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White Light Feedback Interferometry for Aberration Correction, and Near-Real-Time Phase-Difference Amplification

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Abstract

There has recently been much interest in the use of adaptive optics systems for automatic aberration correction in applications such as astronomical and line-of-sight imaging. The background of aberration correction is presented, reviewing applications of adaptive optics systems, and commonly used wavefront sensing and correcting techniques. Aberration correction systems use a variety of correction elements including liquid crystal devices and segmented mirrors. Clearly, characteristics such as response time, phase vs. drive voltage, and sensitivity to wavelength and polarisation state of input light are important to system performance. The characteristics of two liquid crystal spatial light modulators, pertinent to aberration correction, are initially investigated.

Research on a novel white light feedback interferometric technique that can be used for aberration correction is then presented. Using this technique, a phase proportional to the interferometer output intensity is applied to a modulator within the interferometer to achieve automatic phase correction. The interferometer operates in white light. A theoretical analysis of white light feedback interferometry is given along with simulated and experimental results. The results show that aberration correction can, in principle, be achieved using the technique.

Liquid crystal spatial light modulators suffer from relatively slow response times and a limited phase modulation range. Segmented mirrors have much faster response times, and a much larger phase modulation range, but are prohibitively expensive. Therefore, we decided to construct a 3x3 segmented mirror for use in aberration correction systems. The construction, testing, housing and mounting of the segmented mirror purpose built for aberration correction is then discussed.
Phase-difference amplification is a technique that can be used for high accuracy phase measurement. Using this technique a diffraction grating is constructed using the interferogram of a test wavefront. The ±n orders diffracted from the grating are interfered together to form a phase-amplified interferogram from which a phase map of the test wavefront can be recovered. In the future, phase-difference amplification may be a very useful technique for aberration correction systems. Conventional systems use time consuming and inconvenient photographic techniques to construct the grating. The second part of this thesis describes several phase-difference amplification systems where the diffraction grating is written on a liquid crystal spatial light modulator. The liquid crystal devices, which have a high resolution and much faster response times than the photographic development time, make the systems convenient and allow near-real-time operation. A phase map of the test object is retrieved using phase stepping techniques. Experimental results for each system are presented and compared. They show that amplification factors of up to 6 are easily achievable.
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Contents

Abstract ........................................................................................................ iii
Acknowledgements .................................................................................. v
Contents .................................................................................................... vii
Chapter 1. Introduction ............................................................................. 1
Chapter 2. Background of Adaptive Optics .............................................. 5
  2.1 Introduction......................................................................................... 5
  2.2 Applications of Adaptive Optics Systems ......................................... 5
    2.2.1 Atmospheric Aberration Correction........................................... 3
    2.2.2 Imaging Through Fibre Optic Cables ....................................... 6
    2.2.3 Laboratory Based Systems ....................................................... 7
    2.2.4 High Power Lasers ................................................................ 7
    2.2.5 Communication and Imaging Through Scattering or Turbulent Media ........................................................................ 8
  2.3 Phase Aberrations............................................................................. 8
    2.3.1 Atmospheric and Seawater Phase Aberrations.............................. 8
    2.3.2 Phase Aberrations in Fibres ...................................................... 10
  2.4 Wavefront Sensing Techniques ...................................................... 10
    2.4.1 Shack-Hartmann Wavefront Sensor .......................................... 10
      2.4.1.1 Quad Cell Detector .......................................................... 11
      2.4.1.2 Charged Coupled Device Detector .................................... 12
      2.4.1.3 Position-Sensitive Detector ............................................. 13
    2.4.2 Lateral Shearing Interferometer Techniques ............................... 15
      2.4.2.1 Murty Plate Lateral Shear Interferometer ............................ 15
      2.4.2.2 Ronchi Grating Lateral Shear Interferometer ...................... 16
      2.4.2.3 Double Frequency Grating Lateral Shear Interferometer ...... 17
      2.4.2.4 Double Grating Interferometer with Variable Lateral Shear ... 19
    2.4.3 Radial Shearing Interferometer Technique ................................ 20
      2.4.3.1 Radial Shearing Mach-Zehnder Interferometer ................... 21
2.4.3.2 Radial Shearing Sagnac Interferometer

2.4.4 Heterodyne Interferometers

2.5 Wavefront Correcting Techniques

2.5.1 Segmented, Deformable, and Bimorph Mirrors

2.5.2 Deformable Membrane Mirrors

2.5.3 Texas Instruments Deformable Mirror Spatial Light Modulator

2.5.4 Liquid Crystal Television

2.5.5 Other Liquid Crystal Devices

2.5.6 LiNbO3 Crystal Microchannel Spatial Light Modulator

2.5.7 The Hamamatsu PAL-Spatial Light Modulator

2.6 Conclusion

Chapter 3. Practical Issues for Liquid Crystal Spatial Light Modulators in Aberration Correction

3.1 Introduction

3.2 Overview of Experiments

3.3 Requirements for Aberration Correction

3.4 Description of the two LC-SLMs used in these Experiments

3.4.1 LC-SLM with 81 Pixels

3.4.2 Meadowlark LC-SLM

3.5 Experiment 1: Wavelength Dependence of 81 Pixel LC-SLM

3.5.1 Interferometer for Wavelength Dependence Experiment

3.5.2 Phase Retrieval

3.5.3 Wavelength Dependence Results

3.6 Experiment 2: Polarisation Sensitivity of 81 pixel LC-SLM

3.6.1 Rayleigh Interferometer Experimental Setup

3.6.2 Experimental Results for 81 segment LC-SLM configured to be Insensitive to Polarisation State

3.6.3 Variation of the System Configured to be insensitive to Polarisation State, with Wavelength

3.7 Experiment 3: Optical Losses of Meadowlark LC-SLM configured to be Insensitive to Polarisation State of Incident Light

3.7.1 Experimental Setup using Meadowlark LC-SLM Phase Wedge

3.7.2 Phase Wedge Simulation

3.7.3 Experimental Results using Meadowlark LC-SLM Phase Wedge

3.8 Experiment 4: LC-SLM Time Response

3.8.1 Introduction

3.8.2 Transient Nematic effect

3.8.3 Dual Frequency Control Technique

3.8.4 Conventional Time Response

3.8.4.1 Common Path Interferometer to determine LC-SLM Response Time
Chapter 1. Introduction

Aberration correction is the process of measuring and correcting spatial phase aberrations on a light beam. Conventionally this is done using two separate systems, a measurement system and a correction system. Feedback interferometry is a technique in which a phase proportional to the output intensity is applied to a phase modulator within the interferometer. This technique can be used for aberration correction where correction happens as direct result of the detection process, negating the need for separate detection and correction systems.

For many aberration correction techniques it is necessary to measure the phase distribution of the input wave accurately and quickly. Phase-difference amplification is a technique that has potential for use in fast, high accuracy phase measurement. Using this technique, a diffraction grating is constructed from the interferogram of the wavefront under test. The ±n diffracted orders are selected from a plane wave incident upon the diffraction grating and are interfered together. This gives an interference pattern with a phase that is amplified by a factor of 2n.

This thesis is written in two sections. The first section comprises Chapters 2 to 6. In these chapters, issues dealing with aberration correction and white light feedback interferometry are covered. The second section comprises Chapters 7 and 8, where work on phase-difference amplification is presented. The work is a continuation of work done for my Masters thesis on aberration correction using feedback interferometry in monochromatic light. In that work it was shown that a feedback interferometer significantly increased the quality of a plane wave by reducing phase aberrations.

The background of aberration correction is covered in Chapter 2. Applications of adaptive optics systems, then wavefront sensing and correcting techniques that are commonly used are reviewed. Liquid crystal spatial light modulators (LC-SLMs) have recently been
considered for use in many optical information processing applications including adaptive aberration correction systems. Chapter 3 contains work performed on characterising the performance of two LC-SLMs. Experiments and simulations investigating the wavelength dependence of the modulation characteristic, the performance when configured to be insensitive to polarisation state of light, and the time response of the devices are described.

The general principle, theory, and background of feedback interferometry is presented in Chapter 4, before the theory is expanded to include white light in Chapter 5. Chapter 5 also describes experiments carried out using two different feedback interferometers. The first system used two piezo driven mirrors, one to vary the interferometer phase and the other to provide feedback. The second system used two pixels from a LC-SLM to modulate the interferometer and feedback phase in a white light radial shearing interferometer. Results from these experiments show that in principle, aberration correction is possible using white light interferometry and a LC-SLM. Coherence requirements of a white light radial shearing interferometer are then briefly considered.

Benefits of using liquid crystal spatial light modulators are that they are cheap, reliable and easy to use. They do however have some drawbacks. Their response time is relatively slow, and they are limited to about $1.5\lambda$ phase modulation range at 633nm. To overcome these problems, a segmented mirror with 9 segments was built. The design, construction and testing of the segmented mirror which used piezo actuators to drive each segment is presented in Chapter 6. The mirror was constructed by Industrial Research Ltd. in Lower Hutt.

Chapters 7 and 8 contain work done on phase-difference amplification. This technique has the potential to improve aberration correction systems which require fast accurate phase distribution measurement. In conventional phase-difference amplification techniques the diffraction grating is constructed using a photographic plate. The photographic process is time consuming and requires a new plate for each amplification measurement. Work that is presented in Chapter 3 indicated that LC-SLM's would be ideal for use as the diffraction grating in this type of application. An interferogram of a test wavefront may be displayed directly on the LC-SLMs which can be re-used, and have response times that would allow operation in near-real-time.
The background, principle and theory of phase amplification are presented in Chapter 7. In Chapter 8, three phase-difference amplification experiments are discussed. The optical apparatus used for all the experiments contained two parts: a test interferometer, which provided the interferogram of the test object displayed as a grating, and the amplification system, which interfered $\pm n$ orders diffracted from the grating to obtain the output phase-amplified interferogram. Phase stepping techniques were used to reconstruct the phase map of the object. The first experiment was proof-of-principle only, and used a simulated fringe pattern on a liquid crystal TV as the diffraction grating. The second and third experiments used an optically addressed LC-SLM in different interferometric setups. Results from each system are compared and show that phase-difference amplification factors of up to 6 are easily obtainable in near-real-time.
Chapter 2. Background of Adaptive Optics

2.1 Introduction

Recently there has been considerable interest in adaptive optics systems for real-time aberration correction. This chapter describes some applications of these systems, and presents some of the wavefront sensing and correcting techniques that have been used.

2.2 Applications of Adaptive Optics Systems

Adaptive optics systems are used for applications where information is sent and received using light. Aberrations are removed from the light by the adaptive optics system to minimise corruption of the data being transmitted. This section outlines some of the more common applications.

2.2.1 Atmospheric Aberration Correction

Turbulence and large temperature gradients, cause considerable refractive index variation in the atmosphere. Images of astronomical objects viewed through a telescope are blurred by aberrations introduced by these refractive index variations as the light travels through the earth's atmosphere. Adaptive optics systems are becoming more common on telescopes\(^1\) to remove the aberrations so that smaller, more distant objects can be viewed at higher resolution and image quality without recourse to expensive space based telescopes.

The requirements for coherent earth-space communications at optical frequencies instead of conventional radio frequencies have also been discussed\(^2\). It was suggested that these
communications systems might use phased telescope arrays with concomitant signal to noise ratio and cost benefits over a single, large telescope\(^\text{[3]}\). Aberrations introduced by the atmosphere could be removed by adaptive optics systems to further increase the efficiency of the arrays.

Long distance imaging\(^{[4,5]}\) and communication\(^{[6]}\) within the atmosphere, for example between ships and land based observers, could also be improved with the application of adaptive optics. There is considerable military interest in these applications because of the implications for improved far field surveillance and intelligence work.

### 2.2.2 Imaging Through Fibre Optic Cables

Imaging via fibre optics is also important in medicine, industrial inspection and similar fields. Commercially available imaging systems use fibre bundles, where each fibre in the bundle forms a single pixel\(^{[7]}\). However, this method requires many fibres and the fibre ends must be arranged coherently with respect to each other in order to correctly reconstruct the image.

A technique was also proposed\(^{[8]}\), and demonstrated\(^{[9,10]}\), to double pass an image through a single step-index multimode fibre. The modally dispersed image resulting from a single pass down the fibre was reflected back down the same fibre in the opposite direction by a phase conjugate mirror. The modal dispersion on the return pass compensated for the dispersion experienced on the initial pass, so that the original image was restored. In this scheme, however, the image ends up at the same place it started from. If the fibre could be duplicated exactly, then it may be possible to send an image half way to its destination down one fibre, conjugate its phase, then send it the remaining distance down the duplicate fibre oriented in the opposite direction to compensate for the dispersion. However, exact matching of the modal dispersion would be almost impossible and it is highly unlikely that this scheme could be made into a practical system for image transmission down a single fibre.

It would be simpler to transmit the image the whole way through a single fibre, then use an adaptive optics system to correct the dispersion. This would eliminate the need for both the phase conjugate mirror and the exact fibre duplicate.
Image transmission through a single graded index (GRIN) fibre has also been demonstrated\textsuperscript{[11]}. However, experiments show that images can be propagated over short distances only. GRIN fibres, with a quadratic refractive index distribution, show the following characteristics.

- Low mode dispersion so that rays propagate with a characteristic periodicity. This has the result that the image projected on to the input end of the fibre reforms every few mm along its length.

- If the output end of the fibre is cut smoothly at the correct place, the reformed image may be projected through a lens, on to a screen.

However, it is very difficult to construct fibres without residual dispersion and so images are usually corrupted after only short distances in the fibre. Longer propagation distances may be possible with the application of an adaptive optics system in addition to the GRIN fibre, to correct the effects of dispersion.

\textbf{2.2.3 Laboratory Based Systems}

Air turbulence due to heating and air-conditioning units, building vibration, and imperfect optical equipment, all detrimentally affect optical systems in the laboratory. Because of these problems, it has up to now been necessary to take great care with laboratory placement and stabilisation. The application of adaptive optics systems to laboratory optical systems would reduce these effects and reduce environmental control requirements.

\textbf{2.2.4 High Power Lasers}

High power lasers are used for cutting and drilling industrial materials\textsuperscript{[12]}. They are susceptible to housing expansion, and cavity length and mirror shape variation through heating. In order to maximise cutting power it is necessary to minimise waist size, and position the beam waist precisely. This is difficult in an uncontrolled industrial environment. Energy savings and other efficiencies would result from the addition of an adaptive optics system to maintain the size and position of the output beam waist. This has been attempted with deformable resonator mirrors, and intracavity induced thermal lenses\textsuperscript{[13]}. However, because of the enormous power in these lasers (often 1 kW or more\textsuperscript{[12]})
system components must be robust and clean. Sufficiently robust adaptive optics systems are difficult to make and are therefore rarely used in high power applications.

2.2.5 Communication and Imaging Through Scattering or Turbulent Media

Radio waves penetrate only a few centimetres through water. Underwater communication must therefore be carried out using alternative methods. Conventionally, underwater communication has been performed using acoustic sources\cite{14}. Momma and Tsuchiya\cite{15} however, proposed a communication method that used electric current. They showed two technically feasible voice communication systems. One that had a range of 150m using 6W of power and another that had a range of 1km using 280W. The disadvantage of this method was the large power consumption.

Work on underwater communication and imaging through sea water to detect mines using light, has also been proposed\cite{16}. Attenuation due to scattering and absorption is extremely high in seawater, but is least at blue and green wavelengths. Turbulence and temperature gradients also make accurate imaging difficult. The benefits of an adaptive optics system have also been discussed for this situation\cite{17}.

2.3 Phase Aberrations

Information is carried by light through inhomogeneous media in all of the above examples except for the case of light travelling through an optical fibre.

2.3.1 Atmospheric and Seawater Phase Aberrations

In the astronomical case, light travels through turbulence and thermal gradients which continuously vary the refractive index of the earth's atmosphere. Light travelling through seawater experiences a similar variation of refractive index but in this case there is more scattering and attenuation. In the atmospheric and seawater examples, the medium introduces phase aberrations as illustrated in Figure 2-1.
Adaptive optics systems attempt to correct these aberrations, making the wavefronts plane again, as shown in Figure 2-2.

Aberration correction does not increase the total amount of energy coming from the source, but increases the proportion of light contributing to the useable image produced by the system.

Aberration correction is conventionally carried out in two steps.

1. Sensing of the wavefront from a reference object, close to the object being imaged by the system. Here, the phase errors (or spatial phase distortion) introduced into the wavefront from the test object by the aberrating medium are determined.

2. Wavefront correction, during which the measured aberrations are corrected in the wavefronts from both the test and the object being imaged.

Figure 2-3 shows the aberration correction system split into these two steps.
There are many techniques used for both wavefront aberration detection and correction.

2.3.2 Phase Aberrations in Fibres

The aberrations experienced by images transmitted through multimode optical fibres are different. Here the images are corrupted by mode scrambling in the fibre (the time it takes light in each excited mode to travel down the fibre is different) and mode interference at the output of the fibre.

It has been shown\(^{18}\) that an image that has travelled through a single step-index multimode fibre can be restored. This was achieved by diffracting the fibre output from a composite hologram. The composite hologram consisted of several independent component holograms that were superimposed on the same photographic recording medium. Each hologram was a record of a spread function which belongs to a certain input point of the fibre. However, the spatial resolution required to perform this type of correction using currently available aberration correction devices is far too great.

2.4 Wavefront Sensing Techniques

Several techniques that have been used for wavefront sensing, are described below.

2.4.1 Shack-Hartmann Wavefront Sensor

A Shack-Hartmann wavefront sensor\(^{19}\) is shown in Figure 2-4. The Shack-Hartmann wavefront sensor consists of a two dimensional array of small refractive or diffractive lenses, each of which focuses light from an area of the input wavefront to a point on a
detector. When a plane light wavefront is incident normally on the array, each lens will produce a focused spot on its own optical axis.

![Diagram of Shack-Hartmann wavefront sensor]

Figure 2-4 Shack-Hartmann wavefront sensor

However, when the incident wavefront is aberrated, i.e. has a non-uniform tilt variation across the lens array, the focused spot produced by each lens is displaced from the optical axis. For each lens, the magnitude and direction of the displacement of each spot gives information about the average tilt aberration of the incident wavefront across the aperture of that lens. The wavefront incident on the whole system may be reconstructed from this information.

The shape of the aberrated wavefront is usually obtained through extensive calculation using matrix multiplication, collating the information from all detectors in the array. For atmospheric aberration correction it is necessary that this process be carried out up to several hundred times per second so system speed is also an important issue. Most adaptive optics systems currently use a Shack-Hartmann wavefront sensor\(^{20}\).

### 2.4.1.1 Quad Cell Detector

Quad cell detectors are often used to detect the spot positions\(^{21}\). Figure 2-5 shows the principle of operation of this type of detector. By detecting the intensity incident on each sub-cell of the quad it is possible to estimate how far off-axis the centre of gravity of the spot lies. The quad cell on the bottom right of Figure 2-5 for example, would detect almost the same intensity on each cell because the spot is nearly centred.
In each quad cell, the position of the centre of gravity of the spot may be determined from,

\[
\bar{x} = \frac{\int_1fds + \int_2fds - \int_3fds - \int_4fds}{\sum_i fds},
\]

\[
\bar{y} = \frac{\int_1fds + \int_2fds - \int_3fds - \int_4fds}{\sum_i fds}.
\]

2.4.1.2 Charged Coupled Device Detector

CCD (Charged Coupled Device) sensors are also commonly used. Figure 2-6 shows a close up of this type of device which has a large array of small photodiodes (pixels), instead of four large ones as in the quad cell.

The spot is focused on to the CCD array. The centre of gravity of each spot is again calculated from the pixel intensity distribution.
2.4.1.3 Position-Sensitive Detector

Another position sensor is the very high resolution Position-Sensitive Detector (PSD) manufactured by Hamamatsu\cite{22}. An expensive alternative to the discrete element CCD, the PSD, which is an analogue device, provides high position resolution and a fast response time.

![Diagram of a Position-Sensitive Detector](image)

**Figure 2-7 Position sensitive detector**

Figure 2-7 shows a cross section through the PSD, showing the principle of operation in one dimension. The PSD consists of a P-layer with two electrodes, and an N-layer with one electrode, separated by an insulating layer. A focused spot of light on the P-layer creates an electric charge proportional to the intensity of the light. The photocurrents $I_1$ and $I_2$ collected by the two electrodes, arise from charge transport through the P-layer which has uniform resistance. $I_1$ and $I_2$ are therefore inversely proportional to the distance between the position of the spot and each electrode. The following equations are obtained for the photocurrents $I_1$ and $I_2$.

\[
I_1 = I_0 \frac{L-x}{2L}, \quad I_2 = I_0 \frac{L+x}{2L}, \quad \text{and} \quad I_0 = I_1 + I_2.
\]

Here: $I_0$ is the total photocurrent;

$L$ is half the distance between electrodes;

$x$ is the distance from the center of the PSD.

These can be combined to give the position of the incident light spot as
Position Sensitive Detectors come in many configurations. One dimensional detectors are the most common (they are used in camera range finders), but Shack-Hartmann and other position sensing applications require two dimensional detectors. Figure 2-8 shows a Duo-Lateral PSD which has electrodes on both surfaces of a photodiode.

![Figure 2-8 Duo-lateral position sensitive detector](image)

Each surface has its own resistive layer so the photocurrent on each surface is divided into two. The electrode directions for the two layers are at 90°. This device provides high resolution and small error.

Figure 2-9 shows the Tetra-Lateral PSD which has only one resistive layer so that the photocurrent is split four ways. This device has slightly higher position measurement error which is worse near the edges, but has the advantages of a faster response time, more easily applied bias voltage, and a smaller dark current.

![Figure 2-9 Tetra-lateral position sensitive detector](image)
2.4.2 Lateral Shearing Interferometer Techniques

A lateral shearing interferometer splits the input aberrated light beam into two beams, then laterally displaces one with respect to the other, before recombining them\textsuperscript{[23,24]}. This forms an interference pattern which represents the spatial differential of the wavefront slope in the direction of shear. By integrating this, it is possible to reconstruct the phase in one direction. For a complete phase map of the aberrated beam, lateral shear must be applied in two directions separately.

2.4.2.1 Murty Plate Lateral Shear Interferometer

Perhaps the simplest type of lateral shear interferometer is the Murty plate interferometer which consists of just one piece of glass with optically flat, parallel faces. Figure 2-10 shows a Murty plate interferometer for the case where spherical wavefronts are incident upon it.

![Diagram of Murty Plate Lateral Shearing Interferometer](image)

**Figure 2-10 Murty Plate Lateral Shearing Interferometer**

Incident light is reflected from the front and back surfaces of the Murty plate forming two beams that interfere together to form a fringe pattern. When the light is incident at an angle other than 90°, there is a lateral displacement between the two reflected beams. For the case when spherical wavefronts are incident on the plate as shown in Figure 2-10, the output fringes are vertical. When the aberrations are of a higher order the fringes become more
complicated. This type of interferometer only works in laser light due to the inherent path difference between the beams.

In order to produce two fringe patterns (each obtained using lateral shear in different directions) for complete reconstruction of the phase map, it is possible pass the input beam through two Murty plates tilted along different axes.

2.4.2.2 Ronchi Grating Lateral Shear Interferometer

The Ronchi grating interferometer\[^{25}\] as shown in Figure 2-11, is an elegant example of the lateral shearing technique in which white light can be used.

![Figure 2-11 Ronchi white light grating interferometer](image)

Here, the aberrated input beam is passed through a concave lens, and a Ronchi grating (a binary grating having equally spaced transparent and opaque strips) is placed near the rear focal plane. Because the beam entering the grating is not collimated, the zero and first order diffraction beams partially overlap. This results in three identical diverging beams that are projected in slightly different directions. This is equivalent to having one beam laterally displaced from the other. The Ronchi interferometer can be used with white light because diffraction angle is inversely proportional to wavelength, but fringe spacing is directly proportional to wavelength. These effects cancel to produce monochromatic fringes.

A second fringe pattern produced by lateral shear in a different direction may be obtained by rotating the grating about the optic axis. A complete phase map can then be
reconstructed. Note that it is not possible to obtain a second fringe pattern showing lateral shear in a different direction by simply placing a second Ronchi grating whose lines are oriented at a different angle to the first, directly behind the first grating. This is because the first order diffraction beams from each grating overlap when interfering with the zero order beam.

This type of interferometer suffers from several disadvantages. The frequency of the grating lines must be arranged such that the shear produced is greater than half the pupil diameter, otherwise more than two beams will interfere at the output. This limits the range of lateral shear available. The two interfering beams will in general have different intensities due to the diffraction efficiency of the grating, producing low contrast fringes. Also, because the grating must be rotated to produce the second fringe pattern, it is not possible to measure both fringe patterns simultaneously in order to retrieve the complete phase map. It would therefore be inconvenient (but not impossible) to use this interferometer in an adaptive aberration correction system where measurements must be made rapidly.

2.4.2.3 Double Frequency Grating Lateral Shear Interferometer

The Double frequency grating lateral shear interferometer as shown in Figure 2-12 was proposed by Wyant\textsuperscript{[26]}. This is a variation on the Ronchi Grating interferometer.

![Diagram of Double Frequency Grating Lateral Shear Interferometer](image)

**Figure 2-12 Double frequency grating lateral shear interferometer**

Here the input light is passed through a lens and focussed onto a grating which has lines spaced at two different frequencies $v_1$ and $v_2$. Therefore, the grating produces two different
beams for each diffracted order. The two zero order diffracted beams overlap completely with no lateral shear. However, the plus first order diffraction beams are each diffracted through a slightly different angle. They overlap with a lateral shear dependent on the difference between $v_1$ and $v_2$, and form interference fringes. The minus first order diffracted beams overlap in a similar manner. This interferometer has the benefit that the interfering beams are of the same diffracted order and therefore have similar intensities. Thus, high contrast fringes can be obtained. By adjusting $v_1$ and $v_2$ it is also possible to obtain any amount of shear without overlapping diffracted orders.

A second fringe pattern produced by lateral shear in a different direction may be obtained by placing an identical double frequency grating whose lines are oriented at a different angle to the first (generally 90° for simplicity), directly behind the first grating. Both first order diffracted beams of the second grating are then diffracted at this angle instead. This enables both fringe patterns to be measured simultaneously. A complete phase map can then be reconstructed. Figure 2-13 shows the output fringe patterns obtained using this construction. Here the minus first order diffracted beams from each grating are shown also.

![Figure 2-13 Fringe patterns obtained using two double frequency gratings with one rotated by 90°, showing interference between both plus first order beams, and both minus first order beams for each grating](image-url)
Note that the direction of lateral shear for all four fringe patterns is radial, out from the optic axis. A drawback for the double frequency grating lateral shear interferometer is that it requires different gratings to be constructed for each desired value of shear.

2.4.2.4 Double Grating Interferometer with Variable Lateral Shear

Wyant improved on the double frequency grating lateral shear interferometer with the double grating interferometer with variable shear\(^{[27]}\) shown in Figure 2-14.

![Diagram of Double Grating Interferometer with Variable Lateral Shear](image)

**Figure 2-14 Double grating interferometer with variable lateral shear**

Here, the input light is focussed onto two gratings placed close together. The gratings have an identical line frequency. Both gratings produce diffracted orders. The grating frequency is chosen so that the angle of diffraction is large enough to eliminate overlap between the first order diffraction beams and the zero order diffraction beams. When the direction of the lines of both gratings is the same, the plus first order diffraction beams overlap completely with each other, as do the minus first order diffraction beams. However, rotating one grating about the optic axis as shown in Figure 2-14, gives rise to a lateral shear between the plus first order beams which interfere to form a fringe pattern. A similar lateral shear is produced between the minus first order diffracted beams.

The interfering beams are again of the same diffracted order so produce high contrast fringes. This interferometer has the advantage that the lateral shear can easily be varied by
adjusting the angle of rotation between the two gratings. The direction of shear is perpendicular to that of the double frequency grating lateral shear interferometer shown in Figure 2-12.

A second fringe pattern produced by lateral shear in a different direction may be obtained by placing another pair of identical gratings directly behind the first pair. The lines of the second pair are oriented with a small angle between them, and oriented at 90° to the lines of the first pair. Figure 2-15 shows a diagram of the output interference fringes obtained using two pairs of identical gratings in this manner. The zero order beams overlap completely on the optic axis. Note that the direction of lateral shear of the fringe patterns is tangential, around a circle centred about the optic axis.

![Diagram](image)

**Figure 2-15 Interferometer output showing the direction of lateral shear between the two first order diffraction beams for both grating pairs**

### 2.4.3 Radial Shearing Interferometer Technique

A radial shearing telescope may be used to provide a wave approximating to a plane reference wave, derived from the input beam in an interferometer. Two types of radial shearing interferometers are now presented.
2.4.3.1 Radial Shearing Mach-Zehnder Interferometer

Figure 2-16 shows a Mach-Zehnder interferometer containing a radial shearing interferometer.

![Mach-Zehnder radial shearing interferometer](image)

The input beam is split by the input beam splitter. The beam transmitted by the beam splitter is reflected by mirror $M_1$, and then reflected by the output beam splitter. The beam reflected by the input beam splitter is reflected by mirror $M_2$ then passes through the radial shearing telescope. The radial shearing telescope consists of two lenses placed a distance equal to the sum of their focal lengths apart. This expands the beam radially and creates an approximately plane reference beam. This beam is transmitted by the output beam splitter and interferes with the signal beam from the other interferometer arm. Interference fringes occur where the two beams overlap. The phase distribution of the input beam can be determined directly from the phase distribution of the fringes.

2.4.3.2 Radial Shearing Sagnac Interferometer

A radial shearing telescope can also be incorporated in a Sagnac interferometer as shown in Figure 2-17. Here the input beam is split in two by the beam splitter. Both beams travel in opposite directions around the Sagnac, which comprises mirrors $M_1$, $M_2$, and $M_3$. They also both pass through the radial shearing telescope formed by lenses $L_1$ and $L_2$, which are placed the sum of their focal lengths apart. The size of the beam travelling clockwise round the Sagnac is reduced, and the size of the beam travelling anticlockwise is expanded. The
expanded beam forms the reference beam. Both beams exit the interferometer at the beam splitter and interference fringes occur where the beams overlap.

![Figure 2-17 Sagnac interferometer with radial shearing telescope](image)

The advantages of using a radial shearing Sagnac interferometer are that:

1) it is common path and therefore insensitive to vibration and air movement,

2) it can be used with white light,

3) each beam travels through the radial shearing telescope in opposite directions so the overall magnification is twice that experienced in the Mach-Zehnder interferometer shown above.

Much of the feedback interferometry research presented in this thesis involves a radial shearing Sagnac interferometer. Therefore, a more detailed description of the radial shearing telescope is given in Chapter 4.

### 2.4.4 Heterodyne Interferometers

A heterodyne interferometer[^23] can also be used for wavefront sensing. The frequency of the beam in one arm of a heterodyne interferometer, is shifted with respect to the other,
before the two beams are recombined at the output. The frequency shifted beam can be obtained from an acousto-optic modulator, or the diffracted orders of a circular diffraction grating rotated at a constant angular velocity. Heterodyne interferometers are also used to measure surface profiles of test objects such as optical mirrors. A simplified schematic of a Michelson interferometer used for this purpose is shown in Figure 2-18, to show the principle of operation. The input beam is split in two by the beam splitter, and the transmitted beam is reflected from the plane mirror back towards the beam splitter. The input reflected beam is passed through the frequency shifter, reflected from the test object, then passed back through the frequency shifter towards the beam splitter.

Here, \( v_1 \) = input beam frequency,
\[ v_2 \] = frequency of beam returning from test object.

The two beams are recombined at the beam splitter and directed towards a screen at the interferometer output. The fringes of the resulting interference pattern move at a constant speed across the screen because of the frequency difference between the two beams. A scanning detector, and a stationary detector are placed in the fringe pattern. Both detect a sinusoidally varying intensity as the fringes pass over them, and are connected to a phase meter as shown in Figure 2-19.

The phase of the sinewave observed by the stationary detector is used as a reference. The other detector moves incrementally in both the \( x \) and \( y \) directions scanning the whole fringe pattern. The phase meter measures the difference in phase between the sinewaves of the
two detectors as a function of scanning detector position. A phase map of the test object is therefore obtained when the whole fringe pattern has been scanned. This mechanism is used for wavefront sensing by replacing the frequency shifted beam from the test object, with the aberrated beam so that a phase map of the aberrated wavefront is obtained instead.

![Diagram of heterodyne fringe pattern and detectors](image)

**Figure 2-19 Heterodyne fringe pattern and detectors**

The use of a rotating grating to obtain the frequency shifted beam can also provide a simple mechanism for splitting the beam so that the zero, and first orders are used for the two beams of the interferometer. This negates the need for an input beam splitter. Speed is the main problem when using a heterodyne interferometer for wavefront sensing because the whole fringe pattern must be scanned each time the aberrations change.

In principle, the heterodyne technique can be used with most two beam interferometer setups. Wyant\(^{28}\) performed an analysis to determine the accuracy with which an ac heterodyne lateral shear interferometer can measure wavefront aberrations. But to the best of the authors knowledge there has been no experimental implementation of the heterodyne technique in a lateral shear interferometer.

### 2.5 Wavefront Correcting Techniques

Several devices used for wavefront correction, are described briefly below.

#### 2.5.1 Segmented, Deformable, and Bimorph Mirrors

The most common wavefront correctors are segmented, deformable, and bimorph mirrors\(^{21,29}\). Segmented mirrors like those shown in Figure 2-20 are constructed from many small mirrors.
Each mirror segment is controlled separately by piezoelectric or voice coil actuators. Figure 2-20 (a) shows the piston only principle of operation where the movement of each segment is restricted to one dimension and is controlled by one actuator only. Figure 2-20 (b) shows a segmented mirror with both piston and tilt movement where the movement of each segment is controlled by three actuators, which allows tilt as well as piston movement. Segmented mirrors can be scaled to many sizes and shapes, and segments with square, hexagonal, circular and circle segment shapes have all been used.

Deformable mirrors like those shown in Figure 2-21 usually have a continuous reflective surface which is physically distorted. The left diagram in Figure 2-21 shows the principle of operation. Here a thin reflecting surface is supported and deformed by a series of mechanical pistons. The mirror surface must be thin to minimise mechanical fatigue which results from the constant deformation. Therefore, precise control of the structure is necessary to avoid damage\[^{30}\]. This is especially difficult and expensive when constructing large adaptive primary or secondary telescope mirrors\[^{31}\].

![Figure 2-20 Segmented mirrors](image)

**Figure 2-20 Segmented mirrors**

![Figure 2-21 Continuous surface deformable mirrors](image)

**Figure 2-21 Continuous surface deformable mirrors**
The effects of gravity must be taken into account in the design to minimise the energy wasted on correcting inherent mirror deformation which may vary with the position of the telescope. Even if a mirror is built and polished correctly on the construction floor, the effect of gravity will influence the mirror's shape both when it is moved to the telescope site, and when the telescope tracks an object in the sky. This is a problem with any large mirror which is exacerbated by thin plate construction.

The diagram on the right of Figure 2-21 shows the principle of operation of a bimorph deformable mirror. A continuous glass or metal face plate is backed by an electrode and a layer of piezoelectric ceramic material. This front electrode is often contained in the glue used to attach the front glass plate to the ceramic. Back electrodes are placed on the other side of the piezoelectric layer. The local radius of the mirror is altered by applying a voltage between the front and selected back electrodes, expanding and contracting the piezoelectric layer, thereby deforming the face plate by the bimorph principle.

2.5.2 Deformable Membrane Mirrors

Silicon micromachined membrane mirrors are fabricated by OKO Technologies in cooperation with Delft Institute of Microelectronic and Submicron Technology\(^{[32]}\). Figure 2-22 shows a schematic of a Silicon micromachined membrane mirror.

![Silicon micromachined membrane mirror schematic](image)

Figure 2-22 Schematic of Silicon micromachined membrane mirror

The mirror consists of a Gold or Aluminium coated Silicon Nitride membrane mounted over a PCB substrate. The membrane and PCB substrate are held apart by spacers. The Aluminium or Gold coating forms the reflective surface of the mirror. The PCB contains 37
hexagonal control electrodes positioned in a hexagonal array underneath the membrane. The deformation of the membrane is controlled by applying a bias voltage \( V_b \) (0 to 150V) and a control voltage \( V_{1...n} \) (0 to 255V) to each electrode. The applied voltage creates an electrostatic force between the electrode and membrane causing the membrane to deform. Since the membrane is a continuous surface, applying a voltage to one electrode will not only effect the membrane directly above it, but also the membrane above the neighbouring electrodes. This is effectively cross talk between electrodes and occurs with all membrane and continuous surface deformable mirrors.

The maximum deflection of the centre of the membrane is 9μm. The centre to centre distance between the control electrodes on the PCB is 1.75mm, and the mirror aperture is 15mm. The frequency range the mirror can operate over is 0 to 500Hz.

### 2.5.3 Texas Instruments Deformable Mirror Spatial Light Modulator

The characteristics of the deformable mirror device spatial light modulator manufactured by Texas Instruments, are well documented\(^{33,34}\). It consists of a 128 x 128 array of pixels with each pixel constructed with four hinged rectangular reflective surfaces, and a response time in the order of 1μsec. It is a monolithic, line-addressed device capable of modulation at video frame rates. The drive electronics are located on board ensuring the device is compact. A simplified cut away schematic of the device is shown in Figure 2-23.

![Figure 2-23 Deformable mirror spatial light modulator](image)
Four 12.7μm square, cantilever beam deformable mirrors make up each element which are constructed on 51μm centres. An array of electrodes which are connected to the floating MOS transistor sources, are formed beneath the deformable mirror array, with an air gap in between. Each set of four deformable mirrors experiences an electrostatic force, which attracts them to the underlying electrode when a voltage is applied to it. The mirrors are deflected downward, thus modulating the phase of light reflected from the surface. Note also that when the pixels are modulated, diffraction at the phase discontinuities between the pixels and substrate gives rise to some amplitude modulation of the zero order beam reflected from the device.

2.5.4 Liquid Crystal Television

The feasibility of using liquid crystal televisions (LCTVs) such as those found in the Epson Crystal Image Video Projector for aberration correction has also been studied\(^{35,36}\). The LCTV consists of an active matrix addressed 320 x 220 pixel array, with cells containing a twisted nematic liquid crystal material. Phase modulation is obtained by applying a voltage across the liquid crystal layer. The twisted configuration of the liquid crystals, which is necessary for TV projection, is unnecessary and is in many ways disadvantageous in adaptive optics systems. However, it has been shown that in certain operating modes it is possible to achieve a phase change of nearly a full wavelength\(^{36}\). These devices have potential for wavefront correction\(^{37}\).

2.5.5 Other Liquid Crystal Devices

Tests have been performed on the feasibility of using liquid crystal devices for wavefront correction\(^{38}\), and research on new liquid crystal spatial light modulators has recently been carried out by companies such as Meadowlark Optics in the USA\(^{39}\). The Hex69 modulator is specifically designed for adaptive optics applications, but some other products such as the SLM M4-704X512 may also be suitable. The Hex69 spatial light modulator, capable of one wavelength phase modulation, is a parallel aligned, nematic liquid crystal device. The parallel alignment of the liquid crystal layer produces phase only modulation. It operates in 'transmissive mode', which means light passes once through the device. A modulator which reflects light so it passes twice through the liquid crystal layer operates in 'reflective mode'.

Transmissive devices are generally easier to deal with in optical systems but are more difficult to manufacture. These difficulties arise because the device window must be
transparent, yet it must be possible to address each pixel separately and modulate as much of the window as possible. The pixel address wires must also exit the device between pixels. The wires must therefore be not only transparent, but also very thin to minimise the gap between pixels through which light may pass unmodulated and hence uncorrected. However, a reflective device may be addressed by electronics mounted directly behind the pixels making the device smaller and providing better resolution. The SLM M4-704X512 spatial light modulator, also made by Meadowlark is a reflective device. It is a nematic liquid crystal active matrix device with a reflectance of up to 93% and has 704 x 512 pixels on 20μm centres.

2.5.6 LiNbO3 Crystal Microchannel Spatial Light Modulator

The phase modulators discussed previously are all addressed electronically. However, the LiNbO3 crystal microchannel spatial light modulator is addressed optically. Extensive studies on the optical information processing characteristics of this device have been carried out\cite{40}, and it was used for wavefront correction by Fisher and Warde\cite{41,42} in their interference phase loop experiments. A phase distribution is written onto a beam reflected from the front side of the device (the read light), that is proportional to the spatial intensity distribution of another beam incident from behind (the write light). A simplified schematic of the Microchannel Spatial Light Modulator is shown in Figure 2-24.

The write light is incident on a photocathode through a window at the top of the diagram. The number of electrons produced by the photocathode is proportional to the intensity of the light. When a spatially varying intensity distribution is incident, an "electron picture" of the intensity distribution is produced by the photocathode. The "electron picture" is then amplified by a microchannel plate which contains an array of small semiconducting glass pores which each function as electron multipliers. An acceleration grid then accelerates the electrons through an insulating vacuum gap and on to one side of the LiNbO3 crystal electro-optic plate. This surface is also a dielectric mirror which reflects the read light which is incident from below. The refractive index of the crystal plate is varied by the resulting surface charge distribution so that the phase of reflected read light is modulated. The spatial phase distribution of the "processed light" is then proportional to the intensity distribution of the "write light". The surface charge distribution on the electro-optic plate can be removed simply by secondary electron emission.
2.5.7 The Hamamatsu PAL-Spatial Light Modulator

Another optically addressed phase modulator is the PAL-SLM (Parallel Aligned Spatial Light Modulator)\(^\text{[43,44]}\). A simplified schematic of the SLM is shown in Figure 2-25.

**Figure 2-24 Simplified schematic of microchannel spatial light modulator**

**Figure 2-25 Schematic of the PAL-spatial light modulator**
A nematic liquid crystal layer is sandwiched between a front glass substrate and a dielectric mirror. The layer is parallel aligned, i.e., the alignment layers on the front and rear surfaces of the cell are treated in the same way so that the LC molecule directors are aligned in the same (horizontal) direction throughout the thickness of the cell. There is no twist.

Behind the dielectric mirror is a layer of hydrogenated amorphous silicon (α-Si:H) which is the light sensor for the device, and the liquid crystal-dielectric-α-Si:H structure is placed between transparent Indium Tin Oxide electrodes which are supplied with a 0 - 5V amplitude, square wave, AC voltage at 1kHz. Write light falling on the silicon layer changes its resistivity, and this in turn alters the potential across the liquid crystal layer resulting in reorientation of the molecules. Read light (which should be polarised parallel to the liquid crystal molecule directors) entering the front face of the device and reflected from the dielectric mirror, undergoes a phase shift on its double pass through the liquid layer which is dependent on the intensity of the write light. Under those conditions, the polarisation state of the read light is essentially unchanged. The device is therefore an optically addressed spatial light modulator. If the read light is polarised perpendicular to the liquid crystal molecules, there is no detectable phase modulation with changes in write light intensity.

2.6 Conclusion

In this chapter, the background of adaptive optics has been presented. Applications that have benefited from the use of adaptive optics were discussed along with a summary of techniques and devices used for detecting and correcting optical aberrations. In the next chapter some practical issues for the use of two particular liquid crystal spatial light modulators in adaptive aberration correction are discussed in more detail.
Chapter 3. Practical Issues for Liquid Crystal Spatial Light Modulators in Aberration Correction

Practical issues for the use of liquid crystal spatial light modulators (LC-SLMs) in adaptive aberration correction are discussed in this chapter. LC-SLMs introduce a phase delay to transmitted light when an AC voltage is applied across a layer of liquid crystal material. Varying the amplitude of the applied voltage changes the phase delay. Devices with many individually addressable pixels can be used to alter the shape of optical wavefronts.

3.1 Introduction

Low cost, accurate, high resolution spatial light modulators are of increasing interest for adaptive optics applications. Most adaptive optics systems currently use expensive segmented or deformable mirrors. Nematic LC-SLMs have been suggested as possible substitutes for these mirrors as the phase modulating arrays in adaptive optics systems. In this chapter, four experiments that investigate characteristics of two different LC-SLMs are discussed. We carried out three of the experiments here in the Physics Department at the University of Auckland using a nematic LC-SLM with 81 square pixels in a 9x9 array. The other experiment was part of a collaborative project with James Gourlay and Ray Sharples at the University of Durham in the UK. The experimental work using a nematic LC-SLM with 69 hexagonal pixels built by Meadowlark Optics was performed in Durham, while we carried out the theoretical computer simulations here in Auckland. The experimental work carried out in Durham was included in this Thesis in order to compare the results with our theoretical simulations. The results of the four experiments give a good indication of how liquid crystal devices will operate in practical adaptive optics systems.
3.2 Overview of Experiments

First, a brief overview of the experiments contained in this chapter will be given. The first experiment tested the wavelength dependence of the modulation characteristic for the 81 pixel LC-SLM. Results showed that the modulation characteristic varies by less than 20% over a range of input wavelengths from 400 to 900nm.

The second experiment tested the modulation characteristic of the 81 pixel LC-SLM in a system configured to be insensitive to the polarisation state of incident light. A $\lambda/4$ plate was used to swap orthogonal polarisation components of the incident light between two passes through the LC-SLM, thus modulating all polarisation components. Results using this configuration showed that the variation in modulation characteristic was less than 4% for any incident polarisation state. The $\lambda/4$ plate was a zero’th order plate made of quartz. The quartz dispersion curve was then used in a Jones matrix calculation to estimate the variation of LC-SLM modulation with wavelength, when configured to be insensitive to polarisation state.

The third experiment was carried out by James Gourlay and Ray Sharples using the Meadowlark LC-SLM in Durham. The purpose of this experiment was to assess the optical losses associated with using the Meadowlark LC-SLM in a practical system, configured to be insensitive to the polarisation state of incident light. A phase wedge was applied across the LC-SLM, which deflected an incident optical beam. Conclusions about the optical losses were drawn by comparing the point spread function (PSF) of the deflected beam, with the PSF of a non-deflected beam obtained with a uniform phase applied to the LC-SLM. Details of our computer simulations are presented. The simulation results are compared to the experimental results from Durham and show good agreement.

The fourth experiment tested the response time of the 81 pixel LC-SLM using both conventional drive techniques, then an overdriving technique called the 'transient nematic effect'. Results showed that the response time was significantly reduced using the transient nematic effect.

First however, the requirements for spatial light modulators used in aberration correction are discussed. Details of both the 81 segment LC-SLM and the Meadowlark LC-SLM are then given before the four experiments are discussed in detail.
3.3 Requirements for Aberration Correction

In this section, the requirements for modulators to be considered for aberration correction applications are discussed. LC-SLMs and their drive electronics cost less than segmented or deformable mirrors and other conventional phase modulators\textsuperscript{[45]}. Their power requirements are also very low\textsuperscript{[46]}. For these reasons, LC-SLMs would be excellent phase modulators in certain adaptive optics applications\textsuperscript{[47]}, provided they also fulfil certain other requirements. The requirements for LC-SLMs in adaptive optics are that they:

1. operate over a wide wavelength band (say 400 nm to over 1000 nm),
2. are insensitive to the polarisation state of the incident light,
3. have a short response time (\(<10\text{ms}\))
4. have accurate phase modulation (say < 1 percent),
5. have a moderate number of pixels or actuators (100-1000) and,
6. have a high optical throughput (\(>90\text{ percent}\)).

In the four experiments discussed in this chapter, the first three requirements are examined. Some of the other issues are covered by Love\textsuperscript{[47]}.

3.4 Description of the two LC-SLMs used in these Experiments

In this section, the construction and characteristics of both the 81 pixel LC-SLM and the Meadowlark LC-SLM are discussed.

3.4.1 LC-SLM with 81 Pixels

Figure 3-1 shows a simplified expanded view of the 81 pixel LC-SLM, which was provided by Prof. Tschudi at the Technische Hochshule, Darmstadt in Germany. The modulator is constructed with a layer of liquid crystal material sandwiched between two alignment layers deposited on transparent electrodes on the front and rear glass panels of the cell. It consists of 81 square, parallel aligned, nematic liquid crystal, phase modulating pixels.

The transparent indium tin oxide (ITO) electrodes were etched to produce an array of 9 x 9 square pixels together with connection tracks running from each pixel to the edge of the panel. Applying an AC voltage of amplitude between 0 and 5V across a pixel, modulates
the phase of one polarisation component of light passing through the pixel. Maximum modulation is $1.5\lambda$ wavelengths at 633nm. Table 1 gives the modulator characteristics.

![Expanded Construction of Liquid Crystal Phase Modulator](image)

**Figure 3-1  Expanded Construction of Liquid Crystal Phase Modulator**

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of pixels:</td>
<td>81 (9 x 9 array)</td>
</tr>
<tr>
<td>Pixel Pitch:</td>
<td>4.5 x 4.5 mm</td>
</tr>
<tr>
<td>Pixel fill factor:</td>
<td>&gt;90% (Varied slightly for each pixel depending on connection track configuration)</td>
</tr>
<tr>
<td>Liquid crystal material:</td>
<td>Merck ZLI1132 ($\Delta n_{max} = 0.14$)</td>
</tr>
<tr>
<td>Liquid crystal thickness:</td>
<td>9µm</td>
</tr>
<tr>
<td>Phase modulation capability:</td>
<td>$1.5\lambda$ (@633nm, 10Vpp square wave at 1000Hz)</td>
</tr>
<tr>
<td>Response time using conventional drive techniques:</td>
<td>$\sim 50$ ms (0 - $\pi$ phase change)</td>
</tr>
</tbody>
</table>

**Table 1 Characteristics of 81 pixel LC-SLM**
The panel was provided with a dedicated microprocessor-controlled driver circuit, built at the University of Auckland, which provided a 200Hz square wave drive voltage with individually programmable amplitude between 0 and 5V to each pixel. An IBM compatible computer controlled the driver circuit via an RS-232 serial link.

3.4.2 Meadowlark LC-SLM

James Gourlay and Ray Sharples at the University of Durham used a LC-SLM built by Meadowlark in the third experiment discussed in this chapter. Figure 3-2 shows the pixel configuration for the Meadowlark device.

![Figure 3-2 Pixel configuration for Meadowlark LC-SLM](image)

This LC-SLM consists of 69 hexagonal, parallel aligned, nematic liquid crystal phase modulating elements which are addressed individually. The LC-SLM was constructed using optically flat fused silica substrates. The substrates were coated with an ITO transparent conductor that was specially designed for maximum transmission from 450-1800nm. The ITO coating was photolithographically patterned into individual electrodes, creating independently controllable pixels. A thin dielectric alignment layer was then applied over the ITO coating before being gently rubbed, creating parallel micro-grooves. The micro-grooves align the liquid crystal molecules. The two silica substrates were then placed a few microns apart, before the cavity was filled with the birefringent nematic liquid crystal material. Electrical contacts were attached and the device sealed.
Zernike polynomials\textsuperscript{48} can be used to describe different types of optical aberrations eg. tip, tilt, defocus, astigmatism and coma. The performance of the device when producing Zernike wavefronts and static adaptive optics was described by Love\textsuperscript{47}. Its use in a real-time adaptive optics system was described by Gourlay\textsuperscript{49}. An advantage with using this device is that it can easily be incorporated into a system that has either a square or circular aperture. Table 2 shows the Meadowlark modulator characteristics.

<table>
<thead>
<tr>
<th>No. of pixels:</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel size:</td>
<td>Hexagonal, max. diameter 2.08 mm</td>
</tr>
<tr>
<td>Pixel fill factor:</td>
<td>~97%</td>
</tr>
<tr>
<td>Inter pixel spacing:</td>
<td>20 μm</td>
</tr>
<tr>
<td>Liquid crystal material:</td>
<td>Merck E44</td>
</tr>
<tr>
<td>Liquid crystal thickness:</td>
<td>5.5 μm</td>
</tr>
<tr>
<td>Phase modulation capability:</td>
<td>1.75λ (at 633 nm, 20Vpp)</td>
</tr>
<tr>
<td>Transmittance:</td>
<td>90% (at 633 nm)</td>
</tr>
</tbody>
</table>

Table 2 Characteristics of Meadowlark LC-SLM

### 3.5 Experiment 1: Wavelength Dependence of 81 Pixel LC-SLM

An experiment to test phase modulation variation of the 81 pixel LC-SLM with wavelength will now be described. In broad-band adaptive optics applications it is important that the phase modulator modulates all wavelengths by the same amount. An optical wavefront that has travelled through an essentially non-dispersing atmosphere will suffer the same physical wavefront distortion at all wavelengths - but the phase distortion, which is dependent on wavelength will vary, being larger at shorter wavelengths. Thus it is the change in optical path length, or the stroke of the modulator that should be the same at all wavelengths rather than the phase. The experimental results have therefore been expressed in terms of stroke rather than phase modulation.
A deformable mirror lends itself to broad-band adaptive optics because it naturally has the same stroke at all wavelengths. However, this is not necessarily the case for a LC-SLM where the variation of phase modulation with wavelength depends on the dispersion of the liquid crystal material.

3.5.1 Interferometer for Wavelength Dependence Experiment

A schematic of the common path interferometer used to determine the modulator stroke as a function of wavelength for the 81 pixel LC-SLM is shown in Figure 3-3.

![Common path interferometer for characterising the polychromatic operation of the LC-SLM](image)

Light from an incandescent bulb on the left was roughly collimated by lens 1. The light then passed through a Soleil Babinet compensator, the first polariser and the LC-SLM. The modulator was arranged so that the orientation of the liquid crystal molecules was horizontal. Therefore, only the horizontal component of the transmitted light was modulated. The Soleil Babinet compensator provided a variable optical path difference between the horizontal and vertical polarisation components of the input light, which was randomly polarised. This allowed the path difference in the interferometer when zero voltage was applied to the modulator to be set to a convenient value for each wavelength. This made retrieval of modulator stroke by subsequent processing of the interferometer output data far easier. The value of path difference chosen was $\lambda/2$. 

39
The first polariser was oriented at 45° to the horizontal. Therefore, the beam transmitted by this polariser had both vertical and horizontal components. As a result, when the beam passed through the LC-SLM only the horizontal component was modulated. Following the LC-SLM the light passed through a second polariser with an identical orientation to the first, of 45°. This polariser essentially selected and interfered common components of both the horizontal (modulated) and vertical (unmodulated) components of the beam which passed through the LC-SLM. The intensity of the light at the output therefore varied with LC-SLM stroke. For example when the LC-SLM delayed the horizontal component by a half wavelength with respect to the vertical component, the common components of both interfered destructively so the intensity at the output was minimum. When the LC-SLM delayed the horizontal component by a whole wavelength then the common components interfered constructively so the output intensity was maximum.

The beam was then focussed into a spectrometer by Lens 2. The spectrometer could be set to pass any desired spectral component between 400nm and 900nm to a photodiode. The signal from the photo diode passed through a current-to-voltage converter and was logged to file by an A/D board in the host computer.

![Figure 3-4 Wavelength vs. modulator voltage vs. normalised interferometer intensity](image-url)
Since the two beams travelled exactly the same physical path, the interferometer was common path and insensitive to vibrations and air turbulence. Results were taken by logging the output intensity at the photodiode as the LC-SLM drive voltage was varied. Figure 3-4 shows a mesh plot of normalised interferometer output intensity as a function of modulator voltage and wavelength using this setup.

As described earlier, the interferometer path difference was set using the Soleil Babinet compensator, to be \( \lambda/2 \) at zero modulator volts for each wavelength. The output intensity at each wavelength was therefore initially minimum and rose with increasing modulator drive voltage.

### 3.5.2 Phase Retrieval

The procedure for retrieving the phase from the interferometer output intensity data will now be described. The interferometer output intensity at any wavelength \( \lambda \) is given by equation (3-1),

\[
I_{\text{out}} = A \left[ 1 + V \cos \left( \frac{2\pi}{\lambda} (S + S_m) \right) \right],
\]  

(3-1)

where: \( S \) is the optical path difference or stroke introduced by the Soleil Babinet compensator between the horizontal and vertical polarisation components of the input light,

\( S_m \) is the stroke of the LC-SLM,

\( V \) is the visibility or contrast of the fringes, and

\( A = I_1 + I_2 \), the average output intensity

where: \( I_1 \) and \( I_2 \) are the intensities of the two interfering beams at the output.

Equation (3-1) can be rearranged to give equation (3-2) from which the LC-SLM stroke \( S_m \) may be obtained.

\[
S_m = 2n\pi + \frac{\lambda}{2\pi} \cos^{-1} \left( \frac{I_{\text{out}} - 1}{A} \right) - S,
\]  

(3-2)

where \( n \) is an integer.
Note that the modulator stroke varied between 1.5 wavelengths at 900nm, and just over 3 wavelengths at 400nm. Therefore, each value of output intensity corresponded to up to seven possible values of stroke. This caused problems retrieving the LC-SLM stroke \( S_m \) using equation (3-2). The problem was overcome by breaking the interferometer output intensity data set for each wavelength up into monotonically increasing and decreasing sections. Equation (3-2) was then applied to each section. Appropriate phase offsets (integral values of 2\( \pi \)) were then added to the resulting phases to produce a continuous phase characteristic.

Note also that for a given wavelength in Figure 3-4 the intensity of the peaks and valleys varies slightly as the modulator voltage increases. This was due to residual absorption in the LC-SLM. Therefore, the visibility \( V \) and average output intensity \( A \) varied with drive voltage. This caused inaccuracies during phase retrieval. To overcome this problem, \( V \) and \( A \) were calculated separately for each monotonic section of intensity data.

### 3.5.3 Wavelength Dependence Results

Figure 3-5 shows a mesh plot of the reconstructed LC-SLM stroke as a function of applied voltage and wavelength. This shows that the variation of stroke with wavelength is small. A good indication of this is given in Figure 3-6 which shows a plot of modulator stroke vs. wavelength for a modulator voltage of 4.75 \( V \). Figure 3-6 is a cross-section of Figure 3-5 at the maximum modulator voltage and shows that the total variation in stroke over the whole wavelength range is less than 20%.

### 3.6 Experiment 2: Polarisation Sensitivity of 81 pixel LC-SLM

Conventionally, LC-SLMs modulate only one polarisation component of light. For adaptive optics applications it is beneficial if the spatial light modulator is insensitive to the polarisation state of light. Meadowlark Optics produce a device that consists of two orthogonal modulators to achieve polarisation insensitive operation\(^{[50]}\). However, Love\(^{[51]}\) described a configuration containing a single LC-SLM which also achieved this. The configuration included a \( \lambda/4 \) plate that swapped the horizontal and vertical polarisation components of light between two passes through the LC-SLM.
Figure 3-5 Plot of wavelength vs. modulator voltage vs. modulator stroke [r.h1]

Figure 3-6 Cross-section of Figure 3-5 showing modulator stroke vs. wavelength, for a modulator voltage of 4.75 V
The purpose of the experiment described in this section was to quantitatively assess how our 81 pixel LC-SLM operated as an optical wavefront controller, when configured to be insensitive to the polarisation state of incident light in the way Love described. First, the setup is described. Results are then presented which show that light in any polarisation state is modulated almost identically by the LC-SLM using this configuration. Calculations are then shown which estimate how the configuration performance would vary over a range of wavelengths due to dispersion in the quartz $\lambda/4$ plate.

### 3.6.1 Rayleigh Interferometer Experimental Setup

Figure 3-7 shows the Rayleigh interferometer used to characterise the stroke of the LC-SLM for different input polarisation states using the double pass configuration.

![Figure 3-7 Double pass Rayleigh interferometer used to characterise LC-SLM stroke for different input polarisation states](image)

Light from a linearly polarised 10mW HeNe laser passed through a spatial filter. The beam was then collimated by a 120mm focal length lens and passed through a rotatable $\lambda/2$ plate which provided a linearly polarised beam of arbitrary orientation. Two circular apertures formed two separate beams that were both transmitted by a non-polarising beam splitter. One, a control beam passed through a control pixel of the LC-SLM, which was unmodulated. The other, a test beam, passed through a modulated test pixel. The test pixel
initially modulated only the horizontal component of the beam passing through it. However both beams then passed through a $\lambda/4$ plate oriented at 45° and were reflected by a plane mirror back through both the $\lambda/4$ plate and the LC-SLM. The double pass through the $\lambda/4$ plate swapped the vertical and horizontal components of each beam. On the return pass through the LC-SLM, the orthogonal polarisation component of the test beam was modulated by the test pixel but the control beam remained unmodulated. Therefore, full modulation was achieved for all polarisations that passed through the test pixel. The two double passed beams were then reflected by the beam splitter and combined using a 300mm Fourier transform lens. The spatial Fourier transform which consisted of an interference fringe pattern was enlarged by a x40 microscope objective on to a photo diode array. The host computer then logged the output of the diode array.

3.6.2 Experimental Results for 81 segment LC-SLM configured to be Insensitive to Polarisation State

As the test pixel of the LC-SLM was modulated, the interference pattern formed in the Fourier plane moved sideways. Simple Fourier transform techniques were used to determine the stroke of the LC-SLM from the spatial phase of the fringes as they moved. This procedure was carried out for a 180° range of linear input polarisations provided by the rotatable $\lambda/2$ plate.

It was important that both interferometer beams had the same polarisation following the double pass through the modulator otherwise the contrast of the output fringe pattern would have been affected. However, the contrast of the output fringe pattern did not vary significantly as the input polarisation was rotated, indicating that the polarisation of the two output beams was almost identical at all modulator voltages, and all polarisations.

Figure 3-8 shows a graph of input polarisation vs. LC-SLM drive voltage vs. LC-SLM stroke, and shows that the modulation characteristic at each input polarisation was the same. The variation in modulator stroke is less than 4% across the whole range of polarisations. Therefore, we can conclude that the modulator operating in this configuration is insensitive to the polarisation state of incident light.
3.6.3 Variation of the System Configured to be insensitive to Polarisation State, with Wavelength

It would have been interesting to test how the performance of the system configured to be insensitive to the polarisation state of incident light, varied with wavelength. Unfortunately this was not possible because the Rayleigh interferometer experimental setup required a laser input source in order to get interference fringes at the output, and we did not have access to a tunable laser around 633nm. However, it was possible to estimate the wavelength range over which practical satisfactory performance might be obtained. This was done using the dispersion curve of quartz following a standard Jones matrix analysis to calculate the phase errors in the $\lambda/4$ plate.

Phase errors due to dispersion in the $\lambda/4$ plate lead to incomplete swapping of the polarisation components. Residual (unswapped) electric field components will then either be phase modulated twice (residual components parallel to the liquid crystal molecule
directors) or not modulated at all (residual components perpendicular to the directors). Our criterion for determining the allowable wavelength range was that the residual field components should be less than 0.1 of the incoming field component in that direction.

For the purposes of the calculation it was assumed that the light passed once through a $\lambda/2$ plate rather than twice through a $\lambda/4$ plate. These situations are identical when considering effects on polarisation state and phase errors. The Jones matrix $P$ for a $\lambda/2$ plate oriented with one axis vertical and the other horizontal is given by:

$$\begin{pmatrix} \exp\left(\frac{in_1 2\pi d}{\lambda}\right) & 0 \\ 0 & \exp\left(\frac{in_2 2\pi d}{\lambda}\right) \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

where: $n_1$ and $n_2$ are the refractive indices of the two wave plate axes,

$d$ is the thickness of the wave plate,

$\lambda$ is the wavelength of light,

$a = \exp(i\phi_1)$, and $b = \exp(i\phi_2)$

where: $\phi_1 = \frac{n_1 2\pi d}{\lambda}$, and $\phi_2 = \frac{n_2 2\pi d}{\lambda}$

The Jones matrix $P_2$ for a $\lambda/2$ plate rotated by $\theta$ is given by:

$$P_2 = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} (P) \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} a \cos^2 \theta + b \sin^2 \theta & (a-b) \cos \theta \sin \theta \\ (a-b) \cos \theta \sin \theta & a \cos^2 \theta + b \sin^2 \theta \end{pmatrix}.$$ 

For $\theta = 45^\circ$ this becomes

$$P_2 = \frac{1}{2} \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix},$$

We assume that the input light to the $\lambda/2$ plate is polarised in the horizontal direction. Therefore, the Jones vector $L_h$ for the input light is given by
If \( x \) and \( y \) are the components of the electric field at the output after passing through the \( \lambda/2 \) plate, polarised vertically and horizontally respectively, then

\[
\begin{pmatrix}
  x \\
  y
\end{pmatrix} = P_2 L_h
\]

Therefore:

\[
x = \frac{1}{2} \exp(i\phi_1) [1 - \exp[i(\phi_2 - \phi_1)]], \quad \text{and}
\]

\[
y = \frac{1}{2} \exp(i\phi_1) [1 + \exp[i(\phi_1 - \phi_2)]].
\]

Note that for a perfect \( \lambda/2 \) waveplate \( \phi_1 - \phi_2 = \pi \), and the polarisation of the light at the interferometer output would be purely vertical. This would be described by the Jones vector \( L_v \),

\[
L_v = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
\]

However, for a \( \lambda/2 \) plate that has phase error \( \Delta\phi(\lambda) \), \( y \) would not be equal to zero, but would instead be equal to a residual component described by:

\[
y = \frac{1}{2} [1 + \exp[i(\pi + \Delta\phi(\lambda))]].
\]

Note the standing phase \( \exp(i\phi_1) \) has been ignored.

As stated earlier, the criteria for acceptable operation of the configuration is that the horizontal component of the output field is less than 10\% of the original horizontal component ie. \( |y| < 0.1 \). Solving for \( \Delta\phi(\lambda) \) by plotting \( y \) using Matlab shows,

\[
\Delta\phi(\lambda) < 0.2 \text{ rad.}
\]

ie. that the phase errors in the \( \lambda/4 \) plate should be less than 0.2 radians.
Assuming that the plate is a zero’th order plate and is correct for a design wavelength of 632.8 nm (which was the case in our experiments), the dispersion curves for quartz allow calculation of the maximum wavelength range allowable before this limit is exceeded. It was found that a wavelength range of approx. ±40nm about 632.8nm was possible. For a larger allowable wavelength range, an achromatic or super-achromatic waveplate, or an achromatic Fresnel romb retarder could be used instead.

3.7 Experiment 3: Optical Losses of Meadowlark LC-SLM configured to be Insensitive to Polarisation State of Incident Light

Simulations and experiments using the Meadowlark LC-SLM configured to be insensitive to polarisation state of incident light are presented in this section. This work was part of a collaborative project with the University of Durham. We performed the simulations here in Auckland, and James Gourlay and Ray Sharples obtained the experimental results at the University of Durham.

The purpose of this experiment was to assess the optical losses associated with using the Meadowlark LC-SLM in a practical system, configured to be insensitive to the polarisation state of incident light. The technique used to assess the performance of the device was to write a phase wedge across the LC-SLM that would deflect an incident optical beam. When imaged, the Point Spread Function (PSF) was observed to be off-axis. A measurement of the resulting Strehl ratio (ratio of off-axis PSF height to on-axis PSF height obtained with no phase wedge applied) gives a useful measure of any optical losses, and hence a comparison of the device performance to that expected from an ideal phase wedge. The effects of optical losses due to the components (LC-SLM, λ/4 plate), non-ideal LC material parameters, diffractive effects due to the pixellation of the wedge pattern on the SLM, and the finite pixel size, should all be accounted for in the measurement of the Strehl ratio.

First the optical setup used for the experiment will be described. The simulations will then be discussed, before the experimental results are presented and compared with the results from the simulations.
3.7.1 Experimental Setup using Meadowlark LC-SLM Phase Wedge

Figure 3-9 shows the experimental setup used to measure the optical losses associated with the Meadowlark LC-SLM configured to be insensitive to the polarisation state of incident light, using a phase wedge.

Figure 3-9 Apparatus to find PSF resulting from phase wedge written to the Meadowlark LC-SLM

A randomly polarised, collimated laser was used for illumination and first passed through a non-polarising beam splitter. Then, as in the previous section, the light passed through the LC-SLM, a λ/4 plate, reflected off mirror 1, then back through the λ/4 plate and LC-SLM. The beamsplitter then diverted the light through a Fourier transform lens and onto a linear CCD camera for measurement of the PSF. The size of the phase wedge written to the modulator was 1 wave peak-to-peak at 633nm.

3.7.2 Phase Wedge Simulation

The performance of a 'perfect' Meadowlark LC-SLM device configured to be insensitive to the polarisation state of incident light was simulated numerically using Matlab. A matrix of complex numbers was created in the computer. The matrix represented an optical wavefront passing through the hexagonal pixel structure of the LC-SLM. Each complex number contained an amplitude and a phase which could be varied. An image plot of the matrix showing the pixel layout is given in Figure 3-10. For these simulations the phase
distribution across the modulator was arranged to be either a $2\pi$ phase wedge or a $4\pi$ phase wedge. Figure 3-11 shows a mesh plot of the simulated wavefront phase distribution for a $4\pi$ phase wedge. Note that the individual modulator pixels are clearly visible. The amplitude was assumed to be uniform and normalised to 1. Figure 3-12 shows a mesh plot of the wavefront amplitude. Note also that for the simulation the matrix size was 251 by 283. However, for mesh plot clarity only every 5th point has been plotted.

![Mesh plot of simulated wavefront phase distribution](image)

**Figure 3-10 Image plot of simulated pixel layout**

The expected intensity distribution in the output plane was calculated using a two dimensional numerical Fourier transform. Two simulations were carried out for different configurations. These were:

1) No $\lambda/4$ plate – LC-SLM with phase wedge,

2) $\lambda/4$ plate – LC-SLM with phase wedge.
Figure 3-11 Mesh plot of phase component of simulated matrix

Figure 3-12 Mesh plot of amplitude component of simulated matrix
In simulation 1) with a phase wedge, but no $\lambda/4$ plate to swap polarisation components, the LC-SLM modulated only one polarisation component. The other component remained unmodulated. Furthermore, the component that was modulated was modulated twice, once on each pass through the LC-SLM (effectively resulting in a $4\pi$ peak-to-peak wedge for this polarisation component). This resulted in two resolvable spots in the image plane, one on-axis spot from the unmodulated beam and one off-axis spot from the beam modulated twice. This simulation was performed by adding the output intensities of two separate simulations: the first using an input amplitude of 0.5 (instead of 1) and no phase wedge, and the second using an input amplitude of 0.5 and a phase wedge of $4\pi$ across the LC-SLM. Figure 3-13 shows the output intensity resulting from this simulation.

Simulation 2) with the $\lambda/4$ plate and a phase wedge of $2\pi$ applied across LC-SLM, resulted in a single off-axis spot since both polarisation components were modulated.

The height of a single on-axis spot with both the $\lambda/4$ plate and LC-SLM removed, (a diffraction limited spot), was assumed to be 1. Assuming no system aberrations, the height of a single on-axis spot with both the $\lambda/4$ plate and LC-SLM present, but no phase wedge applied across the LC-SLM (a control spot), was also assumed to be 1.

The Strehl ratio $\sigma$ of a modulated spot is defined by:

$$\sigma = \frac{I_m}{I_u},$$

where $I_m$ is the peak intensity of the modulated spot and $I_u$ is the peak intensity of the unmodulated diffraction limited spot.

In simulation 1) the Strehl ratio for the unmodulated spot was calculated to be $\sigma_{u} = 0.5$, and the Strehl ratio for the twice modulated spot was $\sigma_{2m} = 0.38$.

The Strehl ratio for the modulated spot from simulation 2) was calculated to be $\sigma_2 = 0.88$.

Table 3 shows the results of the simulations along with the experimental results for ease of comparison. The experimental results are discussed in the next section.
Figure 3-13 Simulated PSF for phase wedge of one wavelength and no λ/4 plate

Figure 3-14 Experimental PSF for phase wedge of one wavelength and no λ/4 plate
3.7.3 Experimental Results using Meadowlark LC-SLM Phase Wedge

First it was necessary to obtain the experimental results for the diffraction limited case. Therefore, the LC-SLM and the $\lambda/4$ plate were both removed. The height of the PSF was then measured, to use for normalisation in subsequent experimental Strehl ratio measurements. Measurements were then taken of the Strehl ratio when the $\lambda/4$ plate and LC-SLM were reinserted, with and without a $2\pi$ phase wedge written across the LC-SLM.

As an example of experimental results, Figure 3-14 shows the deviated and undeviated spots obtained with no $\lambda/4$ plate, and the wedge applied across the LC-SLM, and Figure 3-13 shows the equivalent spots obtained from the simulation, for comparison. The two plots are very similar, with an on-axis spot, and a deflected off-axis spot easily visible in each.

Table 3 shows the results of the measurements for the various configurations and gives a comparison with the simulation results.

<table>
<thead>
<tr>
<th>Optical Configuration</th>
<th>Measured Strehl Ratio</th>
<th>Simulated Strehl Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffraction limit (no $\lambda/4$ plate – no LC-SLM)</td>
<td>1 (by definition)</td>
<td>1 (by definition)</td>
</tr>
<tr>
<td>No $\lambda/4$ plate – LC-SLM with no phase wedge</td>
<td>0.83</td>
<td>1 (assuming no system aberrations)</td>
</tr>
<tr>
<td>No $\lambda/4$ plate – LC-SLM with phase wedge</td>
<td>Two spots 0.43, 0.31</td>
<td>Two spots 0.5, 0.38</td>
</tr>
<tr>
<td>$\lambda/4$ plate – LC-SLM with no phase wedge</td>
<td>0.80</td>
<td>1 (assuming no system aberrations)</td>
</tr>
<tr>
<td>$\lambda/4$ plate – LC-SLM with phase wedge</td>
<td>0.63</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 3 Results on the measurement of Strehl Ratios
The critical figure of merit for this (Meadowlark) LC-SLM is the Strehl ratio for the device configured to be insensitive to the polarisation state of incident light (ie. λ/4 plate present) and with a phase wedge written to the LC-SLM. Under these conditions the experiment gave a Strehl ratio of 0.63. This is a fair appraisal of the device performance in optical systems, taking into account all device limitations except any chromatic effects described above. It gives the approximate upper limit of the ability of the device to correct for aberrated wavefronts in adaptive optics systems ie. the Strehl ratio can be improved up to 0.63. The Strehl ratio obtained from the simulation under these conditions is 0.88, higher (as expected) than the experimental value. The difference may be ascribed to non-uniformities of phase modulation across individual SLM pixels, connecting wires to the electrode structure, and standing aberrations in the optical system.

The critical figure of merit for comparison with the simulation is the ratio: (Strehl ratio with λ/4 plate and phase wedge) / (Strehl ratio with λ/4 plate and no phase wedge), that is, a comparison of the Strehl ratio degradation which occurred when the phase wedge was written on the SLM, since aberrations introduced by the rest of the components could not be simulated reliably. Table 3 shows that this ratio was: 0.63/0.8=0.79 in the experiment and 0.88/1=0.88 in the simulation.

3.8 Experiment 4: LC-SLM Time Response

Following the introduction in this section, brief descriptions of two non-conventional LC-SLM drive techniques are given. These are techniques using the Transient Nematic Effect\cite{52}, and the Dual Frequency Control\cite{53,54} technique. Methods used to accurately determine the response time of the 81 segment LC-SLM are then discussed, before experiments using the Transient Nematic Effect to decrease the response time of the LC-SLM are presented. Results showed that this technique significantly reduced the response time of the LC-SLM.

3.8.1 Introduction

For good correction of time varying optical phase aberrations introduced by the atmosphere, it is preferable to have a phase modulator capable of modulation rates between 30Hz and 200Hz\cite{55}. This corresponds approximately to a time response of between 16ms and 2.5ms. When driven in the conventional manner ie. by simply applying the voltage
corresponding to the desired phase delay across each LC-SLM pixel, the modulator response time is between 50ms and 200ms depending on where in the driving voltage range, and what direction the device is driven. The response time is much faster when using high values of driving voltage, and also depends on the liquid crystal material, layer thickness, viscosity, temperature, surface treatment and driving waveform\textsuperscript{[56]}. Liquid crystal layer thickness\textsuperscript{[5]} of between 4 and 10 microns are common in recent devices purpose built for use as spatial light modulators\textsuperscript{[47]}. When the LC-SLM drive voltage is increased, the resulting electric field across the liquid crystal layer applies a torque to each liquid crystal molecule. When the drive voltage is switched to a lower value or removed, interactions between liquid crystal molecules provide the dominant forces. The molecule interactive forces are much weaker than the torque caused by the electric field. This leads to a slower relaxation response time (time response for a decreasing phase change) than rise response time (time response for an increasing phase change).

Note that since the response time of the LC-SLM is not wavelength dependent, a single wavelength has been used in the following experiments. Therefore, for convenience the term "phase" instead of "stroke" has been used.

3.8.2 Transient Nematic effect

A brief description of the Transient nematic effect will now be given. The transient nematic effect can be used to reduce the rise time response of a LC-SLM pixel by initially applying a large voltage across the pixel. When the pixel has reached the desired value of phase delay, the voltage across it is reduced to a value that holds the phase delay constant. The LC-SLM is essentially 'kicked' to the desired state as fast as possible. Similarly, but to a lesser extent, the effect can be used to reduce the relaxation time response of a pixel by applying a low voltage (usually zero volts AC) across the pixel until it has reached the desired phase delay. The voltage is then increased to a value that holds the phase delay constant. By making use of the transient nematic effect it is possible to reduce the LC-SLM response time electronically - rather than by physically altering the panel.

Note that since LC-SLMs are driven with AC voltages, it is not possible to apply a voltage lower than zero (since this is just a positive AC voltage with a $\pi$ phase change) across the modulator. The transient nematic effect is more effective at decreasing the rise time response than decreasing the relaxation time response for two reasons.
1) It is possible to apply a very large positive voltage across the LC-SLM (limited by liquid crystal breakdown) for rising phase, compared zero volts which is the lowest possible voltage for relaxing phase.

2) The dominant force for rising phase is the torque due to the applied electric field whereas the dominant force for relaxing phase is the interaction between liquid crystal molecules. The molecule interaction forces are much weaker than the torque caused by the electric field.

Therefore, when using the transient nematic effect (as with conventional drive techniques) the limiting case for LC-SLM operating frequency will always be the relaxation time.

### 3.8.3 Dual Frequency Control Technique

Another technique that can be used to reduce the response time of a LC-SLM is the dual frequency control technique\[53,54\]. The dual frequency control technique was not tested during experiments for this Thesis, but a brief description is incorporated here for interest, since it is likely to play a large role in future LC-SLMs.

The phase delay provided by a liquid crystal material depends on the orientation of the liquid crystals. The dielectric anisotropy $\Delta \varepsilon$ of most liquid crystal material is positive at low frequencies i.e. up to a few hundred Hertz. This results in the liquid crystals orienting themselves in the direction of an applied electric field, which produces maximum phase delay. However, at higher frequencies i.e. in the range of tens of kHz, $\Delta \varepsilon$ changes sign, and the liquid crystals orient themselves perpendicular to the direction of the applied electric field, which produces minimum phase delay. The frequency at which $\Delta \varepsilon$ changes sign is called the crossover frequency $v_c$. The sign reversal of $\Delta \varepsilon$ can be used to orient the liquid crystals in any orientation between perpendicular and parallel to the electric field by applying frequencies on either side $v_c$. This phenomenon is usually much faster than using conventional drive techniques, and results in a similar variable phase delay.

Rise and relaxation times for a $\pi$ phase change were measured by Restino et al\[54\] to be 0.6ms and 2.8ms respectively, using a LC-SLM and drive electronics custom built by Meadowlark Optics. Note that the relaxation time, although much faster than using conventional drive methods, is still the limiting case.
3.8.4 Conventional Time Response

First it was necessary to accurately determine the response time of the liquid crystal phase modulator when driven using conventional drive techniques.

3.8.4.1 Common Path Interferometer to determine LC-SLM Response Time

Figure 3-15 shows a diagram of the common path interferometer used to determine the time response of the LC-SLM.

Figure 3-15 Common path interferometer used to determine LC-SLM response time

A linearly polarised, 10mW HeNe laser was expanded and collimated by two lenses, so that the beam passed through a Soleil Babinet compensator and one pixel of the LC-SLM. The polarisation of the input laser beam was set to be 45° from horizontal. Therefore, the horizontal component (modulated by the LC-SLM) and vertical component (not modulated by the LC-SLM) of the input beam could be thought of as separate beams which propagated independently through the interferometer. The arrows above the components in the diagram are intended to represent the polarisations of the beams (if the beams were passing into the plane of the page) as they passed through the interferometer. Once the two beams had passed through the LC-SLM, they were recombined and made to interfere by selecting a common polarisation component of each with another polariser oriented at 45° from horizontal.
The Soleil Babinet compensator provided a variable phase difference between the horizontal and vertical polarisation components of the input light. This allowed the path difference in the interferometer when the initial voltage was applied to the LC-SLM pixel to be set to a value such that the output intensity was either minimum or maximum before each time response measurement. This made the retrieval of LC-SLM phase far easier during subsequent processing of the interferometer output data. The phase of the LC-SLM was reconstructed from the intensity in the same manner as described in section 3.5.2.

The output intensity was detected with a photodiode. The current produced by the photodiode was amplified and converted into a voltage by a current-to-voltage converter. This voltage was then sampled by an A/D board in the IBM compatible host computer at rates of up to 2MHz and saved to file. The acquisition of data from the photo diode and the LC-SLM modulation were both controlled by programs written in QuickBasic.

Figure 3-16 shows a plot of interferometer output intensity as a function of LC-SLM drive voltage. The amplitude of the drive voltage was varied slowly between 0 and 5V. This plot compares very well to the output intensity at a wavelength of 633nm in Figure 3-4.

Figure 3-16 Interferometer output intensity vs. voltage applied to LC-SLM
The modulator was allowed to reach a steady state after the application of each new voltage step, before the intensity at the photodiode was measured. Figure 3-17 shows the corresponding reconstructed modulator phase in radians vs. LC-SLM applied voltage. Note that there is a small step discontinuity in this plot at about 2.5V. This is due to the piecewise approximation of the visibility \( V \) and average output intensity \( A \) in equation (3-2) used to reconstruct the phase.

![Figure 3-17 Common path interferometer output reconstructed phase vs. applied voltage](image)

3.8.5 Time Response Test Procedure

Optimal results would have been obtained if the intensity at the photo diode were constantly monitored throughout the process of modulating the LC-SLM. However, because the same computer was used to modulate the device and receive information from the photo diode, it was not possible to carry out these two activities at exactly the same time. Therefore the experimental procedure was to first use the computer to set the applied voltage to an initial value \( V_1 \) and wait 5 seconds or so to allow the modulated pixel to reach steady state. The computer then switched to monitoring the photo diode to establish the initial output intensity. The monitoring of the photo diode by the computer was then briefly stopped, long
enough for the computer to send the necessary modulation instructions to the LC-SLM, i.e. a voltage step function from the initial value $V_1$ to a final value $V_2$. As soon as the modulation instructions had been sent, the computer reverted to monitoring the photo diode. Although the time taken for these instructions to be sent was small (less than 10 ms) there were sometimes small discontinuities in the data if the intensity was rapidly changing during the time the photo diode was not monitored. These discontinuities were easily removed later by adding an appropriate constant to the affected data during the data processing.

3.8.6 Conventional Time Response Results

The time constant $\tau$ which is the time it takes the modulator phase to go from an initial phase $\phi_1$ to a phase $\phi_m$ defined by

$$\phi_m = \phi_2 - \left( \frac{\phi_2 - \phi_1}{e} \right) \quad (3-3)$$

where $\phi_1 =$ initial phase,

$$\phi_2 =$ final phase, and

$$e = 2.7183.$$

In the following plots, the voltage step applied to the LC-SLM was applied at time $t=0$. The horizontal lines on the phase plots are positioned so that they intersect the graphs at a time $\tau$, which has been calculated for each plot.

The top of Figure 3-18 shows the interferometer output intensity change as a voltage step from 1.85V to 2.5V was applied across the test pixel of the LC-SLM. Figure 3-17 shows that this corresponds to a phase change of $+\pi$ rad. The corresponding plot of reconstructed phase is shown at the bottom of Figure 3-18, and confirms the size of the phase change. The time constant for the phase change in this plot was $\tau = 0.08$ seconds. Figure 3-19 shows the output intensity as the opposite voltage step i.e. decreasing from 2.5V to 1.85V was applied across the test pixel which corresponds to a decreasing phase change of $\pi$ rad. The corresponding plot of reconstructed phase is shown at the bottom of Figure 3-19.
Figure 3-18 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 1.85V to 2.5V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.08 sec

Figure 3-19 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 1.85V to 1.25V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.083 sec
This shows a time constant of $\tau = 0.083$ seconds. One full $\pi$ rad. phase cycle of the LC-SLM (ie. one going from 0 rad. to $\pi$ rad. and back to 0 rad. again) would therefore take 0.16 sec. which corresponds to an operating frequency of $f_0 = 6.1$Hz.

Figure 3-20 Top shows the output intensity as a voltage step of 2.5V to 3.5V was applied across the test pixel. This again corresponds to an increasing phase change of $\pi$ rad. but further up the LC-SLM phase characteristic curve. Figure 3-20 Bottom shows the corresponding plot of reconstructed phase. Here the time constant was $\tau = 0.025$ seconds. This was just over 3 times faster than the phase plot shown in Figure 3-18 Bottom. This shows that response times due to a small change in high voltage, were much shorter than the response times due to a small change in low voltage.

![Figure 3-20 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 2.50V to 3.50V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.025 sec.](image)

Further, Figure 3-21 Top shows the interferometer output intensity as an increasing voltage step of 1.25V to 4.75V was applied across the test pixel of the LC-SLM. This corresponds
to a phase increase of 3.3π rad. and the plot of reconstructed phase is shown in Figure 3-21 Bottom. This was a much larger increasing voltage step, and therefore a much larger phase change than those shown in the previous graphs. However, the time constant was τ = 0.027 seconds, approximately the same as the time constant for the smaller π phase jump shown in Figure 3-20, and much shorter than the time constant for the phase jump shown in Figure 3-18 and Figure 3-19. Figure 3-22 Top shows the interferometer output intensity as a decreasing voltage step of 3.75V to 0V was applied across the test pixel, and the corresponding plot of reconstructed phase is shown in Figure 3-22 Bottom. Here the time constant was τ = 0.070 seconds, which, despite being a larger jump was shorter than the decreasing phase jump shown in Figure 3-19.

Figure 3-21 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 1.25V to 4.75V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.027 sec
Figure 3-22 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 3.75V to 0V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.070 sec

This shows that the rate of phase change for a large voltage step is substantially faster than the rate of change for a small voltage step at low applied voltages.

3.8.7 Decreased Modulator Response Times using Transient Nematic Effect

Results from the experiments described above confirmed that the Transient Nematic Effect can be used to decrease the response time of the LC-SLM, ie. that it should be possible to 'kick' the modulator to the desired phase. Figure 3-23 shows a plot of interferometer output intensity where the LC-SLM was driven using the Transient Nematic Effect. Here the voltage applied across the LC-SLM pixel was initially 1.90V. The voltage was then briefly increased to the maximum 4.75V (between $t = 0$ sec and $t = 0.01$ sec.) and then reduced to 2.5V for the remaining time. Here the time constant was $\tau = 0.005$ sec., much shorter than the time constant of $\tau = 0.08$ sec. obtained without the 'kick' in Figure 3-18. For simplicity the same definition for the time constant $\tau$ ie., Equation (3-3) is used again.
Figure 3-23 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 1.90V to 2.50V via 4.75V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.005 sec

It was also found that the time constants for decreasing phase jumps were reduced by briefly applying 0V across the LC-SLM pixel. Once the desired phase was reached, the applied voltage was increased to a value that maintained that phase. Figure 3-24 shows this, where the panel was initially modulated with 2.50V, then briefly reduced to 0V, before being increased back up to 1.90V. Here the time constant was $\tau = 0.019$ seconds which was much shorter than the $\tau = 0.083$ seconds shown in Figure 3-19 obtained without using the transient nematic effect. Note however that this was much slower than the 0.005 sec. obtained for an increasing phase of the same magnitude, confirming that even using the Transient Nematic Effect, the relaxation times were longer than rise times. One full $\pi$ rad. phase cycle would therefore take 0.024 sec. which corresponds to an operating frequency of $f_o = 42$Hz. This is above the desired 30Hz necessary to efficiently correct atmospheric induced optical aberrations, and almost 7 times faster than the operating frequency of 6.1Hz, which was achieved without the Transient Nematic Effect.
Figure 3-24 Top: Interferometer output intensity vs. time for LC-SLM applied voltage step from 2.50V to 1.90V via 0V. Bottom: Corresponding reconstructed phase in radians showing a time constant of 0.019 sec

3.9 Conclusion

Four experiments using two LC-SLMs were described in this Chapter. Results for each were presented and discussed. The two LC-SLMs were a device with 81 square pixels, and a 69 pixel device made by Meadowlark Optics. First however, the requirements for spatial light modulators used in aberration correction were discussed. Details of both the 81 segment LC-SLM and the Meadowlark LC-SLM were then given before the four experiments were discussed in detail, and results from each presented.

The first experiment tested the modulation characteristic wavelength dependence of the 81 pixel LC-SLM. Results showed that the modulation characteristic varied by less than 20% over a range of input wavelengths from 400 to 900nm.
The second experiment tested the modulation characteristic of the 81 pixel LC-SLM in a system configured to be insensitive to the polarisation state of incident light. A $\lambda/4$ plate was used to swap orthogonal polarisation components of the incident light between two passes through the LC-SLM, thus modulating all polarisation components. Results using this configuration showed that the variation in modulation characteristic was less than 4% for any incident polarisation state. The $\lambda/4$ plate was a zero'th order plate made of quartz. The quartz dispersion curve was then used in a Jones matrix calculation to estimate the variation of LC-SLM modulation with wavelength, when configured to be insensitive to polarisation state. It was found that practical satisfactory performance would be obtained over a wavelength range of approx. ±40nm about 632.8nm.

The third experiment was carried out by James Gourlay and Ray Sharples using the Meadowlark LC-SLM in Durham as part of a collaborative project and was included in order to compare with simulations we carried out here in Auckland. The purpose of this experiment was to assess the optical losses associated with using the Meadowlark LC-SLM in a practical system configured to be insensitive to the polarisation state of incident light. A phase wedge was applied across the LC-SLM, which deflected an incident optical beam. Conclusions about the optical losses where drawn by comparing the point spread function (PSF) of the deflected beam, with the PSF of a non-deflected beam obtained with a uniform phase applied to the LC-SLM. Details of our computer simulations were presented. The simulation results were compared to the experimental results from Durham and showed good agreement. An upper limit of 0.63 was placed on the expected Strehl ratio, for an adaptive optics system using this LC-SLM, configured to be insensitive to the polarisation state of incident light.

The fourth experiment tested the response time of the 81 pixel LC-SLM using both conventional drive techniques, then an electronic overdriving technique called the Transient Nematic Effect. Here the LC-SLM was essentially kicked to the desired phase. Results showed that the response time was significantly reduced using the transient nematic effect, and that the operating frequency could be increased from 6Hz to over 40Hz.

The experimental results indicate that liquid crystal spatial light modulators are good candidates for high performance phase modulating elements in atmospheric aberration correction applications.
Chapter 4. General Principle, Theory, and Background of Feedback Interferometry

4.1 Introduction

There has recently been much interest in simple low cost adaptive optical aberration correction systems. Adaptive optics systems correct wavefront phase distortions. As discussed in Chapter 1, conventional adaptive optics systems generally consist of two smaller sub-systems. These subsystems are a wavefront sensor (often a Hartmann-Shack wavefront sensor) to determine the shape of the aberrated wavefront, and a wavefront corrector (deformable or segmented mirror) which removes the aberrations. In a feedback interferometer these are effectively combined so that wavefront correction occurs automatically as a part of wavefront sensing. In this chapter, the principle and theory of feedback interferometry using monochromatic light, and the technique of radial shearing to produce a plane reference wave is discussed. The background and development of feedback interferometry is then given.

4.2 General principle of Feedback Interferometry

The general principle of feedback interferometry will now be described with reference to a generic single pixel Michelson interferometer as shown in Figure 4-1. The input beam enters the system from the left, and is split in two by the beam splitter. The beam transmitted through the beam splitter is passed through the phase modulator, reflected from mirror 2 back through the phase modulator, then reflected out off the beam splitter. The input beam that is reflected off the beam splitter is reflected from mirror 1 back through the beam splitter, where it is combined with the first beam forming an interference fringe pattern on a detector. A phase proportional (by the feedback gain $G$) to the intensity at the
detector is applied by the phase modulator to one beam in the interferometer. This feedback causes the interference fringe pattern produced at the output of the feedback interferometer to have a 'saw-tooth' profile as the interferometer phase \( \phi_o \) is varied rather than the usual sinusoidal profile.

![Diagram of feedback interferometer](image)

**Figure 4-1 General feedback interferometer**

### 4.3 Aberration Correcting Feedback Interferometer

Figure 4-2 shows a sketch that gives an overview of an aberration correcting interferometer. It shows a Mach-Zehnder interferometer with a radial shearing telescope in one arm and a spatial phase modulator in the other. The aberrated input wave is divided in two as it enters the system. One beam passes through the radial shearing telescope which creates a semi-plane reference wave. Details of the radial shearing telescope are given in the next section. The other interferometer beam passes through the phase modulator before being interfered with the reference plane wave. A phase proportional to the output intensity is applied to the phase modulator, on a pixel by pixel basis. The beam passing through the phase modulator is also the corrected output wave. This configuration is used so that aberrations introduced externally to the system can be estimated and corrected without the need for a separate external reference wave. The theory of feedback interferometry is given in some detail following the description of the radial shearing telescope which is presented next.
4.4 Radial Shearing Telescope

A radial shearing telescope may be used to provide a semi-plane reference wave, derived from the input beam in an interferometer. Figure 4-3 shows the principle of the radial shearing telescope. A radial shearing telescope consists of two converging lenses separated by the sum of their focal lengths. With this configuration, the incident beam which contains spatial aberrations but is nominally collimated and has width $\omega_1$, is enlarged radially by the input lens. The enlarged beam is then recollimated by the output lens. The output beam in the rear focal plane of the second lens (radius $\omega_2$) is a radially magnified replica of the input aberrated beam in the front focal plane of the first lens.

Figure 4-3 Radial shearing principle
From the diagram it is evident that the central portion of the enlarged output wave approximates a plane reference wave. There is a corresponding loss of intensity in the output beam as a result of expansion. The expansion ratio of the telescope is given by,

$$ R = \frac{\omega_2}{\omega_1}, $$

where $\omega_1$ is the input beam width and $\omega_2$ is the output beam width. Simple geometrical arguments show this can be rewritten as,

$$ R = \frac{f_2}{f_1}, $$

where $f_1$ is the focal length of the input beam expanding lens and $f_2$ is the focal length of the output collimating lens.

### 4.5 Theory of Feedback Interferometry

The theory of monochromatic feedback interferometry will now be presented with reference to the single pixel Michelson interferometer shown in Figure 4-1. The theory will be extended to white light in Chapter 5.

The equation for the output intensity of a feedback interferometer is given by:

$$ I_{out} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_o + \phi_m) $$

where $I_{out}$ is output intensity,

$I_1$ and $I_2$ are the two interferometer beam intensities,

$\phi_o$ is the standing interferometer phase, and

$\phi_m$ is the modulator phase

The modulator phase can be written $\phi_m = Gf(I_{out})$,

where $G$ is the feedback loop gain, and

$f$ is the phase modulator characteristic.

The equation for the output intensity of a feedback interferometer can therefore be rewritten as:
The output intensity $I_{out}$ appears on both sides of the equation, thereby making it difficult to solve analytically. Instead, the solution is found by plotting both the left hand side ($lhs$) and right hand side ($rhs$) of the equation against $I_{out}$ on the same graph as shown in the graphs of Figure 4-4. The solution of $I_{out}$ occurs at the intersection of the two plots. Here the modulator characteristic $f$ is assumed to be linear.

Figure 4-4 Graphical solution to feedback interferometer equation showing, Top: one stable solution, Middle: three possible solutions of which A and C are stable, Bottom: a larger feedback loop gain $G$ with five solutions of which D, F, and H are stable.

### 4.6 Stable Solutions

Figure 4-4 (Top) is plotted with a value of $\phi_0$ such that there is a single solution to the feedback interferometer equation. As the interferometer phase $\phi_0$ is varied, the plot of $rhs$ moves sideways as shown in Figure 4-4 (Middle). This shows that for values of $\phi_0$ where the solution is near the bottom of a $rhs$ fringe, there may be more than one possible solution. It has been shown\[57\] that a solution is stable only when the gradient of the $lhs$ at
the solution is greater than the gradient of the rhs. Therefore the solutions $A$ and $C$ are stable, and solution $B$ is unstable. A stable solution is tracked on a fringe as $\phi_o$ varies until that solution becomes unavailable or unstable. When this occurs the solution jumps to the next nearest stable solution. For every value of $\phi_o$ there is always at least one stable solution.

Increasing the feedback loop gain $G$ effectively decreases the slope of the lhs plot as shown in Figure 4-4 (Bottom). This increases the number of possible solutions. Here solutions $D$, $F$, and $H$ are stable, and $E$ and $G$ are unstable. Increasing $G$ also increases the range over which the solution can track a particular rhs fringe provided the system is not limited by the stroke of the modulator.

Figure 4-5 (top) shows computer simulation results for the solution of how $I_{out}$ varies as a function of interferometer phase $\phi_o$.

![Figure 4-5 Feedback interferometer equation output intensity as a function of increasing (solid line) and decreasing (dotted line) interferometer phase](image)

The solid line is plotted with increasing $\phi_o$, and the dotted line is plotted with decreasing $\phi_o$. They both resemble saw-tooth patterns and differ considerably from the sinusoidal output.
interferogram of a non-feedback interferometer. Note that the plots exhibit hysteresis between the increasing and decreasing graphs. Because of the hysteresis, the range over which the system stays locked to a particular solution i.e. the system capture range, is about 8 rad.

Figure 4-6 shows the computer simulation of the feedback interferometer equation solution as a function of $\phi_0$ with a larger feedback loop gain $G$. Again the solid line shows the solution for increasing $\phi_0$, and the dotted line for decreasing $\phi_0$. Hysteresis is again evident. However, with increased $G$, the capture range has increased to about 10 rad. In theory $G$ can be increased indefinitely, so that the capture range is limited only by the range of the phase modulator. However, in a practical feedback interferometer system, increasing $G$ beyond a critical value puts the system into oscillation. The time response of the system is also very important. A system with a slow time response will go into oscillation at a lower value of $G$.

![Figure 4-6 Feedback interferometer equation output intensity as a function of increasing (solid line) and decreasing (dotted line) aberration phase with increased feedback loop gain, showing increased capture range.](image)

77
For large sections of the capture range in the plots shown in Figure 4-5 and Figure 4-6 (except for a small region at the bottom of each solid line saw-tooth, a small region at the top of each dotted line saw-tooth and the jump between adjacent stable solutions) $I_{out}$ is uniquely defined by $\phi_o$. Further, since $I_{out}$ is related to the phase applied by the modulator $\phi_m$, the graphs show that the modulator phase $\phi_m$ decreases as the standing interferometer phase $\phi$ increases. This is precisely what is required in an aberration correction system and shows that aberration correction occurs automatically in a feedback interferometer without the need for separate detection and correction systems.

4.7 Background and Development of Feedback Interferometry

A brief description of the background and development of feedback interferometry will now be presented. Techniques for correcting images through turbulent media were first suggested by Babcock\cite{58} in the 1950's. Since then, advances in wavefront sensors using techniques such as curvature sensing\cite{59}, lateral shearing interferometry\cite{60} and lenslet arrays in devices such as the Hartmann-Shack sensor\cite{61}, along with the ready availability of wavefront correctors such as deformable and segmented mirrors\cite{62,63}, have improved aberration correction systems greatly. Feedback interferometry is another technique that has potential for aberration correction.

4.7.1 Interference Phase Loop

Most adaptive optics systems consist of separate detection and correction systems, linked together in a final servo loop. However in 1979 Fisher and Ward\cite{64} introduced a self-referencing system called an interference phase loop (IPL). Using the IPL approximate aberration correction is achieved essentially as part of the wavefront detection process. Figure 4-7 shows a diagram of the IPL.

The IPL is effectively a two beam interferometer where the output intensity is fed back to control the phase of an optically addressed spatial light modulator in one arm of the interferometer. The resulting modulator phase is very nearly proportional to the conjugate of the phase difference between the two arms of the interferometer. Fisher and Ward\cite{65} presented results in a second paper, which showed the correction of aberrations introduced into one arm of a Mach Zehnder interferometer, and Fisher\cite{66} later gave a detailed
theoretical analysis of the IPL. It was suggested that a Zernike phase-contrast filter, or a Schlieren device could be used as the all-optical phase sensor.

![Image of interference phase loop]

**Figure 4-7 Interference phase loop.**

### 4.7.2 Radial Shearing Mach-Zehnder Feedback Interferometer

In previous work, it was shown that feedback interferometry may be used to correct aberrations introduced externally to the system using a radial shearing interferometer with an electrically addressed liquid crystal spatial light modulator (LC-SLM). Figure 4-8 shows a simplified diagram of the interferometer.

![Image of Mach-Zehnder feedback interferometer]

**Figure 4-8 Mach-Zehnder feedback interferometer**
The LC-SLM was positioned within one arm of a self-referencing radial shearing Mach-Zehnder interferometer. A phase proportional to the interferometer output intensity was applied to the LC-SLM on a pixel by pixel basis. The input wavefronts were distorted using a non-uniform piece of glass. Results showed that the point spread function of the Fourier transform of the output reduced noticeably when the feedback loop was closed.

4.7.3 Optically Addressed Systems

Barnes et al \cite{69} then demonstrated diffraction-limited real-time aberration correction of arbitrary input wavefronts using a polarisation radial shearing Sagnac interferometer and an optically addressed phase only spatial light modulator (Hamamatsu PAL-SLM). Barnes et al \cite{70} also built a system, using the Hamamatsu PAL-SLM in one arm of a Michelson interferometer. They showed that feedback interferometry can be used for direct, unambiguous, display of fringe phase in measurement applications.
Chapter 5. White Light Feedback Interferometry for Adaptive Optics

5.1 Introduction

White light feedback interferometry is very similar to monochromatic feedback interferometry except white light is used. There are several advantages using white light interferometry for aberration correction. The central fringe of the output fringe pattern of a white light interferometer provides a unique zero path difference reference. The fringes have different peak intensities, which may be used to either determine which fringe the system is tracking, or to ensure the system tracks a known fringe.

Interferometric techniques using broad band sources often require the use of a monochromator or narrow band filter. This reduces the number of useable photons to a small fraction of the total entering the system. The vastly increased bandwidth of a white light system would enable the technique to be used on temporally incoherent sources such as astronomical objects with much improved optical efficiency by allowing the majority of photons to be used.

Two different systems are examined in this chapter. Results for theoretical simulations and experimental implementations of the systems are given and compared. The first system is a single pixel Michelson interferometer with two piezo driven mirrors. One mirror provides the interferometer stroke, and the second provides the feedback. This is a proof-of-principle experiment and the results show that aberration correction using white light feedback interferometry is possible using a linear phase modulator.

The second system is a common path radial shearing Sagnac interferometer, which uses one pixel from a LC-SLM to provide the interferometer (aberration) stroke and another pixel from the same LC-SLM to provide feedback. Again, this is a proof-of-principle experiment
only. The results show that in principle, aberration correction using feedback interferometry is achievable in white light, using a liquid crystal phase modulating device. We believe this is the first time feedback interferometry has been performed using white light with either a liquid crystal spatial light modulator or a segmented mirror.

If a white light radial shearing interferometer were to be built for which feedback were applied to the input aberrated wavefront at positions away from the optic axis, certain spatial coherence considerations would need to be addressed. Some of these considerations are then investigated. An equation relating the size of the input pinhole, the focal length and aperture size of the collimating lens, and the radial shear magnification is determined, such that the degree of coherence between the interferometer beams goes to 0.2 at the edge of the output fringe pattern.

Several techniques to maximise the interferometer power output are then investigated. As a result, an alternative expression for the aperture size of the input pinhole is derived, and the optimum beam splitter transmission-reflection coefficient ratio is shown to be 50-50.

It is then shown that for a matched white light radial shearing interferometer which provides a specified degree of spatial coherence between the two interferometer beams at the edge of the output fringe pattern, the average intensity at the output is dependent only on the colour temperature of the source.

First however, the equation for white light feedback interferometry is given and discussed.

### 5.2 White Light Feedback Interferometer Equation

We assume our interferometer has a path difference of zero so that the temporal coherence is such that we obtain fringes at the output using white light. The concept of ‘phase’ is not appropriate for work using white light so the term “stroke” measured in m is used instead.

The equation for the output intensity of a white light feedback interferometer is given by:

\[
I_{\text{out}} = I_1 + I_2 + 2\sqrt{I_1 I_2} h(x,y) \int_0^\infty \frac{c}{\lambda^2} F(\lambda) \cos \left( \frac{2\pi (d + a + Gf(I_{\text{out}}))}{\lambda} \right) d\lambda
\]  

(5-1)

where: \(I_1\) and \(I_2\) are the two interferometer beam intensities,

\(d\) is the interferometer stroke,

\(a\) is the aberration stroke,
$G$ is the feedback loop gain,
$f$ is the modulator characteristic,
$c$ is the speed of light,
$F$ is the effective power spectral density of the lamp as seen through the system, and
$h$ is a function describing the spatial coherence.

Here we assume $F(\lambda) = M(\lambda)D(\lambda)B(\lambda)$,

Where: $M(\lambda)$ is the normalised power spectrum of a black body, given by Planck's black body equation,

$D(\lambda)$ is the normalised detector characteristic, for which we used standard numerical data for silicon,

$B(\lambda)$ is a function incorporating the normalised characteristics of the remainder of the interferometer optical components such as the beam splitter and mirrors.

Note that $B(\lambda)$ has been assumed to be flat for these simulations.

Figure 5-1 shows the plot of $F(\lambda)$ used for the simulations in this chapter. It was fitted using Planck's black body radiation equation assuming a source colour temperature of $T=2600K$ and standard numerical data for the response of a silicon detector.

![Theoretical effective power spectral density $F(\lambda)$ of lamp as seen through the system](image-url)

Figure 5-1 Theoretical effective power spectral density $F(\lambda)$ of lamp as seen through the system
The simulations described in this chapter were carried out in a similar fashion to those in the previous chapter for monochromatic light. The \( lhs \) and \( rhs \) of equation (5-1) were plotted against \( I_{out} \) on the same plot. The solution of \( I_{out} \) occurred at the intersection of the two plots. The simulations were carried out using Matlab.

5.3 Experiment 1-White Light Feedback using Piezo Driven Mirror

A proof of principle experiment using a white light Michelson feedback interferometer containing a single pixel linear phase modulator will now be described. Figure 5-2 shows a diagram of the optical setup used for this experiment. Here two piezo driven mirrors were used as phase modulators, in a Michelson interferometer with no radial shear. Light from the incandescent bulb was focused through a 15\( \mu \)m pinhole by lens \( L_1 \) and collimated as well as possible by \( L_2 \). The light entered the Michelson interferometer through beam splitter \( BS \). Lenses \( L_3 \) and \( L_4 \) then imaged the interfering beams reflected from mirrors \( M_1 \) and \( M_2 \) in each interferometer arm onto a photodiode detector. Great care was taken to ensure the interferometer path difference was very close to zero in order to obtain fringes at the output.

![Diagram of White light Michelson interferometer using two piezo driven mirrors](image)

Figure 5-2 White light Michelson interferometer using two piezo driven mirrors

Mirrors \( M_1 \) and \( M_2 \) were moved in a piston manner (no tip or tilt) by piezo actuators controlled by a computer, each with a total scan range of about 6\( \mu \)m. Mirror \( M_1 \) was used to scan the interferometer stroke while feedback ie. a stroke proportional to the output intensity, was applied to mirror \( M_2 \) via the computer.
5.3.1 Simulation using White light and a Linear Phase Modulator

A simulation of the system will now be presented. This simulation used a linear phase modulator stroke characteristic \( f \), similar to the characteristic of a piezo controlled mirror. Note that there was no radial shear in this experiment, so \( h=1 \) in equation (5-1). Also, the aberrations are introduced within the interferometer so \( a=0 \), and the results are plotted against interferometer stroke \( d \).

For reference, Figure 5-3 shows a theoretical plot of the white light interferometer equation output as the interferometer stroke is varied, without feedback.

![Graph showing interferometer output intensity vs. interferometer stroke](image)

**Figure 5-3** White light interferometer equation output intensity using a linear modulator, without feedback

The time taken for a piezo to settle to steady state is less than the time for a LC-SLM. In practice this means that the feedback loop gain can be set higher for an interferometer containing a piezo driven mirror than one containing a LC-SLM before the system is driven into oscillation. For this reason the feedback loop gain was deliberately high in this simulation. Figure 5-4 shows the resulting simulation of output intensity vs. aberration
stroke. The sawtooth pattern found in the feedback interferometer simulations using monochromatic light in the previous chapter were again evident. Note however, that here there is a large variation in the size and shape of each “tooth” of the pattern. This variation was caused by the shape of the envelope of the white light fringe pattern and the high feedback loop gain. Within each saw tooth fringe, $I_{out}$ is uniquely defined by aberration stroke $a$. This region also shows that $I_{out}$ (which is related to the modulator stroke by the modulator characteristic) decreases linearly with increasing aberration stroke.

![Graph](image)

Figure 5-4 Simulated white light feedback interferometer equation output intensity using piezo driven mirror with high feedback loop gain

5.3.2 Experimental White Light Results using Piezo Mirror

Figure 5-5 shows a plot of the output intensity as mirror $M_1$ was scanned with no feedback applied to $M_2$. This plot compares well with the theoretical plot shown in Figure 5-3. Note that the fringe pattern is not symmetrical about a central bright fringe as might be expected for a white light fringe pattern. This is due to dispersion in the beam splitter. Figure 5-6 shows the output intensity as $M_1$ was scanned and feedback was applied to $M_2$. Comparing this to the simulated fringe pattern shown in Figure 5-4 we see that the experiment produces
results very similar to those predicted by theory. This shows that in principle aberration correction is possible for aberrations introduced within a Michelson interferometer using white light and a linear phase modulator such as a piezo driven mirror.

Figure 5-5 Michelson interferometer output intensity as one piezo mirror was scanned

Figure 5-6 Michelson interferometer output intensity with feedback
5.4 Experiment 2-White light feedback using LC-SLM

Having shown that white light feedback interferometry was in principle capable of correcting aberrations introduced within a Michelson interferometer using a piezo driven mirror, we next wanted to show it was possible in principle to correct aberrations introduced externally to the system. A radial shearing telescope was therefore incorporated into the system. Also, a Sagnac interferometer was used rather than the Michelson type interferometer used in the previous experiment. Sagnac interferometers are common path and therefore automatically have zero path difference, thereby making them ideal for use with white light. They are also insensitive to vibration. We also wanted to show it was possible to use a liquid crystal device for aberration correction with white light and so the Sagnac system included a LC-SLM as the modulating device.

In this section, the experimental optical setup is described. A simulation of the system is presented then the experimental results are given and discussed.

5.4.1 Optical Setup for White light Sagnac Feedback Interferometer

Figure 5-7 shows a diagram of the optical apparatus used for this experiment.

![Diagram of optical setup](image)

Figure 5-7 White light Sagnac interferometer with LC-SLM, and a radial shearing telescope
Light from an incandescent bulb was imaged onto a 15μm pinhole by lens L_1. The light was then collimated as well as possible by lens L_2 and passed through the LC-SLM before entering the Sagnac interferometer through the non-polarising beam splitter BS. The interferometer comprised mirrors M_1, M_2, and M_3, and contained two counter-propagating beams. Lenses L_3 and L_4, which had focal lengths of 200mm and 500mm respectively, were placed inside the interferometer to form a radial shearing telescope. The size of the beam travelling anti-clockwise around the interferometer was magnified, while the size of the beam travelling clockwise was reduced. This gave a linear magnification of m_1=2.5 and an overall magnification (due to the counter-propagating beams) of m_0=6.25. The resulting interference pattern at the output, which occurred in an image plane of the LC-SLM for both interferometer beams, was imaged onto a CCD camera by L_5 and L_6.

The output image on the CCD camera therefore contained an interference fringe pattern formed between a magnified image of the LC-SLM (from the beam travelling anti-clockwise round the Sagnac) overlapping with an image reduced in size (from the beam travelling clockwise round the Sagnac). The overall magnification of 6.25 was used so that one pixel of the magnified image, which we will call the 'scan pixel', overlapped a 5x5 array of pixels of the smaller image, including the small image of itself. Images from the CCD camera were obtained through a frame grabbing board in the computer. The computer was then used to apply the appropriate stroke to the LC-SLM.

It was found that the spatial coherence obtained by passing the interferometer input light through a 15μm pinhole and using an overall radial shearing magnification of 6.25, provided fringes of sufficient contrast to perform the experiments. The fringe contrast did not quite reduce to zero at the outer edges of the 5x5 pixel array of the radially reduced image of the LC-SLM. A more detailed discussion of spatial coherence requirements for the system is given later in this chapter.

### 5.4.2 Simulation of White light feedback using LC-SLM

A simulation for the white light Sagnac interferometer containing the LC-SLM is now presented. This simulation was performed for light travelling along the optic axis. Therefore, the degree of coherence between the interferometer beams was 1 and we again assume that \( h(x,y) = 1 \) in equation( 5-1 ). Note that since the aberrations are introduced
externally to the system $d$ is assumed to be zero in equation (5-1) and the feedback results are plotted against aberration stroke $a$.

The LC-SLM used in the experiments was the nematic, parallel aligned, transmissive, phase-only device, with 81 pixels that was described in Chapter 3. Figure 5-8 shows experimental data for $f$, the LC-SLM characteristic, stroke vs. applied voltage at a wavelength of 850nm. Again the plot shown in Figure 5-1 was used for $F(\lambda)$ in equation (5-1).

![Figure 5-8 LC-SLM stroke characteristic at 850nm](image)

Figure 5-9 (bottom) shows a plot of output intensity vs. aberration stroke with feedback applied. Note that for interest the range of aberration stroke is much larger than can be obtained in practice with the set up shown in Figure 5-7 since the aberration stroke is introduced using the LC-SLM. Note also that Figure 5-9 (top) shows a plot of white light feedback interferometer equation output intensity without feedback applied, for comparison. The feedback loop gain was reduced from the value used in the previous simulation to account for the slower response time of the LC-SLM. In the three or four feedback fringes close to zero aberration stroke, we again see the characteristic saw tooth pattern. The dotted box indicates the central area with which experimental results are compared in the next section. Within a large section of each of these four central fringes,
$I_{\text{out}}$ is uniquely defined by aberration stroke $a$, and decreases almost linearly with increasing aberration stroke.

![Graph](image)

Figure 5-9 White light feedback interferometer equation output intensity (top) with no feedback, and (bottom) using LC-SLM with low feedback loop gain

5.4.3 Experimental Results of White light feedback using LC-SLM

Experimental results for the white light Sagnac feedback interferometer will now be presented. By varying the stroke produced by the scan pixel, it was possible to scan the fringes across all 25 reduced LC-SLM pixels except where the scan pixel overlapped itself. Here, since the interferometer stroke varied by the same amount in both beams, the fringes remained stationary. Feedback was applied to a ‘feedback pixel’ adjacent to the scan pixel, as the stroke of the scan pixel was varied. Figure 5-10 (top) shows the resulting output intensity at the centre of the feedback pixel. Note that because the aberration stroke was provided by an LC-SLM pixel, the total range of aberration stroke is only about 1μm. This is equivalent to the centre section contained in the dotted box of Figure 5-9 (bottom), a close up view of which is shown in Figure 5-10 (bottom) for comparison. The sawtooth nature of the plot is clearly evident, and the experimental plot agrees well with the theoretical plot. Note however that there is a slight difference in shape. This was because
the aberration stroke for the theoretical plot was provided using a linear stroke characteristic, rather than the non-linear stroke characteristic of the LC-SLM.

Figure 5-10 Sagnac interferometer with radial shear, and LC-SLM output intensity with feedback; Top: experimental results; Bottom: Theoretical simulation

5.5 Spatial Coherence Considerations for a White Light Radial Shearing Interferometer

The simulations and experiments described above were proof-of-principle only. They were performed using light on the optic axis assuming \( h=1 \) (the function describing the spatial coherence of the light) in the white light feedback interferometer equation( 5-1 ). Away from the optic axis, the degree of spatial coherence between the interferometer beams is governed by the diameter of the pinhole in an image plane of the bulb filament and the radius and focal length of the collimating lens. If a white light radial shearing interferometer were to be built for which feedback were applied to wavefronts at positions away from the optic axis, certain spatial coherence considerations would also need to be addressed. Some of these considerations are discussed next.
The interferometer output consists of an image of the input collimating lens that has been radially reduced in size overlapping an image of the input collimating lens that has been radially expanded. Initially the spatial coherence considerations for a radial shearing telescope with infinite radial shear will be investigated. From this, the spatial coherence considerations for a radial shearing telescope with finite radial shear will be derived.

5.5.1 Radial Shearing Telescope with Infinite Radial Shear

Figure 5-11 shows a pinhole of diameter $s$ in front of white light source, and a collimating lens of focal length $f$ and radius $l$ (diameter $A$) positioned with the pinhole at its focal point. For a radial shearing telescope with infinite radial shear, a point $P_2$ at the centre of the expanded image of the collimating lens will overlap with a point $P_1$ at the edge of the reduced image of the collimating lens. We would like to choose a collimating lens of aperture radius $l$ for a given input slit width $s$, such that the degree of coherence between points $P_1$ and $P_2$ stays above a minimum value $h_{\text{min}}$. For these calculations $h_{\text{min}}$ is chosen to be 0.2. However, in practice $h_{\text{min}}$ would be chosen empirically to be the minimum value at which useful information could be gained from the output interferogram.

![Diagram for spatial coherence considerations for radial shearing interferometer with infinite radial shear](image)

In general, where two points $P_1$ and $P_2$ are far enough away from an arbitrary source the degree of coherence between $P_1$ and $P_2$ can be calculated using the van Cittert-Zernike theorem\cite{71}:
\[ h(x, y) = \frac{\iint I(x', y') \exp \left( - \frac{2\pi (xx' + yy')}{\lambda_m D} \right) dx' dy'}{\iint I(x', y') dx' dy'} \]  \hspace{1cm} (5-2)

where: 
- \( x' \) and \( y' \) are the spatial co-ordinates in the plane of the source,
- \( x \) and \( y \) are the components of the distance between points \( P_1 \) and \( P_2 \),
- \( \sigma \) is the area of the source,
- \( I(x', y') \) is the intensity of the source,
- \( \lambda_m \) is mean wavelength of light source, and
- \( D \) is the distance between the source and measurement plane.

By performing a change of co-ordinates and solving the integral it can readily be shown that for a uniformly illuminated pinhole of diameter \( s \), the degree of coherence between points \( P_1 \) and \( P_2 \) is given by:

\[ h = \frac{J_1 \left( \frac{\pi Rs}{D\lambda_m} \right)}{\frac{\pi Rs}{D\lambda_m}} \]  \hspace{1cm} (5-3)

where \( J_1 \) is the first order Bessel function of the first kind,
- \( R \) is the distance between points \( P_1 \) and \( P_2 \).

For the situation shown in Figure 5-11, \( D \) is equal to the collimating lens focal length \( f \), and \( R \) is equal to the radius of the lens \( l \) in equation (5-3).

In this case equation (5-3) is equal to 0.2 when:

\[ s = 0.98 \frac{f\lambda}{l} \]  \hspace{1cm} (5-4)

Therefore, given a collimating lens of focal length \( f \) and aperture radius \( l \), equation (5-4) can be used to find the diameter of the circular input aperture in front of a white light source for which the degree of spatial coherence between points \( P_1 \) and \( P_2 \) is equal to 0.2.
5.5.2 Radial Shearing Telescope with Finite Radial Shear

When a radial shearing telescope that has a finite radial shear is used in the interferometer, the coherence considerations are slightly less stringent. If the magnification of the radial shearing telescope is $M$, then the output of the interferometer consists of a magnified image of the input collimating lens of size $AM$, and a reduced image of size $A/M$. Figure 5-12 shows a diagram of this situation.

![Diagram of spatial coherence considerations for radial shearing interferometer with finite radial shear](image)

Figure 5-12 Diagram for spatial coherence considerations for radial shearing interferometer with finite radial shear

Here, a point at the edge of the reduced image will overlap at a point $P_2$ in the expanded image. We would like to find the degree of coherence between the two images of the collimating lens at point $P_2$. This is equivalent to finding the degree of coherence between points $P_1$ and $P_2$ in the expanded image of the collimating lens. Therefore we would like to choose a collimating lens of diameter $A$ for a given input slit width $s$, such that the degree of coherence between points $P_1$ and $P_2$ is 0.2. Interference fringes occur at the overlap between the two images of the input collimating lens at the output, i.e. over the aperture of the reduced image.

Using simple geometrical arguments it can be seen from Figure 5-12 that:

$$l = \frac{A(M^2 - 1)}{2M} \quad (5-5)$$
Therefore, substituting for $l$ in equation (5-4) gives:

$$s = 1.96 \frac{fAM}{A(M^2 - 1)}$$  \hspace{1cm} (5-6)

Equation (5-6) allows the diameter of the input pinhole for a radial shearing interferometer to be calculated such that the degree of coherence between the two interferometer beams goes to 0.2 at the edge of the output interferogram, given an input collimating lens of diameter $A$ and focal length $l$. Note that in practice this is an order of magnitude calculation only. For an experimental system the pinhole would initially be chosen to be about this size, but then adjusted to optimise the quality of the fringes produced and the overall amount of light.

### 5.6 Output Fringe Intensity Considerations

By careful selection of several key optical components of a white light radial shearing interferometer it is possible to maximise the signal strength at the interferometer output. Two of these, the size of the input pinhole, and the beam splitter transmission-reflection coefficient ratio, will now be considered.

The output intensity of a white light interferometer is given by equation (5-7).

$$I_{\text{out}} = I_1 + I_2 + 2\sqrt{I_1 I_2} h(x, y) \int_0^\infty \frac{c}{\lambda^2} F(\lambda) \cos \left( \frac{2\pi d}{\lambda} \right) d\lambda$$  \hspace{1cm} (5-7)

where: $I_1$ and $I_2$ are the two interferometer beam intensities,
- $d$ is the interferometer stroke,
- $c$ is the speed of light,
- $\lambda$ is wavelength,
- $F$ is the power spectral density, and
- $h$ is a function describing the spatial coherence.

For maximum signal strength at the output of the interferometer it is necessary that the difference ($I_{\text{diff}}$) between the maximum output intensity ($I_{\text{max}}$) and minimum output intensity ($I_{\text{min}}$) is maximised. Figure 5-13 shows a plot of interferometer output intensity vs. interferometer phase $\phi$, showing $I_{\text{diff}}$, $I_{\text{max}}$, and $I_{\text{min}}$.  

96
In equation (5-7), $I_{\text{diff}}$ corresponds to:

$$I_{\text{diff}} = 2\sqrt{I_1 I_2} h(x,y)$$

(5-8)

where $I_1$ and $I_2$ are the intensities of the two interferometer beams and $h$ is the function describing the spatial coherence. Note this is not the fringe visibility since it is possible to have a large visibility and yet a low overall signal level.

The fringe visibility $V$ is given by:

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

(5-9)

Therefore, as $I_{\text{min}}$ tends to zero, $V$ tends to 1, for any value of $I_{\text{max}}$. This is shown in Figure 5-14 where the diagram of output intensity on the left shows a large signal, yet because of the definition of fringe visibility, the diagram on the right has higher visibility.

Figure 5-13 White light interferometer output intensity showing $I_{\text{diff}}$, $I_{\text{max}}$, and $I_{\text{min}}$

Figure 5-14 Output intensity distributions showing a large signal (left) and a high fringe visibility (right)
Obviously the output on the left would be preferable, especially for applications where the amount of light is very important.

5.6.1 Maximise Fringe Intensity Variation with Respect to Pinhole Aperture Size

The size of the input pinhole for which \( I_{\text{diff}} \) is maximised will now be calculated. The total power of the source through the pinhole is proportional to \( s^2 \). Therefore we want to evaluate when:

\[
2\sqrt{I_1 I_2} \frac{\partial}{\partial s} (s^2 h) = 0. \tag{5-10}
\]

Therefore:

\[
\frac{\partial}{\partial s} (s^2 h) = \frac{\partial}{\partial s} \left( \frac{J_1 \left( \frac{\pi d}{f \lambda_m} \right)}{s \left( \frac{\pi d}{f \lambda_m} \right)} \right) = \frac{1}{\left( \frac{\pi d}{f \lambda} \right)^2} J_0 \left( \frac{\pi d s}{f \lambda} \right).
\]

Therefore: \( \frac{\partial}{\partial s} (s^2 h) = 0 \) when \( s = 0 \), or \( J_0 \left( \frac{\pi d s}{f \lambda} \right) = 0 \).

If \( s = 0 \) there is no light in the system. Therefore: \( J_0 \left( \frac{\pi d s}{f \lambda} \right) = 0 \) when \( \left( \frac{\pi d s}{f \lambda} \right) = 2.045 \).

Thus:

\[
s = 2.045 \frac{f \lambda}{\pi d}
\]

Substituting for \( l \) using equation \( (5-5) \) gives:

\[
s = 1.53 \frac{f \lambda M}{A(M^2 - 1)} \tag{5-11}
\]

Note this is 22% smaller than equation \( (5-6) \) found in the previous section which provided a degree of coherence of 0.2 between the two interferometer beams at the edge of the output interferogram. Again however, in practice this is an order of magnitude calculation only.
For an experimental system the pinhole would initially be chosen to be about this size, and then adjusted to optimise the quality of the fringes for the particular application.

5.6.2 Maximise Fringe Intensity Variation using Beam Splitter Ratio

Next we determine the beam splitter transmission-reflection coefficient ratio that provides optimum signal strength at the output of a white light radial shearing Sagnac interferometer. This is done by maximising the difference in power between the fringes in the output interferogram with respect to the beam splitter transmission.

Figure 5-15 contains a diagram of a radial shearing Sagnac interferometer showing the transmission and reflection coefficients for the beam splitter, the mirrors and the lenses.

![Diagram of a radial shearing Sagnac interferometer showing transmission and reflection coefficients of the optical components](image)

Figure 5-15 Radial shearing Sagnac interferometer showing transmission and reflection coefficients of the optical components

If the linear magnification of the radial shearing telescope is $M$, the intensity of the beam travelling anticlockwise round the Sagnac is $I_1$, and the intensity of the beam travelling clockwise round the Sagnac is $I_2$ then:

$$I_1 = I_{in}T_b^2T_1T_2R_1R_2R_3M^2$$
$$I_2 = I_{in}R_b^2T_1T_2R_1R_2R_3M^{-2}$$
where: $I_{in}$ is the input intensity

$T_b$ is the beam splitter transmission coefficient,

$R_b$ is the beam splitter reflection coefficient,

$T_1$ and $T_2$ are the transmission coefficients of lenses $L_1$ and $L_2$,

$R_1$, $R_2$, and $R_3$, are the reflection coefficients of mirrors $M_1$, $M_2$, and $M_3$.

Therefore from equation (5-8):

$$I_{diff} = 2 \sqrt{I_1 I_2} h = 2 I_{in} T_b R_b T_1 T_2 R_1 R_2 R_3 h$$

We also know:

$$R_b = 1 - T_b - A_b,$$

where: $A_b$ is the absorption of the beam splitter. Substituting for $R_b$ gives:

$$I_{diff} = 2 I_{in} T_b (1 - T_b - A_b) T_1 T_2 R_1 R_2 R_3 h$$

The area of the reduced interferometer output beam is proportional to $1/M^2$. Therefore the power difference $P_{diff}$ over the area of the reduced beam is given by:

$$P_{diff} = \frac{2 I_{in}}{M^2} (T_b - T_b^2 - A_b T_b) T_1 T_2 R_1 R_2 R_3 h.$$

We want to maximise $P_{diff}$ with respect to the beam splitter transmission $T_b$, i.e., solve

$$\frac{\partial P_{diff}}{\partial T_b} = 0.$$

This gives:

$$T_b = \frac{1 - A_b}{2}.$$

Substituting into equation (5-11) gives:

$$R_b = \frac{1 - A_b}{2} = T_b.$$

Therefore, the slightly counterintuitive result occurs, that despite the intensity difference between the two interferometer beams at the output due to the magnification of the radial shearing telescope, the power at the output is maximum when a 50-50 beam splitter is used. Note also that best results are obtained using a beam splitter with low absorption.
5.7 Source Colour Temperature

The degree of spatial coherence between the two beams at the edge of the output fringe pattern of a white light radial shearing interferometer is determined by the pinhole size, the focal length and aperture of the input collimating lens, and the radial shearing magnification. This relationship was given earlier in equation (5-6).

Rather than placing the white light source right next to the pinhole, it is more convenient to image the source onto the pinhole using a lens. Figure 5-16 shows a diagram of a white light source imaged onto a pinhole with an imaging lens $L_1$, then collimated using lens $L_2$.

![Figure 5-16 Matched system showing light from a white light source imaged onto a pinhole then collimated](image)

Here lens $L_1$ with aperture $A_1$ is placed a distance $u$ (the object distance) from the source. An image of the source is formed a distance $v$ (the image distance) on the other side of the lens. The solid angle subtended by the area of lens $L_1$ at the source is $\theta$. The width of the source imaged onto the pinhole is $s'$. The pinhole of aperture $s$ is placed in the image plane. Lens $L_2$ with focal length $f_2$ is placed such that the pinhole is at its focal point. To maximise the efficiency of the system, the constraint that lens $L_2$ collects all the light captured by lens $L_1$ is also made. The system is therefore "matched".

We will now show that for a given degree of spatial coherence at the output of a matched white light radial shearing interferometer, the average value of output intensity is dependent only on the colour temperature of the source.
From equation (5-6) we know that:

\[ s \propto \frac{f_2}{A_2} \]  \hspace{1cm} (5-13)

From the geometry of the system shown in Figure 5-16 we see that:

\[ A_1 = \frac{A_2 v}{f_2} \]  \hspace{1cm} (5-14)

and that:

\[ s' = \frac{u}{v} s \]  \hspace{1cm} (5-15)

The solid angle \( \theta \) is given by:

\[ \theta = \frac{\pi A_1^2}{4u^2} \]  \hspace{1cm} (5-16)

and area of the source imaged onto the pinhole is:

\[ a_i = \pi \left( \frac{(s')^2}{2} \right) = \frac{\pi}{4} \left( \frac{us}{v} \right)^2 \]  \hspace{1cm} (5-17)

The power of the source per unit area per unit solid angle is given by:

\[ P_u = \frac{\sigma T^4}{2\pi} \]  \hspace{1cm} (5-18)

where: \( \sigma = 5.67 \times 10^{-8} \) and is the Stefan-Boltzmann constant, and

\( T \) is the colour temperature of the source.

The total power collected by the pinhole \( P_t \) is therefore given by:

\[ P_t = P_u a_i \theta = \frac{\sigma T^4}{2\pi} \frac{\pi}{4} \left( \frac{us}{v} \right)^2 \frac{\pi A_1^2}{4u^2} \]  \hspace{1cm} (5-19)

Substituting for \( s \) and \( A_1 \) using equation (5-13) and equation (5-14) respectively shows that:

\[ P_t \propto T^4. \]  \hspace{1cm} (5-20)
Therefore, for a matched white light radial shearing interferometer which provides a given degree of spatial coherence between the two interferometer beams at the edge of the output field, the average intensity at the output is dependent only on the colour temperature of the source.

5.8 Conclusion

In this chapter, two experiments were performed along with relevant theoretical simulations. The first was a proof-of-principle experiment using white light feedback in a Michelson interferometer with two piezo driven mirrors. One mirror provided a variable interferometer stroke, and the other provided a feedback stroke proportional to the output intensity. The resulting plot of output intensity vs. interferometer stroke showed good agreement with the theoretical simulation. This experiment showed that in principle, aberration correction using white light feedback interferometry was possible using a linear phase modulator.

For the second experiment a radial shearing Sagnac feedback interferometer operating in white light was constructed. In this system, both the aberration stroke and the feedback were provided with the LC-SLM. Experimental results were again in good agreement with the theoretical simulations and showed that in principle automatic aberration correction using feedback interferometry was possible using white light and a LC-SLM, without the need for separate aberration correction and detection systems. We believe this is the first time feedback interferometry has been performed using white light with either a piezo driven mirror or liquid crystal spatial light modulator.

If a white light radial shearing interferometer were to be built for which feedback were applied to wavefronts at positions away from the optic axis, certain spatial coherence considerations would need to be addressed. Some of these coherence requirements were considered. An equation was derived, that related the size of the input pinhole, the focal length and aperture size of the collimating lens, and the radial shear magnification for a white light radial shearing interferometer, such that the degree of coherence between the two output beams went to 0.2 at the edge of the interferometer output.
Techniques to maximise the interferometer power output were then investigated. As a result, an alternative expression for the aperture size of the input pinhole was derived, and the optimum beam splitter transmission-reflection coefficient ratio was shown to be 50-50.

It was then shown that for a matched white light radial shearing interferometer which provides a given degree of spatial coherence between the two interferometer beams at the edge of the output fringe pattern, the average intensity at the output is dependent only on the colour temperature of the source.
Chapter 6. Segmented Mirror

6.1 Introduction

Optical aberrations can be corrected using many different types of phase modulator including liquid crystal phase modulators, deformable mirrors and segmented mirrors. Liquid crystal modulators are relatively cheap, but have slow speeds of operation. In Chapter 3 we showed that by using the transient nematic effect it was possible to increase the operating speed of liquid crystal devices up to frequencies above 40Hz. However, while this frequency of operation is sufficient for low order atmospheric aberration correction operating frequencies well above this, i.e. up to 200–300Hz would be more desirable. If phase stepping methods were used, which require 3 or 4 mirror steps per correction cycle, operating frequencies up to 1kHz would be necessary. Liquid crystal devices are also limited to about $1.5\lambda$ (@ 633nm) phase modulation range. In general, deformable and segmented mirrors can operate at much higher frequencies and have much larger phase modulation ranges than liquid crystal devices. However, these mirrors are also prohibitively expensive. Because of the cost, we decided to attempt the construction of a mirror ourselves rather than buy one.

In this chapter, a relatively cheap segmented mirror with a 3x3 array of segments we designed and had built for us by Industrial Research Ltd. is described. The piezo actuators used to drive the segments, and tests to ensure the piezos were sufficiently waterproof for processes during the mirror construction are presented next. Then the mirror drive electronics built by Paul Harris at Industrial Research Ltd. are discussed. Experiments carried out to test the operation of the electronics driving a single piezo actuator, a light weight test mirror segment, and a heavy test mirror segment over a range of frequencies up to 5kHz are then described. A description of the mirror construction is then given, along
with the mirror clamp and housing design. Frequency tests using a segment of the completed mirror, clamped in the housing are then presented showing that the main concerns when using the mirror in an optical setup are mechanical resonances in the mirror, clamp and housing at frequencies above 1kHz.

### 6.2 Mirror Description

First a description of the segmented mirror is given. Figure 6-1 shows a schematic of the mirror, which has 9 segments in a 3x3 square array. All the glass parts are made from Fine Annealed Pyrex to reduce thermal effects. The total mirror dimensions are 60x60x10mm, with each pixel being 20mm square. The base plate dimensions are 60x60x10mm. Three piezos are attached to three corners of each mirror segment. This allows tip, tilt, and piston movement. The mirror thickness of 10mm was chosen to be rigid enough so that when one of the three piezos expands, the segment rotates about an axis defined by the other two rather than distorts.

![Mirror Segments Diagram](image)

**Figure 6-1 Segmented mirror**

### 6.3 Piezo Actuators

A description of the piezo actuators used to drive the mirror segments is now given. The piezo actuators were made by Thorlabs (AE203D08) and have the dimensions 3.5mm by
4.5mm by 10mm. Conventionally piezo actuators consist of one piezo electric layer with an electrode on either side. Large voltages (up to several thousand volts) are then applied across the piezo layer to produce mechanical expansions of typically 10μm. However, each actuator used to drive our mirror consists of many piezo electric layers stacked between electrodes as seen in Figure 6-2. The layers are connected mechanically in series and electrically in parallel. The benefit of this is that a total mechanical expansion of about 6μm is produced when only 100V is applied between the two sets of electrodes (though this varies slightly between actuators). The piezos have a large capacitance of about 0.2μF.

![Piezo actuator schematic showing the piezo layers connected mechanically in series, and electrically in parallel](image)

**Figure 6-2** Piezo actuator schematic showing the piezo layers connected mechanically in series, and electrically in parallel

### 6.4 Testing Piezo Actuators In Water

During the construction of the mirror it was necessary for water to be used as a coolant and lubricant in the cutting and polishing processes. It was therefore necessary that the piezos be waterproof. Tests that were carried out on two separate piezos to ensure they were sufficiently waterproof will now be described.

Initially the stroke of both piezos was tested using a Michelson speckle interferometer. Figure 6-3 shows a schematic of the interferometer. Light from a HeNe laser entered the interferometer through a beam splitter. One beam was reflected from a mirror, while the other beam was scattered from the front surface of the piezo actuator. The output speckle interferogram was then imaged onto a CCD camera by an imaging lens. The intensity of
individual speckles was then monitored as the voltage across the piezo was varied from 0 to 100V. By counting the resulting intensity fringes it was possible to determine the stroke of the piezo. Initially it was found that the stroke of each piezo was 6μm for an applied voltage of 100V which agreed well with the manufacturers specifications.

![Figure 6-3 Schematic of the Michelson speckle interferometer used to test piezos](image)

The first piezo was soaked in a beaker of water for 4 hours before the stroke was tested again. The piezo was then soaked over night for a period of 14 hours, then boiled in the beaker for 1.5 hours. A second piezo was then added and both were left soaking in cold water for three days. They were then both boiled for a further 3 hours. The stroke of both piezos was again tested. It was found that no detectable physical damage or reduction in stroke resulted from the water tests. We therefore concluded it was unnecessary to further waterproof the piezos during cutting and polishing processes of the segmented mirror construction.

### 6.5 Segmented Mirror Drive Electronics

The control electronics were designed and built by Paul Harris of Industrial Research Ltd in Lower Hutt, New Zealand and went through several stages of development. We tested each upgrade and provided feedback for later versions. The final version of the control electronics was designed to operate safely up to an operating frequency of 5kHz. Figure 6-4 shows a schematic of the positive and negative drive stages for the piezo control electronics.
The driver stage consists of two amplifiers operating as a bridge type driver ie. the piezo element is connected between the two outputs. In this way a power supply of ±55V provides a safe drive of 100V (allowing for 5V output saturation margin in the operational amplifiers) with readily available and relatively cheap Burr Brown OPA445 devices. The two amplifiers are simple inverting amplifiers the first with gain 5, the second acting as a slave off the first with unity gain for a total gain of 10.

The OPA445 actually provides a supply of ±45V, but the piezo elements are damaged if driven in a negative direction. Therefore, protection diodes were incorporated to ensure this never occurs. Thus it is possible to offset the supplies within the common mode operating range in this bridge configuration. The first stage (the negative drive stage) supplies +12V and -60V, and the second slave stage (the positive drive stage) supplies +60V and -12V. Each piezo has a capacitance of approximately 0.2μF, and therefore draws a peak drive current of 120mA at 1kHz. Complementary buffer transistors and a 100Ω resistor connected to the opamp output (for low level signals) source the output load. A cooling fan was also attached to the housing.
6.6 Testing the Electronics using Mirror Segments

The electronics were initially tested by driving three different test objects:

1) an unloaded piezo (no mirror segment attached),

2) a light weight mirror segment driven by three piezos, and

3) a heavy mirror segment driven by three piezos.

The tests were carried out to ensure the electronics could drive loads that would typically be required once the final segmented mirror was built, up to frequency specifications. Figure 6-5 shows the Michelson interferometer used for these tests. The interferometer was essentially the same as the one shown in Figure 6-3 used to test if the piezos were waterproof. However, here the single piezo and the two test mirror segments were driven by the mirror control electronics. Also, the output interference fringe pattern (which was a speckle pattern from the single test piezo, and an ordinary fringe pattern with the light weight and heavy test mirrors) was imaged onto an array of 3 photodiodes rather than a CCD camera. The photodiodes were positioned in the interference pattern at positions that corresponded to the positions of the three piezos. The movement of the individual piezos could therefore be monitored independently. It was necessary to use photodiodes and a large bandwidth amplifier instead of a CCD camera in order to detect the high frequency intensity variation as the piezos were driven at frequencies up to 5kHz. With up to 15 fringes passing the photodiode when the piezos were moved from their minimum to maximum positions, this corresponded to intensity variations up to 75kHz.

Figure 6-5 Michelson interferometer used to test electronics
6.6.1 Single unloaded Test Piezo

First, the stroke of a single unloaded piezo actuator was tested. The unloaded piezo easily provided the stroke of 6µm in line with the manufacturers specifications at all frequencies from 0 to 1kHz with very little variation at all.

6.6.2 Light Weight Test Mirror Pixel

The stroke of the lightweight segment was tested next. It was tested over a range of frequencies from 10Hz to 1kHz with applied voltage of 84Vpp and then for a range of frequencies from 1kHz to 5kHz using an applied voltage of 30Vpp. It was constructed from a 20mm x 20mm x 1mm aluminised segment of a microscope slide and was driven in each of three corners by piezo actuators cemented in place. The other end of each piezo was cemented to a large solid aluminium disk backing plate. The backing plate was constructed to fit into a conventional optics mount for easy adjustment. The response of the lightweight mirror segment and mirror drive electronics was tested as a control for subsequent tests with the heavy segment.

6.6.2.1 Frequency test from 10Hz to 1kHz

A 4.2Vpp sine wave from a digital Hewlett-Packard signal generator was applied to the input of the drive electronics, which then applied 84Vpp to each of the piezo actuators. The frequency of the sine wave was varied between 0Hz to 1kHz. The amplitude of 84Vpp was used rather than the maximum 100Vpp to ensure the piezos did not detach themselves from the mirror at the larger frequencies within the range. Counting the number of fringes passing each photodiode gave the stroke for each piezo individually. The intensity from the three photodiodes was logged to a digital Tektronics oscilloscope and then downloaded via the serial port to a PC computer for analysis. Typical data from the oscilloscope is shown in Figure 6-6.

The two wide fringes in the plot indicate where the piezo was momentarily at rest at both the minimum and maximum scan positions. The stroke of each the piezo was therefore found by counting the fringes between the two wide fringes, to the nearest ¼ of a fringe. For example, the number of fringes the piezo scanned in Figure 6-6 was 13. However, since there is effectively a double pass in the Michelson interferometer this equates to only 6.5 wavelengths of piezo stroke. At a wavelength of 632.8nm this corresponds to 4.1µm.
Figure 6-6 Output intensity data when the lightweight mirror segment was driven with an 84 Vpp sine wave at a frequency of 500 Hz.

Figure 6-7 Lightweight mirror segment stroke for all three piezos driven with 84 Vpp for frequency range 0 to 1 kHz.
Figure 6-7 shows a plot of piezo stroke vs. frequency for all three piezos of the lightweight microscope slide mirror. It is clear that the stroke for all piezos is similar and remains fairly constant around 4μm, rising slightly with frequency up to about 4.6μm at 800Hz. At 800Hz a large resonance occurs and the stroke drops down to about 3.4μm of stroke at about 850Hz.

6.6.2.2 Frequency test from 1kHz to 5kHz

The electronics were then tested with the lightweight mirror segment at a range of frequencies from 1kHz to 5kHz. However, the applied voltage amplitude for this data was reduced to 30Vpp. The amplitude was reduced to keep the temperature of the piezos at a safe level. Since the piezos have a large capacitance, the current drawn increases with frequency, causing the piezo to heat up. The manufacturer’s specifications state that the temperature will rise to unacceptable levels above 1kHz close to the maximum driving voltage of 100Vpp.

However, with the reduced driving voltage amplitude there was a greatly reduced stroke and consequently far fewer fringes (between 2 and 5) passing the photodiodes at the output. Therefore, since the number of fringes could still only be determined to the same accuracy ie. to the nearest ¼ of a fringe, it was more difficult to get accurate results in this range of frequencies. Figure 6-8 shows an example of the interferometer output intensity when the lightweight mirror segment was driven with a 30Vpp sine wave at a frequency of 3kHz. Here, three wide fringes are clearly visible. Therefore the piezo has gone from the minimum scan position, to the maximum scan position, and back again. There are 3.25 fringes between the wide fringes which corresponds to 1.03μm of piezo stroke.

Figure 6-9 shows a plot of the stroke for all three piezos driven with 30Vpp for a frequency range from 1kHz to 5kHz. Here the stroke remains fairly constant around 1.1μm with only a slight decreasing trend. We can conclude that the three piezos remain constant enough so that if necessary the lightweight mirror could safely be driven to strokes of 1μm within this frequency range.
Figure 6-8: Output intensity data when the lightweight mirror segment was driven with a 30Vpp sine wave at a frequency of 3kHz.

Figure 6-9: Lightweight mirror segment stroke with 30Vpp plotted for frequency range 1kHz to 5kHz.
6.6.3 Heavy Test Mirror Pixel

6.6.3.1 Frequency test from 10Hz to 1kHz

A heavy mirror segment 10mm thick was then cemented on top of the lightweight microscope slide segment. The stroke of the heavy segment was then tested over a range of frequencies from 10Hz to 1kHz with applied voltage of 84Vpp and then for a range of frequencies from 1kHz to 5kHz using an applied voltage of 30Vpp.

Figure 6-10 shows a plot of stroke vs. frequency for all three piezos from 0 to 1kHz for the heavy mirror pixel with amplitude 84Vpp.

![Figure 6-10](image.png)

Here we can see the plots look similar in shape to the plots with the lightweight mirror segment, except the resonance that was at about 800Hz has moved down in frequency to about 700Hz, and has become much larger. Here the stroke rises from just over 4μm at 600Hz up to about 5μm at 700Hz, then jumps sharply down to between 2.5μm and 3μm at 800Hz. Above resonance the stroke drops to about half the value at 700Hz. Also, a
resonance at 100Hz which was barely visible in Figure 6-7 has become much more prominent. This showed that the pixel and drive electronics using this experimental setup were well suited to operating at frequencies between 150Hz and 700Hz, but correct operation outside this range may require careful optimisation. However, since the only difference between the previous experiment with the lightweight segment and this experiment was the weight of the segment, we wondered if the resonances were mechanical rather than electrical. If this was the case, the geometry and mass of the optical mount used to hold the mirror would effect the resonances.

6.6.3.2 Frequency Test from 10Hz to 1kHz with differing Mount Rigidity

The experiment was repeated several times with the rigidity of the optical mount increased by attaching large rectangular pieces of metal to the mount, and reinforcing it with other mounts. The mount rigidity was also decreased by using a less stable mount. Figure 6-11 shows a plot of stroke vs. frequency for the three piezos for a less stable mount from 0 to 1kHz for the heavy mirror pixel with amplitude 84Vpp. The plot shows resonances at about 100Hz, 400Hz and 950Hz, and differs considerably to the plots shown in Figure 6-10.

![Figure 6-11 Heavy mirror segment stroke for all three piezos driven with 84Vpp for frequency range 0 to 1kHz, with different mount](image-url)
The results showed that the size of the resonances, and frequencies at which they occurred changed noticeably each time the mount rigidity was changed. It was observed that the resonances were noticeably smaller when the mount was made more stable. Therefore we concluded that the resonances were mechanical in nature rather than electrical. It should be possible to minimise any resonances by ensuring the mount for the final segmented mirror had sufficient mass, was very rigid, and was secured tightly to the optical table.

6.6.3.3 Frequency test from 1kHz to 5kHz

The heavy mirror segment was also tested at frequencies from 1kHz to 5kHz with a reduced amplitude of 34Vpp. Again the amplitude was reduced to keep the temperature of the piezo at a safe level. Figure 6-12 shows the resulting graph.

![Graph showing mirror stroke vs frequency](image)

**Figure 6-12 Heavy mirror segment stroke with 34Vpp plotted for frequency range 1kHz to 5kHz**

Again it was difficult to get accurate results due to the small number of fringes going past the photo diodes as the piezos were scanned. The plots show that the stroke of the each of
the three piezos was similar as the frequency was varied. Also, there does appear to be some structure to the plots, with a peak around 1500Hz, a dip around 2400Hz, and another broad peak around 3500Hz. The plots followed the trend exhibited in the 10Hz to 1kHz plots, that fluctuations are larger with the heavy mirror segment than with the lightweight segment.

6.7 Segmented Mirror Construction

The construction of the segmented mirror will now be described. We designed the mirror in conjunction with Tim Haskell and Dave Cochrane from Industrial Research Ltd. (IRL) in Lower Hutt, New Zealand. We drew up the final mirror design then sent them down to IRL where they constructed the mirror. Figure 6-13 shows an expanded view of the segmented mirror.

A 60x60x5mm template was made to position the piezos correctly on the base plate. Holes of diameter 5.7mm (marginally larger in diameter than the piezos) were drilled through the template in the appropriate positions, i.e. one hole at three corners of each segment. The template was lapped to 6μm flatness and glued to the base plate with UV setting Loctite.
The 4<sup>th</sup> hole of each template segment was then drilled right through the base plate to allow for the wires from the piezo actuators to be threaded through. Figure 6-14 shows the positions of the piezos and template holes under each mirror segment. The bottom left hole has no piezo positioned in it.

![Diagram showing positions of template holes and piezos under each mirror segment](image)

**Figure 6-14** Diagram showing positions of template holes and piezos under each mirror segment

The mirror faceplate was made plane and parallel and lapped to a flatness of 15μm on both the front and back. The faceplate was then faceted into 9 segments in a 3x3 array, with a diamond saw using water as a coolant, to within 2mm of the back surface. Figure 6-15 shows a photo of this process. Both surfaces were lapped to 6 μm flatness with Al₂O₃ and the top surface polished on a pitch lap using a rare earth oxide polishing powder X-OX to a figure of better than 0.05μm. The figure was tested using a Flat Fizeau interferometer.

The piezos were then cemented to the base plate through the holes of the template as shown in Figure 6-16. The faceted mirror faceplate was then positioned on top of the piezos and also cemented into place with epoxy. The overall figure was still measured to be better than 0.05μm. When the epoxy had set the segments were sawn right through with the diamond saw.

Once the segments had been sawn through it was found that the individual segments were still flat but had tilted in relation to one another by up to 6 wavelengths at 633nm. Although applying offset voltages to each of the piezos can align each segment, this is undesirable as it leads to a decrease in overall stroke for that pixel. The complete assembly was therefore figured by hand to return the surface to a figure that was better than 0.05μm. A layer of silver was then vacuum deposited on to the reflecting surface of the mirror segments. Figure 6-17 shows a photo of the completed segmented mirror before the front surface was silvered.
Figure 6-15 Photo of mirror faceplate being faceted into a 3x3 array

Figure 6-16 Piezos cemented in place using the template
Figure 6-17 Complete mirror prior to silvering the reflecting surface

6.8 Mirror Clamp and Housing Design

The design of the segmented mirror clamp and housing will now be described. The design requirements for the clamp and housing of the segmented mirror were that they must:

1) have manual tip-tilt adjustment,

2) be very rigid and stable in order to reduce mechanical resonances,

3) provide protection for the mirror.

It was necessary that the mount was capable of manual tip-tilt control to provide easy adjustment in an optical setup. It was decided that rather than designing and building a mirror clamp with tip-tilt control, it would be easier to buy a standard optics mount with these capabilities, then build a clamp for the mirror that would fit into this mount. A standard Thorlabs 4” kinematic mirror mount (KS4) was bought for this purpose. The base plate of the mirror was square with dimensions 60mm x 60mm. It was therefore necessary
to design a square clamp which attached to a circular front plate that fitted into the Thorlabs mount. It was also necessary to design the clamp so that the front of the segmented mirror was in a plane close to the tip-tilt axes of rotation of the Thorlabs mount so that when the angle of the mirror was adjusted, the lateral displacement of the mirror was minimised. Figure 6-18 shows the design for the clamp from various angles. The designs were all drawn in Corel Draw 7. The designs for these components all contained precise measurements, however these have been removed here for clarity.

Figure 6-18 Design for segmented mirror clamp showing several views

The clamp was designed to clamp the back of the mirror baseplate. When the mirror was sitting back against the bevelled edges the two nuts at the top were tightened on the screws to keep the mirror in place. This clamp was attached to an aluminium circular front plate with a square cut out of it for the rest of the mirror to fit through. Figure 6-19 shows the circular front plate..

Figure 6-19 Circular front plate which inserts into Thorlabs mount
The circular front plate was designed to insert into the Thorlabs 4” mount. The Thorlabs mount was then bolted to the mirror housing. Figure 6-20 shows the design for the mirror housing.

Figure 6-20 Design of segmented mirror housing

The housing consisted of a bottom plate, a base plate, a wire clamp (shown at the back of the base plate), a cover made from 1.2mm thick aluminium sheet, and a 5mm thick Perspex front cover which could be attached when the device was not in use to keep the dust out. The base plate was attached to the bottom plate using four bolts. The holes in the base plate were also designed so that it could be used without the bottom plate if need be. The Thorlabs mount was bolted to the two holes in the middle of the base plate, and the two holes in the vertical brace plate attached to the base plate. The wires coming from the piezo actuators out through the back of the mirror are thin and therefore delicate. The clamp which had rubber buffers was included at the back of the housing to clamp and protect these wires. The bottom plate was designed with slots of width 7mm around the perimeter. The
slots were 25mm long which is the same as the distance between two holes on an optical table. This allowed the mirror housing to be bolted directly to the optical table if necessary, with maximum positional freedom.

For stability and rigidity the mirror clamp and housing were designed out of 120mm thick aluminium. The base plate design had many holes with common optics thread gauges, at commonly used optics spacings, placed across the plate in order to increase the versatility of the housing. Note that many of these holes are not shown in Figure 6-20. The design plans of all components are shown in full in Appendix 1.

The designs were given to the Physical Sciences Workshop of the University of Auckland, and Steve Warrington and John Humphreys built the individual components. Figure 6-21 shows two photos of the completed clamp and housing, holding the segmented mirror.

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**Figure 6-21 Photo of segmented mirror mounted in the clamp and housing, from the front (left) and from the back (right)**

The left photo shows bottom plate with the 25mm long slots underneath the base plate. The vertical brace plate is visible, attached to the base plate on the right of the photo. The black Thorlabs mount holding the square segmented mirror inside the circular front face of the
clamp, is attached to the base plate and vertical brace plate. The photo on the right shows the housing from the back. The wires can be seen protruding from the back of the mirror, through the square mirror clamp. The wires are then clamped to the base plate. The tip-tilt adjustment screws can just be made out at the top left, and bottom right of the black Thorlabs mount. The aluminium housing cover and Perspex front are not shown.

### 6.9 Segmented Mirror Frequency Response Tests

Once the segmented mirror, the mirror clamp, and housing had been constructed, the frequency response of the mirror was tested. The mirror was clamped in the housing then placed securely in the test interferometer used in the previous segment frequency tests shown in Figure 6-5. However, this time only one photodiode was used to detect the intensity at the centre of a segment. The driving voltage was chosen to be 54Vpp so that the piezos could safely be driven from 10Hz to 2.2kHz. The same signal at this driving voltage was applied to the three piezos of one mirror segment. Figure 6-22 shows two plots of piezo stroke vs. frequency for one mirror segment.

![Figure 6-22](image)

**Figure 6-22** Segmented mirror segment stroke for all three piezos driven with 54Vpp for frequency range 0 to 2.2kHz
The lighter solid line shows the stroke vs. frequency with the mirror housing attached to the optical table using 4 posts. A small resonance occurred at about 240Hz, and a much larger one occurred at about 1.2kHz. The experiment was carried out again, this time with another post added to the housing, and an extra brace against the mirror clamp. The darker solid line in Figure 6-22 shows the stroke vs. frequency for this configuration. Differences are clearly noticeable between the light and dark lines. A close-up look at the first resonance shows that on the darker line it has moved down very slightly to about 230Hz. The larger resonance however has decreased markedly in size and has moved down to 1.13kHz. These results supported the idea that the resonances were mechanical in nature and not electrical since the only thing changed was the support of the mount and housing. The results also indicated that the resonances were not just due to the mechanical resonances in the pixels and base plate of the segmented mirror, but also due to resonances throughout the mirror clamp and housing.

However, the plots show that uniform operation can be expected below 200Hz, and then again between 250Hz and 1kHz. It is also possible that with more work done to stabilise the mirror, clamp, and housing, the resonance at 200Hz could be minimised to allow uniform operation from 0Hz to 1kHz.

6.9.1 Investigation into Cross-talk between Segments

In order to investigate whether there was cross-talk between mirror segments, the mirror was imaged onto a CCD camera at the interferometer output instead of the three photodiodes. Images from the CCD camera were then grabbed and stored on the computer. Figure 6-23 shows an example of a CCD camera image. Each segment can clearly be distinguished. The outer corners of the corner segments are missing because the mirror in the other arm of the interferometer was not large enough to enable the whole mirror to be imaged. However, enough of the mirror was visible for the observations described below.

A driving voltage of 54Vpp was again applied to the three piezos of one mirror segment. The segment chosen was the segment in the middle of the bottom row of Figure 6-23. The images on the CCD camera were then observed as the frequency of the driving voltage was varied between 0 and 2.2kHz. Figure 6-24 shows images from the CCD camera at the interferometer output for different frequencies. Again the individual segments, and the fringes on each segment can clearly be seen. The frequencies at which each image were
taken are a) 1054Hz b) 1123Hz c) 1141Hz d) 1149Hz e) 1156Hz f) 1168Hz g) 1196Hz h) 1230Hz i) 1266Hz. These are all frequencies around the large resonance at 1200Hz in Figure 6-22.

Figure 6-23 Image of Segmented mirror at the interferometer output

Figure 6-24 Images of segmented mirror showing segment vibration at different frequencies
Note that the fringes on many of the segments are blurred. The blurred fringes are explained in the following. The refresh rate of the CCD camera was 25Hz. Therefore, the CCD camera was not fast enough to resolve fringes on segments that were moving a distance greater than \( \lambda/2 \), at a frequency higher than 25Hz. When this occurred, the fringes blurred and their contrast reduced. At sufficiently high frequencies the fringes blurred to a uniform grey. This is the reason there are no fringes visible on the segment in the middle of the bottom row in any of the images of Figure 6-24, since this was the modulated segment.

However, it is interesting to note that the fringes in many other segments in the images are also blurred. For example the fringes in the central segment in images c), d) and f) are blurred. Therefore, these segments were moving, even though no modulating voltage was applied to them. Furthermore, many of these segments contain portions of blurred fringes, and portions of high contrast fringes. For example, the top centre segment of image h). Therefore, the segments not only moved when no voltage was applied to them, they moved unevenly. This is clear evidence of cross-talk between mirror segments. Note that the fringes of the segments shown in image a) are all high contrast. This is because the frequency at which this image was taken was well below the resonant frequency so cross-talk did not occur.

We believe these observations indicate that the segment being driven was causing complicated standing waves through the baseplate, the rest of the mirror, and the mirror mount and housing. Even though the mirror was mounted as rigidly as possible, the nature of the clamp and housing meant that it was very hard to stabilise completely. One of the main problems is that the Thorlabs mount contains tension springs in the tip-tilt mechanism. These springs hold part of the mount, and the whole weight of the segmented mirror stable against three contact points. This is the least stable part of the mirror clamp and housing and contributes a large amount to the resonances. Great care should be therefore be taken to ensure the mirror and housing are setup as rigidly as possible to reduce mechanical resonances. It can be concluded that the Thorlabs optics mount is not ideal for this application. In the future the mirror clamp and housing should be redesigned so that a more stable optics mount can be used.
6.10 Conclusion

In this chapter, the construction of a relatively cheap, 3x3 segment mirror and the drive electronics has been described. The piezo actuators used to drive the segments, and tests to ensure the piezos were sufficiently waterproof for processes during the mirror construction were presented next. Experiments carried out to test the operation of the electronics driving a single piezo actuator, a light weight test mirror segment, and a heavy test mirror segment over a range of frequencies up to 5kHz were then described. These experiments showed that resonances observed were mechanical and not electrical in nature. The electronics worked very well. A description of the mirror construction was then given, along with the mirror clamp and housing design. Frequency tests using a segment of the completed mirror, clamped in the housing were then presented showing that the main concerns with using the mirror in an optical setup are mechanical resonances in the mirror and mount at frequencies above 1kHz. However if care is taken to make sure the clamp and housing is set up as rigidly as possible, good operation can be expected.
Chapter 7. Background and Theory of Phase-Difference Amplification

7.1 Introduction

In the next two chapters a new method for achieving phase-difference amplification is described. The method is quick and convenient, and requires no photographic steps. Magnification factors of 2, 4 or 6 are achieved easily in one step. The system operates in real-time. Therefore phase-stepping\textsuperscript{[72]} may be applied to extract the amplified phase distributions. Our method is a variation on longitudinally reversed shearing interferometry, using first- or higher-order diffraction from a grating (hologram) which is in fact the interferogram of the wavefront under test. The grating is derived from a standard two-beam interferometer which is phase-stepped, and displayed in real time on a spatial light modulator in the phase-difference amplification setup. It is illuminated by the two output beams from a Sagnac interferometer, similar to the set up used by Barnes et. al.\textsuperscript{[73]} for spectral resolution enhancement, and a phase-amplified fringe pattern is obtained by spatial filtering using a Fourier transform lens.

We demonstrate operation of the phase amplifier and show amplified phase maps retrieved by phase-stepping. We believe this is the first time that real-time phase amplification without photographic steps and with phase-stepping has been demonstrated. In this chapter the background, principle and theory of phase amplification are given.

7.2 Background

Phase-difference amplification was proposed many years ago by Bryngdahl and Lohmann\textsuperscript{[74]} as a means of improving phase resolution and accuracy in interferometric measurement. The method uses higher diffracted orders from a non-linear hologram which
is essentially an interferogram of the surface under test. Matsumoto and Takashima\textsuperscript{[75]} developed the technique to achieve a high degree of magnification. Bryngdahl\textsuperscript{[76]} later suggested a slightly different technique, longitudinally reversed shearing interferometry, in which interference takes place between the virtual and real images of the test wave reconstructed by a hologram, and Matsuda et. al.\textsuperscript{[77]} improved the method and determined the limits of its application.

In the method presented by Matsuda et. al. a hologram was made by interfering the test wave with a known plane wave. An incoming plane wave was then diffracted from the hologram, and the +1 and -1 diffracted orders (corresponding to real and virtual images - one phase-reversed with respect to the other) were interfered together to produce a second hologram in which recorded phase deviations were twice those in the first. Repeating this procedure several times gave magnification factors of 2, 4, 8, 16, etc.

While this method produced good results, it has significant disadvantages in terms of convenience and speed. For example, the necessity to make a hologram for each factor of two increase in phase magnification is very inconvenient and slow, the system is susceptible to vibration, and it is difficult to apply phase-stepping techniques to automate the phase measurement process. The system can not easily be adapted for automatic pseudo-real-time operation.

One of the main disadvantages of all of the methods described above is the need for a photographic step to produce the hologram from which diffraction occurs as part of the phase amplification process. This prohibits real-time operation, and this, in turn, makes it very difficult to apply phase-stepping techniques to retrieve the amplified phase distribution. The advent of low priced LC-SLM's has changed this situation. It is now possible to write the hologram on a LC-SLM rather than photographic film and achieve phase amplification in real time. Further, the phase of the interference fringes written on the modulator can be quickly changed by scanning the interferometer producing them, enabling the system to be phase-stepped.

### 7.3 Principle of Phase-Difference Amplification

The proposed optical system for phase-difference amplification is shown in Figure 7-1. The system has two parts, the test interferometer that produces an interferogram of the test
surface, and the phase amplifier, which amplifies the phase of the interferogram. Any standard interferometer can be used for the test interferometer. It is important, however, that a tilt component is introduced between the signal and reference arms in the test interferometer so that the output interferogram has 25 or so carrier fringes across it. The interferogram is displayed on a LC-SLM in the phase amplifier.

![Diagram of the phase amplifier and test interferometer](image)

**Figure 7-1 Phase difference amplification system showing the test interferometer and the phase amplifier**

The phase amplifier is the Sagnac interferometer comprising beamsplitter $B_1$, and mirrors $M_1$, $M_2$ and $M_3$. In principle any of the common two-beam interferometers could be used in the phase amplifier, but the Sagnac was chosen for several reasons. The reasons are as follows:
1) Once constructed, it is very easy to adjust the angle of the output beams impinging on the hologram, by adjusting only one mirror $M_2$.

2) The two output beams from the Sagnac are easily arranged to travel at angles symmetric to the optical axis so that when they are adjusted for the correct incident angles onto the hologram, the diffracted beams from the hologram automatically travel along the optic axis, and

3) the Sagnac is a common path interferometer and is therefore very stable.

Mirror $M_2$ is used to adjust the angles of the two output beams from the Sagnac. Lenses $L_3$ and $L_4$ are an afocal imaging system which images mirror $M_2$ onto the LC-SLM, so that the output beams from the Sagnac cross in the plane of the LC-SLM with no shear. The carrier fringes of the interferogram displayed on the LC-SLM diffract the beams from the Sagnac. The angles the Sagnac beams that are entering the LC-SLM make with respect to the optic axis are arranged to correspond to the $\pm 1$ or $\pm 2$ or $\pm 3$, or $\pm n$ etc. order diffraction angles of the grating formed by the carrier fringes in the interferogram displayed on the LC-SLM. This is shown in Figure 7-2.

![Figure 7-2 Two Sagnac beams diffracted by the SLM grating into $\pm n$ orders](image)

Several diffracted beams then emerge from the LC-SLM. Of these, two travel along the optic axis, corresponding to one output beam from the Sagnac having been diffracted through $+n$ orders, and the other output beam diffracted through $-n$ orders. The value of $n$ is selected by adjusting mirror $M_2$ in the Sagnac so that its output beams have the required
angles as they enter the LC-SLM. The phase amplification factor in the final interferogram is then 2n.

The diffracted beams from the LC-SLM are collected by Fourier transform lens L₅, and the on-axis beams selected by a pinhole P at its focus. Lens L₆ is arranged so that it and L₅ together form a filtered image of the LC-SLM (actually an image of the phase-amplified interferogram) on the CCD camera. In our experiments, we used a simple four-step phase-step algorithm to reconstruct the phase of the final interferogram, achieved by moving one of the mirrors in the test interferometer in steps of $\lambda/8n$ to obtain a phase step of $\pi/2$ in the final interferogram. The reconstructed phase is given by,

$$\theta(x,y) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right),$$

where $I_1, I_2, I_3$ and $I_4$ are the spatial intensity distributions given at the system output at each phase step.

### 7.4 Phase Amplification Theory

The interferogram from the test interferometer may be written as:

$$I_\phi(x,y) = K\left[1 + V \cos(\omega x + \phi(x,y))\right]$$

(7-1)

where:

- $\phi(x,y)$ is the phase distribution from the surface under test
- $\omega$ is the spatial frequency of the carrier fringes
- $V$ is the fringe contrast
- $K$ is proportional to the intensity of the illuminating laser
- $x,y$ are Cartesian coordinates in the plane of the test surface

This intensity distribution is written on to the LC-SLM, which is non-linear, to produce a variation in transmittance:

$$T(x',y') = K\left[1 + \sum_n V_n \cos(n(\omega x' + \phi(x',y')))\right]$$

(7-2)
where: $K'$ is a constant dependent on the original illuminating intensity in the test interferometer and the characteristics of the LC-SLM

$V_n'$ are the amplitudes of the fringe-grating spatial harmonics

$x', y'$ are Cartesian coordinates in the plane of the spatial light modulator

In the phase amplifier, the grating on the LC-SLM is illuminated by two waves from the Sagnac interferometer. These are tilted about the $y'$ axis of the LC-SLM coordinate system, travelling at equal angles on opposite sides of the optic axis. For convenience, we assume that these have unity amplitude, and can be represented as:

$$U_1(x', y') = \exp(jm\alpha x') \quad \text{and} \quad U_2(x', y') = \exp(-jm\alpha x')$$  \quad (7-3)$$

Where: $m$ determines the tilt angle of the wavefront, $\theta$, where: $\theta = m\lambda\omega / 2\pi$

These two waves are multiplied by the grating function. Expressing this in terms of complex exponentials, the output waves from the LC-SLM resulting from $U_1$ may be written as:

$$E_{o11}(x', y') = \exp(jm\omega x')K'[1 + \sum_{n=1} \frac{V_n'}{2} \exp(jn(\omega x' + \phi(x', y'))) + \sum_{n=1} \frac{V_n'}{2} \exp(-jn(\omega x' + \phi(x', y')))]$$  \quad (7-4)$$

Expanding:

$$E_{o11}(x', y') = K'\exp(jm\alpha x') + \frac{K'V_k'}{2} \exp(j[(m + k)\omega x' + k\phi(x', y')])$$

$$+ \frac{K'V_k'}{2} \exp(j[(m - k)\omega x' - k\phi(x', y')])$$

$$+ K' \sum_{n=k} V_{n'} \exp(j[(m + n)\omega x' + n\phi(x', y')])$$

$$+ K' \sum_{n=k} V_{n'} \exp(j[(m - n)\omega x' - n\phi(x', y')]),$$  \quad (7-5)$$

where $k$ and $n$ are integers, $k$ is the diffracted order from the fringe-grating that we use to produce the final phase amplified interferogram, and we have abstracted terms corresponding to the $k^{th}$ diffracted orders to the beginning of the expression for clarity.
We adjust the tilt of the waves approaching the LC-SLM so that \( m-k \) is close to zero, or in other words the \(-k^{th}\) diffracted output wave from the fringe-grating travels approximately along the optic axis. The spatial filter P is adjusted to pass this wave while blocking the others.

We get a similar expression for the diffracted waves resulting from \( U_{2} \), and once again the pinhole P allows only the diffracted wave travelling nearly along the optic axis to proceed to the detector.

The net result is that we have two waves travelling towards the detector which interfere to produce the phase-amplified interferogram. These are:

\[
\frac{K'V'_{1}}{2} \exp\left(j\left((m-k)\omega'x'-k\phi(x',y')\right)\right) \quad \text{and} \quad \frac{K'V'_{1}}{2} \exp\left(j\left(-(m-k)\omega'x'+k\phi(x',y')\right)\right)
\]

which add by the principle of superposition to give an output wave, \( E_o(x',y') \):

\[
E_o(x',y') = \frac{K'V'_{1}}{2} \cos\left((k-m)\omega'x'+k\phi(x',y')\right) \quad (7-6)
\]

whose intensity distribution, \( I_o(x',y') \), is:

\[
I_o(x',y') = \frac{(K'V'_{1})^2}{2} \left[1 + \cos\left(2(k-m)\omega'x'+2k\phi(x',y')\right)\right] \quad (7-7)
\]

Note that this represents an interferogram with carrier fringes whose spatial frequency has been heterodyned down to \( 2(k-m)\omega' \) with spatial phase deviations of \( 2k\phi(x',y') \), i.e. the phase distribution of the original test surface has been amplified by a factor \( 2k \) where \( k \) is the diffracted order chosen by adjusting mirror \( M_2 \) in the Sagnac.

Note, too, that the amplified interferogram can be conveniently phase-stepped by scanning the phase of the test interferometer, allowing automated acquisition of the phase map. The phase step applied to the test interferometer should be \( 1/2k \) of that required in the amplified interferogram. For example if a four step phase stepping algorithm is used with a phase amplification factor of 4 (achieved by selecting second order diffraction at the SLM), the test interferometer phase must be stepped by only \( \pi/8 \) to achieve \( \pi/2 \) steps in the amplified interferogram.
The system is very nearly common path because of the Sagnac interferometer and the very small physical displacement of the interfering beams in the rest of the system. It is therefore very stable and easy to set up. Use of an appropriate LC-SLM would also allow real-time operation. It is also worth noting that a reflective-type LC-SLM (such as the Hughes liquid crystal light valve or the Hamamatsu PAL-SLM) could be placed in the position of $M_2$ inside the Sagnac interferometer. In this case, the output of the interferometer may be imaged onto the CCD camera with a single afocal filtering imaging system rather than the double system shown in Figure 7-1.

In order to produce an accurate phase-amplified interferogram it is important to avoid aliasing in the pinhole plane. Aliasing can occur primarily when variations in the gradient of the test surface cause spatial frequency components from other than the $\pm k^{th}$ diffracted orders to pass through the pinhole. The situation is shown in Figure 7-3 where (with a perfectly flat test surface) the nominal position of the $\pm k^{th}$ orders is on-axis.

![Diagram showing arrangement of diffracted orders in the focal plane of the Fourier transform lens](image)

**Figure 7-3 Arrangement of diffracted orders in the focal plane of the Fourier transform lens (shown for a phase amplification factor of 2)**

Other diffracted orders appear at distances corresponding to $l\omega'$ ($l$ is integer) on either side of the optic axis. Variations in the phase gradient of the test surface cause spatial frequency components to appear on either side of all diffracted spot positions, and it is important that these components do not overlap.
The maximum allowable variation in test surface phase gradient is therefore given by:

\[
\left| \frac{d\phi}{dx} \right| \leq \frac{\omega}{2},
\]

which is the usual condition for phase variations displayed on carrier fringes in spatially heterodyned systems. In principle, the size of the pinhole should be chosen to match this condition, but in practice it is wise to choose it slightly smaller to avoid undesirable effects from aberrations and diffraction in the phase amplifier optical system.

The effects of distortions in the LC-SLM should also be considered. Distortion of the fringe pattern corresponds to an additional, spurious phase superimposed on the interferogram from the test interferometer. The easiest way to compensate for this is to first measure the LC-SLM distortions using a known, good optical flat in place of the test surface and then subtract these distortions from subsequent phase measurements. The spurious phase introduced by distortions is amplified in the same way as the phase of the Test Interferometer interferogram, and so this calibration should be performed for each amplification factor. This procedure - which we followed in the experiments described in the next chapter, also compensates for phase errors arising in the phase amplifying interferometer.
Chapter 8. Phase-Difference Amplification Experiments

8.1 Introduction

In this chapter, three phase-difference amplification interferometer systems are described. Results from each are then presented. Each system contains two interferometers, which incorporate phase stepping and operate in near-real-time. The systems are simple to use and require no photographic steps.

The first system is a proof of principle system where the output of the test interferometer is simulated using a computer-generated phase stepped fringe pattern, which modulates a liquid crystal television display (LC-TV). Two beams from a Sagnac interferometer illuminate the LC-TV, which acts as a LC-SLM. Diffraction of these beams at the fringe grating on the LC-TV leads to the final phase-amplified interferogram. Operation of the phase amplifier in real time using computer-generated fringes written on the LC-TV is demonstrated. We also demonstrate that the phase distribution of the phase-amplified interferogram can be retrieved using conventional phase stepping retrieval algorithms, by scanning the fringes written on the LC-TV. Incorporating phase stepping allows accurate phase distributions of the test object to be obtained.

The second system is the Michelson-Sagnac system (where the test object is positioned in a Michelson interferometer, phase amplification achieved with a Sagnac interferometer), and the third system is the Michelson-Michelson system (test object positioned in a Michelson interferometer, phase amplification achieved with a second Michelson interferometer). In these systems, the first interferometer projects an interferogram of the test object onto the write side of an optically addressed LC-SLM. The read side of the LC-SLM is illuminated by two beams from the second interferometer that are adjusted so that their $+n$ and $-n$ order
beams are diffracted back along the optic axis. These produce an output interferogram that is phase-amplified by a factor $2n$. This phase distribution is retrieved using a 4 step, phase stepping routine. Results obtained using a real phase object in these two systems show that amplification factors of up to 6 are easily attainable. A comparison of the results from these two systems is presented. We believe this is the first time that near-real-time phase amplification with phase stepping has been demonstrated.

### 8.2 Phase-Difference Amplification using a Simulated Test Object

#### 8.2.1 Optical Setup for Simulated Test Object

A proof of principle phase-difference amplification experiment using a simulated test phase object is first presented. Figure 8-1 shows the optical setup for phase amplification using a computer simulated test object.

![Optical System Diagram](image)

**Figure 8-1** The optical system used in the experiments
The phase amplifier consisted of a Sagnac interferometer comprising beamsplitter B1 and mirrors M1, M2, and M3. Light from a 5mW HeNe laser entered the system through B1 and was split in two. The two counter propagating beams exited the Sagnac through B1. Lenses L3 and L4 formed an afocal imaging system, which imaged mirror M2 onto the LC-TV.

Mirror M2 was adjusted so that the angles that the Sagnac beams incident upon the LC-TV made with respect to the optic axis corresponded to the ±n order diffraction angles of the hologram displayed on the LC-TV. The carrier fringes of the hologram on the LC-TV diffracted the beams from the Sagnac. The diffracted beams were collected by Fourier transform lens L5, and the two on-axis beams were selected by a pinhole P at its focus. Lenses L5 and L6 were arranged so that they formed a filtered image of the LC-TV (actually an image of the phase-amplified interferogram) on the CCD camera. The two polarisers between L1 and L2 were provided to optimise the output image intensity at the CCD camera.

The LC-TV was a Kopin Corp. miniature VGA LC-TV display with 640 x 480 pixels in a 12mm x 9mm aperture. The simulated test object was a spherical section of wavefront with a maximum deviation of π/2 radians in the centre of a plane wavefront. Figure 8-2 shows a phase map of the simulated test object, with the tilt removed.

The four phase-stepped holograms that would be produced by interference between plane wavefronts, and wavefronts from the test object in a test interferometer were calculated using a computer. Figure 8-3 shows a computer-generated hologram for the test wavefront.

![Figure 8-2 Phase map of simulated test object with tilt removed](image-url)
Figure 8-3 Computer-generated hologram for the test wavefront with a spherical section of maximum deviation $\pi/2$ in the centre of a plane

The four holograms were written to the LC-TV successively and an image of the phase-amplified interferogram produced by each was grabbed from the CCD camera by the computer. The phase step between holograms was set to be $\pi/4k$ as described in Chapter 7. There were approximately 25 carrier fringes across the field of view, which consisted of about 60% of the available LC-TV area. Phase-stepping was then used to reconstruct the phase difference of the simulated test object.

8.2.2 Results using Simulated Test Object

Results using the simulated test object are now given. For each amplification factor, we first measured the phase-amplifier phase distortion by writing holograms of a flat wavefront with the same number of carrier fringes on the LC-TV, and measuring the phase distribution of the amplified interferogram. This distribution was then subtracted from subsequent measurements. Figure 8-4 shows a typical map of phase amplifier errors obtained for an amplification factor of 2 (using the $\pm 1$ diffracted orders from the fringe grating on the LC-TV). The effects of lithography errors in the LC-TV can be clearly seen as a corrugation running down the centre of the field.

To check that the wavefront produced by diffraction from the hologram was approximately correct, we temporarily rotated the beamsplitter $B_1$ causing the $1^{st}$ order from the LC-TV fringe grating (produced by the beam reflected from the beamsplitter) to overlap with the $0^{th}$ order (produced by the beam transmitted through the beamsplitter) in the pinhole plane. This gave a low-contrast interferogram (because of the low diffraction efficiency of the LC-TV fringe grating) with a phase amplification factor of 1 at the output.
Figure 8-4 Phase map of errors in the phase amplifier obtained for an amplification of 2

We phase stepped the hologram in steps of \(\pi/2\) and obtained a phase map of the test object which included the phase-amplifier phase distortion. Figure 8-5 shows the resulting phase map when the phase-amplifier distortion was subtracted. The wavefront shape and deviation can clearly be seen, and compares well with the plot shown in Figure 8-2.

Figure 8-5 Phase map obtained with an amplification of 1 to check that the wavefront produced by the hologram was correct

After the beamsplitter was returned to its original position, we tilted \(M_2\) so that \(m = 1\) thus causing the \(\pm1\) diffracted orders to overlap on-axis in the pinhole plane. We moved the
pinhole to select these orders and obtained an output interferogram with a phase amplification factor of 2. On phase stepping the hologram by $\pi/4$ we obtained the phase map shown in Figure 8-6 which included phase amplifier errors.

![Figure 8-6 Phase map obtained with an amplification of 2, including phase amplifier errors](image)

![Figure 8-7 Phase map obtained with an amplification of 2, after phase amplifier errors had been subtracted including phase amplifier errors](image)

146
On subtracting the previously measured phase errors (shown in Figure 8-4) for this amplification factor, we obtained the map shown in Figure 8-7 in which the phase is amplified by a factor of 2 as expected. We repeated this experiment, each time tilting M₂ in the Sagnac appropriately, to obtain amplification factors of 4 and 6 as shown in Figure 8-8 and Figure 8-9.

Figure 8-8 Phase map obtained with an amplification of 4, after phase amplifier errors had been subtracted

Figure 8-9 Phase map obtained with an amplification of 6, after phase amplifier errors had been subtracted
Note that the phase maps obtained at amplification factors of 4 and 6 show some additional distortion, which increases with amplification factor. We think this is probably due to phase-stepping errors caused by pixelisation in the LC-TV. With 640 pixels horizontally and 40 carrier fringes, each carrier fringe occupies only 16 pixels. At a phase amplification factor of 6, the phase step should be only 1/24 of a fringe - i.e. less than one pixel! We think that the reason the system worked at all at this high magnification was due to averaging caused by the finite bandwidth of the LC-TV video system, but that the phase steps were not uniform giving rise to errors. This would not occur with a non-pixelised spatial light modulator.

8.2.3 System Noise

The phase noise of the system expressed in wavelengths, $\Delta \lambda$, was estimated by subtracting two background phase distortion distributions, such as the one shown in Figure 8-4, from each other. This was done for each of the amplification factors 2, 4, and 6. Figure 8-10 shows a typical noise phase map obtained by subtracting two 1st order (amplification 2) background phase distortion distributions.

![Figure 8-10 Noise phase map for an amplification factor of 2](image)

For an amplification factor of 2, it was found the phase error in wavelengths was $\Delta \lambda_2=0.05\lambda$ with a standard deviation $\sigma_2=0.004\lambda$. For amplification factors of 4 and 6 the
phase error was $\Delta \lambda_A = 0.28 \lambda$, with $\sigma_A = 0.02 \lambda$, and $\Delta \lambda_B = 0.37 \lambda$, with $\sigma_B = 0.025 \lambda$, respectively. Note that these values take into account only background noise such as table and building vibration, and air movement within the lab. The noise does not take into account systematic errors such as phase step errors or CCD non-linearity.

8.2.4 Conclusion for System using a Simulated Phase Object

These results show that in principle, real-time phase-difference amplification by a factor of $4 \times 2^n$ should be possible when the $\pm n$ diffracted orders from a hologram obtained by projecting the interferogram of a test phase object onto a spatial light modulator, are interfered. The spatial phase of the test object can be reconstructed using phase stepping techniques.

8.3 Michelson-Sagnac Phase Amplification System using a Real Phase Object

8.3.1 Optical Setup for Michelson-Sagnac System

The experiment using the Michelson-Sagnac phase-difference amplification system will now be discussed. Figure 8-11 shows a diagram of the Michelson-Sagnac phase amplification system which had a Michelson interferometer on the write side of an optically addressed Hamamatsu PAL LC-SLM\textsuperscript{[78]} and a Sagnac interferometer on the read side. The Hamamatsu PAL LC-SLM was provided for us by Dr Kiyofumi Matsuda from the Japan Science and Technology Co., and the Mechanical Engineering Laboratory (MEL) in Tsukuba-shi, Ibaraki-ken, Japan. Dr Matsuda was working with us at the time these experiments were carried out and provided valuable theoretical and experimental expertise.

The Michelson interferometer (test interferometer) consisted of beam-splitter BS\textsubscript{1} and mirrors M\textsubscript{1} and M\textsubscript{2}. The beam from Laser 1 passed through a microscope objective and spatial filter and was collimated by lens L\textsubscript{1}. The beam was divided at BS\textsubscript{2} as it entered the Michelson. One beam makes a double pass through the phase object, and recombined with the other at the beam splitter. The resulting interferogram was imaged on to the write side of the LC-SLM by lens L\textsubscript{2}. Mirror M\textsubscript{2} was rotated to add the tilt component.
The Sagnac interferometer phase amplifier consisted of beamsplitter BS$_2$, and mirrors M$_3$, M$_4$ and M$_5$. The beam from Laser$_2$ passed through a microscope objective and spatial filter and was collimated by lens L$_3$ before entering the Sagnac interferometer at BS$_2$. The two interferometer beams travelled in opposite directions around the Sagnac, then passed through BS$_3$, lenses L$_4$ and L$_5$, and on to the read side of the LC-SLM. Lenses L$_4$ and L$_5$ formed an afocal imaging system that imaged mirror M$_4$ onto the LC-SLM. The incident angle of each beam was controlled by tilting M$_4$, so that the $+n$ order of one beam, and $-n$ order of the other, were diffracted back from the LC-SLM in the same direction, parallel to
the optic axis. Fourier transform lens $L_6$ and the pinhole selected only the desired orders. The interferogram was imaged onto the CCD camera by lens $L_7$ and recorded by the computer.

Adjusting the tilt of $M_2$ in the test interferometer varied the spacing of the diffracted orders in the Fourier plane of $L_6$. When $M_4$ was tilted, the angles of both beams incident on the read side of the LC-SLM conveniently changed by the same amount, but in opposite directions. The position of the pinhole at the output did not therefore need more than a fine readjustment for each measurement with amplification factors greater than zero.

The $0^{th}$ order diffracted beams contained no phase information about the test phase object since they were plane reflected waves. To get an interferogram whose phase distribution had an amplification factor of one, it was therefore necessary to interfere the $\pm 1$ order of one beam, with the $0^{th}$ order of the other as in the previous experiment using the simulated phase object. This was achieved either by rotating $BS_2$ as well as mirror $M_4$, or by rotating only $M_4$ and moving the pinhole sideways, so that these orders were selected by the pinhole. The contrast of the output interferogram was reduced in this case because the intensity of the different diffracted orders from the LC-SLM were different in general. However, the phase stepping techniques used easily accommodated reductions in contrast of this order.

### 8.3.2 Phase Stepping

A frame from the CCD camera was grabbed by the computer each time $M_1$ in the Michelson interferometer was moved. The phase steps in this setup (where phase stepping occurs before amplification) depended on the diffracted orders selected and therefore the phase amplification. All phases in the test interferometer were amplified, so with an amplification factor of $2n$, the phase step was set at $\lambda/8n$ where $\lambda$ is the wavelength of Laser 1.

A Thorlabs Piezo driven mirror mount and driver were used to provide the phase stepping in the test interferometer. The four step phase step routine discussed in the previous chapter was again used. A low voltage piezo stack in each of 3 corners controlled the mirror mount. The mirror therefore had tip, tilt and piston movement. In these experiments only piston
movement was required so the same voltage was applied across each piezo. The effects of piezo drift and hysteresis were noticeable and were minimised by:

1) always starting the phase step procedure from the same voltage,

2) stepping in the same direction each time, and

3) careful selection of the voltage steps.

Results were obtained by taking measurements with and without the phase object in the system. The measurement without the phase object, measured the background system aberrations. This was subtracted from the measurement taken with the phase object included, to produce the final phase map of the object.

### 8.3.3 Operation Time for Michelson-Sagnac System

The settling time of the liquid crystals in the LC-SLM is approximately 0.1 sec., and the frame time of the CCD camera is 0.04 sec. For this system where phase stepping occurred on the write side of the LC-SLM it was necessary to wait for the LC-SLM to settle before each frame was captured. In practice it was also necessary to allow about 0.02 sec. for the phase stepping mirror and piezo actuators to settle. The total time required for each frame was therefore 0.16 sec. When a four step phase stepping algorithm was used, and we allowed 0.04 sec. for processing, the total time for one measurement was approximately 0.68 sec.

### 8.3.4 Test Phase Object

The test phase object used in this experiment was a set of two Silicon Dioxide (SiO₂) stripes which had been evaporated onto a flat glass substrate. Figure 8-12 shows a diagram of the test object. The step height of the stripes was measured by Talystep at MEL in Japan to be 30nm. Silicon Dioxide has a refractive index of n=1.48. This corresponds to a theoretical optical path step height of 44.4nm. Since there is a double pass through the phase object in the Michelson test interferometer, the phase step measured by the system with an amplification factor of 1 should be 88.8nm or 0.14\(\lambda\) at a test laser wavelength of 633nm.
8.3.5 Michelson-Sagnac System Results

Figure 8-13 shows results using the Michelson-Sagnac phase-difference interferometer. Mirror $M_4$ in the Sagnac interferometer was adjusted to give amplification factors of 1, 2, 4 and 6, and the phase distributions which resulted are shown in Figure 8-13 a-d respectively. It can be seen that the size of the steps in the measured phase distributions increases proportionally with amplification factor.

Figure 8-13 a-d. Phase distribution with Michelson–Sagnac configuration, amplification factors 1, 2, 4 and 6 respectively
A close inspection of the graph in Figure 8-13 a) shows the step size agrees well with the value of 0.14λ predicted from the Talystep measurement. With amplification factors 2, 4 and 6, the predicted step sizes are 0.28λ, 0.57λ, and 0.85λ respectively. Again these compare well with Figure 8-13 b), c) and d).

It can be seen that the dominant noise in Figure 8-13 a) is a series of ripples running in the direction of the carrier fringes, perpendicular to the steps of the phase object. The ripples were caused by small errors in the phase steps which occurred because of piezo drift and hysteresis, table and building vibrations, and air movement within the lab. An estimate of this noise component was made by finding the total range in wavelengths Δλ_n through a cross-section of a nominally flat part of each plot i.e. parallel to the object step. The standard deviation σ_n of the cross-section data was also found. Table 4 contains a summary of the results and errors.

<table>
<thead>
<tr>
<th>Amplification factor</th>
<th>Theoretical step height (λ)</th>
<th>Measured step height (λ)</th>
<th>Background noise range Δλ_n (λ)</th>
<th>Background noise Std. Dev. σ_n (λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.15</td>
<td>0.038</td>
<td>0.0043</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.30</td>
<td>0.16</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.57</td>
<td>0.55</td>
<td>0.17</td>
<td>0.025</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>0.82</td>
<td>0.30</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Table 4 Result summary for Michelson-Sagnac phase-difference amplification set up

It is interesting to note that the size of the ripples increased with the amplification factor. This was expected because the phase amplification occurred after phase stepping, and so any errors in the phase steps were also magnified.

8.4 Michelson-Michelson System using a Real Phase Object

8.4.1 Optical System for Michelson-Michelson System

The experiment using the Michelson-Michelson phase-difference amplification system will now be discussed. Figure 8-14 shows the Michelson-Michelson system which used a
Michelson interferometer to provide the two beams on the read side of the LC-SLM instead of a Sagnac. This Michelson interferometer comprised beam splitter BS2 and mirrors M3 and M4. Lenses L4 and L5 imaged mirrors M4 and M5 onto the LC-SLM. While the common path Sagnac interferometer in the previous experiment provided more stability and the single mirror adjustment made it convenient to select different diffracted orders, it was very difficult to phase step. By using another Michelson interferometer instead of the Sagnac, it was possible to phase step the system after the phase amplification has taken place. The advantages of doing this were that:

1) the phase step size remained constant for any amplification factor, and

2) the phase step errors were not amplified by the system.

Figure 8-14 Phase-difference amplifier set up using Michelson–Michelson configuration
8.4.2 Operation time for Michelson-Michelson System

Using this system it was necessary to wait 0.1 sec. for the LC-SLM to settle only once, before phase stepping. Using a four step algorithm where each CCD camera frame takes 0.04 sec., allowing 0.02 sec. for the mirror and piezo actuators to settle at each step, and allowing 0.04 sec. processing time, the total time for one phase reconstruction measurement was 0.38 sec. This was almost half the time it took using the Michelson-Sagnac system which was 0.68 sec.

8.4.3 Michelson-Michelson System Results

The test phase object used in this experiment was again the object with two SiO₂ stripes shown in Figure 8-12. Results using the Michelson-Michelson system are shown in Figure 8-15 a-d with the Michelson interferometer at the LC-SLM output adjusted to give amplification factors of 1, 2, 4 and 6 respectively.

Figure 8-15 a-d. Phase distribution with Michelson–Michelson configuration, amplification factors 1, 2, 4 and 6 respectively
Again, the results were obtained by subtracting two phase distributions, one with and one without the phase object. It can be seen that the size of the steps in the measured phase distributions increases proportionally with amplification factor and agree well with the step heights predicted by the Talystep measurements. An estimate of the phase errors was again made by finding the total range in wavelengths $\Delta\lambda_n$ through a cross-section of a nominally flat part of each plot, parallel to the object step. The standard deviation $\sigma_n$ of the cross-section data was also found. Table 5 contains a summary of the results and errors.

<table>
<thead>
<tr>
<th>Amplification factor</th>
<th>Theoretical step height $\lambda$</th>
<th>Measured step height $\lambda$</th>
<th>Background noise range $\Delta\lambda_n$ $\lambda$</th>
<th>Background noise Std. Dev. $\sigma_n$ $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.14</td>
<td>0.031</td>
<td>0.0038</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.24</td>
<td>0.037</td>
<td>0.0057</td>
</tr>
<tr>
<td>4</td>
<td>0.57</td>
<td>0.58</td>
<td>0.14</td>
<td>0.015</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>0.84</td>
<td>0.066</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

Table 5 Result summary for Sagnac–Sagnac phase-difference amplification set up

Note that the background noise, and the standard deviation of the noise, are significantly reduced compared to results obtained at the same amplification factors using the Michelson-Sagnac system.

### 8.5 Comparison of Michelson-Sagnac, and Michelson-Michelson Systems

The first system using the Michelson test interferometer and the Sagnac amplifying interferometer was simpler to adjust for different amplifications. However the second Michelson – Michelson set up had far less noise. The first system required that phase stepping be done before the amplification because of the difficulty in phase stepping a Sagnac interferometer, which meant that the phase step errors were also amplified. This is reduced significantly in the second system where the phase stepping took place after amplification.
The total time taken for one measurement using each system shows that the Michelson–Michelson system which takes 0.38 sec. operates almost twice as fast as the Michelson–Sagnac system which takes 0.68 sec. However, compared to previous methods which took at least 30 mins. per measurement due to the photographic processing required, they can both be said to operate in near-real-time.

8.6 Conclusion

In this chapter three experiments were discussed, and their results presented. Amplified phase maps of simulated and real objects were obtained using near-real-time phase stepping interferometers and a LC-SLM.

A proof of principle experiment was carried out first. Here, simulated interferograms of a test object were applied to a LC-TV which was used as a diffraction grating in a second phase amplifying interferometer. The second interferometer provided beams which were incident upon the grating at equal angles, but on opposite sides of the optic axis. The $\pm n$ diffracted orders were selected using a spatial filter and interfered to produce an interferogram whose phase was equal to the phase of the original interferogram amplified by $2n$. Phase stepping techniques were used to retrieve a phase-difference map of the simulated test object. This experiment showed that the technique worked in principle.

The amplifying interferometer for the second experiment was similar to the one used in the first. Here however, the interferogram of a real phase object was imaged onto an optically addressed LC-SLM which was then used as the diffraction grating. The $\pm n$ diffracted orders were collected from the grating and interfered to produce a phase amplified interferogram. Phase stepping techniques were again used to retrieve a phase-difference map of the test object. The magnification of the system was easily changed by tilting one mirror.

In the third experiment, phase stepping was carried out in the readout interferometer, after phase-amplification had occurred. The benefits of this were twofold. The phase steps were the same for any magnification, and phase step errors were not amplified by the system so noise was significantly reduced.

The results showed that amplification factors up to 6 were easily achievable. We believe these techniques will be useful in a variety of optical testing applications where the required
resolution is better than $\lambda/30$. For applications where phase stepping noise is not an issue, and where ease of adjustment is desired, the Michelson-Sagnac system is more suitable because of the inherent stability of the Sagnac interferometer and the ease with which it was adjusted for different amplification factors. However, for systems where most noise comes from the phase stepping, the Michelson-Michelson configuration is more suitable since the phase step errors are not amplified by the system.
Chapter 9. Summary and Conclusion

This thesis was written in two sections, following the introductory Chapter 1. The first section comprised Chapters 2 to 6, in which issues dealing with aberration correction and white light feedback interferometry were covered. The second section comprised Chapters 7 and 8, where work on phase-difference amplification was presented.

This chapter contains a brief summary of the two sections, draws the work in the sections together, and looks forward to the direction future research might take.

9.1 Section 1 Summary

In Section 1 the background of aberration correction, applications of adaptive optics systems, and commonly used wavefront sensing and correcting techniques were presented.

Aberration correction systems use a variety of correction elements including liquid crystal devices and segmented mirrors. Clearly, characteristics such as response time, phase vs. drive voltage, and sensitivity to wavelength and polarisation state of input light are important to system performance. The characteristics of two liquid crystal spatial light modulators, pertinent to aberration correction, were investigated.

Research on a novel white light feedback interferometry technique that can be used for aberration correction was then presented. Using this technique, a phase proportional to the interferometer output intensity is applied to a modulator within the interferometer to achieve automatic phase correction. The interferometer operates in white light. A theoretical analysis of white light feedback interferometry was given along with simulated and experimental results.

Liquid crystal spatial light modulators suffer from relatively slow response times and a limited phase modulation range. Segmented mirrors have much faster response times, and a
much larger phase modulation range, but are very expensive. Therefore, we constructed a 9 segment mirror for use in aberration correction systems. The construction, testing, housing and mounting of the mirror were discussed.

9.2 Section 2 Summary

Phase-difference amplification is a technique that can be used for high accuracy phase measurement. Using this technique a diffraction grating is constructed using the interferogram of a test wavefront. The ±n orders diffracted from the grating are interfered together to form a phase-amplified interferogram from which a phase map of the test wavefront can be recovered. Conventional systems use time consuming and inconvenient photographic techniques to construct the grating. Section 2 of this thesis contained descriptions of several phase-difference amplification systems where the diffraction grating was written on a liquid crystal spatial light modulator.

The liquid crystal devices, which have a high resolution and much faster response times than the photographic development time, make the systems convenient and allow near-real-time operation. A phase map of the test object is retrieved using phase stepping techniques. Experimental results for each system were presented and compared. They showed that amplification factors of up to 6 were easily achievable.

9.3 Conclusions

Aberration correction is the process of measuring and correcting spatial phase aberrations on a light beam. For many aberration correction techniques it is necessary to measure the phase distribution of the input wave accurately and quickly. Results from Section 1 showed that this can, in principle, not only be achieved using feedback interferometry, but be achieved using white light. Phase-difference amplification with a LC-SLM was shown in Section 2 to have potential for fast, high accuracy phase measurement, and is therefore another technique that may be very useful for aberration correction systems.

Once the phase distribution of an aberrated light beam has been measured, it is necessary to perform the phase correction quickly, accurately and if possible cheaply. Liquid crystal spatial light modulators and a purpose built piezo driven segmented mirror were shown to be capable of achieving this.

162
9.4 Future Research

The work presented in this thesis provides the ground work for continued research on white light aberration correction techniques and equipment. We have shown that in principle aberration correction can be achieved in white light using feedback interferometry. A very interesting next step would be to build a working system which corrects the image of an object such as an artificial star. The artificial star might be an intense light with a very high colour temperature placed some distance away from the system. It may also be possible to use a distant but sufficiently bright existing light.

There will also need to be continued research into the effect that intensity variations of the artificial star has on the feedback process, and how the problems this creates with system stability can be overcome, before a reliable system can be built.

It was shown that the 9 segment mirror built for experiments in this thesis was capable of being used as an aberration correction element. The limiting aspects of the mirror if it were to be used in an aberration correction system would be the large size of the pixels and consequently the large size of the mirror itself and the small number of pixels. The rigidity of the mirror mount was also a problem. Research on an improved segmented mirror that addressed these areas would be beneficial. Reducing the size and increasing the number of the pixels may be possible by using smaller piezo actuators placed closer together. It may also be possible to develop a single small piezo stack which has three actuators built in.

There is still a large interest in the use of liquid crystal devices for aberration correction applications primarily because of their low cost. However they suffer from relatively slow response times and a limited phase modulation range. I would love to see continued research into improved liquid crystal spatial light modulators for use in aberration correction systems.
Appendix A

This Appendix contains the designs for the segmented mirror clamp, and housing. These were given to Steve Warrington and John Humphreys in the Physical Sciences Mechanical Workshop who constructed them.
Project: Adaptive mirror housing
Clamp - Aluminium
Geoff Bold ext 8890
page 3 of 8
Project: Adaptive mirror housing
Disk- aluminium
Geoff Bold ext 8890
Page 4 of 8
Project: Adaptive mirror housing
Cover-aluminium
Geoff Bold ext 8890
Page 5 of 8
Project: Adaptive mirror housing
Base plate - Aluminium
Geoff Bold ext 8890
Page 6 of 8

Key for holes:

- m4t = tapped m4 hole
- m6t = tapped m6 hole
- m5 = m5 clearance + countersunk
Project: Adaptive mirror housing
Bottom plate - Aluminium
Geoff Bold ext 8890
Page 7 of 8
Appendix B

Abstract

Low cost, accurate, high resolution spatial light modulators are of increasing interest for adaptive optics applications. Most adaptive optics systems currently use expensive segmented or deformable mirrors. Nematic liquid crystal spatial light modulators have been suggested as possible substitutes for these mirrors as the phase modulating arrays in adaptive optics systems. This paper discusses broad wavelength band, and polarisation insensitive operation of two different liquid crystal spatial light modulators currently available and under investigation as possible phase modulating arrays for adaptive optics systems. The results of our investigations give a good indication of how the devices will operate in practical adaptive optics systems. © 1998 Elsevier Science B.V.

Keywords: Liquid crystal spatial light modulators; Adaptive optics

1. Requirements for adaptive optics

Liquid crystal spatial light modulators (LC-SLMs) and their drive electronics have a lower cost than segmented or deformable mirrors and other conventional phase modulators [1]. Their power requirements are also very low [2]. For these reasons, LC-SLMs would be excellent replacements in certain adaptive optics applications [3], provided they also fulfil certain other device requirements. The requirements for LC-SLMs in adaptive optics is that they:

1. operate over a wide wavelength band (say 400 nm to over 1000 nm),
2. be insensitive to the polarisation direction of incident light,
3. have accurate phase modulation (say < 1%),
4. have a moderate number of pixels or actuators (100–1000),
5. have a high optical throughput (> 90%), and
6. have a fast switching time (< 1 ms).

In this paper, the discussion concentrates on examining the first two requirements i.e. the operation of LC-SLMs in broad band light, and randomly polarised light. Some of the other issues are covered by Love [3].

2. Wavelength dependence of LC-SLM devices

In broad-band adaptive optics applications it is important that the phase modulator affects all wavelengths uniformly. In this sense, it is actually stroke or change in optical path length that should be at least approximately the same at all wavelengths rather than phase modulation (which is usually the parameter of interest in optical information processing applications). An optical wavefront which has travelled through an essentially nondispersing atmosphere will suffer the same physical wavefront distortion at all wavelengths - but the phase distortion which is...
dependent on wavelength will vary, being larger at shorter wavelengths. A deformable mirror lends itself to broadband adaptive optics because it naturally has the same stroke at all wavelengths. However, this is not necessarily the case for a LC-SLM where the variation of phase modulation with wavelength depends on the dispersion of the liquid crystal material. We have therefore expressed the results of our modulation characterisation experiments in terms of stroke rather than phase modulation.

The LC-SLM used for this part of the experiment was provided by Professor Tschudi at the Technische Hochschule, Darmstadt in Germany. It consists of a 9 × 9 array of parallel aligned nematic liquid crystal (Merck ZLI1132) phase modulating pixels [4].

A schematic of the common path interferometer used to determine the modulator stroke as a function of wavelength for this LC-SLM is shown in Fig. 1.

The light from an incandescent bulb on the left of Fig. 1 is roughly collimated by lens 1. It then passes through a Soleil Babinet compensator, the first polariser and the LC-SLM. The Soleil Babinet compensator provides an initial variable optical path difference between the horizontal and vertical polarisation components of the randomly polarised input light. This allows the initial path difference in the interferometer (when zero voltage is applied to the modulator) to be set to a convenient value (λ/2) for each value of wavelength, for subsequent processing of the interferometer output data to produce the modulator stroke characteristic. The polariser is oriented at 45° to the horizontal and the LC-SLM is orientated so that it modulates the optical path length [5] of only horizontally polarised light, or the horizontal polarisation component of light, that passes through it. The beam transmitted by the first polariser has both vertical and horizontal components. Therefore, when this beam passes through the LC-SLM only the horizontal component is modulated. Following the LC-SLM the light passes through a second polariser also oriented at 45°. This polariser essentially selects and interferes common components of both the horizontal (modulated) and vertical (unmodulated) components of the beam which pass through the LC-SLM. The intensity of the light at the output therefore varies with LC-SLM stroke. For example when the LC-SLM delays the horizontal component by a half wavelength with respect to the vertical component, the common components of both interfere destructively so the intensity at the output is minimum. If the LC-SLM delays the horizontal component by a whole wavelength then the common components interfere constructively so the output intensity is maximum. Lens 2 then focuses the beam into a spectrometer which passes any desired spectral component between 400 and 900 nm to a photo diode. The signal from the photo diode passes through a current-to-voltage converter and is logged to file by an A/D board in the host computer. Fig. 2 shows a mesh plot of normalised interferometer output intensity as a function of modulator voltage and wavelength using this experimental setup.

As described earlier, the interferometer path difference is set, using the Soleil Babinet compensator, to be λ/2 at zero modulator volts for each wavelength. The output intensity at each wavelength is therefore initially minimum and rises with increasing modulator voltage. The interferometer output intensity at any wavelength λ is given by

\[ I_{\text{out}} = A \left[ 1 + V \cos \left( \frac{2\pi}{\lambda} (S + S_m) \right) \right]. \]  

where \( S \) is the optical path difference or stroke introduced by the Soleil Babinet compensator between the horizontal and vertical polarisation components of the input light, \( S_m \) is the stroke of the LC-SLM, \( V \) is the visibility or contrast.

![Fig. 1. Common path interferometer for characterising the polychromatic operation of the LC-SLM.](image-url)
of the fringes, and $A = I_1 + I_2$, where $I_1$ and $I_2$ are the intensities of the two interfering beams at the output. The LC-SLM stroke may therefore be obtained from
\[
S_m = 2n\pi + \frac{\lambda}{2\pi}\cos^{-1}\left(\left(\frac{I_1 - I_2}{A}\right)\frac{1}{V}\right) - S, \tag{2}
\]
where $n$ is an integer. The modulator stroke varies between 1.5 wavelengths at 900 nm, and 3 wavelengths at 400 nm, so each output intensity corresponds to up to six possible stroke values. This problem was overcome by breaking the interferometer output intensity data set for each wavelength up into monotonically increasing and decreasing sections. Eq. (2) was applied to each section. Appropriate phase offsets (integral values of $2\pi$) were then added to the resulting phases to produce a continuous phase characteristic. Note that for a given wavelength in Fig. 2 the intensity of the peaks and valleys varies slightly as the modulator voltage increases. This corresponds to a modulator stroke dependent fringe visibility $V$ and average output intensity $A$, and is due to residual absorption in the modulator. Therefore, $A$ and $V$ were experimentally calculated and used in a piece-wise manner for each monotonic intensity section.

Fig. 3 shows a mesh plot of the reconstructed modula-

![Fig. 2. Plot of wavelength versus modulator voltage versus normalised interferometer intensity.](image1)

![Fig. 3. Plot of wavelength versus modulator voltage versus modulator stroke.](image2)
Fig. 4. Plot of modulator stroke versus wavelength, for a modulator voltage of 4.75 V.

Fig. 5. Common path interferometer used for polarisation insensitivity experiments.

Polarisation insensitivity measurement 1: Rayleigh interferometer

For adaptive optics applications the spatial light modulator must also be capable of uniformly modulating light of any polarisation. The purpose of this experiment is to quantitatively assess how an LC-SLM operates as an optical wavefront controller, when configured for polarisation insensitivity as described by Love [6]. Conventionally, LC-SLMs modulate only one polarisation component of light passing through them, in this case horizontal. By swapping the horizontal and vertical polarisation components between two passes through the LC-SLM, uniform modulation should be possible for all polarisation components. Meadowlark has produced a device that consists of two orthogonal modulators to achieve polarisation insensitive operation [7]. However, we are investigating achieving the same operation using a simpler single liquid crystal modulator device. The Rayleigh type interferometer used to characterise the stroke of the LC-SLM for different input polarisations using this double pass configuration is shown in Fig. 5.

A 10 mW linearly polarised HeNe laser passes through a spatial filter. The beam is then collimated and passes through a rotatable λ/2 plate which provides a linearly polarised beam of arbitrary orientation. Two circular apertures create two separate beams which are both transmitted by a beam splitter. One, a control beam, passes through a control pixel of the LC-SLM which is unmodulated. The other, a test beam, passes through a modulated test pixel. The test pixel initially modulates only the horizontal component of the beam passing through it. However both beams then pass through a λ/4 plate oriented at 45° and are reflected by a plane mirror back through both the λ/4 plate and the LC-SLM. The double pass through the λ/4 plate swaps the vertical and horizontal components of each beam. On the return pass through the LC-SLM, the ortho-
The polarisation component of the test beam is modulated by the test pixel but the control beam remains unmodulated. Therefore, full modulation is achieved for all polarisations which pass through the test pixel. The two double passed beams are then reflected by the beam splitter and combined using a Fourier transform lens. The spatial Fourier transform which consists of an interference fringe pattern is enlarged by a microscope objective on to a photo diode array. The host computer then logs the output of the diode array.

As the test pixel of the LC-SLM is modulated, the interference pattern formed in the Fourier plane moves sideways. Fourier transform techniques are used to determine the stroke of the LC-SLM from the spatial phase of the fringes as they move. This procedure was carried out for a 180° range of linear input polarisations. Fig. 6 is a graph of input polarisation versus LC-SLM voltage versus LC-SLM phase, and shows that the modulation characteristic at each input polarisation is the same. The variation in modulator stroke is less than 4% across the whole range of polarisations.

It is also important that both interferometer beams have the same polarisation following the double pass through the modulator. The contrast of the output fringe pattern did not vary significantly as the input polarisation was rotated, indicating that the polarisation of the two output beams was almost identical at all modulator voltages, and all polarisations.

It would be interesting to know how system performance varies with wavelength, but unfortunately our experimental setup required a laser input source and we did not have access to a tunable laser around 633 nm. However, it is possible to estimate the wavelength range over which practical satisfactory performance might be obtained, from a knowledge of the dispersion of quartz. Phase errors due to dispersion in the λ/4 plate lead to incomplete swapping of the polarisation components. Residual (unswapped) electric field components will then either be phase modulated twice (residual unswapped components parallel to the liquid crystal molecule directors) or not modulated at all (residual components perpendicular to the directors). Our criterion for determining the allowable wavelength range was that the residual unswapped field components should be less than 0.1 of the incoming field component in that direction. Using a standard Jones matrix analysis it is easy to show that the phase errors in the λ/4 plate should be less than 0.2 rad to achieve this. Assuming that the plate is a zeroth order plate and is correct for a design wavelength of 632.8 nm (which was the case in our experiments), the dispersion curves for quartz allow calculation of the maximum wavelength range allowable before this limit is exceeded. We found that a wavelength range of approximately ±40 nm about 632.8 nm is possible. For a larger allowable wavelength range, an achromatic or superachromatic waveplate, or an achromatic Fresnel rhomb retarder could be used instead.

4. Polarisation insensitivity measurement II: phase wedge

An experiment using a similar setup was also performed using a different LC-SLM. The LC-SLM was supplied by Meadowlark Optics, and consists of 69 parallel
aligned nematic liquid crystal phase modulating elements [3]. The performance of the device when producing Zernike wavefronts and static adaptive optics was described by Love [3] and its use in a real-time adaptive optics system was described by Gourlay [8].

The purpose of this experiment was to assess the optical losses associated with using the device in a practical system setup, configured to be polarization insensitive. The technique used to assess the performance of the device was to write a phase wedge across the pixel array which would deflect the optical beam. When imaged, the point spread function (PSF) is observed to be off-axis. A measurement of the resulting Strehl ratio (height of the off-axis PSF) gives a useful measure of any optical losses, and hence a comparison of the device performance to what one would expect from an ideal phase wedge. The effects of optical losses due to the components (LC-SLM, \( \lambda/4 \) plate), nonideal LC material parameters, and diffractive effects due to the pixellation of the wedge pattern on the SLM, should all be accounted for in the measurement of Strehl ratio. This result should allow optical system designers to assess the suitability of such a device for their purposes. In addition to measuring the experimentally achievable Strehl ratio, we also simulated the performance of a 'perfect' Meadowlark device numerically. We created in the computer a wavefront with the same phase distribution that would be imparted by the phase wedge across the Meadowlark device – with its hexagonal pixel shape. We then calculated the intensity distribution expected in the output plane using a numerical Fourier transform, and compared experimental and simulated Strehl ratios. The experimental apparatus for measurement of the LC-SLM polarization insensitivity using a phase wedge is shown in Fig. 7.

An unpolarized collimated laser is used for illumination and is first passed through a nonpolarizing beam splitter. Then, as in the previous section, the light passes through the LC-SLM, a \( \lambda/4 \) plate, reflects off mirror 1, then back through the \( \lambda/4 \) plate and LC-SLM. The beamsplitter then diverts the light onto a linear CCD camera for measurement of the PSF. The size of the phase wedge written to the modulator was 1 wave peak-to-peak at 633 nm. Experiments were performed with and without both the \( \lambda/4 \) plate and the LC-SLM in the setup. Table 1 shows the results of the measurements for the various configurations and gives a comparison with the simulation results. To obtain the experimental diffraction limited case, we removed both the LC-SLM and the \( \lambda/4 \) plate and measured the height of the PSF to use for normalization in subsequent experimental Strehl ratio measurements. Measure-

<table>
<thead>
<tr>
<th>Optical configuration</th>
<th>Measured Strehl ratio</th>
<th>Simulated Strehl ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffraction limit (no components)</td>
<td>1 (by definition)</td>
<td>1 (by definition)</td>
</tr>
<tr>
<td>No ( \lambda/4 ) plate-no phase wedge</td>
<td>0.83</td>
<td>1 (assuming no system aberrations)</td>
</tr>
<tr>
<td>No ( \lambda/4 ) plate-phase wedge</td>
<td>two spots 0.43, 0.31</td>
<td>two spots 0.5, 0.38</td>
</tr>
<tr>
<td>( \lambda/4 ) plate-no phase wedge</td>
<td>0.80</td>
<td>1 (assuming no system aberrations)</td>
</tr>
<tr>
<td>( \lambda/4 ) plate-phase wedge</td>
<td>0.63</td>
<td>0.88</td>
</tr>
</tbody>
</table>
ments were then taken of the Strehl ratio when the \( \lambda/4 \) plate and LC-SLM were reinserted, with and without a phase wedge written onto the LC-SLM.

It should be noted that the LC-SLM modulates only one polarisation component when the \( \lambda/4 \) plate is not present. Therefore two spots are observed at the image plane. One unmodulated, resulting in an on-axis spot, and one modulated twice by the phase wedge (effectively resulting in a 2 wave peak-to-peak wedge for this polarisation component) giving an off-axis, resolvable spot. As an example of results obtained, Fig. 8 shows these deviated and undeviated spots obtained with the experimental setup, and Fig. 9 shows equivalent spots obtained from the simulation, for comparison.

The critical figure of merit for this particular LC-SLM is the Strehl ratio for the device operating in polarisation insensitive mode (i.e. \( \lambda/4 \) plate present) and with a phase wedge written to the LC-SLM (1 wave peak-to-peak, at 633 nm). Under these conditions the experiment gave a Strehl ratio of 0.63. This is a fair appraisal of the device performance in optical systems, taking into account all device limitations except any chromatic effects described above. It gives the approximate upper limit of the ability of the device to correct for aberrated wavefronts in adaptive optics systems i.e. the Strehl ratio can be improved up to 0.63. The Strehl ratio obtained from the simulation under these conditions is 0.88, higher (as expected) than the experimental value. The difference may be ascribed to nonuniformities of phase modulation across individual SLM pixels, connecting wires to the electrode structure, and standing aberrations in the optical system.

The critical figure of merit for comparison with the simulation is the ratio: (Strehl ratio with no \( \lambda/4 \) plate no phase wedge)/(Strehl ratio with \( \lambda/4 \) plate and phase wedge), that is, a comparison of the Strehl ratio degradation which occurs when the phase wedge is written on the SLM, since aberrations introduced by the rest of the components cannot be simulated reliably. Table 1 shows that this ratio is: 0.63/0.8 = 0.79 in the experiment and 0.88 in the simulation.

Fig. 8. Experimental PSF for phase wedge of one wavelength.

Fig. 9. Simulated PSF for phase wedge of one wavelength.
5. Conclusion

We have shown that the stroke of a typical liquid crystal spatial light modulator varies by less than 20% over a range of input wavelengths from 400 to 900 nm. It has also been shown that a double pass through the LC-SLM and a correctly oriented \( \lambda/4 \) plate, can provide uniform modulation for all components of randomly polarised light. An upper limit of 0.63 has been placed on the expected Strehl ratio, when low cost components are used in a polarisation insensitive configuration in an adaptive optics system. The experimental results indicate that liquid crystal spatial light modulators are good candidates for high performance phase modulating elements in adaptive optics applications.

References

Appendix C

Phase-difference amplification using a Sagnac interferometer

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Abstract

In this paper, we describe a new method for achieving phase-difference amplification, which is quick and convenient, operates in real time, and requires no photographic steps. Magnification factors of 2, 4 or 6 are achieved easily in one step. Because the system operates in real time, phase stepping may be applied to extract the amplified phase distributions. Our method is a variation on longitudinally reversed shearing interferometry, using first- or higher-order diffraction from a grating (hologram) which is in fact the interferogram of the wavefront under test. The grating is derived from a standard two-beam interferometer which is phase-stepped, and displayed in real time on a spatial light modulator in the phase-difference amplification setup. It is illuminated by the two output beams from a Sagnac interferometer, similar to the set up used by (Barnes et al. Barnes TH, Eiju T, Matsuda K. Appl Opt 1986; 25: 1864). for spectral resolution enhancement, and a phase-amplified fringe pattern is obtained by spatial filtering using a Fourier transform lens. We demonstrate operation of the phase amplifier and show amplified phase maps retrieved by phase-stepping. We believe this is the first time that real-time phase amplification without photographic steps and with phase stepping has been demonstrated. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Phase amplification; Phase step interferometry; Sagnac interferometer

1. Introduction

Phase-difference amplification was proposed many years ago by Bryngdahl and Lohmann [1] as a means of improving phase resolution and accuracy in interferometric measurement. The method uses higher diffracted orders from a non-linear hologram which is essentially an interferogram of the surface under test.
and Matsumoto and Takashima [2] developed the technique to achieve a high degree of magnification. Bryngdahl [3] later suggested a slightly different technique, longitudinally reversed shearing interferometry, in which interference takes place between the virtual and real images of the test wave reconstructed by a hologram, and Matsuda et al. [4] improved the method and determined the limits of its application.

In the method presented by Matsuda et al. a hologram was made by interfering the test wave with a known plane wave. An incoming plane wave was then diffracted from the hologram, and the +1 and −1 diffracted orders (corresponding to real and virtual images — one phase-reversed with respect to the other) were interfered together to produce a second hologram in which recorded phase deviations were twice those in the first. Repeating this procedure several times gave magnification factors of 2, 4, 8, 16, etc.

While this method produced good results, it has significant disadvantages in terms of convenience and speed. For example, the necessity to make a hologram for each factor of two increase in phase magnification is very inconvenient and slow, the system is susceptible to vibration, and it is difficult to apply phase-stepping techniques [5] to automate the phase measurement process. The system can not easily be adapted for automatic pseudo-real-time operation.

One of the main disadvantages of all of the methods described above is the need for a photographic step to produce the hologram from which diffraction occurs as part of the phase amplification process. This prohibits real-time operation, and this — in turn — makes it very difficult to apply phase-stepping techniques to retrieve the amplified phase distribution. The advent of low priced spatial light modulators (SLMs) has changed this situation for it has now become possible to write the hologram on a spatial light modulator rather than photographic film and achieve phase amplification in real time. Further, the phase of the interference fringes written on the modulator can be changed by scanning the interferometer producing them, and so the system can be phase-stepped.

In this paper we propose a novel system for phase amplification based on these principles which is quick, convenient, and requires no photographic steps. Our system uses a phase-stepped test interferometer whose output fringe pattern is used to modulate an SLM which forms the core of the phase amplifier. The SLM is illuminated by two beams from a Sagnac interferometer which gives the system excellent stability because it is nearly common path. Diffraction of these beams at the fringe grating on the SLM leads to the final phase-amplified interferogram. We demonstrate operation of the phase amplifier in real time using computer-generated fringes written on a liquid crystal display which acts as an SLM. We also demonstrate that the phase distribution of the phase-amplified interferogram can be retrieved by phase stepping, by scanning the fringes written on the liquid crystal display and using conventional phase retrieval algorithms. We believe that this is the first time that real-time phase amplification with phase stepping has been demonstrated.

2. Principle

Our proposed optical setup for phase-difference amplification is shown in Fig. 1. The system has two parts: the test interferometer which produces an interferogram of
Fig. 1. Phase difference amplification system showing the test interferometer and the phase amplifier.

the test surface, and the phase-amplifier which amplifies the phase of this interferogram. Any standard interferometer can be used for the test interferometer. It is important, however, that a tilt component is introduced between the signal and reference arms in this interferometer so that the output interferogram has 25 or so carrier fringes across it. The interferogram is displayed on a spatial light modulator (SLM) in the phase amplifier.

The phase amplifier consists of the Sagnac interferometer comprising beamsplitter $B_1$, and mirrors $M_1$, $M_2$ and $M_3$. In principle any of the common two-beam interferometers could be used in the phase amplifier, but the Sagnac was chosen for several reasons as follows:

1. once set up it is very easy to adjust the angle of the output beams impinging on the hologram, by adjusting only one mirror,
2. the two output beams from the Sagnac are easily arranged to travel at angles symmetric to the optical axis so that when they are adjusted for the correct incident angles onto the hologram, the diffracted beams from the hologram are automatically along the optic axis, and
3. the Sagnac is a common path interferometer and so it is very stable.
Fig. 2. Two Sagnac beams diffracted by the SLM grating into $\pm n$ orders.

Mirror $M_2$ is used to adjust the angles of the two output beams from the Sagnac. Lenses $L_3$ and $L_4$ are an afocal imaging system which images mirror $M_2$ onto the SLM, so that the output beams from the Sagnac cross in the plane of the SLM with no shear. The carrier fringes of the interferogram displayed on the SLM diffract the beams from the Sagnac. The angles the Sagnac beams coming into the SLM make with respect to the optic axis are arranged to correspond to the $\pm 1$ or $\pm 2$ or $\pm 3$, or $\pm n$, etc., order diffraction angles of the grating formed by the carrier fringes in the interferogram displayed on the SLM. This is shown in Fig. 2.

Several diffracted beams then emerge from the SLM. Of these, two travel along the optic axis, corresponding to one output beam from the Sagnac having been diffracted through $+n$ orders, and the other output beam diffracted through $-n$ orders. The value of $n$ is selected by adjusting mirror $M_2$ in the Sagnac so that its output beams have the required angles as they enter the SLM. The phase amplification factor in the final interferogram is then $2n$.

The diffracted beams from the SLM are collected by Fourier transform lens $L_5$, and the on-axis beams selected by a pinhole $P$ at its focus. Lens $L_6$ is arranged so that it and $L_5$ together form a filtered image of the SLM (actually an image of the phase-amplified interferogram) on the CCD camera. In our experiments, we used a four-step phase shifting algorithm to measure the phase of the final interferogram, achieved by moving one of the mirrors in the test interferometer in steps of $\lambda/8n$ to obtain a phase step of $\pi/2$ in the final interferogram.

3. Theory

Since it has been more than 20 years since phase amplification was first described, and there has been very little work in the field since then due to the lack of spatial light modulators, we recap the theory here for completeness before describing our experiments. The interferogram from the test interferometer may be written as

$$I_0(x, y) = K[1 + V \cos(\omega x + \phi(x, y))],$$

(1)
where $\phi(x, y)$ is the phase distribution from the surface under test, $\omega$ is the spatial frequency of the carrier fringes, $V$ is the fringe frequency, $K$ is proportional to the intensity of the illuminating laser and $x, y$ are Cartesian coordinates in the plane of the test surface.

This intensity distribution is written on to the SLM, which is non-linear, to produce a variation in transmittance:

$$T(x', y') = K' \left[ 1 + \sum_n V_n \cos(n(\omega'x' + \phi(x', y'))) \right]$$

where $K'$ is a constant dependent on the original illuminating intensity in the test interferometer and the characteristics of the SLM, $V_n$ are the amplitudes of the fringe–grating spatial harmonics and $x', y'$ are Cartesian coordinates in the plane of the spatial light modulator. In the phase amplifier, the grating on the SLM is illuminated by two waves from the Sagnac interferometer. These are tilted about the $y'$-axis of the SLM coordinate system, traveling at equal angles on opposite sides of the optic axis. For convenience, we assume that these have unity amplitude, and can be represented as

$$U_1(x', y') = \exp(jm \omega x') \quad \text{and} \quad U_2(x', y') = \exp(-jm \omega x'),$$

where $m$ determines the tilt angle of the wavefront, $\theta$, where: $\theta = m\lambda\omega/2\pi$. These two waves are multiplied by the grating function. Expressing this in terms of complex exponentials, the output waves from the SLM resulting from $U_1$ may be written as

$$E_{0n}(x', y') = \exp(jm \omega x')K'$$

$$\left[ 1 + \sum_n V_n \exp(jn(\omega'x' + \phi(x', y'))) \right]$$

Expanding

$$E_{0n}(x', y') = K' \exp(jm \omega x') + \frac{K'V_k'}{2} \exp[j((m + k)\omega'x' + k\phi(x', y'))]$$

$$+ \frac{K'V_k'}{2} \exp[j((m - k)\omega'x' - k\phi(x', y'))]$$

$$+ K' \sum_{n \neq k} \frac{V_n}{2} \exp[j((m + n)\omega'x' + n\phi(x', y'))]$$

$$+ K' \sum_{n \neq k} \frac{V_n}{2} \exp[j((m - n)\omega'x' - n\phi(x', y'))].$$

$$T(x', y') = K' \left[ 1 + \sum_n V_n \cos(n(\omega'x' + \phi(x', y'))) \right]$$
where \( k \) and \( n \) are integers, \( k \) is the diffracted order from the fringe-grating that we use to produce the final phase amplified interferogram, and we have abstracted terms corresponding to the \( k \)th diffracted orders to the beginning of the expression for clarity.

We adjust the tilt of the waves approaching the SLM so that \( m - k \) is close to zero, or in other words the \( -k \)th diffracted output wave from the fringe-grating travels approximately along the optic axis. The spatial filter \( P \) is adjusted to pass this wave while blocking the others.

We get a similar expression for the diffracted waves resulting from \( U_2 \), and once again the pinhole \( P \) allows only the diffracted wave traveling nearly along the optic axis to proceed to the detector.

The net result is that we have two waves traveling towards the detector which interfere to produce the phase-amplified interferogram. These are

\[
\frac{K'V_k}{2} \exp\{j[(m-k)\omega'x' - k\phi(x',y')])
\]

and

\[
\frac{K'V_k}{2} \exp\{j[-(m-k)\omega'x' + k\phi(x',y')])
\]

which add by the principle of superposition to give an output wave, \( E_0(x',y') \):

\[
E_0(x',y') = K'V_k \cos((k-m)\omega'x' + k\phi(x',y'))
\]

whose intensity distribution, \( I_0(x',y') \), is

\[
I_0(x',y') = \frac{(K'V_k)^2}{2} [1 + \cos(2(k-m)\omega'x' + 2k\phi(x',y'))].
\]

Note that this represents an interferogram with carrier fringes whose spatial frequency has been heterodyned down to \( 2(k-m)\omega' \) with spatial phase deviations of \( 2k\phi(x',y') \), i.e. the phase distribution of the original test surface has been amplified by a factor \( 2k \) where \( k \) is the diffracted order chosen by adjustment of mirror M2 in the Sagnac.

Note, too, that the amplified interferogram can be conveniently phase stepped by scanning the phase of the test interferometer, allowing automated acquisition of the phase map. The phase step applied to the Test Interferometer should be \( \pi/2k \) of that required in the amplified interferogram, e.g. if a four-step phase-stepping algorithm is used with a phase amplification factor of 4 (achieved by selecting second-order diffraction at the SLM), the Test Interferometer phase must be stepped by only \( \pi/8 \) to achieve \( \pi/2 \) steps in the amplified interferogram.

The system is very nearly common path because of the Sagnac interferometer and the very small physical displacement of the interfering beams in the rest of the system. It is therefore very stable and easy to set up. Use of an appropriate SLM would also
allow real-time operation (although we simulated the test interferometer output in the experiments described below). It is also worth noting that a reflective-type SLM (such as the Hughes LCLV or the Hamamatsu PAL-SLM) can be used if it is placed in the position of M₂ inside the Sagnac interferometer. In this case, the output of the interferometer may be imaged onto the CCD camera with a single afocal filtering imaging system rather than the double system shown in Fig. 1.

In order to produce an accurate phase-amplified interferogram it is important to avoid aliasing in the pinhole plane. Aliasing can occur primarily when variations in the gradient of the test surface cause spatial frequency components from other than the ±kth diffracted orders to pass through the pinhole. The situation is shown in Fig. 3 where (with a perfectly flat test surface) the nominal position of the ±kth orders is on-axis. Other diffracted orders appear at distances corresponding to lω’ (l is integer) on either side of the optic axis. Variations in the phase gradient of the test surface cause spatial frequency components to appear on either side of all diffracted spot positions, and it is important that these components do not overlap. The maximum allowable variation in test surface phase gradient is therefore given by

\[
\frac{d\phi}{dx} \leq \frac{\omega}{2}
\]

which is the usual condition for phase variations displayed on carrier fringes in spatially heterodyned systems. In principle, the size of the pinhole should be chosen to match this condition, but in practice it is wise to choose it slightly smaller to avoid undesirable effects from aberrations and diffraction in the Phase Amplifier optical system.

Fig. 3. Arrangement of diffracted orders in the focal plane of the Fourier transform lens. (Arrangement shown for a phase amplification factor of 2).
The effects of distortions in the SLM should also be considered. Distortion of the fringe pattern corresponds to an additional, spurious phase superimposed on the interferogram from the test interferometer. The easiest way to compensate for this is to first measure it using a known, good optical flat in place of the test surface and then subtract it from subsequent phase measurements. The spurious phase introduced by distortions is amplified in the same way as the phase of the Test Interferometer interferogram, and so this calibration should be performed for each amplification factor. This procedure— which we followed in the experiments below, also compensates for phase errors arising in the Sagnac interferometer.

4. Experiments

We set up the phase-amplifying system shown in Fig. 4 using a 5mW HeNe laser and standard optical components. We used a Kopin Corp. corporation VGA LCTV display with 640 × 480 pixels in a 12 mm × 9 mm aperture for the SLM. This did not allow us to display fringe patterns from a test interferometer directly from a video source as would be possible with an optically addressed SLM or a LCTV, so instead we calculated the phase-stepped fringe patterns that would be produced by interference between plane and spherical wavefronts in a test interferometer by
Fig. 5. Phasemap of errors in the phase amplifier obtained for an amplification of 2.

Fig. 6. Computer-generated hologram for the test wavefront with a spherical section of maximum deviation \( \pi/2 \) in the centre of a plane.
computer. We used a four-step phase-stepping algorithm and wrote the four fringe patterns on the display in succession, grabbing an image of the phase amplified interferogram from each hologram. The phase step between holograms was set to be $\pi/4k$ as described above. There were approximately 25 carrier fringes across the field of view which consisted of about 60% of the available LCTV area.

For each amplification factor, we first calibrated the phase-amplifier phase distortion by writing holograms of a flat wavefront with the same number of carrier fringes on the LCTV, and measuring the phase distribution of the amplified interferogram. This distribution was then subtracted from subsequent measurements. Fig. 5 shows a typical map of phase amplifier errors obtained for an amplification factor of 2 (using the $\pm 1$ diffracted orders from the fringe grating on the LCTV), and the effects of lithography errors in the LCTV can be clearly seen as a corrugation running down the centre of the field.

Fig. 6 shows a computer-generated hologram for the test wavefront (a spherical section of wavefront with a maximum deviation of $\pi/2$ radians in the centre of a plane). Fig. 7 shows the wavefront from which this hologram was calculated, with the tilt removed. To check that the wavefront produced by diffraction from the hologram was approximately correct, we rotated the beamsplitter $B_1$ causing the first order from the LCTV fringe grating (produced by the beam reflected from the beamsplitter) to overlap with the zeroth-order (produced by the beam transmitted through the beamsplitter) in the pinhole plane. This gave a low-contrast interferogram
Fig. 8. Phase map obtained with an amplification of 1 to check that the wavefront produced by the hologram was correct.

Fig. 9. Phase map obtained with an amplification of 2, including phase amplifier errors.
Fig. 10. Phase map obtained with an amplification of 2, after phase amplifier errors had been subtracted.

Fig. 11. Phase map obtained with an amplification of 4, after phase amplifier errors had been subtracted.
(because of the low diffraction efficiency of the LCTV fringe grating) with a phase amplification factor of 1 at the output. We phase stepped the hologram by $\pi/2$ and obtained the phase map shown in Fig. 8. Although this is noisy because of the very low fringe contrast, the wavefront shape and deviation can be seen clearly.

We then tilted M2 more, so that $m = 1$ thus causing the $\pm 1$ diffracted orders to overlap on-axis in the pinhole plane. We moved the pinhole to select these orders and obtained an output interferogram with a phase amplification factor of 2. On phase stepping the hologram by $\pi/4$ we obtained the phase map shown in Fig. 9 which includes phase amplifier errors. On subtracting the previously measured errors for this amplification factor, we obtained the map shown in Fig. 10 in which the phase is amplified by a factor of 2 as expected. We repeated this experiment, each time tilting M2 in the Sagnac appropriately, to obtain amplification factors of 4 and 6 as shown in Figs. 11 and 12.

Note that the phasemaps obtained at amplification factors of 4 and 6 show some additional distortion which increases with amplification factor. We think this is probably due to phase-stepping errors caused by pixilation in the LCTV. With 640 pixels horizontally and 40 carrier fringes, each carrier fringe occupies only 16 pixels. At a phase amplification factor of 6, the phase step should be only $1/24$ of a fringe —ie. less than one pixel! We think that the reason the system worked at all at this high magnification was due to averaging caused by the finite bandwidth of the SLM video system, but that the phase steps were not uniform giving rise to errors. This would not occur with a non-pixelised SLM.

![Phase map obtained with an amplification of 6, after phase amplifier errors had been subtracted.](image-url)
5. Noise

The phase noise of the system expressed in wavelengths, \( \Delta \lambda \), was estimated by subtracting two background phase distortion distributions, such as the one shown in Fig. 5, from each other. This was done for each of the amplification factors 2, 4, and 6. Fig. 13 shows a typical noise phase map obtained by subtracting two first-order (amplification 2) background phase distortion distributions. For an amplification factor of two, it was found \( \Delta \lambda_2 = 0.05 \lambda \) with a standard deviation \( \sigma_2 = 0.004 \lambda \). For amplification factors of 4 and 6 the phase error was \( \Delta \lambda_4 = 0.28 \lambda \), with \( \sigma_4 = 0.02 \lambda \), and \( \Delta \lambda_6 = 0.37 \lambda \), with \( \sigma_6 = 0.025 \lambda \), respectively. Note that these values take into account only background noise such as table vibration and air movement within the lab. The noise does not take into account systematic errors such as phase step errors or CCD non-linearity.

6. Conclusion

Theory and a proposed optical setup for a new real time, phase stepped, phase amplification system using a Sagnac interferometer have been presented. The first part of the system is a phase stepping test interferometer which produces an interferogram.
of a test mirror. The interferogram is written to a spatial light modulator to act as a hologram in the second part of the system, a phase amplifying Sagnac interferometer. The two Sagnac beams are imaged to form an interference fringe pattern in the plane of the SLM. When the exit angles of the two Sagnac beams are arranged so the $\pm n$th diffracted orders from the SLM overlap in the Fourier plane filtered by a pinhole, a phase-amplified interferogram of amplification $2n$ is produced and imaged onto a CCD camera.

A demonstration system similar to this was built, using computer-generated holograms representing flat wavefronts with spherical deformations rather than holograms produced by a test interferometer. A liquid crystal display was used as the SLM. Phase stepping the holograms on the display using appropriate step sizes of $\lambda/4n$ produced results giving phase amplifications of 1, 2, 4 or 6 ($n = 0, 1, 2, 3$) indicating that the system works as theoretically predicted. The system noise for amplification factors of 1, 2, 4 and 6 was found to have a standard deviation of $\sigma_2 = 0.004\lambda$, $\sigma_4 = 0.02\lambda$, and $\sigma_8 = 0.025\lambda$, respectively.

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References

Appendix D

Real-time phase-difference amplification with a liquid-crystal spatial light modulator

Kiyofumi Matsuda, Geoffrey T. Bold, Thomas H. Barnes, Tomoaki Eiju, and Colin J. R. Sheppard

We describe a simple system for achieving real-time phase-difference amplification of interferograms. We arrange the interferogram such that it contains high-spatial-frequency carrier fringes and project it onto the write side of an optically addressed phase-only spatial light modulator. The resultant phase pattern on the modulator is read out by two readout beams, and diffraction by the carrier fringes provides the spatial heterodyning that is necessary for achieving phase-difference amplification. We present results that demonstrate real-time phase-difference amplification by as much as a factor of 10. © 2000 Optical Society of America

OCIS codes: 120.3180, 120.6660, 120.5050, 120.2830, 120.2880.

1. Introduction

Phase-difference amplification is a useful technique that has a potential for improving accuracy in determining the phase distribution in interference experiments. With this technique, fringes that carry the phase distribution of interest are recorded on a nonlinear material, and two light beams are then diffracted from the recorded pattern. The system is arranged such that light from the +n and −n diffracted orders interfere to produce a new fringe pattern whose contours occur at intervals of λ/2n, i.e., the same as those that would have been produced had light of wavelength λ/2n been used to produce the original fringes. It should be possible to combine phase-difference amplification with phase stepping1 to produce a system that is capable of making measurements with a high degree of accuracy. Phase-difference amplification was proposed and demonstrated by Bryngdahl and Lohmann,2 who used interference between high-diffracted-order beams from a nonlinear hologram. Matsumoto and Takashima3 achieved phase contours at intervals of λ/14 by using a special photographic plate with strong nonlinearities to record the hologram. Bryngdahl4 also proposed the use of longitudinally reversed shearing interferometry to achieve phase-difference amplification but did not determine the maximum phase-magnification factor that is possible in practice. Matsuda et al5 made some improvements to Bryngdahl’s method and examined the factors that set the limits of useful phase-difference amplification by use of holograms.

In all the phase-difference amplification methods discussed above, the hologram is recorded on film, which provides the nonlinear medium but requires lengthy and inconvenient chemical processing. It is worth noting that interesting electronic methods for achieving phase-difference amplification have also been developed,6 but these require complex electronic processors for video-rate operation.

In this paper we describe a phase-difference amplification system in which the hologram is written directly upon a liquid-crystal spatial light modulator (LCSLM), so system therefore operates in nearly real time. We use a parallel aligned phase-only modulator, so the main nonlinearity that gives rise to higher-order diffracted beams occurs naturally because the intensity distribution of the original interference fringes is converted into a phase distribution from which the light coming into the hologram is diffracted. If the phase modulation is cosineoidal, the relative

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intensities of different diffracted orders are determined by the Bessel functions that appear in standard communications phase-modulation theory and which one can control by changing the input writing intensity to the modulator and the driving voltage. We believe that this is the first time that near-realtime operation has been demonstrated in a phase-difference amplification system.

2. Theory

Figure 1 is a general schematic that shows the principle of phase-difference amplification with an optically addressed phase-only LCSLM. An interference fringe pattern from an interferometer operating with light of wavelength \( \lambda \) and containing the test object (the test interferometer), \( f(x,y) \), is imaged onto the write side of the LCSLM. We assume a fringe contrast of 1 and write the fringe pattern intensity distribution as

\[
I(x, y) = 2A^2[1 + \cos\phi(x, y)],
\]

where the two interfering beams from the test interferometer have electric field amplitude \( A \) and the phase difference between them is \( \phi(x, y) \). We assume that \( \phi(x, y) \) has two components, \( \phi(x, y) \) and \( \chi \), where

\[
\phi(x, y) = \phi(x, y) + \alpha \chi,
\]

\( \phi(x, y) \) is the phase distribution that arises from the test object (i.e., the phase distribution that is to be measured), and \( \alpha \) represents a tilt introduced between the interfering wavefronts in the test interferometer that gives rise to carrier fringes.

The phase-only LCSLM converts the write-side intensity distribution into a phase distribution that is impressed onto the light beams reflected from the read side of the device. These beams approach the LCSLM at equal angles on opposite sides of the optic axis such that their phase distributions across the read side are \( \pm \beta \chi \). Assuming unity reflected amplitude, the electric field distributions in the reflected beams are then

\[
E_1(x, y) = \exp(j\beta \chi)\exp(j2A^2K[1 + \cos\phi(x, y) + \alpha \chi])\]

\( \exp(j2A^2K[1 + \cos\phi(x, y) + \alpha \chi]) \) (3)

\[
E_2(x, y) = \exp(-j\beta \chi)\exp(j2A^2K[1 + \cos\phi(x, y) + \alpha \chi])\]

(4)

where \( j = \sqrt{-1} \).

Following the normal analysis that is made for phase modulation in communication theory, we can expand the second exponential in each of these expressions in terms of Bessel functions:

\[
\exp(j2A^2K[1 + \cos\phi(x, y) + \alpha \chi]) = \exp(j2A^2K) \sum_{n=-\infty}^{\infty} J_n(2A^2K)\exp(jn\phi(x, y) + \alpha \chi + \pi/2)\]

(5)

The electric field distributions of the reflected beams then become

\[
E_1(x, y) = \exp(j2A^2K)\exp(j\beta \chi) \sum_{n=-\infty}^{\infty} J_n(2A^2K)\]

\[
\times \exp(jn\phi(x, y) + \alpha \chi + \pi/2)\]

(6)

\[
E_2(x, y) = \exp(j2A^2K)\exp(-j\beta \chi) \sum_{n=-\infty}^{\infty} J_n(2A^2K)\]

\[
\times \exp(jn\phi(x, y) + \alpha \chi + \pi/2)\]

(7)

In each of these equations, the \( n \)th-order Bessel function corresponds to the \( n \)th diffracted beam from the carrier fringes written on the LCSLM. Each diffracted beam leaves the LCSLM traveling at a different angle to the system’s optic axis. By adjusting the angles of the input beams symmetrically on either side of the optic axis it is possible to arrange that, for one input beam, \( n = -\beta/\alpha \) [in Eq. (6)] and, for the other input beam, \( n = \beta/\alpha \) [in Eq. (7)]. Two diffracted beams then appear that are traveling along the optic axis [\(-n \)th-order beam from Eq. (6) and \( n \)th-order beam from Eq. (7)] and can be written as

\[
E_1(x, y) = J_n(2A^2K)\exp(-jn\phi(x, y) + \pi/2)\]

\( \exp(-jn\phi(x, y) + \pi/2) \) (8)

\[
E_2(x, y) = J_n(2A^2K)\exp(jn\phi(x, y) + \pi/2)\]

\( \exp(jn\phi(x, y) + \pi/2) \) (9)

These two beams then combine interferometrically to produce a fringe pattern whose intensity distribution is

\[
I_{Out} = |E_1 + E_2|^2 = 2[J_n(2A^2K)]^2[1 + \cos(2n\phi(x, y))].
\]

(10)

This is a fringe pattern that corresponds to a phase distribution that is \( 2n \) times the phase distribution that originally arises from the test object. We can think of this fringe pattern as being the one that would have been produced by the test interferometer if it were operated with light of wavelength \( \lambda /2n \) rather than \( \lambda \). This result is the same as that obtained from conventional phase-difference amplification systems by use of holograms written upon photographic plates whose fringe contours occur at path-difference intervals of \( \lambda /2n \), except that use of the phase-only LCSLM permits real-time operation of the system. Further, one can adjust the phase-modulation depth by changing the input light intensity at the write side of the modulator (and also by adjusting the modulator drive voltage and frequency) such that, in principle, the balance of intensities in

Fig. 1. General arrangement for real-time phase-difference amplification with a LCSLM.

5126 APPLIED OPTICS / Vol. 39, No. 28 / 1 October 2000

203
the diffracted orders can be adjusted to optimize system operation for a particular order of diffraction (n) and phase-amplification factor.

3. Experiment

We demonstrated phase-difference-amplification with a hologram written upon a phase-only optically addressed LCSLM by using the system shown in Fig. 2. The system contains two interferometers: the test interferometer that was used to produce the original fringe pattern written onto the LCSLM and the readout interferometer that produced the final phase-amplified interference fringes.

The test interferometer was a Michelson interferometer that was driven by a He–Ne laser (Laser 1) and comprised beam splitter BS₁, mirrors M₁ and M₂, and imaging lens L₀, which imaged the interference fringes onto the write side of the LCSLM. A known phase distribution was introduced into the interferometer by use of a test object that consisted of a transparent glass plate with accurately plane surfaces upon which was deposited a pattern of stripes in SiO₂ of known thickness. The stripes were 3 mm wide, with 3-mm gaps. Their height was measured with a Talystep profilometer (the result is shown in Fig. 3) and was found to be 50 nm. The test object was placed just in front of one of the Michelson mirrors in a plane that was conjugate (via lens L₀) to the write side of the LCSLM. One of the Michelson mirrors was tilted to produce carrier fringes [corresponding to phase gradient a required in Eqs. (3) and (4)].

The LCSLM used was a parallel aligned spatial light modulator (X5641 series made by Hamamatsu Photonics K.K.), which has a light-sensitive layer of amorphous silicon to control the amplitude of an ac electric field across a parallel aligned liquid-crystal (LC) layer placed in front of a multilayer dielectric mirror. The ac field comes from transparent electrode plates placed on the outside of the α-silicon–mirror–LC layer stack. Write light to the device impinges upon the amorphous layer and reduces its local resistivity. This reduced resistivity causes a rise in the electric field across the LC layer, changing the orientation of the molecules and the refractive index. Read light enters through the LC layer, is reflected off the dielectric mirror, and leaves the device after passing through the LC layer again. The double pass through the modulated layer causes spatial phase modulation to appear on the polarization component of the outgoing light beam parallel to the LC directors. The device has a spatial resolution of ~50 line pairs/mm over an 18 mm × 18 mm square aperture with a maximum phase modulation of −1.5 λ at 633 nm.

The reading beams for the LCSLM were derived from a system similar to a Sagnac interferometer.
that comprised a He–Ne laser (Laser$_2$), beam splitter BS$_2$, and mirrors M$_3$–M$_5$ (Fig. 2). Tilting M$_4$ about a vertical axis caused the two output beams from the system to travel toward the read side of the LCSLM at equal angles on opposite sides of the optic axis, yielding appropriate carrier fringes, that is, yielding the required phase gradients, which are close to but slightly different from the −β and β required for selection of the phase-amplification factor [Eqs. (6) and (7)].

On reflection (and diffraction) at the LCSLM, the output beams from the system were picked off by a third beam splitter, BS$_3$, and passed through a Fourier-transform lens (L$_4$). An array of spots that correspond to the diffracted orders from the two Sagnac beams appeared in the focal plane of L$_4$, and we used a pinhole to select only the two orders that appeared to be closest to the optic axis. Lens L$_5$ formed an afocal imaging system with L$_4$ to image the output face of the LCSLM onto the CCD camera, where phase-amplified output fringes appeared. Two polarizers in front of the CCD camera were used to adjust the intensity to appropriate levels to prevent overloading.

Figure 4 shows interference fringes from the object (the transparent glass plate) with suitable carrier fringe components obtained directly from the Michelson interferometer on the write side of the LCSLM. The optical path difference introduced by the stripes is rather small, ~48 nm, or λ/13 (assuming that the refractive index of SiO$_2$ is 1.48, with a double pass through the sample in the interferometer), so the fringe deviation at the edges of the stripes is rather small.

Figure 5 shows interference patterns obtained on the CCD camera, with the angle of M$_4$ adjusted such that different diffracted orders passed through the pinhole to produce several different phase-amplification factors. The input intensity to the LCSLM and its driving voltage were adjusted such that it operated on the linear part of its intensity–phase transfer characteristic. Figure 5(a) is the result obtained with the 0th orders from both read beams that have passed through the pinhole. Note that the phase-amplification factor is zero, as expected, with no fringe deviation. Figures 5(b), 5(c), 5(d), and 5(e) show the results obtained when M$_4$ was adjusted such that ±1st, ±2nd, ±3rd, and ±4th orders, respectively, interfered at the CCD camera. Note that no recognizable fringes were obtained with the ±4th orders because the intensity of the diffracted light from the LCSLM was too low.

We therefore increased the intensity at the write side of the LCSLM to increase the phase-modulation depth and the nonlinearity, and therefore the intensities of the higher-order diffracted beams, and obtained the results shown in Fig. 6. Figures 6(a), 6(b), 6(c), and 6(d) show the results for the ±3rd, ±4th, ±5th, and ±6th diffracted orders interfering at the CCD camera. Clearly, the maximum useful amplification available with this system was obtained with ±5th orders interfering, i.e., with an amplification factor of 10×.

4. Discussion

Although the results presented in Figs. 5 and 6 clearly demonstrate the phase-amplification capability of our system, we found some interesting effects on close examination of the fringes obtained without a phase object in the Michelson interferometer. Figure 7 shows fringes obtained with phase-amplification factors of 2 and 4, with no phase object in the interferometer. We expected to find uniformly spaced fringes but learned that such is not in fact the case. We believe that this outcome stems from a combination of several effects:

- Shear at Fourier-transform lens L$_4$ and at the read-side face of the LCSLM;
- Flatness errors in the mirrors of the Sagnac interferometer;
- Shear (the effect of which is amplified) at imaging lens L$_5$ at the Michelson output;
- Flatness errors in the Michelson mirrors.

We consider each of these in turn.

A. Shear between the Two Beams at Lens L$_4$ and at the Read-Side Face of the LCSLM

When mirror M$_4$ is tilted, the two output beams from the Sagnac interferometer travel at equal angles on opposite sides of the optic axis. The beams separate as they travel toward the LCSLM and are laterally sheared at the read-side face. The shear, combined with aberrations introduced by the lenses and mirrors in the Sagnac system, gives rise to a spatially varying phase difference between the two beams that distorts the fringes produced at the CCD camera. Further, the diffracted beams from the LCSLM also pass through slightly different areas of Fourier-transform lens L$_5$, and are therefore affected slightly differently by aberrations of this lens. This outcome gives rise to additional distortion of the output fringes.

5128  APPLIED OPTICS / Vol. 39, No. 28 / 1 October 2000
Interference fringes obtained at the CCD camera with the following phase-amplification factors: (a) 0 (0th orders); (b) 2 (±1st orders); (c) 6 (±3rd orders); (d) 8 (±4th orders).

B. Flatness Errors in the Mirrors of the Sagnac Interferometer

Whereas flatness errors in the Sagnac mirrors cause phase errors in the output interference pattern because of the presence of unwanted shear in the Sagnac output optical system, they can also directly introduce phase errors. Note that, although counterpropagating rays in the Sagnac interferometer traverse the same areas of the beamsplitter, they are reflected from areas on opposite sides of each mirror. Mirror surface errors can therefore cause phase errors.

C. Shear (the Effect of Which Is Amplified) at Imaging Lens L2 at the Michelson Output

The two output beams from the Michelson interferometer travel at different angles to the optic axis (they must be tilted with respect to each other to produce the carrier fringes that are necessary for this phase-amplification scheme). They are therefore laterally sheared with respect to each other at lens L2 and are affected differently by the lens aberrations, giving greater nonlinearity.
causing them to have a phase difference that is amplified by the phase-amplification factor to produce a phase error on the phase-amplified output fringes.

At first sight it may seem that the shear at this lens is small and that therefore this effect should be negligible. For example, the separation of 8th and 1st orders in the focal plane of \( L_1 \) (750-mm focal length) was 4 mm, from which the angle between the interfering beams from the Michelson interferometer at the LCSLM may be calculated to be 0.0052 rad. The distance from the Michelson mirrors to lens \( L_2 \) was 240 mm, so the shear at \( L_2 \) was only 1.28 mm. However, the phase difference that arises from this shear will be amplified in the same way as the phase difference that come from the Michelson interferometer itself and so may have a significant effect on the output fringes from the system. For example, for a magnification of 10X (±5th-order interference), the effective shear in the plane of this lens would be very roughly 12 mm, which is rather large.

D. Flatness Errors in the Michelson Mirrors

Phase differences that arise from flatness errors in the Michelson mirrors will be amplified in exactly the same way as for the phase distribution from the test object. It is worth noting that vibration and air turbulence effects in the Michelson mirrors will also be amplified. We found that the LCSLM had a fairly slow response time (~300 ms), so rapid changes in the Michelson mirrors caused the phase patterns that were written on the LCSLM to become smeared out, and the diffracted orders at the output disappeared temporarily until the interferometer had settled down again. The whole system was therefore mounted upon an antivibration table, and the usual precautions were taken to avoid undue air movement.

5. Conclusions

We have demonstrated near-real-time phase-difference amplification with a hologram generated by a Michelson interferometer that contains the test object and is written on a phase-only optically addressed spatial light modulator. The modulator was read out with two beams derived from an arrangement similar to that of a Sagnac interferometer, and we used Fourier filtering techniques to select the required interfering orders from the hologram and determine the phase-amplification factor. With this method we were able to demonstrate phase-difference amplification of as much as 10X.

Note that aberrations in the test interferometer are magnified in the same way as the phase differences introduced by the test object. As the amplification factor increases, it is necessary to ensure that lens aberrations, vibration, and air turbulence are minimized. Nonetheless, we believe that this is a useful technique that has the potential to provide significant improvements in accuracy and resolution in manufacturing-inspection situations in which the time taken to make measurements with a conventional phase-stepping or heterodyne interferometer cannot be justified.

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References

Appendix E

Phase-difference amplification in near-real-time phase-stepping interferometers

Geoffrey T. Bold, Kiyofumi Matsuda, and Thomas H. Barnes

We present results from two interferometer systems incorporating phase amplification and phase stepping that operate in near real time. Each system contains two interferometers. The first interferometer projects an interferogram of the test object onto the wide side of an optically addressed phase-only liquid-crystal spatial light modulator (LCSLM). The read side of the LCSLM is illuminated by two beams from the second interferometer that are adjusted so that their +n- and -n-order beams are diffracted back along the optic axis. These produce an output interferogram that is phase amplified by a factor 2π. This phase distribution is retrieved by phase stepping. © 2001 Optical Society of America

OCIS codes: 120.3180, 120.6660, 120.5050, 120.2830, 120.2080

1. Introduction

Phase-difference amplification uses higher-order beams diffracted from a nonlinear diffraction grating that is an interferogram of the test object, to improve accuracy and resolution in phase-measurement interferometers. The method was first proposed by Bryngdahl and Lohmann,1 andMatsumoto and Takashima2 further developed the technique to achieve a high degree of magnification. A variation of the technique, longitudinally reversed shearing interferometry, was later suggested by Bryngdahl3 and improved on by Matsuda et al.4

In the experiments above, a photographic plate was used to record intensity modulation holograms. However, photographic plate processing is slow and inconvenient. This problem was overcome by Bold and Barnes5 by means of recording simulated holograms on a liquid-crystal TV with a CCD camera. This technique was further developed by Matsuda et al.6 by means of writing the hologram directly to an optically addressed liquid-crystal spatial light modulator (LCSLM). With this phase-modulation method the intensity distribution hologram was converted to a phase distribution by the LCSLM. Therefore, even though the intensity distribution was sinusoidal, higher-order diffracted beams were obtained. This enabled high phase-amplification factors in real time.

In this paper, phase stepping,7 which operates in near real time, is introduced into the phase-modulation method described above. Incorporating phase stepping allows accurate phase distributions of the test object to be obtained. First, the phase-difference amplification method is briefly discussed; then the two interferometric systems are described.

The two systems are the Michelson–Sagnac system (test object used in a Michelson interferometer, phase amplification achieved with a Sagnac interferometer) and the Michelson–Michelson system (test object used in a Michelson interferometer, phase amplification achieved with a second Michelson interferometer). Both systems are simple to use and operate in near real time. Results and a comparison of the two systems are then presented.

2. Phase-Difference Amplification

The principle and the theory of phase-difference amplification have been described in previous studies,8 so here we give only a brief review. There are two parts to the system: a test interferometer, which produces an interferogram of the test object, and the amplification interferometer, which amplifies the phase of the interferogram. One of these interferometers must also be capable of phase stepping. The phase-amplification mechanism is shown in Fig. 1 with a transmission grating for simplicity.
practice, the interferogram of the test object is imaged onto the write face of the optically addressed LCSLM, which is a reflective device. A tilt component must be introduced between the two beams in the test interferometer to give a large number of carrier fringes in the interference pattern. The LCSLM converts the fringe pattern intensity distribution to a phase distribution on the read side, which acts as a phase-diffraction grating. This is the grating shown in Fig. 1. The amplifying interferometer provides two read beams incident on the read side of the LCSLM at different angles. The beam angles can be arranged so that the +n and the −n orders are diffracted parallel to each other, along the optic axis. It has been shown that the phase distribution of the interferogram formed by these two beams, together with judicious spatial filtering, is equal to the phase distribution of the original interferogram amplified by a factor of 2n. Phase-stepping techniques on either the write or the read side can be used to reconstruct the phase distribution of the test object.

3. Experiments and Discussion

A. Michelson-Sagnac System

1. Optical Setup

Figure 2 shows the first setup, which has a Michelson interferometer on the write side of the LCSLM and a Sagnac interferometer on the read side. The Michelson interferometer consists of a beam splitter BS1 and mirrors M1 and M2. The beam from Laser1 passes through a microscope objective and spatial filter and is collimated by L1 before entering the Sagnac interferometer at BS2. The two interferometer beams travel in opposite directions around the Sagnac passing through BS2 and on to the read side of the LCSLM. The incident angle of each beam is controlled by means of tilting M4 so that the +n order of one beam and the −n order of the other are diffracted back from the LCSLM in the same direction, parallel to the optic axis. Fourier transform lens L4 and the pinhole select only the desired orders. The interferogram is imaged onto the CCD camera and recorded by the computer.

The spacing of the diffracted orders in the Fourier plane of L4 can be varied by means of adjusting the tilt of M4 in the test interferometer. When M4 is tilted, the angle of both beams incident on the read side of the LCSLM changes by the same amount but in opposite directions. The position of the pinhole at the output does not therefore need more than a fine readjustment for each measurement with amplification factors greater than zero.

The zero-order diffracted beams contain no phase information about the test phase object, since they are plane reflected waves. To get an interferogram whose phase distribution has an amplification factor of 1, it is therefore necessary to interfere the ±1 order of one beam with the zero order of the other. This is achieved either by rotation of BS1 and mirror M4 or by rotation of only M4 and movement of the pinhole sideways so that these orders are selected by the
A pinhole. The contrast of the output interferogram is reduced in this case, because the intensity of the different diffracted orders from the LCSSL will in general be different. However, the phase-stepping techniques easily accommodate reductions in contrast of this order.

2. Phase Stepping

A frame from the CCD camera was grabbed by the computer each time M1 in the Michelson interferometer was moved. A simple four-step, phase-step algorithm was used to reconstruct the phase difference of the test phase object. The phase steps of M1 in this setup (where phase stepping occurs before amplification) depend on the diffracted orders selected and therefore the phase amplification. All phases in the test interferometer are amplified, so with an amplification factor of \( 2n \) the phase step must be set at \( \lambda / 8n \) where \( \lambda \) is the wavelength of Laser 1.

A Thorlabs piezodriven mirror and driver were used to provide the phase stepping. The mirror mount was controlled by a low-voltage piezostack in each of three corners. The mirror therefore has tip, tilt, and piston movement. In these experiments only piston movement was required, so the same voltage was applied across each piezostack. The effects of piezodrift and hysteresis were noticeable, and we minimized them by always starting the phase-step procedure from the same voltage, stepping in the same direction each time, and carefully selecting the voltage steps. We obtained results by taking measurements with and without the phase object in the system, then subtracting the two phase distributions. The measurement without the phase object measures the background system aberrations, which must be removed from the final phase map.

3. Operation Time for Michelson–Sagnac System

The settling time of the liquid crystals in the LCSSL is approximately 0.1 s, and the frame time of the CCD camera is 0.04 s. For this system in which phase stepping occurs on the write side of the LCSSL it is necessary to wait for the LCSSL to settle before each frame is captured. In practice it is also necessary to allow 0.02 s for the phase-stepping mirror and piezoactuators to settle. The total time required for each frame is therefore 0.16 s. If a four-step phase-stepping algorithm is used, and if we allow 0.04 s for processing, the total time for one measurement is approximately 0.68 s.

4. Results

Phase-difference amplification results using the Michelson–Sagnac system are shown in Fig. 3. The Sagnac interferometer on the read side of the LCSSL was adjusted to give amplification factors of 1, 2, 4, and 6, shown in Figs. 3(a), 3(b), 3(c), and 3(d), respectively. It can be seen that the size of the steps in the measured phase distributions increases proportionally with amplification factor.
An estimate of the background system and measurement noise was obtained by subtraction of two phase-difference measurements of the background aberrations without the phase object. The total peak-to-peak range in wavelengths $\Delta \lambda_o$ and the standard deviation $\sigma_o$ of the background noise were then calculated.

The height of the phase object steps in these experiments was 30 nm. The phase object was made of silicon dioxide ($\text{SiO}_2$), which has a refractive index of $n = 1.48$. This corresponds to a theoretical optical path-step height of 44.4 nm. Since there is a double pass through the phase object in the Michelson test interferometer, the phase step measured by the system with an amplification factor of 1 should therefore be 90 nm or 0.14A at a test laser wavelength of 633 nm. We can see that this corresponds well with the step size in Fig. 3. With amplification factors 2, 4, and 6, the theoretical step sizes are 0.28A, 0.57A, and 0.85A, respectively. Again these compare well with the corresponding figures. It can be seen that the dominant noise in Fig. 3(a) is a series of ripples running in the direction of the carrier fringes, perpendicular to the steps of the phase object. The ripples are caused by small errors in the phase steps that occur because of piezodrift and hysteresis, table and building vibrations, and air movement within the lab. It is interesting to note that the size of the ripples increases with the amplification factor. This is expected, because the phase amplification occurs after phase stepping, and so any errors in the phase steps are also magnified. A summary of the errors is shown in Table 1.

### Table 1. Result Summary for Michelson-Sagnac Phase-Difference Amplification Setup

<table>
<thead>
<tr>
<th>Amplification Factor</th>
<th>Theoretical Step Height (A)</th>
<th>Measured Step Height (A)</th>
<th>Background Noise Range $\Delta \lambda_o$ (nm)</th>
<th>Background Noise Std. Dev. $\sigma_o$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.15</td>
<td>0.038</td>
<td>0.0043</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.30</td>
<td>0.16</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.57</td>
<td>0.55</td>
<td>0.17</td>
<td>0.025</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>0.82</td>
<td>0.30</td>
<td>0.055</td>
</tr>
</tbody>
</table>

2. **Operation Time for Michelson–Michelson System**

With this system it is necessary to wait 0.1 s in the LCSLM to settle only once, prior to phase stepping. Using a four-step algorithm in which each camera frame takes 0.04 s, allowing 0.02 s for the mirror and piezoactuators to settle at each step, and allowing 0.04 sec. processing time, we obtain a total time for one measurement of 0.38 s.

3. **Results**

Results using the Michelson–Michelson system are shown in Figs. 5(a), 5(b), 5(c), and 5(d) with the Michelson interferometer at the LCSLM output adjusted to give amplification factors of 1, 2, 4, and 6, respectively.

Again, the results were obtained by subtraction of two phase distributions, one with and one without the phase object. It can be seen that the size of the steps in the measured phase distributions increases proportionally with amplification factor. In addition to this, the noise and phase-step-error ripples are

### B. Michelson–Michelson System

1. **Optical System**

Figure 4 shows the second setup, which uses a Michelson interferometer to provide the two beams on the read side of the LCSLM instead of a Sagnac. This Michelson interferometer consists of beam splitter $BS_2$ and mirrors $M_4$ and $M_4$. Although the common-path Sagnac interferometer provides more stability and the single mirror adjustment makes it convenient to select different diffracted orders, it is difficult to phase step. Using another Michelson interferometer instead of the Sagnac has the advantage that it is possible to phase step the system after the phase amplification has taken place. The phase-step size therefore remains constant for any amplification factor, and the phase step errors are not amplified by the system.

Fig. 4. Phase-difference amplifier setup using Michelson–Michelson configuration.
significantly reduced compared with results obtained at the same amplifications with use of the first setup. The results are given in Table 2.

C. Comparison

The first system using the Michelson test interferometer and the Sagnac amplifying interferometer is simpler to adjust for different amplifications. However, the second Michelson–Michelson setup has far less noise. The first system requires that the phase stepping be done before the amplification, because of the difficulty in phase stepping a Sagnac interferometer, which means that the phase-step errors are also amplified. This is reduced significantly in the second system, where the phase stepping takes place after the amplification.

The total time taken for one measurement using each system shows that the Michelson–Michelson system, which takes 0.39 s, operates almost twice as fast as the Michelson–Sagnac system, which takes 0.68 s. However, compared with previous methods, which took at least 30 mins per measurement owing to the photographic processing required, they can both be said to be operating in near real time.

4. Conclusions

In this paper we have presented two systems in which amplified phase maps of a real object are obtained with near-real-time phase-stepping interferometers and an optically addressed LCSLM. The interferogram from a test interferometer is imaged onto the LCSLM and used as a diffraction grating. A second interferometer provides beams that are incident on the grating at equal angles but on opposite sides of the optic axis. The ±n diffracted orders are selected by use of a spatial filter and interfere to produce an interferogram whose phase is equal to the phase of the original interferogram amplified by 2n. Phase-stepping techniques are used to retrieve a phase-difference map of the test object. The results show that amplification

<table>
<thead>
<tr>
<th>Amplification Factor</th>
<th>Theoretical Step Height ((\lambda))</th>
<th>Measured Step Height ((\lambda))</th>
<th>Background Noise (\Delta\lambda_{\text{noise}} (\lambda))</th>
<th>Background Noise Std. Dev. (\sigma (\lambda))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.14</td>
<td>0.031</td>
<td>0.0038</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.24</td>
<td>0.037</td>
<td>0.0057</td>
</tr>
<tr>
<td>4</td>
<td>0.57</td>
<td>0.58</td>
<td>0.14</td>
<td>0.015</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>0.84</td>
<td>0.066</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

Fig. 5. Phase distribution with Michelson–Michelson configuration, amplification factors (a) 1, (b) 2, (c) 4, and (d) 6.
factors up to 6 are easily achievable. They also show that noise is significantly reduced if the phase stepping is carried out in the readout interferometer.

We believe that these techniques will be useful in a variety of optical testing applications in which the required resolution is better than \( \frac{\lambda}{30} \). For applications in which phase-stepping noise is not an issue, and for which ease of adjustment is desired, the Michelson–Sagnac system is more suitable because of the inherent stability of the Sagnac interferometer and the ease with which it is adjusted for different amplification factors. However, for systems in which most noise comes from the phase stepping, the Michelson–Michelson configuration is more suitable, since the phase-step errors are not amplified by the system.

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References
References


