



Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand). This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.

<http://researchspace.auckland.ac.nz/feedback>

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library

[Thesis Consent Form](#)

Non-Invasive Electrical Imaging of the Heart

Leo Cheng

l.cheng@auckland.ac.nz

Supervised by

Associate Professor Andrew Pullan

a.pullan@auckland.ac.nz

A thesis submitted in partial fulfilment of the
requirements for the degree of Doctor of Philosophy



THE UNIVERSITY OF AUCKLAND
NEW ZEALAND

Department of Engineering Science
School of Engineering
The University of Auckland
New Zealand

December 2001

Abstract

Non-invasive electrical imaging of the heart aims to quantitatively reconstruct information about the electrical activity of the heart from multiple thoracic ECG signals. The computational framework required to produce such electrical images of the heart from non-invasive torso surface signals is presented. It is shown reliable electrical images of the heart can be obtained under a controlled environment. This has been demonstrated using an anatomically realistic boundary element porcine torso model.

The procedures required to create a subject specific model using a small number of control points and to create a specific heart model from three-dimensional ultrasound images using a linear fitting procedure are presented. From discrete ECG electrodes a continuous representation of the potential field over the entire torso surface can also be produced using this linear fitting procedure.

The construction of the transfer matrices for the two predominant electrocardiographic sources (epicardial potentials and myocardial activation times) are described in detail. The transfer matrices are used to compute activation times within the heart and epicardial potentials on the heart surface. Myocardial activation times are computed using an algorithm based on the Critical Point Theorem while epicardial potentials are computed using standard Tikhonov and Truncated SVD spatially regularised methods as well as Greensite's spatial and temporal regularisation method. The regularisation parameters for the epicardial potentials are determined using a variety of methods (*e.g.*, CRESO criterion, L-curve, zero-crossing).

The potential and activation based formulations are compared in a comprehensive inverse simulation study. To try and capture the dynamic and variable nature of cardiac electrical activity, the study is performed with three different types of cardiac sources with a realistic porcine model. These simulations investigate the effect on the computed solutions of individual and combinations of modelling errors. These errors include corruption in the torso surface signals, changes in material properties and geometric distortion.

In general, the activation based formulation is preferred over the epicardial potential formulations, with Greensite's method found to be the best method for reconstructing epicardial potentials. Under optimal conditions, the activation approach could reconstruct the activation times to within 4 ms RMS. Both potential and activation based formulations were found to be relatively insensitive to changes in material properties such as lung conductivities and activation function shapes. When

examining individual errors, the geometry and positions of the torso and heart had the greatest effects on the inverse solutions. The relative heart position needed to be determined to within 5 mm to obtain results within 2 ms of the solutions obtained under control conditions. When the modelling errors are combined to produce errors which can be expected in a clinical or experimental situation the activation based solutions were consistently more accurate than potential based solutions.

The next necessary step in this project is the detailed validation of the results against *in-vivo* data. This step is necessary before such algorithms can be reliably used to aid in the assessment of heart function in a clinical environment.

Acknowledgements

There are large number of people to thank for the work presented throughout this thesis. A large portion has been based upon the foundations provided by **CMISS** – of which a large number of people (past and present) have contributed. For that reason I would like to deeply thank Professor Peter Hunter for having the vision, so long ago, to establish this framework which we have all learnt to love (and hate). Without this unique environment, the goals of this thesis would not have been able to be met.

Also, thanks to my supervisor, Associate Professor Andrew Pullan. His guidance and support leading up to and throughout the duration of this thesis have been indispensable while his enthusiasm has been an outstanding source of encouragement. I would also like to acknowledge Dr David Budgett for his stellar supervision during the first year of my studies while Andrew was on sabbatical.

Special mention should go to close colleagues who have directly aided in this project. In Auckland, John Bodley and Dr Greg Sands. Your contributions and friendship has been invaluable throughout the last year. In Oxford, Drs Martyn Nash, Chris Bradley and David Patterson for supplying the experimental data without which a large portion of this thesis would not exist.

Shane Blackett, Dr Richard Christie and Dr David Bullivant for developing such a wonderful canvas for us all to analyse and visualise our results. It has been invaluable for many of the figures included in this thesis. Also, many thanks to Dr Karl Tomlinson for sharing with me his vast knowledge of **CMISS** (and everything else for that matter).

I am grateful to Auckland UniServices Ltd., the Graduate Research Fund and the Bioengineering Research Group for their financial support. Without their assistance I would not have been able to attend valuable conferences in Auckland, Atlanta, Pakatoa Island, Waitangi, Waiheke, Hutchinson Island, Boston, Queenstown and Christchurch. Its tough but somebody had to do it!

My fellow students (and the best of friends) who have endured and shared the ups and downs for the past four years. The few that managed to leave, those that left but came back, and the majority that could not quite bring themselves to leave this wonderful family. Special mention to Richard ‘*Munce*’ Boyes, David ‘*Andre*’ Nickerson, Warren ‘*Java Boy*’ Hedley, Carey ‘*Bad Place*’ Stevens and Martin Buist for their valued friendship for the better part of a decade.

I also would like to thank Mum and Dad for their love and support throughout my life. Finally, to Phong, the dearest of friends. Your distractions from work have been a refreshing delight.

Good, Better, Best ...

Don't Ever Stop Until,

Good is Better, and Better is Best.

Contents

Abstract	iii
Acknowledgements	v
List of Figures	xv
List of Tables	xv
Notation	xxiii
Glossary of Symbols & Abbreviations	xxv
1 Introduction & Historical Overview	1
1.1 Cardiac Anatomy & Function	3
1.2 Electrocardiology	7
1.2.1 The Problems of Electrocardiology	8
1.2.2 Electrocardiograms	8
1.2.3 The Standard 12-lead ECG	11
1.2.4 Invasive Mapping Techniques	14
1.2.5 Body Surface Potential Mapping	15
1.2.6 Technical Requirements	16
1.3 Data Visualisation	17
1.4 Thesis Overview	19
2 The Finite Element & Boundary Element Methods	21
2.1 Mesh Division	22
2.2 One-Dimensional Basis Functions	22
2.2.1 Lagrange Basis Functions	24
2.2.2 Hermite Basis Functions	26
2.3 Two and Three Dimensional Basis Functions	28

2.3.1	Bilinear Lagrange	28
2.3.2	Bicubic Hermite	28
2.4	Sector Elements	30
2.5	Mesh Refinement	32
3	Geometric & Field Fitting with the Finite Element Method	33
3.1	Interpolation and Fitting	34
3.2	Field Fitting	35
3.2.1	Data Projection	35
3.2.2	Linear Field Fitting	36
3.2.3	Error Metrics	37
3.2.4	Sobolev Smoothing	38
3.3	Applications of Geometric and Field Fitting	42
3.3.1	Activation Time Fitting	42
3.3.2	Signal Fitting	43
3.3.3	Geometric Fitting	45
3.4	Model Customisation	48
3.4.1	Host Mesh Customisation	50
3.4.2	Heart Customisation	52
4	Geometric Torso & Cardiac Source Models	57
4.1	Previous Geometric Models for Electrophysiology	58
4.2	Model Generation	59
4.3	Generic Male Torso Model	61
4.4	Generic Porcine Model	64
4.4.1	Fitting Results	67
4.5	Material Properties	68
4.5.1	Passive Electrical Conductivities	68
4.5.2	Bidomain Conductivities	70
4.6	Cardiac Sources	70
4.6.1	Normal Ventricular Activation Sequence	72
4.6.2	Single Dipole Source	73
4.6.3	Eikonal Activation Profile	74
4.6.4	Point Stimulus Activation Profile	75
4.6.5	Experimentally Derived Activation Profile	76

5	The Forward Problem of Electrocardiology	81
5.1	Governing Equations	82
5.1.1	Generalised Laplace's Equation	83
5.1.2	Poisson Equation	84
5.1.3	Boundary Conditions	85
5.2	Coupling Regions	85
5.3	Transfer Matrices From a Boundary Element Point of View	88
5.4	Activation Function	95
5.5	Analytic Test Cases	97
5.5.1	Laplace's Equation Analytic Test Problem	98
5.5.2	Poisson Equation Analytic Test Problem	104
5.6	Convergence Analysis of Generic Model	118
5.6.1	Mesh Resolution	119
5.6.2	Potential Based Convergence	119
5.6.3	Activation Based Convergence	123
5.6.4	Convergence Summary	125
6	The Inverse Problem of Electrocardiology	127
6.1	Singular Value Decomposition	129
6.2	Epicardial Potential Formulation	131
6.2.1	Tikhonov Regularisation	131
6.2.2	Truncated SVD Regularisation	133
6.2.3	Greensite Temporal & Spatial Regularisation	134
6.2.4	Determining the Regularisation Parameter	135
6.3	Myocardial Activation Time Formulation	138
6.3.1	Determining & Refining the Activation Sequence	139
6.3.2	Estimation of the Activation Sequence	140
6.3.3	Critical Points & Times	141
6.3.4	Critical Point Function	142
6.3.5	Myocardial Activation Regularisation Constraints	145
6.3.6	Critical Point & Times Constraints	148
6.4	Control Inverse Simulations	149
6.4.1	Double Point Stimulus Activation Profile	150
6.4.2	Eikonal Activation Profile	151
6.4.3	Experimentally Derived Activation Profile	152
6.4.4	Summary for Control Inverse Simulations	153

6.4.5	Effect of Refinement Level	154
7	The Sensitivity of Inverse Solutions to Modelling Errors	155
7.1	Previous Investigations	156
7.2	Simulation Study Overview	157
7.3	Effect of Signal Errors	160
7.3.1	Effect of Uncorrelated Electrical Noise	161
7.3.2	Effect of Uncorrelated Electrode Displacement	166
7.3.3	Effect of Correlated Electrode Displacement	168
7.4	Effect of Material Property Errors	170
7.4.1	Effect of the Transmembrane Jump Magnitudes	170
7.4.2	Effect of the Width of the Activation Upstroke	171
7.4.3	Effect of Lung Masses & Conductivities	172
7.5	Effect of Geometric Errors	174
7.5.1	Effect of Heart Translation in the Lateral Directions	174
7.5.2	Effect of Heart Translation in the Posterior-Anterior Directions	176
7.5.3	Effect of Heart Translation in the Superior-Inferior Directions	178
7.5.4	Effect of Heart Rotation About the Coronal Plane	180
7.5.5	Effect of Heart Rotation About the Sagittal Plane	182
7.5.6	Effect of Heart Size	184
7.5.7	Effect of Torso Size	186
7.6	Effect of Realistic Experimental Errors	188
7.7	Comparisons Between Activation & Potential Formulations	196
7.7.1	Effect of Individual Errors	197
7.7.2	Effect of Combined Errors	199
7.7.3	General Comparisons	199
8	Conclusions, Validation & Future Developments	201
8.1	Validation of Numerical Methods	202
8.2	Future Developments	204
A	Laplace's & Poisson Equation Analytic Test Solutions	207
A.1	Convergence of Analytic Laplace's Equation Problem	207
A.2	Convergence of Analytic Poisson Equation Problem	209
A.2.1	Convergence Analysis of Potential and Normal Current	209
A.2.2	Comparison of Potential and Normal Current with Analytic Solution	210

B	Convergence Analysis of Generic Porcine Model	211
B.1	Porcine Model Surface Information	212
B.2	Forward Simulation with Dipole Source	213
B.3	Forward Simulation with an Activation Source	220
C	Inverse Simulation Results	227
C.1	Effect of Signal Errors	228
C.2	Effect of Material Properties	231
C.3	Effect of Geometric Errors	233
C.4	Typical Experimental Error	242
D	CMISS Routines & Command Files	245
D.1	Computer Systems	246
D.2	Commonly Used Routines in CMISS	247
D.2.1	General Routines	247
D.2.2	Fitting Routines	248
D.2.3	Forward Problem Routines	248
D.2.4	Inverse Problem Routines	249
D.3	Signal Fitting Comfile	250
D.4	Host Mesh Customisation Comfile	252
D.5	Geometric Fitting Comfile	254
D.6	Forward Problem With Dipole Source Comfile	256
D.7	Activation Inverse Problem Comfile	258
D.8	Activation and Potential Simulation Comfile	264
	References	273

List of Figures

1	Introduction & Historical Overview	1
1.1	Cross-section of a human heart	3
1.2	Fibre distribution on the Auckland canine heart model	4
1.3	Electrical activity, pressure and ventricular volume through the cardiac cycle	5
1.4	Relationship between the action potential, intracellular calcium and force in a ventricular cell	6
1.5	Waller’s first demonstration of non-invasive electrical recordings from a human	7
1.6	Einthoven’s triangle formed from three bipolar leads	9
1.7	Temporal relationships between a body surface ECG and a cardiac action potential	10
1.8	Traces from a standard 12-lead ECG	12
1.9	Electrode placements for a standard 12-lead ECG and the Auckland BSPM setup	13
1.10	Two-dimensional and three-dimensional potential maps	18
2	The Finite Element & Boundary Element Methods	21
2.1	Mapping between global space and local ξ space	23
2.2	Linear and quadratic Lagrange basis functions	25
2.3	Cubic Hermite basis functions	27
2.4	Schematic of a bicubic Hermite element	30
2.5	Formation of a sector elements from a collapsed quadrilateral element	31
2.6	Refinement of a bicubic Hermite element	32
3	Geometric & Field Fitting with the Finite Element Method	33
3.1	Adjusting the nodal parameters of a mesh to match data points	34
3.2	Orthogonal projection of data points onto a finite element surface	35
3.3	Effect of Sobolev smoothing on a fitted surface	41
3.4	Fitted epicardial activation field from epicardial electrodes	43
3.5	Schematic torso showing a subset of the full recording electrodes	44

3.6	Traces comparing recorded and fitted signals	46
3.7	Body surface potential map created by fitting data from 240 electrodes	47
3.8	MRI and ultrasound images with the traced heart surfaces	48
3.9	Geometric fitting of an epicardial surface to ultrasound data	49
3.10	Host mesh customisation example	51
3.11	Host mesh fitting of the generic porcine model to a specific subject	52
3.12	Ultrasound transducer with magnetic receiver	53
3.13	Three-dimensional ultrasound calibration process	55
3.14	Two-dimensional ultrasound image from the calibration process	55
3.15	Constructing a three-dimensional model from two-dimensional ultrasound images .	56
4	Geometric Torso & Cardiac Source Models	57
4.1	Generic human and canine heart models	62
4.2	Generic human male torso model	63
4.3	Digitised CT slices used to create the generic porcine model	65
4.4	Generic porcine torso model	66
4.5	Cardiac source derived from an eikonal equation model of ventricular activation . .	75
4.6	Cardiac source generated from two activation seed points	76
4.7	Experimentally recorded and fitted epicardial signals	78
4.8	Experimentally derived cardiac source	79
5	The Forward Problem of Electrocardiology	81
5.1	A transverse slice through the torso at heart level	82
5.2	Coupling regions to provide conservation of potential and current	87
5.3	Transmembrane action potential compared with the Heaviside step approximation .	95
5.4	Approximation of the transmembrane potential jump with a sigmoidal function . .	96
5.5	Schematic of Laplace's equation analytic solution	99
5.6	Convergence of potential for Laplace's equation problem	104
5.7	Convergence of normal current for Laplace's equation problem	104
5.8	Analytic potential solutions for Laplace's equation problem	105
5.9	Schematic of Poisson equation analytic solution	106
5.10	Extracellular potential convergence analysis for Poisson equation problem	115
5.11	Normal current convergence analysis for Poisson equation problem	115
5.12	Convergence of potential and normal currents for Poisson equation problem	116
5.13	Analytic extracellular potential solutions for the Poisson equation problem	117

5.14	Refinement of the endocardial surfaces	118
5.15	Schematic of electrode locations of the signals used for convergence analysis . . .	120
5.16	Torso surface signals using the dipole source	121
5.17	Potential maps of reference simulation with dipole cardiac source	122
5.18	Convergence comparison between bicubic Hermite and linear Lagrange basis functions with the dipole source	123
5.19	Torso surface signals using the eikonal source	124
5.20	Potential maps of reference simulation with eikonal cardiac source	125
6	The Inverse Problem of Electrocardiology	127
6.1	Singular value spectra from an SVD decomposition of a signal matrix	130
6.2	Generic L-curve for determining a regularisation parameter	137
6.3	Critical points and times of an activation wavefront	142
6.4	Comparison of critical point functions between critical and non-critical points . . .	144
6.5	Comparison of critical point algorithm estimates between clean and noisy signals .	145
6.6	Central difference approximation to the second derivative	147
6.7	Surface Laplacian for a two-dimensional grid	148
6.8	Activation maps of double point source inverse simulation with control conditions .	150
6.9	Activation maps of eikonal source inverse simulation with control conditions . . .	152
6.10	Activation maps of experimentally derived source inverse simulation with control conditions	153
7	The Sensitivity of Inverse Solutions to Modelling Errors	155
7.1	Schematic of the positions of the electrodes illustrated in Figure 7.2	162
7.2	Simulated signal traces with Gaussian noise	163
7.3	Effect of Gaussian electrical noise on inverse solutions	165
7.4	Effect of uncorrelated electrode displacement on inverse solutions	167
7.5	Effect of correlated electrode displacement on inverse solutions	169
7.6	Effect of the transmembrane jump magnitude on the activation based formulation .	170
7.7	Effect of the width of the activation upstroke on the activation based formulation .	171
7.8	Effect of lung conductivities on inverse solutions	173
7.9	Effect of lateral heart translations on inverse solutions	175
7.10	Effect of posterior-anterior heart translations on inverse solutions	177
7.11	Effect of superior-inferior heart translations on inverse solutions	179
7.12	Effect of heart rotations about the coronal plane on inverse solutions	181

7.13	Effect of heart rotations about the sagittal plane on inverse solutions	183
7.14	Effect of heart size on inverse solutions	185
7.15	Effect of torso size on inverse solutions	187
7.16	Activation maps of inverse simulation with a double point source and realistic errors	190
7.17	Effect of realistic errors with double point source	191
7.18	Activation maps of inverse simulation with eikonal source and realistic errors . . .	192
7.19	Effect of realistic errors with eikonal source	193
7.20	Activation maps of inverse simulation with experimentally derived source and realistic errors	194
7.21	Effect of realistic errors with experimentally derived source	195
7.22	Comparison between activation times and those derived from potentials	197

List of Tables

1	Introduction & Historical Overview	1
1.1	Electrode placements for a standard 12-lead ECG	12
1.2	Mapping system setups from different research groups	17
2	The Finite Element & Boundary Element Methods	21
3	Geometric & Field Fitting with the Finite Element Method	33
3.1	Effects of Sobolev smoothing on a fitted surface	42
3.2	Comparison measures between recorded and fitted signals	45
4	Geometric Torso & Cardiac Source Models	57
4.1	Fitting results for generic male torso model	62
4.2	Fitting results for generic porcine model	67
4.3	Summary of passive material conductivity values	69
4.4	Summary of bidomain conductivity values	70
5	The Forward Problem of Electrocardiology	81
6	The Inverse Problem of Electrocardiology	127
6.1	Double point source inverse simulation with control conditions	151
6.2	Eikonal source inverse simulation with control conditions	151
6.3	Experimental source inverse simulation with control conditions	153
7	The Sensitivity of Inverse Solutions to Modelling Errors	155
7.1	Noise levels applied to four simulated electrodes	161
7.2	Heart scaling parameters	184
7.3	Torso scaling parameters	186

7.4	Comparison between activation times and those derived from corresponding potentials	196
7.5	Summary of preferred inverse algorithms for typical levels of modelling error . . .	198
A	Analytic Test Solutions	207
A.1	Convergence analysis of potential for Laplace's equation problem	208
A.2	Convergence analysis of current for Laplace's equation problem	208
A.3	Convergence analysis of the extracellular potential for Poisson equation problem .	209
A.4	Convergence analysis of the normal current distributions for Poisson equation problem	209
A.5	Convergence of extracellular potential distributions to the Poisson equation problem	210
A.6	Convergence of current distributions to the Poisson equation problem	210
B	Convergence Analysis of Generic Porcine Model	211
B.1	Mesh statistics for porcine model with different refinement levels	212
B.2	Convergence of epicardial surface by examining the epicardial potentials	214
B.3	Convergence of epicardial surface by examining the torso potentials	215
B.4	Convergence of the lung surfaces by examining the epicardial potentials	216
B.5	Convergence of the lung surfaces by examining the torso potentials	216
B.6	Convergence of torso surface by examining the epicardial potentials	217
B.7	Convergence of torso surface by examining the torso potentials	217
B.8	Convergence of epicardial surface by examining the epicardial potentials with linear interpolation	218
B.9	Convergence of epicardial surface by examining the torso potentials with linear interpolation	219
B.10	Convergence of the epicardial surface by examining the epicardial potentials	221
B.11	Convergence of the epicardial surface by examining the torso potentials	221
B.12	Convergence of the left ventricular surface by examining the epicardial potentials .	222
B.13	Convergence of the left ventricular surface by examining the torso potentials	222
B.14	Convergence of the right ventricular surface by examining the epicardial potentials	223
B.15	Convergence of the right ventricular surface by examining the torso potentials . . .	223
B.16	Convergence of the lung surfaces by examining the epicardial potentials	224
B.17	Convergence of the lung surfaces by examining the torso potentials	224
B.18	Convergence of the torso surface by examining the epicardial potentials	225
B.19	Convergence of the torso surface by examining the torso potentials	225

C	Inverse Simulation Results	227
C.1	Effect of Gaussian electrical noise on RMS error metric	228
C.2	Effect of Gaussian electrical noise on the SI error metric	228
C.3	Effect of Gaussian electrode displacement on the RMS error metric	229
C.4	Effect of Gaussian electrode displacement on the SI error metric	229
C.5	Effect of correlated electrode displacement on the RMS error metric	230
C.6	Effect of correlated electrode displacement on the SI error metric	230
C.7	Effect of transmembrane jump on the RMS error metric	231
C.8	Effect of transmembrane jump on the SI error metric	231
C.9	Effect of the width of the activation upstroke on the RMS error metric	231
C.10	Effect of the width of the activation upstroke on the SI error metric	231
C.11	Effect of lung conductivities on the RMS error metric	232
C.12	Effect of lung conductivities on the SI error metric	232
C.13	Effect of heart translation in the lateral directions on the RMS error metric	233
C.14	Effect of heart translation in the lateral directions on the SI error metric	233
C.15	Effect of heart translation in the posterior-anterior directions on the RMS error metric	234
C.16	Effect of heart translation in the posterior-anterior directions on SI error metric . . .	234
C.17	Effect of heart translation in the superior-inferior directions on the RMS error metric	235
C.18	Effect of heart translation in the superior-inferior directions on the SI error metric . .	236
C.19	Effect of heart rotation in the sagittal plane on the RMS error metric	237
C.20	Effect of heart rotation in the sagittal plane on the SI error metric	238
C.21	Effect of heart rotation in the coronal plane on the RMS error metric	239
C.22	Effect of heart rotation in the coronal plane on the SI error metric	239
C.23	Effect of heart size on the RMS error metric	240
C.24	Effect of heart size on the SI error metric	240
C.25	Effect of torso size on the RMS error metric	241
C.26	Effect of torso size on the SI error metric	241
C.27	Typical experimental errors with double point source on the RMS error metric	242
C.28	Typical experimental errors with double point source on the SI error metric	242
C.29	Typical experimental errors with eikonal source on the RMS error metric	243
C.30	Typical experimental errors with eikonal source on the SI error metric	243
C.31	Typical experimental errors with experimentally derived source on the RMS error metric	244
C.32	Typical experimental errors with experimentally derived source on the SI error metric	244

Notation

- Mathematical variables represented by bold lower case symbols are vector quantities (*e.g.*, \mathbf{x}) and bold upper case symbols refer to tensor quantities (*e.g.*, \mathbf{X}).
- Einstein summation is used, where repeated indices imply summation over the individual components. For example a vector dot product may be written as

$$a_i b_i = \mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^N a_i b_i$$

where N is the length of the vector.

- An inner product of two column vectors is denoted by \langle , \rangle .

Glossary of Symbols & Abbreviations

Scalar Symbols

Symbol	Description
a	resting potential
b	transmembrane potential jump
d	data point number
D	total number of data points
e	element number
h	characteristic element size
n	local node number
N	global node number or full rank of matrix
s_e	element based scale factors
S_n	nodal based scale factors
t	time
T	total number of time steps
u_n	dependant variable at local node n
α	Sobolev weighting on the first derivative terms
β	Sobolev weighting on the second derivative terms
ϕ_e	extracellular potential
ϕ_i	intracellular potential
ϕ_m	transmembrane potential
ξ	local element coordinates
λ	regularisation parameter
λ_t	regularisation parameter at time t
ω	activation window
ε	an unspecified basis function
φ_n	Lagrange basis function
Ψ_n^d	Hermite basis function
ζ_n	Hermite sector basis function collapsed at local node 1
η_n	Hermite sector basis function collapsed at local node 3
\mathcal{L}_n	surface Laplacian at node n

Vector & Matrix Symbols

Symbol	Description
a	column of transfer matrix
A	transfer matrix
$A_{\lambda_i}^\dagger$	pseudo inverse of transfer matrix regularised at i
B	magnetic induction
d	dipole source vector
D	electric displacement
E	electric field intensity
E_1	Gaussian noise
E_2	correlated noise
G	first-order Tikhonov regularisation matrix (surface gradient)
H	magnetic field intensity
I	zero-order Tikhonov regularisation matrix (identity matrix)
J	current density
J_i	impressed source current density
L	second-order Tikhonov regularisation matrix (surface Laplacian)
n	unit outward normal
P	boundary element global potential matrix
q	flux vector
Q	boundary element global flux matrix
R	reduced global matrix or Tikhonov regularisation matrix
s	solution vector
U	spatial SVD decomposition of a matrix
V	temporal SVD decomposition of a matrix
x	node on heart surface
y	node on torso surface
z_d	global position of data point d
σ_e	extracellular conductivity tensor
σ_i	intracellular conductivity tensor
$\hat{\phi}_B$	calculated torso potentials from a cardiac source
ϕ_B	measured torso potentials
τ	activation field on the heart
Γ	surface mesh domain
Ω	mesh domain
Φ	signal data matrix
Σ	singular values from an SVD

Abbreviations

Abbreviation	Description
AHA	American Heart Association
ARI	Activation Recovery Interval
AT	Activation Time
AV node	Atrioventricular node
BEM	Boundary Element Method
BSM	Body Surface Map
BSPM	Body Surface Potential Mapping
CMISS	Continuum Mechanics, Image analysis, Signal processing and System identification
CPA	Critical Point Algorithm
CT	Computed Tomography
ECG	ElectroCardioGram
FEM	Finite Element Method
LA	Left Atria
LV	Left Ventricle
MRI	Magnetic Resonance Imaging
RA	Right Atria
RV	Right Ventricle
SA node	Sinoatrial node
UnEmap	Universal Electrophysiological Mapping System
WCT	Wilson Central Terminal