Non-Invasive Electrical Imaging of the Heart

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Abstract

Non-invasive electrical imaging of the heart aims to quantitatively reconstruct information about the electrical activity of the heart from multiple thoracic ECG signals. The computational framework required to produce such electrical images of the heart from non-invasive torso surface signals is presented. It is shown reliable electrical images of the heart can be obtained under a controlled environment. This has been demonstrated using an anatomically realistic boundary element porcine torso model.

The procedures required to create a subject specific model using a small number of control points and to create a specific heart model from three-dimensional ultrasound images using a linear fitting procedure are presented. From discrete ECG electrodes a continuous representation of the potential field over the entire torso surface can also be produced using this linear fitting procedure.

The construction of the transfer matrices for the two predominant electrocardiographic sources (epicardial potentials and myocardial activation times) are described in detail. The transfer matrices are used to compute activation times within the heart and epicardial potentials on the heart surface. Myocardial activation times are computed using an algorithm based on the Critical Point Theorem while epicardial potentials are computed using standard Tikhonov and Truncated SVD spatially regularised methods as well as Greensite's spatial and temporal regularisation method. The regularisation parameters for the epicardial potentials are determined using a variety of methods (*e.g.*, CRESO criterion, L-curve, zero-crossing).

The potential and activation based formulations are compared in a comprehensive inverse simulation study. To try and capture the dynamic and variable nature of cardiac electrical activity, the study is performed with three different types of cardiac sources with a realistic porcine model. These simulations investigate the effect on the computed solutions of individual and combinations of modelling errors. These errors include corruption in the torso surface signals, changes in material properties and geometric distortion.

In general, the activation based formulation is preferred over the epicardial potential formulations, with Greensite's method found to be the best method for reconstructing epicardial potentials. Under optimal conditions, the activation approach could reconstruct the activation times to within $4\ \mathrm{ms}$ RMS. Both potential and activation based formulations were found to be relatively insensitive to changes in material properties such as lung conductivities and activation function shapes. When

examining individual errors, the geometry and positions of the torso and heart had the greatest effects on the inverse solutions. The relative heart position needed to be determined to within $5\,\mathrm{mm}$ to obtain results within $2\,\mathrm{ms}$ of the solutions obtained under control conditions. When the modelling errors are combined to produce errors which can be expected in a clinical or experimental situation the activation based solutions were consistently more accurate than potential based solutions.

The next necessary step in this project is the detailed validation of the results against *in-vivo* data. This step is necessary before such algorithms can be reliably used to aid in the assessment of heart function in a clinical environment.

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Good, Better, Best ...

Don’t Ever Stop Until,

Good is Better, and Better is Best.

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Notation

- Mathematical variables represented by bold lower case symbols are vector quantities (*e.g.*, $\boldsymbol{x}$) and bold upper case symbols refer to tensor quantities (*e.g.*, $\boldsymbol{X}$).
- Einstein summation is used, where repeated indices imply summation over the individual components. For example a vector dot product may be written as

$$a_i b_i = \boldsymbol{a} \cdot \boldsymbol{b} = \sum_{i=1}^{N} a_i b_i$$

  where N is the length of the vector.
- An inner product of two column vectors is denoted by $\langle\ ,\ \rangle$.

Glossary of Symbols & Abbreviations

Scalar Symbols

Symbol	Description
a	resting potential
b	transmembrane potential jump
d	data point number
D	total number of data points
e	element number
h	characteristic element size
n	local node number
N	global node number or full rank of matrix
s_e	element based scale factors
S_n	nodal based scale factors
t	time
T	total number of time steps
u_n	dependant variable at local node n
α	Sobolev weighting on the first derivative terms
β	Sobolev weighting on the second derivative terms
ϕ_e	extracellular potential
ϕ_i	intracellular potential
ϕ_m	transmembrane potential
ξ	local element coordinates
λ	regularisation parameter
λ_t	regularisation parameter at time t
ω	activation window
ε	an unspecified basis function
φ_n	Lagrange basis function
Ψ_n^d	Hermite basis function
ζ_n	Hermite sector basis function collapsed at local node 1
η_n	Hermite sector basis function collapsed at local node 3
$\mathfrak{L}_n$	surface Laplacian at node n

Vector & Matrix Symbols

Symbol	Description
$\boldsymbol{a}$	column of transfer matrix
$\boldsymbol{A}$	transfer matrix
$\boldsymbol{A}^{\dagger}_{\lambda_i}$	pseudo inverse of transfer matrix regularised at i
$\boldsymbol{B}$	magnetic induction
$\boldsymbol{d}$	dipole source vector
$\boldsymbol{D}$	electric displacement
$\boldsymbol{E}$	electric field intensity
$\boldsymbol{E}_1$	Gaussian noise
$\boldsymbol{E}_2$	correlated noise
$\boldsymbol{G}$	first-order Tikhonov regularisation matrix (surface gradient)
$\boldsymbol{H}$	magnetic field intensity
$\boldsymbol{I}$	zero-order Tikhonov regularisation matrix (identity matrix)
$\boldsymbol{J}$	current density
$\boldsymbol{J}_i$	impressed source current density
$\boldsymbol{L}$	second-order Tikhonov regularisation matrix (surface Laplacian)
$\boldsymbol{n}$	unit outward normal
$\boldsymbol{P}$	boundary element global potential matrix
$\boldsymbol{q}$	flux vector
$\boldsymbol{Q}$	boundary element global flux matrix
$\boldsymbol{R}$	reduced global matrix or Tikhonov regularisation matrix
$\boldsymbol{s}$	solution vector
$\boldsymbol{U}$	spatial SVD decomposition of a matrix
$\boldsymbol{V}$	temporal SVD decomposition of a matrix
$\boldsymbol{x}$	node on heart surface
$\boldsymbol{y}$	node on torso surface
$\boldsymbol{z}_d$	global position of data point d
$\boldsymbol{\sigma}_e$	extracellular conductivity tensor
$\boldsymbol{\sigma}_i$	intracellular conductivity tensor
$\hat{\boldsymbol{\phi}}_{\boldsymbol{B}}$	calculated torso potentials from a cardiac source
$\boldsymbol{\phi}_{\boldsymbol{B}}$	measured torso potentials
$\boldsymbol{\tau}$	activation field on the heart
Γ	surface mesh domain
Ω	mesh domain
Φ	signal data matrix
$\boldsymbol{\Sigma}$	singular values from an SVD

Abbreviations

Abbreviation	Description
AHA	American Heart Association
ARI	Activation Recovery Interval
AT	Activation Time
AV node	Atrioventricular node
BEM	Boundary Element Method
BSM	Body Surface Map
BSPM	Body Surface Potential Mapping
CMISS	Continuum Mechanics, Image analysis, Signal processing and System identification
CPA	Critical Point Algorithm
CT	Computed Tomography
ECG	ElectroCardioGram
FEM	Finite Element Method
LA	Left Atria
LV	Left Ventricle
MRI	Magnetic Resonance Imaging
RA	Right Atria
RV	Right Ventricle
SA node	Sinoatrial node
UnEmap	Universal Electrophysiological Mapping System
WCT	Wilson Central Terminal