## Two-pixel computer-generated hologram with a zero-twist nematic liquid-crystal spatial light modulator

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We present a method of producing a computer-generated hologram by use of a zero-twist linear nematic liquid-crystal spatial light modulator. A  $2 \times 1$  macro pixel method is used; one pixel represents the real data, and one, the imaginary. A method is shown that produces both positive and negative analog amplitude modulation. © 2000 Optical Society of America

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Zero-twist nematic liquid-crystal (ZTNLC) (also known as planar nematic) spatial light modulators (SLM's) are usually used to modulate light in a phase-only mode. When the correct input polarization state is selected, the device will act as if its refractive index were electronically modulated. Such devices have been used in adaptive optics, diffractive optical elements, and optical correlators. However, because the devices are birefringent they can also be used as amplitude modulators if the input polarization state is linear and at 45° to the extraordinary axis. With this configuration is it also possible to obtain positive and negative amplitude modulation; i.e., the device can modulate along the real axis. By combining two pixels, a phase detour technique makes full complex modulation possible. This method of complex modulation is similar to that used by Lee<sup>1</sup> and Burckhardt<sup>2</sup> but requires only two pixels instead of four; thus it is more suitable for computergenerated hologram production on a SLM for which the number of pixels are limited. Other techniques, such cascading two SLM's<sup>3</sup> and using deformable mirror devices,<sup>4</sup> have been used.

If the device is set up in single-pass mode with its extraordinary axis vertical, a SLM pixel is represented by the Jones matrix

$$\mathbf{L} = \begin{bmatrix} \exp(-i\Gamma/2) & 0\\ 0 & \exp(i\Gamma/2) \end{bmatrix}, \quad (1)$$

where  $\Gamma$  is the retardance of the SLM pixel. If the input polarization state is linear and at 45° and the output polarizer is also at 45°, we obtain

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \exp(-i\Gamma/2) & 0 \\ 0 & \exp(i\Gamma/2) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} \cos\Gamma/2 \\ \cos\Gamma/2 \end{bmatrix}, \quad (2)$$

which gives amplitude modulation from 1 to -1 for  $\Gamma = 0-2\pi$ . However, the current trend in the manufacture of a ZTNLC SLM is to build it onto a silicon backplane. This makes the device double pass. It is not adequate merely to double the value of  $\Gamma$  in Eq. (1) because a coordinate transform has also occurred. The correct Jones matrix for the SLM in reflection is given by<sup>5</sup>

$$\mathbf{J} = \begin{bmatrix} -1 & 0\\ 0 & 1 \end{bmatrix} \mathbf{L}^t \mathbf{L}, \qquad (3)$$

where  $\mathbf{L}^t$  is the transpose of  $\mathbf{L}$ .

The system in double pass is now given by

$$\frac{1}{2} \begin{bmatrix} 1 & 1\\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\exp(-i\Gamma) & 0\\ 0 & \exp(i\Gamma) \end{bmatrix} \begin{bmatrix} 1\\ 1 \end{bmatrix} = \begin{bmatrix} i\sin\Gamma\\ i\sin\Gamma \end{bmatrix},$$
(4)

which is still equivalent to modulating along the real axis because the overall phase shift introduced by i in Eq. (4) can be neglected. The intensity and phase modulation are shown in Fig. 1.

We achieved complex modulation by placing the real data on a pixel next to the imaginary data. The data are thus arranged into a  $2 \times 1$  macro pixel. The sign of every second macro pixel must be reversed (see Fig. 2) to produce a constant phase delay. The complex reconstruction then occurs at an angle such that there is  $\pi/2$  phase lag between the pixel that represents the real data and the pixel that contains the imaginary data. The sign reversal is necessary to remove the  $\pi$  phase step that would otherwise occur.



Fig. 1. Left, the phase shift (in radians) created by a double-pass ZTNLC SLM as the retardance (in radians) increases. Right, the corresponding intensity transmission.

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Fig. 2. Method of producing a full complex modulation from two pixels.

The SLM used was a  $128 \times 128$  ZTNLC SLM from Boulder Nonlinear Systems, Inc. The device has a pixel size of 40  $\mu$ m, a fill factor of 60%, 8-bit (256 levels) resolution, and a liquid-crystal response time of  $\sim 2$  ms (depending on temperature).

We calibrated the SLM by measuring the intensity throughput of the SLM for various voltages applied with the system shown in Fig. 3. A 638.2-nm He-Ne laser was used as the light source. The SLM is capable of producing a  $2\pi$  rad phase shift at this wavelength. The results for the intensity calibration are shown in Fig. 4. A polynomial was then fitted to the data from SLM value 0 to the first turning point, and another polynomial was fitted to the data from the last turning point in Fig. 4 to the SLM value 256. Then, as each region was  $\pi$  out of phase with each other region, the first polynomial was used to represent the negative values, and the second, the positive values. A fast Fourier transform of the letter P (slightly off axis) was then performed on a PC. These complex data were then arranged into  $2 \times 1$  macro pixels as described above. This means that there is a different sampling rate for each direction and that the output field is not a square. The data were then passed through the calibration polynomials and written to the SLM. The CCD camera in Fig. 3 digitized the resultant reconstruction shown in Fig. 5. Blanking the imaginary data leads to the appearance of a conjugate image in the reconstruction (see Fig. 6).

Full complex modulation has been demonstrated with the ZTNLC SLM. The reconstruction appears off axis in Figs. 5 and 6 because of the phase wedge introduced into the system. The reconstruction is in fact one quarter of the way between the zeroth and the first orders.

There is also a dc term visible. The dc term is not inherent in the method but comes from calibration errors and unmodulated reflections from the front glass plate of the SLM. The on-axis dc term can be calculated by the integral of the input function. Because this method can produce positive and negative modulation, this integral can be zero. This technique differs from that of Lee,<sup>1</sup> in which there is a large dc bias. The dc terms could probably be reduced by careful calibration of each pixel of the SLM. Interferometric analysis of the SLM shows that there is a nonuniform response across the device, which is likely to be caused by the device's being of nonuniform thickness. The faceplate over the SLM has a broadband antiref lection coating upon it; the amount of reflections from it could be reduced by use of a narrow-band coating, suitable for the specific laser used.

The  $2 \times 1$  macro pixel format means that there is a reduction in resolution in one dimension. The low



Fig. 3. Experimental setup: BS, beam splitter; POL, polarizer.



Fig. 4. Calibration graph for the ZTNLC SLM: PD, normalized photodiode voltage.



Fig. 5. Reconstruction of the hologram.



Fig. 6. Reconstruction of the real-only part of the hologram.

quality of the reconstruction is likely to come from the nonuniform response of the SLM and the small number of pixels used. The quality of the reconstruction could be improved if a larger-resolution device were used. A  $512 \times 512$  format will shortly be available from Boulder Nonlinear Systems.

Full complex modulation with a nematic liquidcrystal spatial light modulator has been demonstrated. Two pixels, one representing the real data and one the imaginary data, were combined. Methods for producing positive and negative modulation have been shown.

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