation of model parameters.

The direction of the current, solute fluxes and fluid flow that were predicted by the model were consistent with experimental data but the magnitude of the surface current was a tenth to a third of

the values measure by the vibrating probe.

To demonstrate the application of the lens model, the two-dimensional model was used to simulate age-related changes in lens physiology. This was done by increasing the radius of the lens to simulate growth with age. The model predicted an increase in the intracellular Na^+ concentration, Cl^- concentration and potential, and a decrease in the intracellular K^+ concentration with age. These trends were consistent with those observed by Duncan et al. (1989), except for the intracellular K^+ concentration, where they reported no change with age.

The two-dimensional model forms a foundation for future developments and applications.

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Finally, I would like to acknowledge three important people, my parents Kura and Roger, and my partner Sally.

My father has a PhD in Physics and as a dyslexic kid I remember saying "I want a DhP when I grow up", only to be hassled about wanting to be a "Damn Hopeless Person". Fortunately, I learnt to spell and chose the better path. The point of that story is that my parents have been an inspiration to me, they have supported me in whatever I've wanted to do and gave me a world where I could

play with an inquisitive mind. I am like I am because of their loving support and guidance, for which I will alway be grateful.

Sally has spent seven of the last ten years putting up with, and supporting, a student boyfriend. This is testament of the love, support and guidance she has given. I will alway be grateful for that.

Naku Mataora

No tai mai au te karo iakoe
Te roa i to tere
Na te Kuere, Kuere, Ku-ooo-o-i
Oki mai oki mai

Ea'a te tau i Taunganui e
E tau ora te tau i Taunganui
E tini e mano te ano maira i te moana
E Aitu kake ki te akau e
Oro mai, Oro mai e Punua e

— Chant from Ngaputoru

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Glossary of Symbols & Abbreviations

Scalar Symbols

Symbol	Description
a	fibre cell height
A	area
b	fibre cell width
c	cross-fibre direction
C	concentration
D	diffusion coefficient
e	electron charge
E_α	Nernst potential
f	fibre direction
F	Faraday constant
F	body force
g	membrane ion conductance
h	linearised extracellular cleft width
I_{max}	maximum Na^+/K^+ pump current density
I_p	Na^+/K^+ pump current density
$j_{\alpha,m}$	cell membrane flux
$j_{\alpha,s}$	lens surface flux
J_w	fluid mass flow
k	electro-osmotic coefficient
k_B	Boltzmann constant
K	hydraulic conductivity
$K_{\frac{1}{2}\alpha}$	half-maximal concentrations
L_m	transmembrane hydraulic permeability
L_p	intercellular hydraulic permeability
L_s	surface membrane hydraulic permeability
Os	osmolarity
N_0	Avagadro number
p	pressure
P_m	membrane solute permeability
R	radius

Symbol	Description
s	source
T	temperature
V	volume
V_m	transmembrane potential
z	valency
α	solute species
ε_0	permittivity of vacuum
ε_r	dielectric constant
ζ	zeta potential (cell membrane potential)
η	fibre coordinate system
$\tilde{\eta}$	cleft coordinate system
λ_D	Debye length
Λ_e	extracellular volume fraction
Λ_i	intracellular volume fraction
μ	dynamic viscosity
ν	kinematic viscosity
ξ	wiggle factor
ρ	mass density
ρ_c	charge density
ρ_m	membrane density
σ	membrane reflectance
τ	tortuosity
ϕ	potential

Vector & Matrix Symbols

Symbol	Description
$\boldsymbol{i}$	current density vector
$\boldsymbol{j}$	solute flux vector
$\boldsymbol{n}$	unit normal vector
$\boldsymbol{u}$	fluid velocity vector

Abbreviations

Abbreviation	Description
AP	Anterior Pole
AQP	Aquaporin
EDL	Electric Double Layer
EQ	Equator
FVM	Finite Volume Methods
PP	Posterior Pole