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Nonlinear Rotational Behaviour of Shallow Foundations on Cohesive Soil

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Supervised by Professor Michael J. Pender

The University of Auckland
Department of Civil and Environmental Engineering
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The most recent version of the New Zealand design and loadings standard eliminated a clause for the design of rocking foundations. This thesis addresses that clause by presenting a strong argument for rocking shallow foundations in earthquake resistant design. The goals of the research were to perform large scale field experiments on rocking foundations, develop numerical models validated from those experiments, and produce a design guide for rocking shallow foundations on cohesive soil. Ultimately, this thesis investigates the nonlinear rotational behaviour of shallow foundations on cohesive soil.

Field experiments were performed on an Auckland residual soil, predominantly clay. The experiment structure – a large scale steel frame – was excited first by an eccentric mass shaker and second by a quick release (snap-back) method. The results show that rocking foundations produce highly nonlinear moment-rotation behaviour and a well defined moment capacity. A hyperbolic equation is proposed in Chapter 4 utilising the initial stiffness and moment capacity to predict nonlinear pushover response. The results show that the initial stiffness should be based on an ‘operational soil modulus’ rather than a small strain soil modulus. Therefore, the reduction factor from the small strain modulus was around 0.6 for the experiment testing. Additionally, the experiments showed that rocking foundations demonstrate significant damping; snap-back experiments revealed an average damping ratio of around 30%.

Experiment data validated two numerical models developed for this study: one, a finite element model in Abaqus and the other, a spring bed model in OpenSEES. The models showed that both forms of nonlinearity in rotating shallow foundations – geometric nonlinearity and material nonlinearity – should be considered in shallow foundation
Abstract

analysis. These models also confirmed the need for an ‘operational soil modulus’ on shallow foundation rocking, and analysis of varying vertical loads suggested that this reduction factor is dependent on the vertical factor of safety of the foundation.

Lastly, two design methods are presented, a displacement-based method and a force-based method, and two examples of rocking shear walls are given. The displacement-based method is the recommended option, and it is shown that design displacements and rotations compare well to time history analyses performed using the validated OpenSEES model.
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NOTATION

**Roman**

\( A_b \)  
base area of a footing

\( A_c \)  
critical contact area

\( Axial_{North} \)  
axial force on the north column recorded in experiments

\( A_s \)  
side wall area of a footing

\( Axial_{South} \)  
axial force on the south column recorded in experiments

\( Axial_{Strut} \)  
axial force on the diagonal strut recorded in experiments

\( a \)  
area ratio of CPT probe (Chapter 3)

\( a \)  
parameter of moment-rotation equation relating to initial stiffness (Chapter 4 and 6)

\( a_{fnd} \)  
foundation radius

\( a_g \)  
horizontal ground acceleration beneath the foundation

\( a_o \)  
dimensionless parameter to determine dynamic stiffness and damping values

\( B \)  
foundation width

\( B' \)  
effective foundation width

\( b \)  
parameter of moment-rotation equation relating to moment capacity (Chapter 4 and 6)

\( b \)  
rate the yield surface changes as plastic straining develops (Chapter 5)

\( C \)  
damping of a single degree of freedom system (Chapter 2)

\( C \)  
initial kinematic hardening modulus (Chapter 5)

\( C_d \)  
ratio of maximum drag to ultimate resistance for qz spring calculations

\( C_d(T_1) \)  
horizontal design action coefficient

\( C_d(T) \)  
spectral shape factor

\( C_r \)  
parameter controlling the range of the linear elastic region for qz spring calculations
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<td>base shear coefficient</td>
</tr>
<tr>
<td>$k_{central}$</td>
<td>stiffness of the central region for a shallow foundation in FEMA 356</td>
</tr>
<tr>
<td>$k_{end}$</td>
<td>stiffness of the end region for a shallow foundation in FEMA 356</td>
</tr>
<tr>
<td>$k_\mu$</td>
<td>inelastic spectrum scaling factor</td>
</tr>
<tr>
<td>$L$</td>
<td>foundation length</td>
</tr>
<tr>
<td>$L'$</td>
<td>effective foundation length</td>
</tr>
<tr>
<td>$L_c$</td>
<td>critical foundation length</td>
</tr>
<tr>
<td>$LI$</td>
<td>liquidity index</td>
</tr>
<tr>
<td>$LL$</td>
<td>liquid limit</td>
</tr>
<tr>
<td>$LVDT_{North}$</td>
<td>vertical measurement on the north LVDT during experiments</td>
</tr>
<tr>
<td>$LVDT_{South}$</td>
<td>vertical measurement on the south LVDT during experiments</td>
</tr>
<tr>
<td>$L_{wall}$</td>
<td>wall length</td>
</tr>
<tr>
<td>$M$</td>
<td>general moment on a foundation</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>dimensionless moment load parameter</td>
</tr>
<tr>
<td>$M^*$</td>
<td>maximum moment exerted on a foundation</td>
</tr>
<tr>
<td>$M_{fnd}$</td>
<td>experimental moment recorded on the foundation</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>ultimate moment capacity of a foundation</td>
</tr>
<tr>
<td>$Mom_{North}$</td>
<td>Moment recorded on the north column during experiments</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>Moment recorded on the south column during experiments</td>
<td></td>
</tr>
<tr>
<td>$M_s$</td>
<td>moment on a foundation in the width direction</td>
</tr>
<tr>
<td>$M_y$</td>
<td>moment on a foundation in the length direction</td>
</tr>
<tr>
<td>$m$</td>
<td>mass of a single degree of freedom system</td>
</tr>
<tr>
<td>$m_{wall}$</td>
<td>wall mass</td>
</tr>
<tr>
<td>$m_i$</td>
<td>mass at each level of a MDOF system</td>
</tr>
<tr>
<td>$N$</td>
<td>vertical load on a foundation</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>dimensionless vertical load parameter</td>
</tr>
<tr>
<td>$N_c$</td>
<td>bearing capacity factor</td>
</tr>
<tr>
<td>$N_{fnd}$</td>
<td>vertical load of a foundation</td>
</tr>
<tr>
<td>$N_k$</td>
<td>plasticity factor in CPT interpretation</td>
</tr>
<tr>
<td>$N_q$</td>
<td>bearing capacity factor</td>
</tr>
<tr>
<td>$N(T,D)$</td>
<td>near fault factor</td>
</tr>
<tr>
<td>$N_u$</td>
<td>ultimate vertical load that may be applied to the foundation in the absence of shear and moment loading</td>
</tr>
<tr>
<td>$N_{wall}$</td>
<td>vertical load of a wall</td>
</tr>
<tr>
<td>$N_q$</td>
<td>bearing capacity factor</td>
</tr>
<tr>
<td>$n$</td>
<td>correction factor for soil type index in CPT interpretation (Chapter 3)</td>
</tr>
<tr>
<td>$n$</td>
<td>rocking foundation impact number (Chapter 4)</td>
</tr>
<tr>
<td>$n$</td>
<td>parameter controlling the post yield shape in qz spring calculations (Chapter 5)</td>
</tr>
<tr>
<td>$P_a$</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>$P_{act}$</td>
<td>active earth pressure</td>
</tr>
<tr>
<td>$PL$</td>
<td>plastic limit</td>
</tr>
<tr>
<td>$P_o$</td>
<td>at rest earth pressure</td>
</tr>
<tr>
<td>$P_p$</td>
<td>passive earth pressure</td>
</tr>
<tr>
<td>$p$</td>
<td>parameter used in Housner’s equations of motion (Chapter 2)</td>
</tr>
<tr>
<td>$p$</td>
<td>normal stress (Chapter 3)</td>
</tr>
<tr>
<td>$p(t)$</td>
<td>the forcing function for a single degree of freedom system</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>function of original signal in SASW testing</td>
</tr>
<tr>
<td>$Q$</td>
<td>shear stress for hyperbolic stress strain behaviour</td>
</tr>
<tr>
<td>$Q_{n}$</td>
<td>normalised cone penetration resistance for CPT testing</td>
</tr>
<tr>
<td>$Q_u$</td>
<td>ultimate load capacity for a foundation</td>
</tr>
</tbody>
</table>
### Notation

- $Q_{y,\text{spring}}$: ultimate load for the springs within the OpenSEES spring bed models
- $Q_\infty$: maximum change in size of the yield surface
- $q$: surcharge pressure exerted on a foundation (Chapter 2)
- $q$: deviatoric stress (Chapter 3)
- $q$: load in qz spring calculations (Chapter 5)
- $q_c$: measured cone resistance in CPT testing
- $q^c$: closure part of the qz spring elements
- $q^d$: drag part of the qz spring elements
- $q_t$: corrected cone resistance for CPT testing
- $q_u$: gross ultimate bearing pressure
- $q_{ult}$: ultimate load for qz spring calculations
- $q_0$: load at yield point in qz spring calculations
- $q_0^d$: initial drag force of the current cycle for qz spring calculations
- $R$: distance to the centre of mass in Housner’s rocking block (Chapter 2)
- $R$: force reduction factor in the Canadian code (Chapter 6)
- $R$: return period (Chapter 6)
- $r$: energy reduction factor (Chapter 2)
- $r$: plate radius for WAK testing (Chapter 3)
- $S_p$: structural performance factor
- $s_u$: undrained shear strength of soil
- $T$: period of a system
- $T_e$: system effective period
- $t$: time
- $t(f)$: travel time between SASW receivers
- $t_{\text{wall}}$: wall thickness
- $u$: displacement of a single degree of freedom system
- $u_g$: ground displacement
- $\ddot{u}_s$: equivalent system ground displacement
- $u2$: measured pore water pressure
- $\dot{u}$: velocity of a single degree of freedom system
- $\ddot{u}$: acceleration of a single degree of freedom system
- $V$: horizontal shear on a foundation
- $\ddot{V}$: dimensionless horizontal shear parameter


**Notation**

- **$V_b$**: base shear exerted on a foundation
- **$V_R$**: Rayleigh (or wave) velocity
- **$V_s$**: shear wave velocity of the soil
- **$v$**: generalised velocity of shear waves
- **$W$**: vertical load for Housner's rocking block (Chapter 2)
- **$W_i$**: seismic weight of a structure
- **$W_{wall}$**: wall weight
- **$w_n$**: natural water content of soil
- **$X$**: initial amplitude of generalised single degree of freedom oscillating system
- **$X(f)$**: converted frequency domain signal in SASW testing
- **$x$**: generalised amplitude for logarithmic decrement calculations
- **$x(t)$**: input signal from accelerometer or geophone in SASW testing
- **$x_{wall}$**: height to the centre of mass of a wall
- **$Y(f)$**: converted frequency domain signal in SASW testing
- **$y(t)$**: input signal from accelerometer or geophone in SASW testing
- **$Z$**: hazard factor
- **$z$**: vertical displacement of OpenSEES springs
- **$z^g$**: displacement of the gap spring for qz spring calculations
- **$z^p$**: plastic deformations for qz spring calculations
- **$z_{0g}$**: initial displacement of the gap spring on the current cycle for qz spring calculations
- **$z_{0p}$**: displacement at the yield point for qz spring calculations
- **$z_{50}$**: displacement where 50% of the ultimate load is mobilised for qz spring calculations

**Greek**

- **$\alpha$**: tipping angle of a rocking structure (Chapter 2)
- **$\alpha$**: soil backstress (Chapter 5)
- **$\alpha_s$**: backstress at saturation
- **$\dot{\alpha}$**: backstress rate of change
- **$\gamma$**: unit weight of the soil
- **$\gamma$**: parameter that defines the rate the kinematic hardening decreases with increasing plastic strain (Chapter 5)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{\text{concrete}}$</td>
<td>unit weight of concrete</td>
</tr>
<tr>
<td>$\gamma^2$</td>
<td>coherence function of signal</td>
</tr>
<tr>
<td>$\Delta_d$</td>
<td>design displacement</td>
</tr>
<tr>
<td>$\Delta_f$</td>
<td>foundation displacement for design</td>
</tr>
<tr>
<td>$\Delta_H$</td>
<td>foundation horizontal displacement for design</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>structure displacement profile</td>
</tr>
<tr>
<td>$\Delta_{\text{relative}}$</td>
<td>relative displacement in relation to the structures initial position</td>
</tr>
<tr>
<td>$\Delta_s$</td>
<td>structural displacement for design</td>
</tr>
<tr>
<td>$\Delta_{\text{static}}$</td>
<td>equivalent static deflection</td>
</tr>
<tr>
<td>$\Delta_{\text{total}}$</td>
<td>total system static deflection under the force based design</td>
</tr>
<tr>
<td>$\delta$</td>
<td>logarithmic decrement</td>
</tr>
<tr>
<td>$\delta(T)$</td>
<td>site hazard spectral displacement</td>
</tr>
<tr>
<td>$\delta_v$</td>
<td>vertical settlement of foundations during experiments</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>axial strain</td>
</tr>
<tr>
<td>$\epsilon_s$</td>
<td>shear strain</td>
</tr>
<tr>
<td>$\dot{\epsilon}^{pl}$</td>
<td>plastic flow rate within soil</td>
</tr>
<tr>
<td>$\dot{\epsilon}^{pl}$</td>
<td>equivalent plastic strain rate within soil</td>
</tr>
<tr>
<td>$\theta$</td>
<td>generalised form of rotation</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>design rotation of a foundation</td>
</tr>
<tr>
<td>$\theta_n$</td>
<td>normalised angle of rotation</td>
</tr>
<tr>
<td>$\theta_{n0}$</td>
<td>initial normalised angle of rotation</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>initial rocking block rotation</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>rotational velocity</td>
</tr>
<tr>
<td>$\lambda_{cd}$</td>
<td>depth adjustment factor for cohesive resistance of soil</td>
</tr>
<tr>
<td>$\lambda_{ci}$</td>
<td>inclination adjustment factor for cohesive resistance of soil</td>
</tr>
<tr>
<td>$\lambda_{cs}$</td>
<td>shape adjustment factor for cohesive resistance of soil</td>
</tr>
<tr>
<td>$\lambda_{qd}$</td>
<td>depth adjustment factor for surcharge</td>
</tr>
<tr>
<td>$\lambda_{qi}$</td>
<td>inclination adjustment factor for surcharge</td>
</tr>
<tr>
<td>$\lambda_{qs}$</td>
<td>shape adjustment factor for surcharge</td>
</tr>
<tr>
<td>$\lambda_{gd}$</td>
<td>depth adjustment factor for frictional resistance of soil</td>
</tr>
<tr>
<td>$\lambda_{gi}$</td>
<td>inclination adjustment factor for frictional resistance of soil</td>
</tr>
<tr>
<td>$\lambda_{gs}$</td>
<td>shape adjustment factor for frictional resistance of soil</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\mu & \quad \text{ductility} \\
\mu_f & \quad \text{coefficient of friction} \\
\nu & \quad \text{poisons ratio of soil} \\
\zeta & \quad \text{damping ratio} \\
\zeta_f & \quad \text{total foundation damping ratio} \\
\zeta_H & \quad \text{damping ratio of the foundation horizontal response} \\
\zeta_g & \quad \text{equivalent damping ratio of the soil (material damping)} \\
\zeta_s & \quad \text{damping ratio of the structural response} \\
\zeta_{sys} & \quad \text{system damping ratio} \\
\zeta_{\varphi} & \quad \text{damping ratio of the foundation rotational response} \\
\rho & \quad \text{density of soil} \\
\sigma & \quad \text{soil normal stress} \\
\phi & \quad \text{phase of the cross spectrum in SASW testing} \\
\phi_s & \quad \text{soil friction angle} \\
\sigma_{vo} & \quad \text{total vertical stress} \\
\sigma_y & \quad \text{ultimate yield stress} \\
\sigma_0 & \quad \text{stress of point of first yield} \\
\sigma_{y0} & \quad \text{yield stress at zero plastic strain} \\
\omega & \quad \text{generalised frequency} \\
\omega_h & \quad \text{foundation horizontal natural frequency} \\
\omega_n & \quad \text{natural frequency of a system} \\
\omega_s & \quad \text{structural natural frequency} \\
\omega_{sys} & \quad \text{system natural frequency} \\
\omega_{\varphi} & \quad \text{foundation rotational natural frequency}
\end{align*}
\]
Notation
INTRODUCTION

The prevailing thought on shallow foundations is to design them strong enough so yielding and nonlinear behaviour occurs in structural components and the foundation remains elastic. This is a result of an endemic communication problem in design offices between structural and geotechnical engineers. Often a structural engineer will provide axial, shear, and moment loads to a geotechnical engineer who will then perform a foundation design with the aim of catering to those loads. Separating a structural system as such creates inefficiency in foundation design. Therefore, a more holistic approach should be taken.

Often, the issue with shallow foundations is not enough vertical load to preclude some uplift and rocking during an earthquake. Thus, if a geotechnical engineer is only provided with loads from the structure above, then foundations will, more often than not, be over-designed. An over-designed foundation will not only cost more in construction, but will make the structure stiffer and susceptible to greater seismic loading.
Shallow foundations have the potential to reduce demands on a structure during an earthquake through rocking. The rocking increases the period of a structure and increases damping of the system significantly. Figure 1.1 presents the concept of a rocking shear wall. The wall on the left is under static conditions with vertical loading only. The wall on the right is a snapshot of how a rocking foundation would rotate during the rocking motion.

A problem with rocking foundations, and the reason why they are not widely used at the moment, is that loss of contact between foundation and soil results in reduced stiffness and bearing capacity. This plus the material yielding of the soil during rocking means the problem becomes ‘double nonlinear’. Double nonlinearity is when two coupled forms of nonlinearity occur; in this case there is geometric nonlinearity, from uplift and loss of contact, and material nonlinearity, from yielding within soil caused by rocking.

This thesis presents large scale field experiments, as well as numerical models of these experiments and design guidelines for shallow foundations that rock during earthquake excitation. The experiments were performed on a site in the Pinehill subdivision of Albany on Auckland’s North Shore, and were excited first by forced-vibration, and second by snap-back tests to induce rocking. The numerical models were developed in software programs Abaqus and OpenSEES respectively. The design guidelines were

*Figure 1.1 A shear wall and shallow foundation sitting statically (left) and rocking (right)*
developed based on a displacement-based philosophy and a force-based philosophy, and design examples are given.

1.1 RESEARCH MOTIVATION

In the past, Engineers in New Zealand had scope to specifying rocking foundations in design. The New Zealand design and loadings code from 1992, NZS 4203: 1992, stated foundations could be designed for the reduced forces associated with a ductility of 2. However, this scope is removed in the newer version, NZS 1170.5, stating that any designs of rocking foundations had to be completed by a special study.

The research motivation is to provide insight into rocking foundation behaviour to address the clause in the new standard, mentioned above. Industry professionals suggested that quality experimental data be obtained through testing of realistic structures sitting on actual ground, coupled with numerical models developed and validated by them. The aim was to take these two sets of information, experimental and numerical, and develop a design guide for rocking foundations.

Additionally, many researchers say there is a lack of experimental data for rocking foundations, which needs to be addressed. Thus, there is a need not only to contribute to the wider research community, by providing a good set of experimental data as well as validated numerical models, but also to contribute to the engineering community in New Zealand through design methodologies for rocking shallow foundations.

The research was also undertaken in order to develop a set of numerical models validated by experimental data. These models can be used for design purposes, and will ultimately help determine if rocking foundations can be introduced back into the New Zealand design standard.

1.2 THESIS OUTLINE

This thesis covers three main topics in the seismic response of shallow foundations, and each topic is covered in one or two chapters:
1. Experiments
2. Numerical modelling
3. Design

Chapter 2 is a broad literature review, chapters 3 and 4 cover the experimental configuration and results respectively, chapter 5 covers the development of the two numerical models, chapter 6 includes rocking foundation design guidelines, and chapter 7 is a summary of all the discussions and conclusions made throughout this research. There are six appendices titled A though F that cover plots from testing, calculation sheets, and model codes.

The literature review goes into detail about general shallow foundation characteristics such as bearing capacity and stiffness. It covers other topics such as soil-structure-interaction as well as a brief history of rocking foundations. Additionally, each chapter contains another literature review related directly to the chapter’s subject. These are more specific reviews included at the beginning of each chapter rather than in a separate section.

Chapter 3, titled Experimental Configuration, describes the experiment concept, design, construction, and test procedures. The two types of experiments conducted were dynamic forced-vibrations tests and dynamic snap-back tests. Prior to the experiment execution, several types of geophysical and geotechnical tests were performed to understand the soil properties. The soil was classed as an Auckland residual soil, mainly Waitemata Clay. This chapter is supplemented with a small literature review on past research into Auckland residual soil properties. It correlates past properties of Auckland residual soil to data gathered through the geotechnical and geophysical testing.

Chapter 4 presents the key findings and results from the experiments. It follows on from Chapter 3 with the experimental section of this thesis. The chapter starts with a literature review on past experimental works undertaken on rocking foundations with research at the University of Auckland by Bartlett, Wiessing, and Taylor. The review details the comprehensive centrifuge modelling both on shear walls and bridge footings at the University of California, Davis. The chapter goes on to describe the result of both the forced-vibration and snap-back tests, including an equation modelling the pushover
response recorded during the snap-backs. The layout of results is similar in both series of experiments. The chapter ends with discussion and conclusions obtained throughout the experiments.

Chapter 5 describes the numerical models developed to predict the behaviour of rocking shallow foundations during earthquakes. Numerical modelling comprises the second section of this thesis. Two models were developed: a finite element model developed in Abaqus, and a spring bed model developed in OpenSEES. The Abaqus model was developed in stages from initial elastic conditions to a nonlinear constitutive soil model. The OpenSEES model was similarly developed, initially based on FEMA 356 guidelines for spring bed models. Both models were then compared to experimental data: pushover curves from Abaqus and OpenSEES were calibrated to pushovers during the snap-back testing.

Chapter 6 presents rocking foundation design guidelines that were developed as part of this research. The chapter begins with a look at previous papers describing design guidelines both within New Zealand and internationally. A new guideline is developed using the results from the preceding two chapters on numerical modelling and experimental results. It presents two different design methods: a displacement-based design, and a more traditional, force-based design. Next are examples of a single story shear wall and a six story shear wall. The single story shear wall is designed three times, once with each of the guidelines and the third is a design to preclude rocking. This emphasises the effects rocking foundations have on foundation design. The two examples are then subjected to time history analysis using the OpenSEES model and the results are compared.

Chapter 7 is a summary of main-discussions and conclusions made throughout the thesis. It details them chapter by chapter and indicates further research needed in the area of rocking foundations.

The five appendices in order cover: CPT test data; critical plots from the forced-vibration tests; critical plots from the snap-back tests; the OpenSEES code used to generate the model for the design example; and the time history analyses performed on two rocking shear wall examples.
An extensive literature review on the seismic response of shallow foundations is broken into separate parts with each chapter containing a review relevant to its topic. This chapter contains a broader look at shallow foundations. It begins with shallow foundation stiffness and bearing capacity, giving fundamental ideas and equations that will ultimately be used later on. The Terzaghi bearing capacity equation (Terzaghi 1943) is outlined and displayed as a vertical-shear-moment 3D bearing strength surface. For comparison, a definition of the bearing strength surface from Eurocode 8 (European Committee for Standardization 1998) is included. The elastic response of a simple soil-foundation-structure interaction (SFSI) framework is also outlined. Plastic SFSI is introduced, however this is described in depth in Chapter 5 on the numerical modelling of shallow foundations. The chapter concludes with a history of rocking foundation research detailing the original work and the progression to current research today.
2.1 SHALLOW FOUNDATION STIFFNESS

In ‘Elastic Solutions for Soil and Rock Mechanics’ by Poulos and Davis (1974), a range of solutions are presented for shallow foundation stiffness on an elastic continuum. Equations for horizontal, vertical, and rotational displacements are provided for a variety of different foundation conditions that include:

- Circular and rectangular foundations
- Flexible and rigid foundations
- Embedded foundations
- Homogenous or layered soil deposits

Various modifications were made to the work by Poulos and Davis to account for foundations of any shape and embedment. Gazetas and his co-workers created modified equations for vertical stiffness (Gazetas et al. 1985), horizontal stiffness (Gazetas and Tassoulas 1987), rocking stiffness (Hatzikonstantinou et al. 1989), and torsional stiffness (Dobry and Gazetas 1986). The universal equation for shallow foundation stiffness in any direction is:

\[ K_{\text{embedded}} = K_{\text{basic}} I_{\text{shape}} I_{\text{depth}} I_{\text{sidewall}} \]  

(2.1)

where \( K_{\text{basic}} \) = the stiffness of an infinite strip footing resting on the ground surface. The other three factors account for contributions from the shape of the foundation, depth of embedment, and vertical sides of the foundation respectively.

2.2 BEARING CAPACITY

2.2.1 Terzaghi Bearing Capacity

One of the best known theories on bearing capacity is from ‘Theoretical Soil Mechanics’ by Terzaghi (1943). The theory consists of three components contributing to shallow foundation bearing capacity: a component from the cohesive resistance of soil, a surcharge component from stresses generated at the underside of the footing, and
a component from frictional resistance underneath the foundation. The general equation is:

\[ q_u = cN_c + qN_q + 1/2 \gamma BN \]  \hspace{1cm} (2.2)

where \( q_u \) = gross ultimate bearing pressure; \( c \) = cohesion; \( q \) = surcharge pressure; \( \gamma \) = unit weight of the soil; and \( N_c, N_q, \) and \( N_\gamma \) = bearing capacity factors, all functions of the friction angle \( \phi_s \) of the soil.

Due to an inaccurate definition of the failure of the soil, the bearing capacity factors developed by Terzaghi were not precise, especially at low \( \phi_s \) values. More adequate factors were developed by Prandtl (1921) and Reissner (1924) for \( N_c \) and \( N_q \) and Brinch Hansen (1961) for \( N_\gamma \). The new factors defined in those papers are:

\[ N_q = e^{\sqrt{\tan \phi_s}} \tan \left(45^\circ + \frac{\phi_s}{2}\right) \] \hspace{1cm} (2.3)

\[ N_c = (N_q - 1)cot \phi_s \quad \text{for} \quad \phi_s > 0 \quad N_c = 5.14 \quad \text{for} \quad \phi_s = 0 \] \hspace{1cm} (2.4)

\[ N_\gamma = 2(N_q - 1)tan \phi_s \] \hspace{1cm} (2.5)

Meyerhof (1953) considered the effect of moment on the bearing capacity of shallow foundations by reducing the contact area between soil and footing. The two formulas in Equation 2.6 calculate the eccentricity (\( e \)) in each direction based on moment (\( M \)) and vertical loading (\( N \)) for the foundation given in Figure 2.1.

\[ e_x = \frac{M_x}{N} \quad \text{and} \quad e_y = \frac{M_y}{N} \] \hspace{1cm} (2.6)

The effective dimensions of the footing (\( L' \) and \( B' \)) are:

\[ L' = L - 2e_y \quad \text{and} \quad B' = B - 2e_x \] \hspace{1cm} (2.7)

Subsequently, modification factors (\( \lambda \)) dependent on the shape of the foundation, depth of embedment, and the application of shear and moment were developed. These can
either have beneficial or detrimental effects on bearing capacity. Therefore, the bearing capacity equation is modified to:

$$q_u = c\lambda_{cs}\lambda_{cd}\lambda_{ci}N_c + q\lambda_{qs}\lambda_{qd}\lambda_{qi}N_q + 1/2\gamma B\lambda_{qs}\lambda_{qd}\lambda_{qi}N_y$$  \hspace{1cm} \text{(2.8)}

### 2.2.2 Terzaghi Bearing Strength Surface

A bearing strength surface in moment-axial-shear ($M$-$N$-$V$) space can be derived from the Terzaghi bearing capacity equation given in Equation 2.8. It uses a set of dimensionless parameters: one for vertical load, another for horizontal shear, and a third for moment applied to the foundation. These dimensionless parameters are defined as:

$$\bar{N} = \frac{N}{N_u}, \quad \bar{V} = \frac{V}{N_u}, \quad \bar{M} = \frac{M}{N_uB}$$  \hspace{1cm} \text{(2.9)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Plan view of a shallow foundation showing the effective dimensions $L'$ and $B'$}
\end{figure}
where $B$ = foundation width; $N_u$ = ultimate vertical load that may be applied to the foundation in the absence of shear and moment loading, calculated using conventional bearing capacity equations; and $N, V$ and $M$ are the actions on the foundation.

Equations for the bearing strength surface for clay and sand are given in Equations 2.10 and 2.11 respectively. The derivation for these functions can be found in (Pender 2010b). The equations given are for strip foundations, and adjustments should be made to account for foundation shape and embedment.

\[
\begin{align*}
    f_{\text{clay}}(N, V, M) &= \left[ 2N - \left( 1 - \frac{2|M|}{N} \right) \right]^2 - \left( 1 - \frac{2|M|}{N} \right)^2 + N\sqrt{\frac{2|M|}{N}} = 0 \\
    f_{\text{sand}}(N, V, M) &= N - \left( 1 - \frac{2|M|}{N} \right)^2 \left( 1 - \frac{|V|}{N} \right)^3 = 0
\end{align*}
\]

Figures 2.2 and 2.3 show the bearing strength surfaces based on the equations above. Only positive $N$ is shown as a foundation that must always have a compressive load on it. The $V$ values go both positive and negative, and the $M$ can be both positive and negative during an earthquake; however, only positive values are given for clarity. Outside of these surfaces the soil will yield and begin to behave inelastically. The bearing strength surfaces take on a ‘rugby ball’ shape, with the greatest moment occurring when the foundation is loaded to 0.5 of its ultimate bearing capacity ($N_u$).

### 2.2.3 Eurocode 8 Bearing Strength Surface

An additional bearing strength surface in moment-axial-shear ($M-N-V$) space was included in Eurocode 8, part 5 (European Committee for Standardization 1998). It uses the same dimensionless parameters as defined in Equation 2.9, however has a further dimensionless parameter ($\bar{F}$) to account for the effect of the soil inertia force applied to a foundation. The equation’s for this inertia effect is defined as:

\[
\begin{align*}
    \bar{F}_{\text{undrained}} &= \frac{\rho a g B}{s_u}, \quad \bar{F}_{\text{drained}} = \frac{a_g}{g \tan \phi}
\end{align*}
\]
where \( a_g \) = horizontal ground acceleration beneath the foundation; and \( \rho \) = density of soil.

The bearing strength surface specified in EC8 is:

\[
f(\bar{N}, \bar{V}, \bar{M}, \bar{F}) = \frac{(1 - e \bar{F})^{C_f} \left( \beta \bar{V} \right)^{C_f}}{\bar{N}^{\alpha} \left(1 - m \bar{F}^{\frac{1}{b}}\right)^{\omega} - \bar{N}^{\beta}} + \frac{(1 - f \bar{F})^{C_m} \left( \chi \bar{M} \right)^{C_m}}{\bar{N}^{\alpha} \left(1 - m \bar{F}^{\frac{1}{b}}\right)^{\omega} - \bar{N}^{\beta}} - 1 = 0 \tag{2.13}
\]

where the numeric values for the fourteen parameters, \( a-f, m, k, k', C_f, C_m, C'M, \beta, \) and \( \chi \) are given in Annex F of EC8, Part 5.

The background to this bearing strength surface can be found in Salençon and Pecker (1995b, 1995a) and Pecker (1997). Some design examples are given in Chapter 10 of Fardis et al. (2005). The bearing strength surfaces for purely cohesive (clay) and purely cohesionless (sand) soil are given in Figures 2.4 and 2.5 respectively.

Figure 2.6 compares the \( V-N \) and \( M-N \) sections of the surfaces. They show that the shallow foundation on clay has greater shear and moment capacity than on sand (Pender 2008). This is also consistent with the bearing strength surfaces from Terzaghi’s equations.

Figures 2.7 and 2.8 display the effect of the dimensionless parameter \( \bar{F} \) (the parameter to account for seismic inertia in the soil beneath a foundation). Consequently the bearing strength surface reduces with increasing \( \bar{F} \), (an increase in seismic acceleration). The figures show that sand is much more sensitive than clay to the increased seismic acceleration (Pender 2008). Thus, shallow foundations on sand, whilst having a potentially greater static bearing capacity, are more vulnerable to seismic attack.
2.2 - Bearing Capacity

Figure 2.2 Bearing strength surface for a cohesive soil based on Terzaghi’s bearing strength equation

Figure 2.3 Bearing strength surface for a cohesionless soil based on Terzaghi’s bearing strength equation
Figure 2.4 EC8 bearing strength surface for a cohesive soil

Figure 2.5 EC8 bearing strength surface for a cohesionless soil
2.2 - Bearing Capacity

Figure 2.6 Comparisons of the H-V (top), and M-V (bottom), sections of the EC8 bearing strength surface of clay (cohesive) and sand (cohesionless)

Figure 2.7 The effect of the dimensionless parameter F, which accounts for soil inertia, on the response of shallow foundations on clay (undrained) in the H-V (top) and M-V (bottom) space
Figure 2.8 The effect of the dimensionless parameter $F$, which accounts for soil inertia, on the response of shallow foundations on sand (drained) in the $H-V$ (top) and $M-V$ (bottom) space

2.3 SOIL FOUNDATION STRUCTURE INTERACTION

Mylonakis et al. (2006) offers an explanation of what occurs when a soil-foundation-structure system is vibrated by an earthquake. The paper explains that SFSI can be broken into kinematic interaction and inertial interaction:

*During earthquake shaking, soil deforms under the influence of the incident seismic waves and “carries” dynamically with it the foundation and supported structure. In turn, the induced motion of the superstructure generates inertial forces which result in dynamic stresses at the foundation that are transmitted into the supporting soil.*

Kinematic interaction results from the earthquake incident waves acting on the foundation and supporting structure that leads to a foundation input motion. Kinematic interaction in some cases may be neglected; seismic building codes will often ignore it, and kinematic interaction is far more difficult to evaluate than inertial interaction (Pecker and Pender 2000). Inertial interaction refers to the response of the complete soil-foundation-structure system to the excitation associated with a superstructure.
2.3.1 Elastic SFSI

John P Wolf’s ‘Dynamic Soil-Structure Interaction’ (Wolf 1985) covers elastic methods for assessing soil-structure-interaction and is developed from a background of designing nuclear power plants. The example given in section 3.4 of his book is a simple way of explaining soil-structure interaction. Figure 2.9 is a generic representation of a single degree of freedom (SDOF) system along with the common soil-foundation-structure system.

The system has a spring of stiffness $K$ acting in parallel to a dashpot of damping $C$, since most vibrating systems exhibit some form of damping. Equation 2.14 gives the equation of motion for Figure 2.9.

\[ m\ddot{u} + C\dot{u} + Ku = p(t) \]  \hspace{1cm} (2.14)

where $m$ = mass of the system; $\ddot{u}$ = acceleration; $C$ = damping of the system; $\dot{u}$ = velocity; $K$ = stiffness of the system $u$ = displacement; and $p(t)$ = the forcing function.

The undamped natural frequency and period of the system are:

\[ \omega_0 = \sqrt{\frac{K}{m}} \quad \text{and} \quad T = \frac{2\pi}{\omega_0} \]
Resonance occurs when the excitation of the system corresponds to the natural frequency ($\omega$) and damping controls the amplitude of motion. Without damping, the solution of the differential equation (2.14) becomes infinite. At critical damping the structure does not oscillate at all, this is given by:

$$c_c = 2\sqrt{KM} = 2M\omega$$

(2.16)

Thus, the damping ratio is:

$$\xi = \frac{c}{c_c}$$

(2.17)

Damping of a soil-foundation-structure system can come from three different mechanisms: energy dissipating within the structural elements of the system, called structural damping; energy radiating in the soil away from the source of vibration, called radiation damping; and inertial dissipating of energy within the soil, referred to as material damping. For foundation rocking, there is a fourth kind of damping that arises from impact of the foundation on the soil, called impact damping.

Wolf’s example only considers radiation damping on an elastic soil layer, but demonstrates the effect of damping on a soil-foundation-structure system. Brown (1975) further explains the significance of soil-foundation-structure interaction.

Figure 2.10 shows the amplitude of the steady state dynamic response of an arbitrary foundation normalised with respect to the static displacement against the frequency response normalised against the natural frequency. It shows that at resonance ($\omega/\omega_n = 1$) and zero damping the solution is infinite. Note that another effect of damping is to move the frequency with the peak response away from $\omega_n$. This is particularly evident in Figure 2.10 when $\xi$ is greater than 0.30. Richart, Hall and Woods (1970) calculates equations for foundation stiffness and damping based on soil properties for a rigid circular disk. Their equations for spring stiffness and damping values for horizontal vibration are:
And for rocking vibration:

\[
K_{f\phi} = \frac{8G\rho a_{fnd}^3}{3(1-\nu)}
\]  \hspace{1cm} (2.20)

\[
c_{\phi} = \frac{0.4\rho V_s a_{fnd}^4}{1-\nu}
\]  \hspace{1cm} (2.21)

where \( K \) = stiffness value; \( c \) = damping value; \( G \) = shear modulus of the soil; \( a_{fnd} \) = foundation radius; \( \nu \) = poisons ratio; and \( V_s \) = shear wave velocity of the soil.

\[\nu \leq \frac{2}{\pi_0 f_{nd} H_f G a K \rho \phi - \frac{2}{6.4^2 f_{nd} s h a V c)} \phi - \frac{1}{14.4 f_{nd} s a V c \phi}}\]

\[\omega_f = \frac{\omega}{\omega_n} \]

\[\xi = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1.0\]

\[\text{Figure 2.10 Normalised natural frequency against displacement magnification}\]
The above damping terms are the static stiffness and damping parameters. Veletsos and Wei (1971) present curves that extend the given equations to cover dynamic response. The dynamic coefficients \( K(a_o) \) and \( c(a_o) \) in the paper relate to the property \( a_o \), a dimensionless parameter defined as:

\[
a_o = \frac{\omega \cdot a}{V_s}
\]  

(2.22)

The dynamic stiffness and damping values are calculated using the dynamic coefficients and the static stiffness value:

\[
K_{\text{dynamic}}(a_o) = K_{\text{static}} \cdot K(a_o) \quad c_{\text{dynamic}}(a_o) = K_{\text{static}} \cdot c(a_o)
\]  

(2.23)

The system natural frequency and damping can be found from the stiffness and damping values by:

\[
\omega^2_{\text{sys}} = \frac{\omega_i^2}{1 + \frac{K_s + \xi_h^2}{K_{f_H}} + \frac{K_s}{K_{f_\phi}}}
\]  

(2.24)

\[
\xi_{\text{sys}} = \xi_s \frac{\omega_{\text{sys}}^2}{\omega_i^2} + \xi_{H} \frac{\omega_{\text{sys}}^2}{\omega_{H}^2} + \xi_{\phi} \frac{\omega_{\text{sys}}^2}{\omega_{\phi}^2} + \xi_{s} \left(1 - \frac{\omega_{\text{sys}}^2}{\omega_i^2}\right)
\]  

(2.25)

where \( K_s \) = structural stiffness; \( K_{f_H} \) = horizontal foundation stiffness (Equation 2.18 for a rigid circular disk); \( K_{f_\phi} \) = rotation foundation stiffness (Equation 2.20 for a circular disk); \( \omega_{\text{sys}} \) and \( \xi_{\text{sys}} \) = natural frequency and the damping ratio of the total system; \( \omega_i \) and \( \xi_s \) = natural frequency and damping ratio of the structural response; and \( \xi_g \) = equivalent damping ratio of the soil (material damping).

The equivalent ground motion that the system experiences can be calculated by:

\[
\tilde{u}_g = \frac{\omega_{\text{sys}}^2}{\omega_i^2} u_g
\]  

(2.26)

where \( \tilde{u}_g \) = equivalent system ground displacement; \( u_g \) = ground displacement.
The magnitude of the total displacement of the mass with respect to the initial position is found using the equation:

$$\Delta_{\text{Total}} = \Delta_H + \Delta_{\phi} + \Delta_s$$  \hspace{1cm} (2.27)

where $\Delta_{\text{Total}}$ = total horizontal displacement of the structure; $\Delta_H$ = horizontal displacement of the structure caused by horizontal movement of the foundation (sliding); $\Delta_{\phi}$ = horizontal displacement of the centre of mass caused by foundation rotation; and $\Delta_s$ = horizontal displacement of the structure (structural deflection). The three horizontal displacement components (foundation horizontal, foundation rotation, and structural) can be defined as:

$$\Delta_H = \frac{\omega_h^2}{\omega_s^2} \left( 1 + 2 \xi_s i - 2 \xi_s i - 2 \xi_s i \right) \Delta_s,$$  \hspace{1cm} (2.28)

$$\Delta_{\phi} = \frac{\omega_{\phi}^2}{\omega_s^2} \left( 1 + 2 \xi_s i - 2 \xi_s i - 2 \xi_s i \right) \Delta_s,$$ and \hspace{1cm} (2.29)

$$\Delta_s = \frac{\omega_s^2}{\omega_{\text{sys}}^2} \tilde{u}_g \sqrt{1 - \frac{\omega_s^2}{\omega_{\text{sys}}^2} \gamma_s^2 + 4 \xi_{\text{sys}}^2}$$  \hspace{1cm} (2.30)

A simple parametric study on how the response of a single degree of freedom structure changes with soil conditions is a good way to illustrate the sensitivity of soil-foundation-structure interaction. Figure 2.11 gives the total displacement against the natural frequency (normalised with respect to the structural natural frequency) with varying shear modulus values. It shows how the peak displacement is a combination of structural and foundation displacements: at a low soil modulus the foundation displacement is greater, and at a high soil modulus the structural displacement is greater. This illustrates how soil-structure-interaction can be either beneficial or detrimental to structural performance.
Figure 2.11 Normalised natural frequency against total displacement for a single degree of freedom structure – it shows how the response changes with varying soil conditions.

Figure 2.12 The structural displacement against shear modulus for a single degree of freedom structure

Figure 2.12 presents the structural displacement against the natural frequency for a single degree of freedom structure with different shear modulus values. It is evident that as the shear modulus increases the structural displacement increases, causing greater
2.3 - Soil Foundation Structure Interaction

damage to a superstructure. The shape of the foundation in this example was square. This ratio was chosen as it demonstrates the greatest effects on the natural frequency.

The difference between Figure 2.11 and 2.12 is that the former shows the total deflection of the structure, whereas the latter only shows the structural deflection. Figure 2.12 demonstrates that as the soil modulus diminishes the deflection of the structure ($\Delta_s$) also becomes less, and the deflections of the foundation ($\Delta_H + \Delta_\phi$) consequently increase.

### 2.3.1.1 Correspondence principle

Wolf briefly explains the correspondence principle, a very useful concept in considering the dynamics of foundations on and within elastic media. The static response of a system can be converted to a dynamic response by replacing static stiffness values with complex quantities called impedances. For example, material damping occurring within soil is independent of frequency, but it can be introduced into the solution when working in the frequency domain by use of the correspondence principle. Chapter 15 on Foundation Vibrations of the Foundation Engineering Handbook, 2nd edition (Gazetas 1990), gives values for converting static stiffness into dynamic stiffness (called the dynamic stiffness coefficients) for a range of embedment and soil conditions.

### 2.3.2 Elastic Plastic SFSI

Extending the response to include inelastic behaviour is the next step in assessing the response of soil-foundation-structure. Muir Wood (2004) outlines the general process for elastic plastic modelling:

1. Elastic Laws – The elastic behaviour of soil can be defined by the Gazetas stiffness equations (Equation 2.1) that are governed by shear modulus ($G$) and Poisson’s ratio ($\nu$).

2. Yield Function – The combination of loads that will create yielding of a soil. This can be thought of as the bearing strength surfaces in Figures 2.2 and 2.3, sometimes referred to as a ‘bounding surface’.
3. Plastic Flow – Describes the mechanism of plastic deformation, otherwise referred to as the flow rule or the plastic potential function. This can be associative, meaning that the direction of plastic flow is normal to the bounding surface, or non-associative. The direction of plastic flow is not normal to the bounding surface and is defined by the user.

4. Evolution Laws – These define how a bounding surface changes with time; either hardening or softening.

There are many inelastic models of soil-foundation behaviour, and some of these are described in detail in Chapter 5, Numerical Modelling Development.

2.4 ROCKING FOUNDATION HISTORY

Allowing a structure to rock provides base isolation by reducing moments and shear at the base, ultimately protecting the structure. The rocking concept was first researched when post earthquake inspection revealed that rocking foundations sometimes occurred despite not being designed to do so. Some examples of foundations rocking without intentional design to do so are:

Several inverted pendulum structures survived the Chilean earthquake of 1960, whereas more stable structures were severely damaged (Housner 1963).

During the 1952 Arvin-Tehachapi earthquake in California, tall, slender, petroleum-cracking towers stretched their anchor bolts and rocked back and forth on their foundations (Housner 1956).

It has also been noted that free-standing stone pillars supporting heavy statues in Indian cities remained intact while more stable structures did not (Campbell et al. 1977).

After the 1977 Tongan Earthquake, it was observed that many walls could be rocked by hand, but had not fallen; and monuments that did not fall despite their relatively small size, thus indicating a strong case for rocking (Campbell et al. 1977).
Naturally, foundation rocking can have negative effects on structural performance. When the soil is weak, it may itself undergo large deformations caused from bearing capacity failure (Gazetas et al. 2003), evident by the large displacements at low soil moduli in Figure 2.11. This could result in catastrophic failure if stability is lost, a clear example of this is the Terveler building that toppled into a neighbouring building in Adapazari during the 1999 Kocaeli earthquake (Gazetas and Anastasopoulos 2007). Figure 2.13 shows a photo of this failure, taken from the paper.

Housner (1963) performed one of the first investigations into the mechanics of rocking. He considered the rocking motion of a rigid block, shown in Figure 2.14, for free vibration, constant acceleration, sinusoidal acceleration, and earthquake motion.

Housner showed that the frequency of rocking increased with decreasing amplitude and derived the following equation of motion for a rocking block:

$$I_0 \frac{d^2 \theta}{dt^2} = -WR \cdot \sin(\alpha - \theta)$$  \hspace{1cm} (2.31)

where $I_0$ = moment of inertia; $\frac{d^2 \theta}{dt^2}$ = angular acceleration and $W$ = vertical load.
Based on the blocks initial angle ($\theta_0$), the period of a structure rocking on a rigid surface can be calculated by:

$$T = \frac{4}{p} \cosh^{-1} \left( \frac{1}{1 - \frac{\theta_0}{\alpha}} \right)$$  \hspace{1cm} (2.32)

where

$$p = \sqrt{\frac{WR}{I_0}}$$  \hspace{1cm} (2.33)

Housner concluded that energy is dissipated through the impact of the block rocking on a rigid surface. He stated that if the impact was assumed to be inelastic (no bouncing), then the moment of momentum about point $0/0'$ is conserved and the energy reduction factor can be calculated by:

$$r = \left( \frac{\dot{\theta}_1}{\dot{\theta}_2} \right)^2 = \left[ 1 - \frac{mR^2}{I_0} (1 - 2\cos 2\alpha) \right]^2$$  \hspace{1cm} (2.34)

where $r = \text{energy reduction factor}$; and $m = \text{mass of rocking rigid block}$.

Housner found that the larger the value of $R$ (the radius from the centre of mass to the
point of rotation), the greater the stability. Thus a bigger block is more stable than a smaller block in an earthquake and there is an unexpected scale effect – because the ground motion is not scaled with the block. He showed the stability of a tall slender block is much greater in an earthquake than the same block under constant horizontal acceleration, which is why the taller slender inverted pendulum structures survived whereas other structures did not.

From this pioneering research, a number of studies into the motion of rocking have been carried out (Aslam et al. 1980, Ishiyama 1982, Rutenberg and Heidebrecht 1985, Yim et al. 1980, Bailey et al. 1973). Yim et al. (1980) investigated the rocking response of rigid blocks and studied the overturning of a block from a probabilistic point of view. They found that the rocking response of a block is very sensitive to small changes in its size and slenderness ratio, and the stability of a block to a particular ground motion does not necessarily increase with increasing intensity (Yim et al. 1980).

Aslam et al. (1980) performed a similar study using a computer program to model the rocking response of blocks. The blocks were of various sizes and aspect ratios and were subjected to several different earthquake motions. The study primarily investigated the response of solid concrete block stacks used as radiation shields in particle accelerator laboratories.

Ishiyama (1982) classified the motions of a rigid body on a rigid floor into six types: rest, slide, rotation, slide-rotation, translation-jump, and rotation-jump. He found that the coefficient of friction must be greater than the breadth-height ratio in order for a body to continue to rock, or sliding occurs. He found that for overturning to occur, both the horizontal acceleration and velocity must be greater than certain threshold levels. Therefore, it is possible to estimate the lower limits of maximum acceleration, velocity, and displacement depending on the size of the rocking object (Ishiyama 1982).

A simplified analysis using a two-spring model and a Winkler spring bed model is described in (Chopra and Yim 1985). The analysis computes base shear and deformation of an uplifting structure directly from earthquake response spectra. It demonstrates a useful degree of accuracy for practical structural design, though the foundation parameters are difficult to evaluate in practice (Chopra and Yim 1985).
In New Zealand there were two early design applications of rocking structures. The first is a 35 m tall industrial chimney made from reinforced concrete designed to rock at the Christchurch airport. Figure 2.15 is a photo of this chimney. It was constructed as a cruciform shape and could rock in two directions, accommodating in and out of plane shaking. It had additional steel dampers for extra protection against more violent shakes.
The other is the South Rangitikei Rail Bridge, a 70 m tall railway viaduct with twin-legged piers that sit on rubber bearings. The bridge was designed to allow the piers to ‘step’ during an earthquake (Beck and Skinner 1974). Figure 2.16 shows photos of the bridge.

The design of rocking foundations in New Zealand was first detailed in (Priestley et al. 1978). The paper suggests the advantages of rocking foundations may be compared to those of base-isolation and proposes a simple design approach for estimating the maximum displacements. The design guide limits analysis to the structural response and ignores the effects of soil-structure-interaction. The experimental study that supplemented the paper reported much lower accelerations resulting from impact for flexible base conditions (foundation sitting on rubber pads) than for rigid base conditions.

Around the same time, the University of Auckland performed cyclic loading tests of rocking foundations on both clay and sand (Bartlett 1976, Wiessing 1979). Consequently, Taylor and Williams (1979) published another design guideline for New Zealand. This paper is discussed in greater depth in Chapter 6, Rocking Foundation Design Guideline.

2.5 DISCUSSION

The literature review included in this chapter covers a wide array of shallow foundation behaviour. Elastic stiffness of shallow foundations has a general equation for footings resting on the ground surface, which includes additions for shape, depth, and sidewall effects. Stiffness is critical in estimating the deformations of foundations, and they can be calculated with the simple elastic stiffness equations (Equations 2.1-2-7).

The Terzaghi bearing capacity equation is useful for estimating the ultimate bearing capacity of any foundation shape. However, it doesn’t include the effects of the acceleration of an earthquake, which is something that the EC8 bearing strength surface includes. Sand potentially has a much greater static bearing capacity, but it has much greater sensitivity to earthquake acceleration on the bearing capacity than clay.
Chapter 2 - Literature Review

Soil-foundation-structure interaction can have positive and negative effects on a structure during an earthquake. Equations for evaluating simple elastic SFSI were given. Figures 2.11 and 2.12 show the effects of the shear modulus with respect to displacement.

The occurrence of rocking foundations has been observed in the past, even though such foundations were not designed to rock. From these observations a great deal of research was undertaken into the mechanics of the rocking motion. Although there have been structures designed to rock, soil-structure effects were generally ignored.

The next chapter delves into the experiments conducted for this thesis. Chapter 3 details the experimental configuration, and Chapter 4 explains the results.
The next two chapters investigate the earthquake response of shallow foundations subject to rocking through large scale field testing. Since experiments of this type were only attempted infrequently before, an original concept, design, and methodology were developed for these experiments. A frame, made from structural steel, was designed, fabricated, and assembled onsite along with several reinforced concrete foundations. A thorough ground investigation classed the soil as an Auckland residual soil, mainly Waitemata Clay. Furthermore, geotechnical and geophysical tests were performed insitu to obtain soil properties such as the soil type, the small strain shear modulus ($G_{max}$), and the undrained shear strength ($s_u$). Laboratory tests were conducted to determine properties such as the natural water content ($w_n$) and Atterberg Limits ($LL$ and $PL$). Two methods of dynamic excitation were used throughout the experiments: forced-vibration and free-vibration. In both series of tests the structure was equipped with up to 42 instruments to capture the total behaviour of the system. This chapter walks through the design and construction process, as well as the geotechnical details and data acquisition.
It concludes with tables summarising the tests performed. The subsequent chapter details the results obtained in the experiments.

## 3.1 Concept

The goal of the experiments was to test shallow foundations and provide data to be implemented into a design solution. Many of the retrofit design applications for this research in New Zealand apply to rocking foundations sitting under concrete shear walls. However, there are problems with testing reinforced concrete shear walls. These issues include: testing dynamically may damage the concrete, making a wall viable for only one test; loading the wall vertically with enough mass to be realistic and retaining stability; and mounting the shaker, which has a footprint around one metre square.

The experiments required a structure versatile enough to conduct experiments yet realistic enough to produce useful results. It had to be demountable and reusable, making it possible to test on different soils around New Zealand, and easy to transport. The structure had to be built to allow rocking foundations but remain essentially elastic; a yielding structure would create difficulties for subsequent use. Lastly, it had to be large enough to accommodate sufficient mass to achieve realistic soil pressures, and to

---

*Figure 3.1 A 3D representation of the experiment concept, the ground surface is not shown here but went up to the top of the foundations*
support the earthquake shaker. A structural steel frame could accommodate all the above requirements. Figure 3.1 shows a 3D representation of the resulting structure. The concept was to conduct tests with varying masses on different foundations. Consequently four foundation locations were planned, totalling eight foundations.

### 3.2 DESIGN

A full design was performed on the structure following the proposed concept. It was broken into three parts: design of the structural frame, geotechnical design of the foundations, and structural design of the foundations.

#### 3.2.1 Structural Frame

Professional steel fabricators constructed the truss design structurally capable of a 100 kN horizontal load as a fixed base system. The frame was composed of three parts, two vertical end frames and a top frame, that enabled the structure to be transported with relative ease. The end frames connected to the foundations by anchor bolts. The top frame bridged the two end frames and supported the shaker and additional mass.

A 2100 by 1530 mm steel plate was welded onto the middle of the top frame and the shaker was bolted to this using eight M24 8.8 structural bolts. Two universal column sections, a 200 UC 42.6 and a 100 UC 14.8, were used in the design, and the three

![Figure 3.2 Plan detail of the top frame of the structure, all dimensions are in mm](image-url)
components were bolted together using M20 8.8 structural bolts. Bracing made the structure as stiff as possible. A 100 mm circular hollow section (CHS) ran horizontally between the two end frames close to the ground surface.

**Figure 3.3** Elevation detail of the end frames – both end frames were made identical, all dimensions are in mm

**Figure 3.4** An elevation view of the structure showing the diagonal braces and the horizontal CHS running close to the ground surface
In addition, 20 mm diameter steel braces were fixed diagonally across the structure. Figure 3.2 shows a plan detail of the top frame, Figure 3.3 shows an elevation detail of the end frames, and Figure 3.4 shows an elevation view from the side of the structure.

3.2.2 Foundations

The foundations design was split into the structural design and the geotechnical design. The geotechnical design was a simple calculation of bearing capacity using Equation 2.2 to determine how large the foundations needed to be. Consequently, the dimensions of the foundations were 2 m long by 0.4 m wide by 0.4 m deep with the top of the foundations at the ground surface. The foundations were designed for static loading, neglecting any dynamic effects. The design originally called for a foundation with a factor of safety close to 3, but the strong soil strength meant the stability of the structure was a concern as the foundation size to produce a factor of safety of 3 was very small. In the interest in safety, the foundations were sized, mass determined, and factor of safety back calculated from this. The fully loaded structures had factors of safety from 6.6 to 8.0.

Figure 3.5 shows an elevation design of the foundations detailing the dimensions and reinforcement. The concrete specified for the foundations had a compressive yield strength ($f_c$) of 30 MPa after 28 days. The longitudinal reinforcing was two D20 bars top and bottom and the transverse reinforcement was D10 hoops spaced at 150 mm centres. A total of eight foundations were cast into the soil making four different frame positions. The foundations were somewhat closely spaced but far enough apart (around 3 m) not to interact with each other.
Figure 3.5 Detail of the foundation elevation

Figure 3.6 The site before the experimental program began

3.3 SITE

Figure 3.6 shows the site prior to any construction or testing. The site was located east of the Auckland northern motorway in the Pinehill subdivision of Albany suburb. The land owners along with their consultants facilitated access to the site. Upon initial inspection, the soil was a silty clay deemed to be an Auckland residual soil. Past research on the properties of Auckland residual soil is detailed below. To determine soil
properties accurately (a key part in understanding the mechanics of rocking foundations) several geotechnical and geophysical tests were performed onsite and in the lab. Cone penetration tests, seismic cone penetration tests, wave activated stiffness tests, and spectral analysis of surface wave tests were conducted, as well as simple tests onsite such as shear vane tests. Using collected samples, laboratory tests determined the natural water content and Atterberg limits.

### 3.4 AUCKLAND RESIDUAL SOIL PROPERTIES

A residual soil is defined as a soil deposit that has had insitu weathering and is not formed by stress history processes such as glacial deposits. An excellent resource on the properties of Auckland residual soil is a Ph.D. thesis done at the University of Auckland by Vaughan Meyer titled Stress-Strain and Strength Properties of an Auckland Residual Soil (Meyer 1997). The soil samples used for his laboratory testing were taken from a site in Pinehill around 500 m away from the site for this research. The soil tests in Meyer’s research revealed silty clay derived from the Waitemata series, much like the soil from this site. He wanted to determine the characteristics of the soil under very low effective stress, which is applicable to shallow foundations because the depth of influence is often less than one foundation length below the bottom of the foundation. Meyer found the average properties of the soil he sampled were: natural water content of 45.5%; initial bulk density of 1707 kg/m$^3$; density of soil particles of 2630 kg/m$^3$; plastic limit of 32; and liquid limit of 60.

One major characteristic of residual soil is variability. Figure 3.7 shows drained and undrained shear strengths against water content at failure for Auckland residual clay. It shows no defined relationship, unlike the remoulded material plotted on the same graph. Several sites around Auckland reached the same conclusion (Pender 2010b, Pender 2010a).
3.4.1 Shear Strength

Even at low confining pressures, the Mohr-Coulomb failure criterion accurately defined the observed peak shearing resistance (Meyer 1997). Figure 3.8 shows triaxial test data gathered by Meyer for drained and undrained compression and extension tests. The results produced an angle of shearing resistance that is different in compression than extension, but the strength parameter values are the same for drained and undrained conditions (Pender 2010a).

The undrained shear strength, $s_u$, is commonly measured by unconsolidated undrained (UU) tests or insitu using hand shear vane tests. Meyer did not perform UU tests because if the sample is not completely saturated prior to testing, compression of air voids can result in an inaccurate $s_u$. However, saturating the samples would also produce inaccurate results since the water content in the sample would not be the same as onsite. By taking $s_u$ as half the measured deviator stress, $q/2$, he made a correlation with the void ratio at failure, $e_{\text{peak},q}$, in consolidated undrained (CU) and consolidated
drained (CD) tests. The data demonstrates a significant amount of scatter but has a general trend of decreasing $s_u$ with an increasing void ratio (Meyer 1997).

### 3.4.2 Stiffness

The soil tested in Meyer’s research did not display common yielding characteristics, rather it demonstrated continuous yielding behaviour up until the peak deviator stress. He did however select points of maximum curvature in deviator stress shear strain plots ($q$ vs. $\varepsilon_s$) to define a yield criterion for the Waitemata Clay. These points cannot be thought of as the transition from elastic to plastic behaviour, but they are associated with the greatest change towards plastic soil behaviour (Meyer 1997).

Bender element tests were performed by Meyer to determine the small strain shear modulus of Auckland residual soil. Comparisons of $G_{\text{max}}$ to $s_u$ produced a linear correlation that was lower than expected:

$$G_{\text{max}} = 284s_u$$  \hspace{1cm} (3.1)
For saturated clays, Seed & Idriss (1970) developed the following correlation between $G_{\text{max}}$ and $s_u$:

$$\frac{G_{\text{max}}}{s_u} \equiv 1000 \text{ to } 2500$$ (3.2)

Comparatively Meyer’s equation is an order of magnitude different from Equation 3.2, though it is more comparable to research by Hara et al. (1974) who developed:

$$G_{\text{max}} = 516s_u^{1.012}$$ (3.3)

From Meyer’s research there are insights into the behaviour of the Waitemata Clay in terms of strength and stiffness. One advantage was the proximity of his soil samples to the site for this research. However, soil properties used in this work were taken from the tests conducted both onsite and in the laboratory, and Meyer’s findings are used only as a reference.

### 3.5 CONE PENETRATION TESTS

Cone penetration tests were performed extensively onsite over a two day period. A CPT was done at the end of each foundation for a total of 16 CPT’s on eight foundations. Figure 3.9 shows a layout of the site and the foundation and labelled CPT numbers, and Figure 3.10 shows the CPT rig used. The set-up, limited by the size of the site, meant the foundations overlapped each other, so foundations 1 and 3 made up the footings for one test, foundation 2 and 4 made up another, and so on.

Figure 3.11 gives the Cone Resistance plots for CPT03 and CPT06 (foundation 3, a forced-vibration test) and for CPT12 and CPT13 (foundation 8, a snap-back test). The figures show that the soil is similar from one end of a foundation to the other. The CPT graph on the left is for foundation 3 and shows that the upper layers of soil are composed of a very stiff fine grained material down to a depth of around 1.6 m. There is a thin layer of sand at around 1.6 m which could account for the spike in the plots. The next layer of soil comprises clay to silty clay or clayey silt to silty clay.
Figure 3.9 Layout of the site showing the numbering of the CPT’s and foundations

Figure 3.10 The CPT rig used for all CPT’s and SCPT’s, the CPT probe can be seen on the right-hand side of the photo
Chapter 3 - Experimental Configuration

The graph on the right shows the data from foundation 8. The results from both ends of the foundation are comparable, but CPT12 has a spike around 1.4 m whereas CPT13 does not. The soil is very stiff and fine-grained in the upper section, and the spike in CPT12 is a thin layer of sand similar to the previous foundation. The material becomes clay to silty clay and progresses into clayey silt to silty clay further down. The CPT pore pressure response for every CPT put the water table at 5.1 m below the ground surface, but the soil was considered saturated to near the surface because the tests were conducted near the end of the wet season.

The correlations of soil properties were done using the report ‘Guide to Cone Penetration Testing for Geotechnical Engineering’ by Robertson and Cabal (2010). Each correlation is outlined below. Appendix A lists the results from all CPT tests.

Robertson and Cabal’s (2010) procedures identifies soil type and computes various correlations that are a useful alternative for geotechnical engineers to the CPT soil type identification plot developed by Douglas and Olsen (1981). First, a small correction to
the cone resistance is required. This is to account for pore pressure generated at the intersection of the sleeve and the cone:

$$q_t = q_c + u2(1 - a)$$  \hspace{1cm} (3.4)

where $q_t$ = corrected cone resistance; $q_c$ = measured cone resistance; $u2$ = measured pore water pressure; and $a$ = area ratio usually determined by laboratory calibration.

The normalised friction ratio is given by:

$$F_r = \left( \frac{f_s}{q_t - \sigma_{vo}} \right) \cdot 100$$  \hspace{1cm} (3.5)

where $f_s$ = measured sleeve friction; and $\sigma_{vo}$ = total vertical stress.

From Equation 3.4 a normalised cone penetration resistance can be calculated by:

$$Q_m = \frac{q_t - \sigma_{vo}}{P_a} \left( \frac{P_a}{\sigma'_{vo}} \right)^n$$  \hspace{1cm} (3.6)

where $P_a$ = atmospheric pressure and $n$ is discussed below.

The soil behaviour type, $I_c$, and the $n$ value used above are classified by:

$$I_c = \left[ (3.47 - \log (Q_m))^2 + (\log (F_r) + 1.22)^2 \right]^{0.5}$$  \hspace{1cm} (3.7)

and:

$$n = 0.381 \cdot I_c + 0.05 \left( \frac{\sigma'_{vo}}{P_a} \right) - 0.15 \hspace{1cm} n \leq 1.0$$  \hspace{1cm} (3.8)

Some iteration is required to achieve the right $n$ value. Table 3.1 gives the $I_c$ value corresponding to the different soil types, and Figure 3.12 gives the soil classification type chart for CPT06 and CPT12. Both soil type indexes show that the soil had an $I_c$ value of 5 in the upper layers. This corresponds to sand mixtures: silty sand to sandy silt. The description given above classifies the material as a silty clay – not a sandy silt – so there is some discrepancy between the onsite description and the CPT results.
Although the $I_c$ index classed the soil more in the sandy range, laboratory tests were performed on samples from hand augers, and the results are discussed further below. The results indicate that the soil is more clayey than sandy, confirming the initial onsite observations.

The final correlation applicable to the soil onsite is to achieve an estimate of the undrained shear strength $s_u$. The formula for calculating $s_u$ from $q_t$ is:

$$ s_u = \frac{q_t - \sigma_{vo}}{N_k} \tag{3.9} $$

where $N_k$ depends on the plasticity of the clay. Robertson and Cabal (2010) suggests that $N_k$ should be between 10 and 18; higher with increasing plasticity and lower with increasing soil sensitivity. For deposits where little experience is available, a range between 14-16 is recommended, while the upper value should be used for a more conservative estimate (Robertson and Cabal 2010).

Figure 3.13 shows the $s_u$ results from CPT06 and CPT12 using an $N_k$ value of 15. A spike similar to the cone resistance plots occurs around 1.5 m due to the sandy layers. For analysis purposes, the undrained shear strength was calculated between 100 and 200 kPa. This is an appropriate range considering the depth of influence for a shallow foundation (roughly one foundation length below the bottom of the foundation, or down to 2.0 m). These values also correspond well to shear vane tests performed onsite. Appendix A gives the results as plots for all of the processes above.

### Table 3.1 Soil type index based on $I_c$ values, taken from Robertson and Cabal (2010)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Soil Behaviour Type</th>
<th>$I_c$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sensitive, fine grained</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>Organic soils – clay</td>
<td>&gt; 3.6</td>
</tr>
<tr>
<td>3</td>
<td>Clays – silty clay to clay</td>
<td>2.95-3.6</td>
</tr>
<tr>
<td>4</td>
<td>Silt mixtures – clayey silt to silty clay</td>
<td>2.60-2.95</td>
</tr>
<tr>
<td>5</td>
<td>Sand mixtures – silty sand to sandy silt</td>
<td>2.05-2.60</td>
</tr>
<tr>
<td>6</td>
<td>Sands – clean sand to silty sand</td>
<td>1.31-2.05</td>
</tr>
<tr>
<td>7</td>
<td>Gravelly sand to dense sand</td>
<td>&lt; 1.31</td>
</tr>
<tr>
<td>8</td>
<td>Very stiff sand to clayey sand*</td>
<td>NA</td>
</tr>
<tr>
<td>9</td>
<td>Very stiff, fine grained*</td>
<td>NA</td>
</tr>
</tbody>
</table>

*Heavily overconsolidated or cemented
Figure 3.12 The soil behaviour type for CPT06 (left) and CPT12 (right) – refer to Table 3.1 for classification.

Figure 3.13 The $s_u$ results from CPT06 (left) and CPT12 (right) calculated using the techniques from Robertson and Cabal (2010).
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3.6 LABORATORY TESTS

Table 3.2 shows data from standard laboratory tests performed on samples collected onsite to determine water content, Atterberg limits, and undrained shear strength. The data was collected by a fellow researcher investigating the seismic response of piles (M.Sa’don 2010), and the samples were taken about ten metres away from the foundation locations. The depth of the samples taken by M.Sa’don were around 0.5m, close to the foundation depth. The average test values were: a natural water content of 32.8%, a plastic limit of 28, a liquid limit of 54, and an undrained shear strength of 152 kPa. Table 3.3 shows a comparison between Meyer’s findings and the laboratory results.

Table 3.2 The natural water content, Atterberg limits, plasticity index, liquid index, and undrained shear strength from samples taken from site – taken from M.Sa’don (2010)

<table>
<thead>
<tr>
<th>Natural Water Content, $w_n$ (%)</th>
<th>Atterberg Limits</th>
<th>Plasticity Index</th>
<th>Liquidity Index</th>
<th>Undrained Shear Strength, $s_u$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.6</td>
<td>40.7</td>
<td>23.9</td>
<td>16.8</td>
<td>191.4</td>
</tr>
<tr>
<td>20.7</td>
<td>35.2</td>
<td>25.6</td>
<td>9.6</td>
<td>103.0</td>
</tr>
<tr>
<td>34.0</td>
<td>61.2</td>
<td>28.8</td>
<td>32.4</td>
<td>173.0</td>
</tr>
<tr>
<td>30.3</td>
<td>61.2</td>
<td>27.0</td>
<td>34.2</td>
<td>184.0</td>
</tr>
<tr>
<td>39.9</td>
<td>51.7</td>
<td>31.3</td>
<td>20.4</td>
<td>154.6</td>
</tr>
<tr>
<td>33.4</td>
<td>54.5</td>
<td>28.5</td>
<td>26.0</td>
<td>191.4</td>
</tr>
<tr>
<td>22.4</td>
<td>34.6</td>
<td>20.1</td>
<td>14.6</td>
<td>136.2</td>
</tr>
<tr>
<td>43.5</td>
<td>61.2</td>
<td>31.3</td>
<td>29.8</td>
<td>158.2</td>
</tr>
<tr>
<td>31.8</td>
<td>46.9</td>
<td>25.5</td>
<td>21.4</td>
<td>132.5</td>
</tr>
<tr>
<td>37.2</td>
<td>62.7</td>
<td>30.4</td>
<td>32.3</td>
<td>154.6</td>
</tr>
<tr>
<td>39.3</td>
<td>67.5</td>
<td>31.4</td>
<td>36.1</td>
<td>125.1</td>
</tr>
<tr>
<td>38.8</td>
<td>66.8</td>
<td>31.7</td>
<td>35.1</td>
<td>150.9</td>
</tr>
<tr>
<td>34.5</td>
<td>54.8</td>
<td>28.8</td>
<td>26.0</td>
<td>147.2</td>
</tr>
<tr>
<td>31.6</td>
<td>55.3</td>
<td>27.9</td>
<td>27.4</td>
<td>128.8</td>
</tr>
</tbody>
</table>

Table 3.3 Comparisons of Meyer’s findings and the lab tests performed

<table>
<thead>
<tr>
<th>Natural Water Content, $w_n$ (%)</th>
<th>Meyer</th>
<th>Lab Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic Limit, PL</td>
<td>45.5%</td>
<td>32.8%</td>
</tr>
<tr>
<td>Liquid Limit, LL</td>
<td>32</td>
<td>28</td>
</tr>
</tbody>
</table>
Figure 3.14 shows the natural water content plotted against the undrained shear strength of the samples. Just as in Figure 3.7 there is no apparent correlation between the two for the Auckland residual clay samples.

**Figure 3.14** Undrained shear strength plotted against natural water content, like Figure 3.7 it shows no apparent relationship between the two properties

**Figure 3.15** Atterberg limits for samples taken from the Auckland residual clay at Pinehill (M. Sa’don 2010)
Figure 3.15 shows the plasticity index plotted against the liquid limit in the laboratory tests. The results reveal dispersion along the Casagrande A-line, ranging from low to high plasticity. Most samples lie on the clay side of the line, but some samples were classified as silt. This was consistent with observations made onsite.

Because both the initial observations and the lab tests indicated that the material was clayey, further calculations assume that the soil onsite is clay, rather than sand as the $I_c$ index suggested.

### 3.7 SEISMIC CONE PENETRATION TESTS

Seismic cone penetration tests (SCPT’s) were performed between 2 and 3 m deep simultaneously with the CPT tests. A SCPT is also known as a down hole test; it requires an accelerometer at depth and a vibration source at the surface (in this case an instrumented hammer). Travel times are recorded from the S waves generated by the vibration between two known depths, so the shear wave velocity, $V_s$, can be calculated.

Figure 3.16 is a schematic of a down hole set up taken from Kramer (1996a). Table 3.4 gives the shear wave velocities between 2 and 3 m deep. Small strain shear modulus values are also given, using a mass unit weight, $\rho$, of 1700 kg/m$^3$. The shear wave velocities recorded a maximum of 196 and minimum of 135 m/s. These corresponded to shear modulus values of 65 MPa and 31 MPa respectively. The average shear wave velocity was 155.6 m/s, or 41.1 MPa.

Figure 3.17 states the relationship between the small strain shear modulus (calculated using the SCPT results) and the undrained shear strength. The line of best fit shows that $G_{max}$ equals 610 times the undrained shear strength, $s_u$, given in Equation 3.10. In his research, Meyer outlines the relationship: $G_{max}$ equals 286 times $s_u$. Past research with the most comparable equation came from Hara et al. (1974) who developed a relationship of $G_{max}$ roughly equal to 516 times $s_u$. (see Equation 3.3).

$$G_{max} = 610s_u$$  \hspace{1cm} (3.10)
3.7 - Seismic Cone Penetration Tests

Figure 3.16 Schematic of a down hole test, showing the vibrating source, the S waves, and the receiver, taken from Kramer (1996a)

Table 3.4 The shear wave velocity and corresponding $G_{\text{max}}$ values from the SCPT tests

<table>
<thead>
<tr>
<th>CPT No.</th>
<th>Foundation No.</th>
<th>Depth (m)</th>
<th>$V_s$ (m/s)</th>
<th>$G_{\text{max}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>2</td>
<td>2-3</td>
<td>173</td>
<td>51</td>
</tr>
<tr>
<td>03</td>
<td>3</td>
<td>2-3</td>
<td>150</td>
<td>38</td>
</tr>
<tr>
<td>04</td>
<td>4</td>
<td>2-3</td>
<td>152</td>
<td>39</td>
</tr>
<tr>
<td>05</td>
<td>4</td>
<td>2-3</td>
<td>135</td>
<td>31</td>
</tr>
<tr>
<td>06</td>
<td>3</td>
<td>2-3</td>
<td>144</td>
<td>35</td>
</tr>
<tr>
<td>07</td>
<td>2</td>
<td>2-3</td>
<td>159</td>
<td>43</td>
</tr>
<tr>
<td>08</td>
<td>1</td>
<td>2-3</td>
<td>196</td>
<td>65</td>
</tr>
<tr>
<td>09</td>
<td>5</td>
<td>2-3</td>
<td>167</td>
<td>47</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>2-3</td>
<td>147</td>
<td>37</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>2-3</td>
<td>147</td>
<td>37</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>2-3</td>
<td>147</td>
<td>37</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>2-3</td>
<td>155</td>
<td>41</td>
</tr>
<tr>
<td>14</td>
<td>7</td>
<td>2-3</td>
<td>149</td>
<td>38</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>2-3</td>
<td>143</td>
<td>35</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>2-3</td>
<td>170</td>
<td>49</td>
</tr>
</tbody>
</table>
3.8 WAVE ACTIVATED STIFFNESS TESTS

Briaud and Lepert (1990) first defined wave activated stiffness tests, or WAK tests, as a simple experiment to determine the small strain shear modulus near the ground surface. A 500 mm diameter plate is bedded onto the ground surface with accelerometers on either side. An instrumented hammer strikes the middle of the plate and acceleration readings are taken. To obtain the shear wave modulus, the acceleration signals must first be integrated to obtain velocity. This is done by taking the Fast Fourier Transform (FFT) and integrating the transform in the frequency domain using Equation 3.11 (Kreyszig 2006).

\[
\int FFT(p(x)) = \frac{FFT(p(x))}{i\omega}
\]

(3.11)

where \( p(x) \) = FFT function of the original signal and \( \omega \) = frequency of the FFT function (in rads/sec).

![Figure 3.17 Undrained shear strength plotted against the small strain shear modulus showing the linear relationship given in Equation 3.10](image-url)
3.8 - Wave Activated Stiffness Tests

A velocity/force modulus versus frequency function must be generated, and the idealised equation is:

\[
\left| \frac{v}{F} (\omega) \right| = \frac{\omega}{\left( \left[ K - m \omega^2 \right] + C^2 \omega^2 \right)^{1.5}}
\]

(3.12)

where \( v \) = velocity time history recorded; \( F \) = force time history recorded; \( \omega \) = natural frequency (rads/sec); \( K \) = stiffness; \( m \) = mass; and \( C \) = damping.

This function is plotted in Figure 3.18. The initial vertical stiffness, \( K_{ini} \), is taken as the inverse of the slope of the curve at the origin. From the initial stiffness, the small strain shear modulus, \( G_{max} \), is found by (Pender 2010b):

\[
G_{\text{max}} = \frac{K_{\text{ini}} (1 - \nu)}{4r}
\]

(3.13)

where \( r \) = plate radius.

Table 3.5 gives the average \( G_{\text{max}} \) taken from multiple whacks of the hammer in each WAK test. Two tests, orthogonal to each other, were performed at each foundation location. Throughout the testing there were issues with accelerometer readings, and one accelerometer began producing faulty results. This is labelled on the table as ‘Instrument problem’. The maximum and minimum shear modulus values obtained from the WAK tests were 48.7 and 15.7 MPa respectively, and the average was 32.6 MPa. These values are slightly less than those obtained from the SCPT tests, but they are values at the surface where the SCPT tests were values obtained between 2 and 3 m deep. The higher shear modulus values at depth can be attributed to increased confining stress. The results also show that the shear modulus values were similar from North-South as compared to East-West.
Figure 3.18 Idealised velocity/force vs. frequency function as defined by Equation 3.12

Table 3.5 The $G_{\text{max}}$ values obtained from all the WAK tests

<table>
<thead>
<tr>
<th>Foundation No.</th>
<th>Direction</th>
<th>$G_{\text{max}}$ – average (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>North-South</td>
<td>31.4</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>33.8</td>
</tr>
<tr>
<td>2</td>
<td>North-South</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>29.8</td>
</tr>
<tr>
<td>3</td>
<td>North-South</td>
<td>41.5</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>Instrument problem</td>
</tr>
<tr>
<td>4</td>
<td>North-South</td>
<td>Instrument problem</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>Instrument problem</td>
</tr>
<tr>
<td>5</td>
<td>North-South</td>
<td>Instrument problem</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>Instrument problem</td>
</tr>
<tr>
<td>6</td>
<td>North-South</td>
<td>48.7</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>26.7</td>
</tr>
<tr>
<td>7</td>
<td>North-South</td>
<td>37.7</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>31.4</td>
</tr>
<tr>
<td>8</td>
<td>North-South</td>
<td>29.8</td>
</tr>
<tr>
<td></td>
<td>East-West</td>
<td>Instrument problem</td>
</tr>
</tbody>
</table>
Spectral Analysis of Surface Waves (SASW) tests were performed onsite in conjunction with the WAK tests. Figure 3.19 shows the set up of both experiments. A single dispersion curve (i.e. the plot of Rayleigh wave velocity versus frequency) can be obtained for impulse loading on a plate. Based on procedures outlined in (Kramer 1996a, Stokoe II et al. 2006, M.Sa’don et al. 2010, Stokoe II et al. 1994) shear wave velocities are obtained. The procedure requires signals from two receivers – either accelerometers or geophones, \( x(t) \) and \( y(t) \). These are converted into the frequency domain \( (X(f) \) and \( Y(f) \)) by a FFT. The power spectra for both signals \( (G_{XX} \) and \( G_{YY} \)) and the phase of the cross spectrum \( (\phi(f)) \) are calculated using equations 3.14 through 3.16.

The coherence function, calculated by Equation 3.17, is used to determine the quality of the received signal. It takes on a value between 0 and 1 with a weaker signal being closer to 0 and a stronger signal closer to 1. Figure 3.20 shows the phase of the cross spectrum and coherence function for the SASW test on foundation 4. The clean phase signal plot suggests a good quality test, and the coherence plot affirms this for a frequency above 50 Hz.

\[
G_{XX} = \overline{X(f)} \cdot X(f) \quad \text{and} \quad G_{YY} = \overline{Y(f)} \cdot Y(f) \tag{3.14}
\]

\[
G_{XY} = \overline{X(f)} \cdot Y(f) \tag{3.15}
\]

\[
\phi(f) = \arctan \left( \frac{\text{Im}(G_{XY})}{\text{Re}(G_{XY})} \right) \tag{3.16}
\]

\[
\gamma^2 = \frac{G_{XY} \cdot \overline{G_{XY}}}{G_{XX} \cdot G_{YY}} \tag{3.17}
\]

where \( \overline{X(f)} \) = the complex conjugate of the quantity; \( \text{Im} \) = the imaginary part of the expression; and \( \text{Re} \) = the real part of the expression.

The travel time between receivers can be calculated for each frequency \( (f) \) and therefore velocity by Equations 3.18 and 3.19.
Because the distance between the receivers \( (d) \) is known, the wave velocity \( (V_R) \) is calculated as:

\[
V_R = \frac{d}{t(f)}
\]  

(3.19)

A small adjustment is required for the difference between surface wave velocity and shear wave velocity. For all calculations, the velocity was multiplied by 1.09 to get the shear wave velocity (Stokoe et al. 1994). Table 3.6 gives the velocity, small strain shear modulus, and depth calculated at each foundation location. The table shows that the data is variable across the different foundation positions, as was the case for the SCPT and WAK tests. The minimum shear wave velocity calculated was 127.3 m/s that corresponded to a \( G_{\text{max}} \) of 27.5 MPa. Subsequently the maximum shear wave velocity was 162.4 m/s, which corresponded to a \( G_{\text{max}} \) of 44.8 MPa. The table suggests this method was the most consistent in calculating the small strain shear modulus. The \( G_{\text{max}} \) for modelling and analysis purposes was between 30 – 40 MPa, and most often at the higher end. Stokoe’s paper suggests that the depth of the calculated shear modulus is one third of the spacing between the source and the second geophone, or 0.33 m. This correlates well to the 0.4 m depth from ground surface to the underside of the foundation. The SASW tests also have the ability to measure shear wave velocity at depth, but the signal strength was too poor to measure deeper than around 0.5 m.

\[f = \frac{\phi(f)}{2\pi} \]  

(3.18)
3.10 - Hand Shear Vane Tests

Hand shear vane tests were performed along each foundation position. This occurred after digging the foundation trenches but before pouring the concrete on the day of foundation construction. Five peak and two residual readings were taken for each foundation.

The tests were positioned at even distances along the 2 m foundation length. Table 3.7 shows the results from the vane readings taken, and the top row lists the distance from the North edge of the foundation. Table 3.8 shows a second set of shear vane readings around foundations 5-8. At the time, the ground had become substantially more saturated, and readings from the altered soil conditions were necessary. All values were

---

Table 3.6 The shear wave velocity and corresponding $G_{\text{max}}$ values from the SASW tests

<table>
<thead>
<tr>
<th>Foundation No.</th>
<th>Depth (m)</th>
<th>$V_s$ (m/s)</th>
<th>$G_{\text{max}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.33</td>
<td>127.3</td>
<td>27.5</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>162.4</td>
<td>44.8</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>134.6</td>
<td>30.8</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>151.9</td>
<td>39.2</td>
</tr>
<tr>
<td>5</td>
<td>0.33</td>
<td>154.3</td>
<td>40.5</td>
</tr>
<tr>
<td>6</td>
<td>0.33</td>
<td>135.7</td>
<td>31.3</td>
</tr>
<tr>
<td>7</td>
<td>0.33</td>
<td>142.1</td>
<td>34.3</td>
</tr>
<tr>
<td>8</td>
<td>0.33</td>
<td>No Result</td>
<td>No Result</td>
</tr>
</tbody>
</table>

---

Figure 3.20 Plots of the phase (left) and coherence function (right) for the SASW test on foundation 4
taken at the foundation depth (cut to 0.4 m below the ground surface) and are given in kPa.

The results show the soil was very stiff to hard in most readings with only a few readings below 100 kPa. The average ultimate undrained shear strengths from each foundation were: foundation 1 = 118 kPa, foundation 2 = 182 kPa, foundation 3 = 182 kPa, foundation 4 = 176 kPa, foundation 5 = 211 kPa, foundation 6 = 234 kPa, foundation 7 = 218 kPa, and foundation 8 = 190 kPa. The average ultimate shear strength values from in the second set of readings were: foundation 5 = 152 kPa, foundation 6 = 171 kPa, foundation 7 = 135 kPa, and foundation 8 = 103 kPa. The soil was still very stiff though the ground was substantially saturated in the second set of readings.

Table 3.7 The ultimate and residual shear strength values obtained using the hand shear vane for each foundation location, distances are to the edge of the north face of the foundation trenches

<table>
<thead>
<tr>
<th>Foundation</th>
<th>333 mm</th>
<th>666 mm</th>
<th>1000 mm</th>
<th>1333 mm</th>
<th>1666 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fnd 1</td>
<td>Ultimate</td>
<td>99</td>
<td>94</td>
<td>129</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 2</td>
<td>Ultimate</td>
<td>169</td>
<td>184</td>
<td>188</td>
<td>178</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 3</td>
<td>Ultimate</td>
<td>191</td>
<td>118</td>
<td>200</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 4</td>
<td>Ultimate</td>
<td>162</td>
<td>&gt;239</td>
<td>180</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 5</td>
<td>Ultimate</td>
<td>224</td>
<td>234</td>
<td>188</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>88</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 6</td>
<td>Ultimate</td>
<td>&gt;239</td>
<td>&gt;239</td>
<td>&gt;239</td>
<td>&gt;239</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 7</td>
<td>Ultimate</td>
<td>193</td>
<td>&gt;239</td>
<td>&gt;239</td>
<td>&gt;239</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fnd 8</td>
<td>Ultimate</td>
<td>195</td>
<td>191</td>
<td>221</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.8 The ultimate shear strength values for foundations 5, 6, 7 and 8, done prior to Test 7

<table>
<thead>
<tr>
<th>Foundation</th>
<th>North</th>
<th>South</th>
<th>East</th>
<th>West</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fnd 5</td>
<td>158</td>
<td>121</td>
<td>142</td>
<td>188</td>
</tr>
<tr>
<td>Fnd 6</td>
<td>173</td>
<td>134</td>
<td>145</td>
<td>232</td>
</tr>
<tr>
<td>Fnd 7</td>
<td>121</td>
<td>98</td>
<td>175</td>
<td>144</td>
</tr>
<tr>
<td>Fnd 8</td>
<td>144</td>
<td>77</td>
<td>81</td>
<td>109</td>
</tr>
</tbody>
</table>
3.11 Instrumentation

Bearing capacities for each foundation were calculated using the undrained shear strength and Equation 2.8 for undrained behaviour. The equation is a modified version specific for undrained behaviour. Equation 3.20 shows the modified bearing capacity equation that was used to calculate ultimate bearing pressures.

\[ q_u = c \lambda_d \lambda_p \lambda_c C_0 + q \]  

(3.20)

3.11 Instrumentation

The structure and foundations were highly instrumented to ensure all behaviour was captured. Linear variable differential transducers (LVDT’s), strain gauges, accelerometers, pressure sensors, geophones, and a load cell were used throughout both the forced-vibration and snap-back tests.

3.11.1 Linear Variable Differential Transformers

Linear variable differential transformers (LVDT’s) were attached to the foundation to measure vertical and horizontal movement. The vertical movement was used to calculate settlement and rotation of the foundation, and the horizontal movement was used to calculate horizontal displacement. These LVDT’s were in turn attached to a 6 m long 5” by 2” piece of timber that was anchored to the ground at each end but remained off the ground along its length. This was to ensure minimal vibrations from the rocking were transferred to the LVDT recordings. The span of the beam (6 m) was distant enough from the foundations for the readings only to record the foundation movement and was not affected by ground vibrations.

3.11.2 Strain Gauges

Strain gauges attached to the steel columns recorded the forces transferred into the foundations. Axial and bending strains in the members were converted into axial forces and bending moments through elastic strain principles. Each side of a vertical column had an orthogonal 4 active gauge system bridge. Two gauges were vertical and two horizontal, and through Poisson’s ratio (taken as 0.28), the strain in the member was
obtained. The diagonal struts fixed into the base of the columns were also instrumented with an orthogonal 2 active gauge system bridge for the same measurement.

Figure 3.21 shows the calibration of the strain gauges with static lateral push over tests conducted in the laboratory. Each end of the frame was fixed to the strong floor and an actuator applied cyclic motion on top. The data obtained by the strain gauges were compared to theoretical strains calculated using elastic strain principles. On both end frames the theoretical and measured strains were comparable.

3.11.3 Accelerometers

Accelerometers were placed on the foundation and the structure to measure the acceleration in all three directions: vertical, longitudinal, and transverse. The accelerometers in the force-vibration tests had a range of +/- 2g, but the accelerometers attached to the top of the structure were changed to +/- 4g for the snap-back tests because the recorded accelerations were greater than 2g.
3.11.4 Pressure Sensors

Pressure sensors are paper thin instruments that can record change of pressure of a defined bearing area. They were placed at intervals on the underside of the foundation. Under constant loads these sensors tend to drift, making calibration, in this case, nearly impossible because they were sitting under concrete foundations. However, the overall purpose of the sensors was to achieve a ‘before and after’ picture of what was happening to the soil underneath the footing. The gap between the soil and the foundation was measured by recording the pressures on the underside of the footing before, during, and after excitation. Unfortunately, problems arose with the pressure sensors because they had to withstand a concrete pour and a considerable constant load. Few produced results, and the rest were deemed unusable.

3.11.5 Geophones

Geo Space Technologies geophones were placed on the ground surface at predetermined distances from the east foundation to measure the impact that occurred during the snap-back tests. Calibration of the geophones is nonlinear depending on frequency: the data collected was converted to the frequency domain by a FFT and calibrated, and the force was converted back to the time domain to provide velocity.

3.11.6 Load Cell

A load cell was instrumented during the snap back tests to record the force required to pull the structure over, and the data was then converted into moments to give the static moment rotation curves. Figure 3.22 presents a photo of the snap back test layout. The load cell measured the force in both chains. Initially it was assumed the load was distributed evenly between the two chains, but this is almost impossible to achieve because of the nature and size of the equipment involved. Acceleration records comparing the frame acceleration at each end were used to mitigate this error as much as possible. The difference in acceleration records were used to calculate torsion of the frame. However this difference between one end and the other was minimal and torsion was calculated as very small.
3.11.7 Instrument List’s

The next two pages have the complete instrument lists for the forced-vibration tests and the snap-back tests respectively. Table 3.9 gives the instrument list – 42 total – for the forced-vibration tests. Table 3.10 gives the instrument list – 31 total– for the snap-back tests. The label, type, description, range, calibration factor, and units for each instrument are provided.

Table 3.9 The instrument list for the forced vibration tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Type</th>
<th>Location Description</th>
<th>Range</th>
<th>Calibration Sensitivity</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil - West</td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
<tr>
<td></td>
<td>pressure</td>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom</td>
<td>kPa/V</td>
</tr>
</tbody>
</table>

Figure 3.22 Side view of the test set up showing the chains wrapped round the tops of the columns, the actuator, the load cell and the snap shackle
### Instrumentation

<table>
<thead>
<tr>
<th>Pressure</th>
<th>Location</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre of E Footing</td>
<td>539.55 N</td>
<td>Custom kPa/V</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration (acc)</th>
<th>Location</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>South end - vertical</td>
<td>2g</td>
<td>0.996446 g/V</td>
<td></td>
</tr>
<tr>
<td>North end - vertical</td>
<td>2g</td>
<td>0.989313 g/V</td>
<td></td>
</tr>
<tr>
<td>South end - x direction</td>
<td>2g</td>
<td>0.2287335 g/V</td>
<td></td>
</tr>
<tr>
<td>North end - x direction</td>
<td>2g</td>
<td>0.2297636 g/V</td>
<td></td>
</tr>
<tr>
<td>Centre - y direction</td>
<td>2g</td>
<td>0.2294841 g/V</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Axial Strain (ε)</th>
<th>Location</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>South gauge of South column</td>
<td>2.8% ε</td>
<td>0.369 ε/V</td>
<td></td>
</tr>
<tr>
<td>North gauge of South column</td>
<td>2.8% ε</td>
<td>0.369 ε/V</td>
<td></td>
</tr>
<tr>
<td>South gauge of North column</td>
<td>2.8% ε</td>
<td>0.369 ε/V</td>
<td></td>
</tr>
<tr>
<td>North gauge of North column</td>
<td>2.8% ε</td>
<td>0.369 ε/V</td>
<td></td>
</tr>
<tr>
<td>Upper side of strut</td>
<td>2.8% ε</td>
<td>0.738 ε/V</td>
<td></td>
</tr>
<tr>
<td>Lower side of strut</td>
<td>2.8% ε</td>
<td>0.738 ε/V</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LVDT</th>
<th>Location</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>South end - vertical</td>
<td>100mm</td>
<td>9.979427 mm/V</td>
<td></td>
</tr>
<tr>
<td>North end - vertical</td>
<td>100mm</td>
<td>9.90527 mm/V</td>
<td></td>
</tr>
<tr>
<td>Centre - y direction</td>
<td>100mm</td>
<td>22.138645 mm/V</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LVDT</th>
<th>Location</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>South end - vertical</td>
<td>50 mm</td>
<td>5.002554 mm/V</td>
<td></td>
</tr>
<tr>
<td>North end - vertical</td>
<td>50mm</td>
<td>4.884273 mm/V</td>
<td></td>
</tr>
<tr>
<td>Centre - y direction</td>
<td>50mm</td>
<td>4.930432 mm/V</td>
<td></td>
</tr>
</tbody>
</table>
### Table 3.10 The instrument list for the snap back tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Type</th>
<th>Location Description</th>
<th>Range</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>axial strain</td>
<td>South gauge of South column</td>
<td>2.8% $\varepsilon$</td>
<td>0.369</td>
</tr>
<tr>
<td>Frame - West</td>
<td>axial strain</td>
<td>North gauge of South column</td>
<td>2.8% $\varepsilon$</td>
<td>0.369</td>
</tr>
<tr>
<td>Frame - West</td>
<td>axial strain</td>
<td>South gauge of North column</td>
<td>2.8% $\varepsilon$</td>
<td>0.369</td>
</tr>
<tr>
<td>Frame - West</td>
<td>axial strain</td>
<td>North gauge of North column</td>
<td>2.8% $\varepsilon$</td>
<td>0.369</td>
</tr>
<tr>
<td>Frame - West</td>
<td>axial strain</td>
<td>Upper side of strut</td>
<td>2.8% $\varepsilon$</td>
<td>0.738</td>
</tr>
<tr>
<td>Frame - West</td>
<td>axial strain</td>
<td>Lower side of strut</td>
<td>2.8% $\varepsilon$</td>
<td>0.738</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>LVDT</td>
<td>South end - vertical</td>
<td>100 mm</td>
<td>9.979427 mm/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>LVDT</td>
<td>North end - vertical</td>
<td>100 mm</td>
<td>9.90527 mm/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>LVDT</td>
<td>South end - y direction</td>
<td>100 mm</td>
<td>22.138645 mm/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>LVDT</td>
<td>South end - vertical</td>
<td>50 mm</td>
<td>5.002554 mm/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>LVDT</td>
<td>North end - vertical</td>
<td>50 mm</td>
<td>4.884273 mm/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>LVDT</td>
<td>South end y direction</td>
<td>50 mm</td>
<td>4.930432 mm/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>acc</td>
<td>South end - vertical</td>
<td>2g</td>
<td>0.996446 g/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>acc</td>
<td>North end - vertical</td>
<td>2g</td>
<td>0.989313 g/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>acc</td>
<td>South end - x direction</td>
<td>2g</td>
<td>0.2300543 g/V</td>
</tr>
<tr>
<td>Fnd – West</td>
<td>acc</td>
<td>Centre - y direction</td>
<td>2g</td>
<td>0.2298956 g/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>acc</td>
<td>South end - vertical</td>
<td>2g</td>
<td>1.011964 g/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>acc</td>
<td>North end - vertical</td>
<td>2g</td>
<td>0.990884 g/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>acc</td>
<td>North end - x direction</td>
<td>2g</td>
<td>0.2292631 g/V</td>
</tr>
<tr>
<td>Fnd – East</td>
<td>acc</td>
<td>Centre - y direction</td>
<td>2g</td>
<td>0.2299591 g/V</td>
</tr>
<tr>
<td>Load force</td>
<td>Load through chain</td>
<td>200 kN</td>
<td>176.5 kN/V</td>
<td></td>
</tr>
<tr>
<td>Top frame</td>
<td>acc</td>
<td>East end - y direction</td>
<td>2g</td>
<td>0.2289063 g/V</td>
</tr>
<tr>
<td>Soil velocity</td>
<td>East Fnd - 1.0 m away</td>
<td>5.08 cm/s</td>
<td>Custom m/s/V</td>
<td></td>
</tr>
<tr>
<td>Soil velocity</td>
<td>East Fnd - 2.0 m away</td>
<td>5.08 cm/s</td>
<td>Custom m/s/V</td>
<td></td>
</tr>
</tbody>
</table>
The foundation excavation and construction was completed in one day to reduce the impact of the weather, either drying and cracking in hot sun or flooding due to heavy rain. The foundation dimensions were 0.4 m by 0.4 m by 2 m. They acted as strip footings beneath each frame end and were embedded to 0.4 m, with the top of the concrete at the ground surface.

An excavator cut the foundation trenches to the required width (shown in Figure 3.23), and additional work was completed by hand with a spade. Next, the pressure sensors were attached along the ground surface. A layer of geotextile was layered atop the pressure sensors to give them extra protection from the concrete, and the steel reinforcing cages were placed. The concrete was cast directly into the foundation trenches and not cast against formwork and backfilled as would be expected in normal construction. This ensured full contact between the sides of the foundation and the natural soil.
The structure’s anchor bolts were located within the reinforcing steel using templates made from plywood, ensuring that the bolt pattern was accurate within the structure’s base plates. The distance between the two foundations, especially the foundation bolts, was carefully planned so the structure would connect to the foundations without issue.

Once the steel was placed and the anchor bolts carefully positioned, concrete was poured until it rose to the underside of the plywood templates or the ground surface. Figure 3.24 is a photo of the concrete pour taking place. Once the foundations were cast, the concrete was left to cure for 28 days before testing.

### 3.13 EXCITATION

A substantial excitation was necessary in these experiments to achieve adequate results with such a large structure. There were two forms of excitation throughout the experimental program: one from an eccentric mass shaker (forced-vibration) and one from pulling the structure over and quickly releasing (snap-back).
3.13.1 Eccentric Mass Shaker

The dynamic excitation in the forced-vibration tests came from an eccentric mass shaker attached to the top of the structure. The shaker has two counter-rotating fly wheels with various masses bolted to half of each wheel. As they rotate, the inertia force from the masses adds in one direction but cancel in the perpendicular direction, delivering a unidirectional sinusoidal forcing function. Figure 3.25 depicts the top frame with the foundations and shaking direction. The force the shaker delivers is dependent on frequency and the quantity of mass. Figure 3.26 shows force versus frequency plots for the shaker running at minimum and maximum, up to 12 Hz. At full capacity the

![Diagram of structure with foundations, top frame, and shaking direction](image1)

*Figure 3.25 A plan view of the structure, showing the top frame, foundations, and shaking direction*

![Force vs. Frequency Plot](image2)

*Figure 3.26 A force vs. frequency plot, showing the curves for the minimum and maximum weights for the eccentric mass shaker*
shaker can deliver 98 kN dynamic force at a frequency of 7 Hz.

The tests sought to explain how the response of the foundations changes as the dynamic force increases. As explained above the magnitude of the dynamic force depends on the shaker frequency and the number of masses attached to the flywheels. Therefore, each test run began and ended with a frequency sweep containing only small masses. The initial small excitation response of the system was recorded and compared to high-level shaking.

### 3.13.2 Snap-Backs

The snap back tests were conducted by pulling the structure over using chains attached to a hydraulic jack and a quick release mechanism. Chains were secured around the top of each column on the north side of the structure and fastened to a 50 tonne crawler crane through a quick release mechanism, a 100 kN actuator, and a load cell. The crane was used only as an anchor point for the chains and hydraulic jack. At the desired rotation the device was released and the structure would rock in free vibration. Figure 3.27 shows the setup used for the snap back tests: the chains are wrapped around the top of the columns and the frame is in a rotated position. The figure shows the chains extending downwards away from the frame. Although not ideal because it adds vertical load into the frame, this was the only way the chains could be safely secured to the crane. The vertical component on the frame was considered in the moment calculations. The lever arm was the distance from the middle of the end frames to where the chains were fixed (0.85 m).

Figure 3.28 presents photos of the quick release mechanism. The device, a snap shackle, is used for quick spinnaker release on large sailing boats. It has a working load of 110 kN and a breaking load of 220 kN.
Figure 3.27 Set up for a typical snap back test: the crane was used as an anchor point and a hydraulic jack was used to pull the frame over

Figure 3.28 Photos of the snap shackle used as the quick release mechanism

3.14 VERTICAL LOAD

The structure alone is relatively light, around 50 kN of vertical load, and thus affects minimal stresses in the soil beneath the foundations. Tests were conducted with varying vertical loads to achieve different static factors of safety in bearing. An approximate
amount of mass was determined using the results from the hand shear vane tests to achieve different factors of safety in bearing.

### 3.14.1 Forced-Vibration Vertical Load

The additional mass for the forced vibration tests was achieved with road construction plates, i.e. plates that are used to cover up temporary holes during road construction and repair. The plates were 22 mm thick by 2 m by 3.5 m, and weighed around 1200 kg each. A total of 12 plates, 6 each side of the shaker, weighing around 150 kN were placed on top of the structure. Because of the eccentric mass shaker in the centre of the structure, two stacks of road plates on either side were lifted into place.

The plates were secured by sandwiching channel sections that ran across the top of the plates and the bottom of the beams. Figure 3.29 shows the loaded plates for the forced vibration tests. The road plates and eccentric mass shaker can be seen on top of the structure, as well as the four sets of channel sections used to clamp the plates down.

*Figure 3.29 Set up for a typical forced vibration test, the road plates and eccentric mass shaker are attached to the top of the structure and the channel sections can be seen sandwiching the plates to the top frame*
3.14.2 Snap-Back Vertical Load

The vertical load for the snap back tests was changed from road plates to steel billets, sections of steel used for rolling into bars or beams. Mass was stretched across the whole frame top because there was no shaker for this test series. The billets were 150 mm square by 8.5 m long and weighed 1500 kg each. A steel fabrication yard loaned 14 billets, for around 210 kN of vertical load. These were fixed to the top frame by wrapping several sets of strops around them and the structure.

3.15 DATA ACQUISITION

The Data Acquisition System (DAQ) used in all the tests was written in LabVIEW (NI 2010). Inputs such as sample rate and calibration files were submitted into the program and data outputted into a single text file. The sampling rate used for most tests was 256 samples per second, however test 1 had a sampling rate of 200 samples per second.

3.16 DATA PROCESSING

Data was processed in both MathCad (PTC 2007) and Matlab (MathWorks 2009). First, the raw data was converted using the calibration factors for each signal and then centred back to zero. In some cases the data was filtered with lowpass, highpass, or bandwidth Butterworth filters to remove unwanted noise (Ewins 2000).

Strain gauge data was converted into stresses and subsequently into forces by means of elastic principles. Fully fixed connections were assumed between the columns and the foundations, meaning the load recorded in the strain gauges was transferred into the foundations. This is not always the case, but the anchor bolts were tightened as much as possible to minimise error. The moment on the foundations was calculated by the equation:

\[ M_{\text{fnd}} = (\text{Axial}_{\text{North}} + \text{Axial}_{\text{South}} + \text{Axial}_{\text{Strut}} \cos(45))x + M_{\text{Mom North}} + M_{\text{Mom South}} \]  

(3.21)

where \( M_{\text{fnd}} \) = the moment on the foundation; \( \text{Axial}_{\text{North}} \) = the axial force on the north column; \( \text{Axial}_{\text{South}} \) = the axial force on the south column; \( \text{Axial}_{\text{Strut}} \) = the axial force
coming into the foundations from the diagonal strut (only the vertical component of this load was used); \( x \) = the lever arm to the centre of the foundation (850 mm); \( \text{Mom}_{\text{North}} \) = the moment in the north column; and \( \text{Mom}_{\text{South}} \) = the moment in the south column. Figure 3.30 presents a close up elevation drawing of the foundation and frame showing the positions of the strain gauge bridges.

The moments recorded by the strain gauges were verified with accelerometer data. The accelerations were multiplied by the mass of the structure to obtain force, and moments were calculated by assuming an SDOF structure and using the lever arm as the centre of mass. In all cases these was similar to the strain gauge recordings.

The settlements and rotations were calculated from the LVDT data. The settlement was taken as the average of the two readings, one from each end of the foundation. The rotation was taken as the inverse tangent of the difference between the readings divided by the distance between them (1700 mm). Equations 3.22 and 3.23 give these calculations of both settlement and rotation:

\[
\delta_s = \frac{\text{LVDT}_{\text{North}} + \text{LVDT}_{\text{South}}}{2}
\]

(3.22)
\[ \theta = \alpha \tan \left( \frac{LVDT_{\text{North}} - LVDT_{\text{South}}}{1700} \right) \] (3.23)

where \( \delta_v \) = vertical settlement; \( LVDT_{\text{North}} \) and \( LVDT_{\text{South}} \) are the two LVDT readings from each end of the footing; and \( \theta \) = foundation rotation.

### 3.17 TEST SUMMARY

Tables 3.10 and 3.11 gives an outline of the forced-vibration and snap-back tests respectively, including the foundations used, the vertical load on the structure (including self weight), the factor of safety, and the maximum force that was reached during the forced-vibration tests, or the angle of rotation reached during the snap-back tests.

#### Table 3.11 Test summary for series 1 – the forced-vibration tests

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Test No.</th>
<th>Foundations</th>
<th>Vertical Load, N (kN)</th>
<th>Vertical factor of safety, ( FS_v )</th>
<th>Maximum Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBA01</td>
<td>1-a</td>
<td>1 &amp; 3</td>
<td>50</td>
<td>29</td>
<td>( &gt;0^\dagger )</td>
</tr>
<tr>
<td></td>
<td>1-b</td>
<td>1 &amp; 3</td>
<td>50</td>
<td>29</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>1-c</td>
<td>1 &amp; 3</td>
<td>50</td>
<td>29</td>
<td>15</td>
</tr>
<tr>
<td>TBA02</td>
<td>2-a</td>
<td>1 &amp; 3</td>
<td>200</td>
<td>7.2</td>
<td>( &gt;0^\dagger )</td>
</tr>
<tr>
<td></td>
<td>2-b</td>
<td>1 &amp; 3</td>
<td>200</td>
<td>7.2</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>2-c</td>
<td>1 &amp; 3</td>
<td>200</td>
<td>7.2</td>
<td>( &gt;0^\dagger )</td>
</tr>
<tr>
<td>TBA03</td>
<td>3-a</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>N/A\dagger</td>
<td>N/A\dagger</td>
</tr>
<tr>
<td></td>
<td>3-b</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>N/A\dagger</td>
<td>N/A\dagger</td>
</tr>
<tr>
<td></td>
<td>3-c</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>N/A\dagger</td>
<td>N/A\dagger</td>
</tr>
<tr>
<td>TBA04</td>
<td>4-a</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>8.6</td>
<td>( &gt;0^\dagger )</td>
</tr>
<tr>
<td></td>
<td>4-b</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>8.6</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>4-c</td>
<td>2 &amp; 4</td>
<td>200</td>
<td>8.6</td>
<td>( &gt;0^\dagger )</td>
</tr>
</tbody>
</table>

* Tests were performed with zero masses in the shaker – so exerted just above zero force on the structure

\dagger This test did not produce any results due to human error
### Table 3.12 Test summary for series 2 – the snap-back tests

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Test No.</th>
<th>Foundations</th>
<th>Vertical Load, N (kN)</th>
<th>Vertical factor of safety, FS</th>
<th>Rotation angle (rads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBA05</td>
<td>5 – snap 1 through 3</td>
<td>2 &amp; 4</td>
<td>50</td>
<td>54</td>
<td>5.2 x 10^{-1}</td>
</tr>
<tr>
<td>TBA06</td>
<td>6 – snap 1 through 7</td>
<td>6 &amp; 8</td>
<td>200</td>
<td>8.7</td>
<td>8.7 x 10^{-3} to 0.026</td>
</tr>
<tr>
<td>TBA07</td>
<td>7 – snap 1 through 9</td>
<td>6 &amp; 8</td>
<td>260</td>
<td>6.6</td>
<td>3.5 x 10^{-3} to 0.031</td>
</tr>
<tr>
<td>TBA08</td>
<td>8 – snap 1 through 7</td>
<td>5 &amp; 7</td>
<td>50</td>
<td>44</td>
<td>3.5 x 10^{-3} to 0.026</td>
</tr>
<tr>
<td>TBA09</td>
<td>9 – snap 1 through 9</td>
<td>5 &amp; 7</td>
<td>260</td>
<td>6.9</td>
<td>3.5 x 10^{-3} to 0.026</td>
</tr>
<tr>
<td>TBA10</td>
<td>10 – snap 1 through 10</td>
<td>5 &amp; 7</td>
<td>50</td>
<td>44</td>
<td>3.5 x 10^{-3} to 0.026</td>
</tr>
</tbody>
</table>

### 3.18 DISCUSSION AND CONCLUSIONS

This chapter presented the experimental configuration of the large scale field tests that were performed on site in Auckland. A new concept, design, and methodology were developed because tests of this type had been attempted only infrequently before. The frame was designed to be used for future testing on different sites and configurations.

Soil testing developed an understanding of the soil – an Auckland residual soil, mainly Waitemata Clay – and findings are used in the next chapter where the experiment results are explained in detail. The soil was very stiff, having high undrained shear strengths and a small strain shear modulus of between 30 - 40 MPa. One main issue with residual soil is its variability, which is seen in prior research and the geotechnical and geophysical tests performed onsite.

The structure was outfitted with up to 42 instruments throughout the testing program to capture the total system behaviour. Because of the extensive testing program, some
instruments performed better than others, but each test yielded enough information to make valid conclusions.

There were two methods of excitation throughout the testing, forced-vibration tests and snap-back tests. The excitation of the structure came from an eccentric mass shaker during the forced-vibration tests and from releasing the structure in free vibration during the snap-back tests. Both test series were dynamic tests.

A substantial amount of mass was required in order to create factors of safety in bearing that were comparable to what a realistic situation might have. Therefore, road plates and then steel billets were used as a vertical load during the experiments.

The next chapter details the experiment results for both the forced-vibration tests and the snap-back tests.
4 EXPERIMENTAL RESULTS

This chapter goes into detail on the results from the large scale field tests that were performed on residual soil on a site in Auckland. It begins by describing past experiments from the pioneering laboratory experiments done at the University of Auckland through to the latest centrifuge test at The University of California, Davis. The review highlights findings, including moment capacity equations, rotational stiffness degradation, and high levels of damping.

The results section following the literature review begins with the forced-vibration tests and progresses to the snap-back tests. The chapter ends with conclusions and observations made from the experiment results. Throughout the results it is evident that data gathered supports past conclusions, mentioned above, and the benefits and detriments of rocking foundations become apparent. The presentation of results is similar for both sets of tests with a few additions specific to either forced-vibration or snap-back tests. The topics covered in the results are:

- Bearing capacity prior to testing
- Input forcing function (forced-vibration tests only)
• Moment-rotation – including the static and dynamic moment-rotations (snap-back tests only), the moment capacity equation, and fitting of static pull back moment-rotations to a hyperbolic curve (snap-back tests only).
• Rotation stiffness – the initial stiffness and the degradation throughout the testing
• Shear (forced-vibration tests only)
• Damping
• Frequency response
• Settlements
• Energy Analysis (for only the snap-back tests)

Appendices B and D cite the critical plots for all the forced-vibration tests and the snap-back tests respectively.

4.1 OVERVIEW OF PREVIOUS RESEARCH

4.1.1 Laboratory Experiments

Bartlett (Bartlett 1976) conducted 1g experiments by testing model footings (0.50m by 0.25m) on clay subjected to slow cyclic and dynamic rocking. He presented the relationship between the vertical load and overturning moment that acts on a shallow foundation, and the influences of soil properties and foundation stiffness on the amount of energy dissipated. The dynamic experiments agreed with Housner’s conclusion that foundation rocking elongates the period of a structure (Housner 1963). They also showed large rotations, which partially separated the soil from footing, induced soil yielding, resulting in a loss of foundation stiffness.

Wiessing (Wiessing 1979) conducted similar 1g tests on shallow foundations (0.50m by 0.25m) sitting on sand and subjected to slow cyclic and dynamic rocking. He found that large energy dissipation, and subsequently progressive settlement, occurred during rocking cycles. Wiessing, alongside Bartlett, described a rounding of the soil that occurs due to yielding under the edges of a foundation, which reduces the stiffness of the system and causes nonlinearity in moment-rotation behaviour.
The paper by Taylor et al. (1981) summarises the two research experiments by Bartlett and Wiessing. Figures 4.1 and 4.2 give moment-rotation and settlement-rotation plots for the experiments done on clay and sand respectively. The rocking foundations induced nonlinear moment-rotation behaviour even when the underlying material was elastic because of changing contact between soil and foundation. This relationship became highly nonlinear with hysteretic damping when material yielding underneath the footings occurred. Research revealed that the yielding of soil might occur beneath a foundation without any serious affect on the bearing capacity if the initial vertical factor of safety is high. They suggested that spread footings may be designed to yield during high intensity earthquakes to capitalise on the benefits of rocking foundations.

Georgiadis and Butterfield (1988) performed 1g tests on model footings (0.40m by 0.05m) resting on sand and subjected the models to eccentric and inclined loads. The tests investigated relationships between the ultimate vertical, horizontal, and moment for combined loads. They helped verify the size and shape of the interaction diagrams for $N-V$ and $N-M$ space. They also investigated the coupling between the vertical displacement, horizontal displacement, and rotation. They found that by increasing the eccentricity of inclined load, the vertical displacements reduce while the horizontal displacement and rotation increase.

Gottardi and Butterfield (1993) carried out experiments on model footings (0.50m by 0.10m) to further analyse the interaction diagrams and failure conditions in all three loading planes. They found simple expressions to interpret the experimental data, and the experiments confirmed what previous research proposed about the failure planes for $M-N$ and $V-N$ space.
Chapter 4 - Experimental Results

Figure 4.1 Moment-rotation and settlement-rotation plots from rocking foundation experiments on clay by Bartlett, taken from Taylor et al. (1981)

Figure 4.2 Moment-rotation and settlement-rotation plots from rocking foundation experiments on sand by Wiessing, taken from Taylor et al. (1981)
Gottardi et al. (1999) performed tests with model circular footings (0.10 m diameter) on dry dense sand. Combinations of vertical, horizontal, and moment loadings were applied to the footings. The results provided insight to bearing capacity of footings in cases other than vertical loading. He also obtained data about the hardening law and flow rule appropriate for a plasticity model, as well as an elastic response within the yield surface.

Shirato et al. (2008) performed 1g large scale shake table tests and cyclic loading of shallow foundation models (0.50m by 0.50m). They tested a range of soil densities and aspect ratios of foundations, and it is to date one of the most complete experimental data sets for the nonlinear behaviour of shallow foundations. The foundations rested in a large container filled with sand with relative densities, $D_r$, of 60% or 80% and were subject to either cyclic lateral loading or dynamic base shaking. During the experiments they observed a change in the contact length between the foundation and soil due to soil rounding. Consequently, there was a decrease in stiffness of the foundation altering the predominant vibrating frequency. They assumed the rotation of a shallow foundation can be broken into three parts: elastic rotation, plastic rotation, and rotation from uplift, and they found the uplift component can be as dominant as the plastic component. The research also demonstrated that yielding shallow foundations dissipate energy well, and the residual displacement is dependent on the number of cycles during a test as well as the base excitation intensity (Shirato et al. 2008).

### 4.1.2 Centrifuge Experiments

Several series of centrifuge tests have been performed on foundations sitting under shear walls, many of which were at the University of California, Davis. The first set of experiments was the KRR01 tests (Rosebrook and Kutter 2001a). The model consisted of a rigid structure sitting on strip footings made from aluminium. Two shear walls connected by a rigid floor diaphragm were tested at 20g. The diaphragm was included to maintain stability during testing. Figure 4.3 shows the model used in the testing program. The footings were embedded into dry Nevada Sand with a relative density, $D_r$, of either 60 or 80%.
In total, 35 static events, including vertical and lateral push tests, and 46 dynamic events were applied to the structure. The dynamic events included a variety of motions such as frequency step waves, sine waves, and scaled earthquake motions. The footing size, the mass, or the relative density of the sand altered the static factor of safety ($FS_v$) in bearing throughout the testing program. The values for the static $FS_v$ ranged from 1.3 to 6.2. After these first experiments, the KRR02 test series continued on a similar track (Rosebrook and Kutter 2001b). However, differences between the two testing series include:

- The KRR02 models rested on the ground surface and were not embedded as the KRR01 models were.
- A small amount of WD40 (lubricating oil) was applied to the sand around the foundations. This was to provide some cohesion to stop sand collapsing into the footprint before and after testing.
- KRR02 tested footings on dry sand with a relative density, $D_r$, of 60%, unlike KRR01 that tested on both 60 and 80%.
- Only two values of Static $FS_v$ were tested; 1.6 and 4.1.

The KRR03 series was the final series performed by Rosebrook and Kutter, an investigation into shear wall structures sitting on clay (Rosebrook and Kutter 2001c). The soil was made up of, in prototype scale, a 2.72m layer of dry Nevada Sand (90% +
relative density), underlying a 1.70m layer of San Francisco Bay mud. A thin layer of Monterey Sand (0.26m) was applied over the clay for ease of placing the structure on the ground surface. The clay was consolidated over time using a hydraulic press until the shear strength at the surface was around 180 kPa. However, this value decreased with each event, and, for the static lateral and dynamic tests, $s_u$ was around 100 kPa. The soil was saturated in flight prior to testing; the container was spun, and valves opened to allow any pore water to drain out of the bottom. Once drained, the valves were closed and water was added from above. This technique was adopted for a more accurate estimate of water content. Apart from the difference in soil type, the testing was similar to the previous two. The static $FS_v$ values in KRR03 were 2.8 and 4.8.

The fourth series of tests to study the behaviour of rocking shear wall footings (SSG02) went back to having strip footings resting on dry Nevada Sand with a relative density, $D_r$, of 80% (Gajan et al. 2003a). However, this series tested single walls and footings, unlike the previous series which tested dual walls for stability purposes. The vertical weight on the structure varied along with the lateral load heights. A range of safety factors were recreated including 3.4, 5.3, 6.8, and 9.6. All of the footings in this series rested on the ground surface.

The series SSG03, which came after the SSG02 series, was altered to include foundation embedment (Gajan et al. 2003b). Five walls were tested with varying vertical load, where the static $FS_v$'s for SSG03 were 1.1, 4.0, 6.4, 8.2, and 11.5.

A summary of the above five tests series (KRR01, KRR02, KRR03, SSG02, and SSG03) can be found in Gajan et al. (2005) describing the testing program, the data processing, and outlines important test results – mostly on sand. The paper discusses the rotational stiffness degradation displayed by a rocking foundation and gives a recommended stiffness reduction for shallow footings rocking on sand, given in Equation 4.1:

$$\frac{K_{f, \theta}}{K_{\theta, \text{max}}}=3.0 \times 10^{-3} \cdot (\theta_f^{-0.6})$$  \hspace{1cm} (4.1)
where $K_{\theta_f} = \text{rotational stiffness}$; $K_{\theta,\text{max}} = \text{maximum (or initial) rotational stiffness}$; and $\theta_f = \text{rotation (radians)}$.

This equation was accurate within ±1 standard deviation of all the centrifuge results for foundation rocking on sand, as well as with results from 1g tests performed at the University of Auckland (Wiessing 1979). The paper goes on to discuss the relationship between permanent settlement and the static vertical factor of safety, $FS_v$. It plots the experimental failure points in the moment-vertical ($F_m-F_v$) plane against two theoretical failure envelopes from Cremer et al. (2001), and Houlsby and Cassidy (2002). Consequently, an equation for the moment capacity of a foundation was given:

$$M_{\text{max}} = \frac{N \cdot L}{2} \left[ 1 - \frac{L_c}{L} \right] \quad (4.2)$$

where $M_{\text{max}} = \text{moment capacity}$; $N = \text{vertical load}$; $L = \text{length of foundation in shaking direction}$; and $L_c = \text{critical contact length – length of foundation where the bearing capacity factor of safety equals 1}$.

For the footings resting on the ground surface, it was shown that:

$$\frac{L_c}{L} = \frac{1}{FS_v} \quad (4.3)$$

where $FS_v = \text{vertical factor of safety in bearing}$. The critical contact length, $L_c$, for embedded footings took into account the passive, active, and sidewall effects (Gajan et al. 2005):

$$M_{\text{max}} = \frac{L \cdot N}{2} \left[ 1 - \frac{1}{FS_v} \right] + P_p \frac{D}{3} + 2P_p k \frac{D}{3} - P_{act} \frac{D}{3} \quad (4.4)$$

where $P_p$, $P_{act}$, and $P_o$ are the passive, active, and at rest earth pressures; $D = \text{depth of embedment}$; and $k = \text{base/side shear coefficient}$. 
Figure 4.4 A rocking foundation shear wall setup, the critical length, $L_c$, can be seen on the diagram on the right

Figure 4.4 shows the concept of a critical length, this is further discussed in (Gajan and Kutter 2008b) where it uses a more generic term of $L_c$, regarding it as an area ($A_c$) rather than a length. This allows multi-direction, or shaking outside the longitudinal plane of the foundation, to occur. Gajan and Kutter (2008b) also alter the equation to account for embedment, changing Equation 4.2 to:

$$M_{\text{max}} = \frac{N \cdot L}{2} \left(1 - \frac{A_c}{A}\right) + \frac{F_{\text{side}}}{2} \frac{L_c}{A} + \frac{P_p D}{3}$$

(4.5)

where $A$ = foundation area; $A_c$ = critical contact area; $F_{\text{side}}$ = side friction resistance; and $P_p$ = passive earth pressure resistance. The paper mentions that the last two terms in Equation 4.5, the terms with $F_{\text{side}}$ and $P_p$, count for less than 5% of the moment capacity for relatively shallow embedded footings (footing height < $D$). Equation 4.5 is similar to Equation 4.4, but it includes the frictional sidewall resistance and simplifies the end wall resistance from three terms (containing $P_p$, $P_o$ and $P_{\text{act}}$) to one term (containing just $P_p$).

Research by Gajan & Kutter (2008a, 2009b) discuss the SSG04 test series, the rocking shear wall tests performed on sand. The load-displacement behaviour of shallow foundations depends on the moment to shear ratio (related to the aspect ratio of the
wall). For large moment to shear ratios, where the height of centre of gravity is larger than the footing length, a foundation tends to rotate more than slide. As this ratio increases, so does the energy dissipated by rocking as opposed to sliding. The damping ratio ($\xi$) was calculated according to the following definition for estimating the amount of energy dissipated during the rocking (Kramer 1996b):

$$\xi = \frac{1}{4\pi} \frac{\text{Area of moment rotation hysteresis loop}}{1/2(\text{Max moment}) \cdot (\text{Max rotation})} \quad (4.6)$$

The calculated energy dissipation due to rocking can have equivalent damping ratios between 20-30% (Gajan and Kutter 2008a). The capacity of a rocking foundation is dependent on the $A/A_c$ ratio, and from this ratio well defined moment capacities can be assumed. It suggests the design of foundations should accommodate a large $A/A_c$ ratio that has the following positives:

- Allowing uplift to enable mobilisation of a well defined moment capacity that is insensitive to $A/A_c$.
- Considerable amount of energy dissipation from a rocking footing, even from such high $A/A_c$ ratios.
- Minimal permanent settlement.

Figure 4.5 The model used in the JMT02 centrifuge tests, taken from Chang et al. (2006)
Figure 4.5 shows the test setup for the JMT02 centrifuge test series at Davis that combined frame-wall-foundation systems (Chang et al. 2006). The test series focused on an idealised two story two bay planar reinforced concrete frame with an attached shear wall. They were among the first experiments to have reasonably accurate simulations of building nonlinearity with foundation nonlinearity. Test analysis indicated the frame-wall systems have highly asymmetric hysteretic loops due to the asymmetry of the lateral force resisting system. In addition, the moment capacity of a footing is dependent on the vertical load (Equation 4.2), and therefore, a shear wall at the end of a building will behave differently to one in the middle (Chang et al. 2006).

Recently, the research focus at Davis shifted to rocking bridge foundations. Ugalde et al. (2006) tested embedded bridge footings that were attached to an elastic column and then fixed to a lumped mass. The system is considered a typical ‘lollipop’ structure. Similar conclusions about the foundation moment capacity were reached in that it is heavily dependent on vertical load and on foundation length.

Figures 4.6 and 4.7 show the test setups for the LJD01, and LJD02 test series (Deng et al. 2008, Deng et al. 2009). The first series, LJD01 was similar to JAU01 in that it tested ‘lollipop’ structures on shallow foundations. However the second series, LJD02, tested holistic bridge systems where foundations were attached to a dual column bent that was attached to a deck that sat on abutments. In addition, both series introduced yielding columns as well as yielding foundations. Columns were made to a required strength and the foundations were designed to be either stronger, weaker, or match that strength to get an array of different yielding mechanisms. All the columns were notched at one diameter about the foundation to create a weak point where the rotation would occur. This coincided with the distance to the centre of a plastic hinge region of a reinforced concrete column.

Figure 4.8 shows moment-rotation loops from two separate tests during the LJD02 series (Deng et al. 2010). The plots on the left correspond to a smaller foundation and the ones on the right to a larger foundation. In addition, the top row shows the footing rotation where the bottom row shows the column rotation. The dotted line in the column moment-rotation is the capacity of the column recorded during a slow cyclic test. The
smaller footing rotates more and dissipates more energy than the larger footing, and the opposite is happening in the column rotation plots; the larger column rotates more and is closer to capacity. Overall more rotation was observed in the larger footing due to the larger rotation of the column. The plots show that a smaller footing can protect the structure by reducing demands in the column, and also can dissipate greater energy.

Figure 4.6 The setup for the SDOF ‘lollipop’ model in LJD01 centrifuge tests, taken from Deng et al. (2008)

Figure 4.7 The setup for the holistic bridge systems model in the LJD02 centrifuge tests, taken from Deng et al. (2009)
In general researchers performing experiments on rocking foundations have seen the attractiveness, as an alternate design solution, in them. Some of the major findings from past research have been the moment capacity equation that has been tested against numerous experiments. The high damping rocking foundations provide, and the shift in natural frequency relate the benefits of rocking foundations to base isolation systems (Priestley et al. 1978). The chapter will now explore the results obtained through the experiments, and the correlations between the results and past research becomes apparent.

### 4.2 FORCED-VIBRATION TEST RESULTS

The first series of tests performed were forced-vibration tests that used the eccentric mass shaker as excitation. The shaker was placed at the top of the structure and there

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*Figure 4.8 Moment-rotation plots of one test in the LJD02 series, the first row is the footing moment-rotation and the second row is the column moment-rotation, while the first column is the small footing and the second column is the large footing, taken from Deng et al. (2010)*
were three levels of shakes for each test: an initial low-level shake with no eccentric mass in the shaker, a high-level shake with all the eccentric mass in the shaker, and another low-level shake with no eccentric mass in the shaker. In total there were four forced vibrations tests undertaken, however test 3 did not produce any results due to human error. Test 1 was done with no extra vertical load; test 2 and 4 both had 150 kN of extra vertical load. Figure 4.9 shows the test set up for the forced-vibration series. It shows the shaker attached to the top middle of the structure, and the road plates – used as mass – attached either side.

### 4.2.3 Bearing Capacity

Table 4.1 gives the bearing capacity for each forced-vibration test based on the $s_u$ values obtained during construction. The tests were in dryer weather, and subsequently the soil strength and stiffness were high. The average $s_u$ value of foundation 1 was around 118 kPa, and around 180 kPa for the other three foundations. The table shows that the factors of safety for test 1, where no extra mass was added, were 23 and 35. The factors of safety for the other two tests, both with around 200 kN of vertical load, ranged from 5.7 to 8.7. The factors of safety were calculated by calculating the ultimate bearing capacity using Equation 3.20. The soil conditions were treated as undrained, and
therefore the friction angle was ignored and only the undrained shear strength considered.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Foundation</th>
<th>Undrained shear strength, $s_u$ (kPa)</th>
<th>Vertical load, N (kN)</th>
<th>Vertical factor of safety, $FS_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>118</td>
<td>50</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>182</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>118</td>
<td>200</td>
<td>5.7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>182</td>
<td>200</td>
<td>8.7</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>182</td>
<td>200</td>
<td>8.7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>176</td>
<td>200</td>
<td>8.4</td>
</tr>
</tbody>
</table>

### 4.2.4 Input Forcing Function

The eccentric mass shaker was limited to a sinusoidal force input and was controlled by the input frequency and the number of masses inside the shaker. At maximum the shaker could deliver around 100 kN of dynamic load at around 7 Hz. The time it took to complete a test was long (over 10 minutes on average) because the shaker was ramped up slowly through the range of frequencies. Figures 4.10 and 4.11 give the time history input force of the high-level shakes for tests 1 and 4 respectively. Figure 4.10 shows that test 1 had an input force that reached around 14 kN. Ideally this should have been higher but the shaking machine restricted the input force from increasing any further; possibly because of the energy the machine was delivering was at a maximum due to the amount of rotation the structure was undergoing.

Figure 4.11 shows that test 4 had an input force that reached around 100 kN; the maximum capacity of the shaker. However it was observed that during this test the range that produced the peak rocking response was at lower forces. At higher forces, and subsequently higher frequencies, the frame began to behave in a torsional mode, and the rocking was minimal. The time it took to complete the tests were; over 4 minutes for test 1, and over 35 minutes for test 4.
Figure 4.10 The input time history for test 1

Figure 4.11 The input time history for test 4

4.2.5 Moment Rotation

Figures 4.12 and 4.13 give examples of the moment rotation curves for tests 1 and 4. They show only the portion of the time history where the most yielding took place. The red lines on the figures show the predicted moment capacities using Equation 4.2. The green line gives the rotational stiffness based on the small strain soil modulus.
Figure 4.12 Moment-rotation plot of the yielding section for test 1, the red line shows the moment capacity and the green line shows the small strain stiffness.

Figure 4.13 Moment-rotation plot of the yielding section for test 4, the red line shows the moment capacity and the green line shows the small strain stiffness.
Figure 4.12 shows that theoretical prediction of moment capacity matched the experimental moment capacity well during test 1, and was around 48 kNm. The areas of the loops indicate good hysteresis, and during the test the structure had a maximum rotation of around 0.008 radians. The small strain stiffness line shows that, at rotations that are applicable to earthquake engineering, the stiffness of a rocking foundation had reduced a significant amount.

Figure 4.13 shows that test 4 had a slightly greater moment capacity than the prediction, 100 kNm capacity compared to a prediction of 90 kNm. The hysteresis is not as prominent either with the area of the loops a little smaller than test 1. The amount of damping calculated using the moment rotation loops is discussed further below. The maximum rotation reached in test 4 was around 0.003 radians. The maximum rotation is less due to the extra vertical load of test 4 compared to test 1, requiring much larger moments to rotate the heavier structure. Again the theoretical small strain elastic stiffness is substantially greater than the experimental recording at those rotations – this is discussed further below. Both plots show the many cycles this type of test has – due to the eccentric mass shaker having to wind up through frequency ranges. This was one of the reasons why the test methodology was changed to snap-back tests.

### 4.2.6 Rotational Stiffness

Figures 4.14 and 4.15 show the initial recordable part of the moment-rotation curves for test 1 and test 4 respectively. The plots have been smoothed using a Butterworth filter in the frequency domain, but as they show very small rotations, there is a lot of electrical noise present. Figure 4.14 is right on the edge of what could be considered quality data, however more filtering would create more error and so it was decided to show the data as is. Plotted in red is the small strain elastic rotational stiffness calculated by the Gazetas equations described in Chapter 2, and plotted in green is the stiffness of best fit. The plots are from the first definable recordings on the moment rotation curve, previous to this the data was too small and the noise too great to interpret. Figure 4.14 shows that for test 1 the predicted rotational stiffness is greater than the experimental readings. The initial recordable reading has the rotational stiffness at 50% of the theoretical value.
4.2 - Forced-Vibration Test Results

**Figure 4.14** The initial (recordable) stiffness of test 1, with the small strain rotational stiffness and line of best fit.

**Figure 4.15** The initial (recordable) stiffness of test 4, with the small strain rotational stiffness and line of best fit.
Figure 4.16 The steady state moment-rotation of test 1, with the small strain rotational stiffness and line of best fit.

Figure 4.17 The steady state moment-rotation of test 4, with the small strain rotational stiffness and line of best fit.
Figure 4.15 shows similar trends for test 4; however the experimental stiffness is closer to the theoretical value, around 80% of the Gazetas formula.

Figures 4.16 and 4.17 show the steady state moment-rotation plots after yielding had occurred. Again the theoretical stiffness is plotted in red and the fraction of that value that best fits the data in green. The figures both show that there is a substantial reduction in rotational stiffness during the tests. This is consistent with observations made by other researchers; for example Gajan et al. (2005) and Taylor et al. (1981).

During test 1, the stiffness reduced from 50% of the theoretical value at the smallest recordable value to 2.7% at the maximum rotation. During test 4 the stiffness reduced from 80% of the theoretical value at the smallest recordable value to 18% at the maximum rotation. These plots show that for a working rotation level the rotational stiffness is reduced from elastic values, and this should be accounted for in design. Further discussion on rotational stiffness degradation is detailed in the snap-back tests below. Also evident is the fact that the damping was higher for test 1 than it was for test 4, this phenomenon is discussed below.

**4.2.7 Shear**

The measured moment to shear ratio of the structure was around 3.7:1, and as a result the shear was relatively low compared with the moment. In addition the foundations were embedded, so passive earth pressures resisting shear were high. Figure 4.18 shows the shear force against horizontal displacement in test 4. Shear was measured using accelerometers, and the horizontal displacement was measured with the horizontal LVDT’s. Additionally the shear was checked by calculating it from the strain-gauge measurements on the diagonal strut. The graph shows the shear-displacement relationship was linear, and no failure, or nonlinearity, occurred in this mode.

The maximum shear recorded was around 25 kN and the maximum horizontal displacement around 2.2 mm. Dividing the maximum moment by the maximum shear (100 kNm divided by 25 kN) gives a lever arm of 4 m – similar to what the measured lever arm was. The linear relationship is interesting because it shows that all the
nonlinear behaviour occurred in the rocking mode, not a horizontal displacement, or sliding, mode. This result is acceptable when thinking of a realistic foundation; typically embedded with high shear resistance also. A rocking mode is more desirable for a designer than a sliding mode as it has a natural self-centring mechanism that sliding does not provide. Only one sample result is included here because all the tests had similar shear-horizontal displacement behaviour.

4.2.8 Damping

The method for calculating the amount of damping during the tests was taken from Kramer (1996b). This method has been used by researchers in the past to calculate damping in centrifuge experiments (Gajan 2006, Gajan and Kutter 2008a). It uses the area of the moment rotation curves divided by the maximum moment and rotation, and is defined in Equation 4.6. Table 4.2 shows the damping ratios calculated for each foundation of each test. The table shows that damping for the steady state parts of the forced-vibration tests were around 8-15%, with one outlier at 32%. This is encouraging.
because it shows that the damping is high even at a steady state response. Damping that would occur during a seismic event may be greater than these values because foundations will rock at their natural frequency; not forced to rock at a frequency driven by a shaker. This point is evident below where damping of the snap-back tests is presented.

**Table 4.2 The damping calculated for the three forced-vibration tests using the method based on the area of the moment-rotation loops**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Foundation</th>
<th>Damping, $\xi$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>East</td>
<td>31.6</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>13.6</td>
</tr>
<tr>
<td>2</td>
<td>East</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>13.4</td>
</tr>
<tr>
<td>4</td>
<td>East</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>11.2</td>
</tr>
</tbody>
</table>

### 4.2.9 Frequency Response

Figures 4.19 and 4.20 represent the frequency response functions from tests 1 and 4 respectively. They were calculated using a p-Welch spectrum and show the fundamental frequencies of the structure during forced-vibration testing. The plots were normalised with respect to the top of the frame, with the greatest amplitude in the first mode shape (which is the rocking mode).

The left-hand graph of figure 4.19 shows that the initial low-level shake of test 1 had a fundamental frequency around 10.7 Hz. An analysis of a fixed base structure in SAP2000 (CSI 2004) revealed that the structure in test 1 had a fixed base frequency of 7.7 Hz – lower than the fundamental frequency recorded in the tests. This frequency reduced to 2.2 Hz with violent shaking, shown on the right-hand side of Figure 4.19, because the structure was vibrating in the first rocking mode.

Figure 4.20 explains that the fixed base frequency of the structure during test 4 was around 5.9 Hz. The SAP2000 analysis on this structure revealed a theoretical fixed base frequency of 5.2 Hz. Again, the SAP2000 analysis was lower than the tests results suggesting it could be due to different modes being excited that SAP2000 doesn’t consider. For example, the structure could be vibrating in a slightly torsional mode, a higher frequency than the SAP200 model assumed. With the more violent shake, the
natural rocking frequency came in around 2.2 Hz – similar to the previous test. This suggests the frequency of the rocking mode was less dependent on the mass of the structure and more dependent on the vibration force of the eccentric mass shaker.

The Figures 4.19 and 4.20 show a decrease in natural frequency when rocking occurs: from 10.7 to 2.2 Hz, and from 5.9 to 2.2 Hz. This could be a beneficial tool for design when considering spectral accelerations. Lower spectral accelerations could be designed for based on a reduced natural frequency, and therefore design forces would be less. However, if a displacement-based design methodology was used, lower natural frequencies (and subsequently higher natural periods) will increase the spectral design

![Figure 4.19 The frequency response of test 1; left = test 1a (low-level shake); right = test 1b (high-level shake)](image)

![Figure 4.20 The frequency response of test 4; left = test 4a (low-level shake); right = test 4b (high-level shake)](image)
displacement, and design for higher displacements will be necessary.

Figure 4.21 shows the frequency response functions from test 2, with around 200 kN of vertical load on the structure. Again both plots have both been normalised against the displacement of the top of the frame. The plot on the left shows the first small level excitation, prior to the large level excitation, and the plot on the right shows the second small level excitation, which followed the large level excitation. They confirm that the fundamental frequency of the system changes slightly due to the large level shake in between. The natural frequency reduces from 6.1 Hz to 5.8 Hz. Therefore, the large level excitation results in small material yielding, which alters system characteristics. This reduction in natural frequency will result in a reduction of stiffness (of around 10%) that could be detrimental to the performance of the structure and should be accounted for.

4.2.10 Settlements

Historically, settlements and rotations have been a limiting factor to the performance of rocking foundations both during rocking and with residual amounts at the end of rocking (Gajan et al. 2005). Residual rotations are typically minimized by the natural self centring mechanism of rocking foundations, but large settlements can cause problems for a superstructure.
Both static and dynamic settlements were measured throughout the testing program. A theodolite-type level recorded static settlements, while dynamic settlements were calculated using the average vertical LVDT readings from each foundation. The time for static settlement varied depending on test schedules, but the structure was left to settle for more than 3 days after each test. Observation suggested most settlement took place in the first 24 hours.

Settlement before and after test 1 was minimal because the $FS_v$ was sufficiently high. The static settlement that occurred prior to the test two was around 12 mm, after the 150 kN was loaded onto the structure but before the test began. The static settlement prior to test three (this test did not produce accurate results) was around 5 mm. The settlement prior to test 4 was only around 2 mm. Test 3 and test 4 occurred on the same
foundations so the total static settlement for that set was 7 mm.

Figures 4.22 and 4.23 outline the settlement time history and settlement-rotation curves for tests 1 and 4. The dynamic settlement in both tests was less than 1 mm, an order of magnitude less than static settlement. The settlement-rotation curves are typical of a rocking foundation that does not induce much soil yielding: the rounded, upward-tilting edges suggest a large amount of uplift and little material yielding. The settlement and rotation was calculated around the centre of the foundation. The settlement behaviour depicted here can be beneficial for rocking foundation design. Designers will be more inclined to use rocking foundations as a viable design option if they are certain dynamic settlement (that occurs during a seismic event) will be less than static settlement.

4.2.11 Pressure Sensors

The pressure sensors did not produce any satisfactory results, as was mentioned in Chapter 3. Figure 4.24 shows the time history of two working sensors in test 1. The graph indicates a clear difference in the change in pressure between the two sensors after the test. However, these were the only sensors that worked on that particular foundation (out of 12), and it was impossible to determine the gap that opened between the soil and foundation. The results from the pressure sensors in subsequent tests deteriorated and the results were deemed unsatisfactory.

![Figure 4.24 Two time history plots of the pressure sensors for test 1 – these were the only two working sensors](#)
4.3 SNAP-BACK TEST RESULTS

While the forced vibration tests do excite the foundations enough to rock, they were not the most realistic representation of an earthquake. First, this is because of the hundreds it took for the machine to vibrate at the desired force, thus requiring long test periods. This point is illustrated in Figures 4.12 and 4.13 where a multitude of cycles were recorded. Second, the shaker vibrated at relatively constant frequencies, which are unrealistic in seismic behaviour. Therefore, snap-back tests were conducted to generate results that circumvent these issues.

A snap-back is a comparatively short dynamic test, which allows the structure to rock at its natural rocking frequency. The test summary table in Chapter 3 (Table 3.11) details all the tests performed. Several key issues from the results are discussed below. The photo in figure 4.25 shows the frame setup on new foundations. From the photo it is evident that tests were performed in the wet season, and the saturated soil left pools of water around the frame.

![Figure 4.25 The setup of the frame on a new set of foundations; the site was saturated, shown by the water pooling on the ground surface](image)

4.3.12 Bearing Capacity

Factors of safety were calculated based on shear vane results using the modified bearing capacity equation (Equation 3.20). The unit weight of the soil, $\gamma$, was considered as 17
kN/m³ and the soil friction angle, $\varphi_s$, as zero. The original shear vane tests were inappropriate for these calculations because soil conditions were different, and a second set of hand shear vane results were used to generate factors of safety. However, the vane results show that the soil was still very stiff and had average $s_u$ strengths of around 150 kPa. From these $s_u$ values, the $FS_v$ for bearing capacity were calculated based on the vertical load of the structure. These are given in Table 4.3 below.

**Table 4.3 The vertical static factor of safety in bearing calculated based on hand shear vane values**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Foundations</th>
<th>$s_u$ - kPa (average)</th>
<th>Vertical load, N (kPa)</th>
<th>$FS_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>182</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>176</td>
<td>50</td>
<td>53</td>
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<tr>
<td>6</td>
<td>6</td>
<td>171</td>
<td>200</td>
<td>10.8</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>103</td>
<td>200</td>
<td>6.5</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>171</td>
<td>260</td>
<td>8.2</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>103</td>
<td>260</td>
<td>4.9</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>152</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>135</td>
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<tr>
<td>9</td>
<td>5</td>
<td>152</td>
<td>260</td>
<td>7.3</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>135</td>
<td>260</td>
<td>6.5</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>152</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>135</td>
<td>50</td>
<td>41</td>
</tr>
</tbody>
</table>

The table shows that the factors of safety ranged from 55 to 4.9. Factors in the 40s and 50s (test 5, 8, and 10) were considered elastic. These tests were performed with no extra mass, so the vertical load came from the structure’s own weight, around 50 kN. Test 6 had 10 billets sitting atop the structure, around 200 kN of vertical load. The corresponding factors of safety were 10.8 and 6.5 respectively. Tests 7 and 9 had around 260 kN of vertical load with 14 billets. These tests had the lowest factors of safety, ranging from 8.2 to 4.9.

The undrained shear strengths calculated from CPT data (see Chapter 3) had a positive correlation with the hand shear vane results. The plots for all the CPT data, including the $s_u$ values, can be found in Appendix A. Figure 4.26 gives two example $s_u$ plots with depth. They are from CPT10 and CPT15 and correspond to foundation 6 (tests 6 and 7). The average $s_u$ value calculated by hand shear vane for that foundation was 171 kPa, and the $s_u$ values indicated on the graphs are between 100 and 200 kPa around 0.4 m deep.
4.3.13 Moment Rotation

Two types of moment capacities were obtained in snap-back testing: one from the load cell recording during the initial pull back, and one from the strain gauges during rocking. The load cell calculation can be thought of as a static moment-rotation, and the strain gauges give the dynamic moment-rotation curves.

4.3.13.1 Static Moment Rotation

Figures 4.27 and 4.28 depict the static moment rotation curves of test 7 and test 9 during the application of the pull-back. There is considerable nonlinearity in the moment rotation curves, and the initial stiffness is reduced from one snap to the next.

Figure 4.27 shows that the snaps leading up to the largest (snap 6 – the black line) lose stiffness with each consecutive snap, but they remain relatively constant on the way back down after the largest snap (snaps 7 through 9). This suggests there is rounding of
the soil under the foundation, which increases with increasing rotation and leads to a
decrease in the initial rotational stiffness. Snaps 4 and 5 apparently are exceptions as
both have less rotation stiffness than snap 6. This is yet another example the variability
of residual soil. The maximum snap (snap-back 6) follows a hyperbolic shape to a
capacity of around 90 kNm, and rotates to around 0.0276 radians.

Figure 4.28, the static pull over moment-rotations for test 9, demonstrates some
important differences with Figure 4.27. The snap-backs leading up to the largest snap-
back (snap-back 6) again show decreasing rotational stiffness as a consequence of
material yielding under the edges of the foundation during previous snaps. However
snap-backs 7 and 8 show similar decrease in rotational stiffness, unlike what was seen
in test 7 where snap-backs 7 and 8 displayed the same stiffness. Snap-back 9 appears to
be stiffer, following the same curve of snap-back 6, however the test was very small and
so is fairly inconclusive. The shape of these curves is similar to a typical backbone
curve for nonlinear soil response (Boulanger et al. 1999). These backbone curves are
subsequently used for numerical models of shallow foundations and can be used to
calibrate other, more complex, finite element models.

4.3.13.2 Dynamic Moment Rotation

Figures 4.29 and 4.30 are dynamic moment-rotations recorded by strain gauges attached
to the frame’s steel columns alongside the static moment-rotations for each test. The
comparison between the static and dynamic moment-rotation is good and they both
have similar moment capacities. The dynamic plots show the motion was damped out
within a couple of cycles. The gyre indicates the movement of the structure and shows
how the moment decreases with each rotation. The loop area indicates a large amount of
energy dissipation, which is discussed further below. The scale of each test was left
unchanged for comparison. Unfortunately, the strain gauges became ‘tired’ throughout
the extensive experimental program, so the later snap-back tests started to produce
faulty results. Consequently, not all dynamic moment-rotations were accurate, but the
tests that did produce accurate moment-rotations are included in Appendix D: critical
plots of the snap back test series.
Figure 4.27 The pull back moment-rotation curves from all snaps in test 7

Figure 4.28 The pull back moment-rotation curves from all snaps in test 9
4.3 - Snap-Back Test Results

Figure 4.29 Dynamic moment-rotation for test 7, snap back 4

Figure 4.30 Dynamic moment-rotation for test 7, snap back 5
4.3.13.3 Curve Fitting

The static moment-rotation curves were fitted with curves using Matlab’s nonlinear regression function that enables an equation to be fitted to an arbitrary curve (MathWorks 2009). Producing equations that represent the static moment rotation curves given in Figures 4.27 and 4.28 would allow a designer to choose a rotational stiffness based on what rotations are expected, and thus proceed with designing for rocking foundations. Sullivan et al. (2010) say this particular topic needs to be researched further, and the results from the static moment rotation curves gives us an insight into how much stiffness degradation takes place during rocking.

Kondner (1963) gives an equation for a hyperbolic stress strain relationship for cohesive soils and describes the factors contributing to the curve. The form of the stress strain curve is:

\[ Q = \frac{\varepsilon}{a + b\varepsilon} \]  

(4.7)

where \( Q \) = shear stress; \( \varepsilon \) = axial strain; and \( a \) and \( b \) are parameters for the hyperbolic curve, given below.

Figure 4.31 The stress strain curves that give the parameters \( a \) and \( b \). The actual curve (left) transformed to estimate parameters for stress strain behaviour (right)
Although these parameters are for stress-strain behaviour they can be applied to moment-rotation behaviour. The parameter with initial slope $a$ is related to the initial stiffness of the system, and the parameter that specifies the horizontal asymptote $b$ relates to the moment capacity of the system.

Figure 4.32 presents the static moment-rotation curves from test 7 (same as Figure 4.27) along with upper and lower bounds from the curve fitting. Figure 4.33 is a close up of those upper and lower bounds compared with snap-backs 1 and 9 respectively. The curves were created with an initial elastic part followed by actuating the nonlinear regression at some defined moment – the point when initial nonlinearity occurs. The curve functions capture the static moment rotation distribution recorded in the experiments. The initial elastic part of the curve was around 60% of the Gazetas formula for the upper bound and around 20% for the lower bound. Therefore, the ultimate moment capacities were 110 kNm for the upper bound and around 85 kNm for the lower bound.

![Figure 4.32 The upper and lower bound of the fitted curves, with the 9 snap-backs of test 7 plotted in grey](image-url)
Figure 4.33 A close up with of the upper (left) and lower (right) bound fitted curves for test 7

Figure 4.34 The upper and lower bound of the fitted curves, with the 9 snap-backs of test 9 plotted in grey

Figure 4.34 displays the upper and lower bounds of the curve fitting for test 9 alongside all test 9 snap-backs. Once again the nonlinear behaviour can be captured by the equation. The upper bound had an initial stiffness around 80% of the Gazetas value and a moment capacity of 110 kNm – the same as test 7. The lower bound had an initial
stiffness around 50% of the Gazetas value and a moment capacity of around 85 kNm – also aligning with test 7.

4.3.13.4 Moment Capacity Equation

The $s_u$ values were back calculated using moment capacities predetermined by the static moment-rotations. It was previously shown that the moment capacity equation works well both for the forced-vibration tests and other research (Deng et al. 2010, Gajan 2006, Gajan and Kutter 2008a, Gajan and Kutter 2008b, Gajan et al. 2005). In addition, $s_u$ values obtained from hand shear vanes were highly variable and were not considered a reliable measure of shear strength. Table 4.4 lists the moment capacity of each test determined with Equation 4.2, including the curve fitting function, the factor of safety, and measured undrained shear strength. Tests 5, 8, and 10 had moment capacities close to the overturning moment for rocking on a rigid surface. Thus the factors of safety were very high, and the infinity symbol is used here to show the $FS_v$ in these cases were not calculated.

The curve fitting calculated the moment capacity of test 7 and test 9 as 110 kNm, and the maximum moments recorded were 90 and 85 kNm respectively. However, the maximum recorded moment came from snap-back 6, and permanent soil deformation had already occurred, accounting for the difference. As mentioned above, the moment capacity reduced to 85 kNm by the end of each test, meaning a loss of strength around 23%. A 1984 version of the New Zealand earthquake standard, NZS 4203:1984, stated that plastic hinges may form if the horizontal load carrying capacity of the structure was not reduced by more than 20% (Standards New Zealand 1984). The strength reduction from rocking foundations on bearing capacity is assumed to be less than 20% because the 23% reduction occurred across 9 snaps, inducing more damage than an earthquake.
Table 4.4 The moment capacities determined by the curve fitting, the vertical factor of safety, and the corresponding undrained shear strengths

<table>
<thead>
<tr>
<th>Test</th>
<th>Moment Capacity, Mult (kNm)</th>
<th>Vertical Factor of Safety, $F_S$</th>
<th>Undrained Shear Strength, $s_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
<td>$\infty$</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>10.0</td>
<td>165</td>
</tr>
<tr>
<td>7</td>
<td>110</td>
<td>6.5</td>
<td>140</td>
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<tr>
<td>8</td>
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<td>9</td>
<td>110</td>
<td>6.5</td>
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<td>10</td>
<td>25</td>
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</tbody>
</table>

4.3.14 Rotational Stiffness

Figure 4.35 compares the static moment rotation curves to the theoretical elastic rotational stiffness predicted with the small strain shear modulus for test 7. The figure shows six curves representing the six snaps from test 7, and a straight line that represents elastic rotational stiffness. The plots are enlarged sections of the moment rotation curves in Figure 4.27 above. The yellow line is the unfiltered data; the blue line is filtered with a low pass Butterworth filter to remove the noise; and the black line is the theoretical stiffness predicted by the Gazetas equations.

The initial stiffness reduces throughout the snaps because of soil rounding. This phenomenon was mentioned above but can be seen more clearly in these plots. The stiffness of each black line was reduced by reducing length $L$ in contact with the footing. Each plot uses the value $L$ to calculate stiffness. The initial contact length was 2.0 m – the total length of the foundation. A constant shear modulus of 30 MPa was used throughout, the lower end of the measured small strain modulus. This suggests that even at the initial stages of foundation rotation an ‘operational’ shear modulus may be necessary to capture foundation behaviour. The plots indicate the contact length decreases to around 1.5 m after the final tests are completed. Given that the rotational stiffness is related to the second moment of area $I$, the stiffness reduced to 52% of the original stiffness—a substantial reduction.
Figure 4.35 Close up views of 6 of the 9 snap-backs from test 7 with the small strain rotation stiffness. The snap-back number is labelled on each one along with the foundation length used to calculate the small strain rotation stiffness.

The soil rounding apparent in the snap-back tests may be greater than expected for foundation behaviour during an earthquake. This is because the static nature of the pull...
backs are much more damaging to the soil underneath the edges of the foundation than brief instances of bearing strength failure that occur in an earthquake. Pender et al. (2009) suggested that brief instances of bearing strength failure during an earthquake may not be detrimental to foundation characteristics. Soil rounding is a concern to the designer as it decreases the contact between soil and foundation, thus reducing both strength and stiffness.

Figure 4.36 is a closer view of the initial rotational stiffness from snap 3 and snap 7 during test 7. At the origin the theoretical stiffness corresponds well with the experimental data. However, degradation occurs rapidly during the pullback, indicating the inclusion of an operational shear modulus may be used for design. This trend was evident in all slow pull backs from the tests.

### 4.3.15 Damping

Throughout the snap-back tests the structure, once released, only rocked for one or two oscillations before it came to rest, which is evident in the dynamic moment rotation plots. This indicates a high-level of damping, therefore various methods for calculating damping coefficients were used. Out of four methods to calculate damping, Coulomb damping, viscous damping, and damping from moment rotation loops and damping
from the logarithmic decrement, only the last method was useful in the snap back tests. The other methods, and why they were not applicable, are explained below.

### 4.3.15.1 Coulomb Damping

Coulomb damping results from sliding friction on two dry surfaces. The friction force $F_f = \mu_f N$, where $\mu_f$ is the coefficient of friction and $N$ is the force normal to the surface. The friction force is assumed to be independent of velocity once the motion is initiated (Chopra 2007). Coulomb damping results in a linear decay of amplitude with time. Figure 4.37 explains how Coulomb damping captures the system damping, taken from Chopra (2007). Thus, Figure 4.38 shows the decay of one rotational time history, where a linear line from one peak to the next can be seen in red. The line establishes that the damping of the system cannot be captured with a single line, and therefore estimating the damping by Coulomb’s criteria is inappropriate.

![Figure 4.37 Coulomb damping (linear decay)](image)
Figure 4.38 Rotational time history of test 7, snap-back 6. It shows that Coulomb damping is an inappropriate way of calculating damping for these tests.

4.3.15.2 Viscous Damping

The viscous damping technique is a little more applicable to rocking foundations because it follows an exponential decay of amplitude. The damping curve can be seen in Figure 4.39 (Thomson 1993) and follows the equation:

\[ X \cdot e^{-\xi \omega_n t} \]  \hspace{1cm} (4.8)

where \( X \) = initial amplitude; and \( \omega_n \) = natural frequency of the system.

A viscous damping criterion was attempted for the same rotational time history plot. The match is better than Coulomb damping in Figure 4.40, but the viscous damping technique cannot capture the amount of damping after the first cycle. The viscous damping method is best for a system with a constant natural frequency (\( \omega_n \)), but the mechanics of rocking is such that the frequency of the system is dependent on the amplitude. Every quarter-cycle has a different frequency than the next, and the frequency increases with decreasing amplitude.
Figure 4.39 Idealised viscous damping (exponential decay)

Figure 4.40 Rotational time history of test 7, snap-back 6. It shows that viscous damping is also an inappropriate way of calculating damping for these tests

4.3.15.3 Damping Based on Moment-Rotation Area

Measuring damping based on moment-rotation area is accurate for calculating the damping during the forced-vibration tests, but less accurate for the snap back tests. The nature of the motion during the snap-back tests means there is no clear loop on the
moment-rotation plot where the area can be calculated. Consequently, this method was not considered in calculating the damping for the snap-back tests.

### 4.3.15.4 Logarithmic Decrement

The logarithmic decrement was judged the best way to calculate damping for the snap-back tests. The equation uses the peaks from two concurrent cycles, and therefore can be used on a system with changing natural frequency. The formula for calculating the damping is given below (Chopra 2007, Thomson 1993):

\[
\delta = \ln \frac{x_1}{x_2}
\]

and:

\[
\delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}
\]

From these the damping coefficient, \( \zeta \), can be calculated by solving Equation 4.10.

Figure 4.41 plots the damping values against the amplitude of rotation from all snap-back tests using the logarithmic decrement method. The most important feature is the significant amount of damping, ranging from around 0% to around 50%. Another notable feature of this plot is the degree of scatter; there is no definable relationship between the amount of damping and the amplitude of rotation. Gajan and Kutter (2008) reached similar conclusions in their work:

*Though there is significant scatter in data, it is interesting to observe that as long as yielding is significant (i.e., rotations are greater than 0.002), the damping ratio is not very sensitive to the amplitude of rotation for slow lateral cyclic tests.*

They deduced that that any rotation greater than 0.002 radians had significant damping and was not sensitive to the amplitude of rotation. A similar conclusion can be drawn from Figure 4.41, but significant damping was found from rotations of around 0.0003 radians (seven times smaller than Gajan and Kutter found).
Figure 4.41 Damping values against half amplitude of rotation of all the snap-back tests

Figure 4.42 The damping ratio that occurred on each impact during the snap-back tests
Figure 4.42 gives the damping values against the impact number that they were calculated on. The figure shows that there was significantly more damping occurring during impact 0 and 2 than impact 1 and 3. The reason for this is believed to come from soil damage that occurred during the initial pull back part of the test that would reduce the damping. As the structure is being pulled over, that particular edge of the foundation induces a large amount of damage to the soil underneath. When let go the foundation rocks back and the impact is occurring on the other, non damaged, edge. This would correspond to the values for impact 0. Subsequently the foundation rocks up onto that edge and the falls back down on the more damaged side – impact 1. Thus all the odd impacts occur when the foundation is impacting on the particular edge that was more damaged through the pull back of the structure, which would account for this difference in damping. During a real earthquake the damping will behave more like what occurred on impact 0 and 2 – because the soil beneath the edges of a foundation will not have been damaged previously.

Figure 4.43 shows examples of the rotational time history for tests 8 and 10. The fact that the system is critically damped can be seen as no oscillations occur. This is the reason why damping from these tests were not included in the previous plots. During the testing, it took quite a lot of time to pull the structure over statically because this was being done by a hand pump. As a consequence water that was pooled on the surface flowed into the cracks between the foundation and the soil. When released, the structure
had to eject this from the underside of the footing, and the water behaved in a similar manner to what a shock absorber in a car might. This is somewhat unrealistic due to the amount of water that would flow into the foundation compared to what would occur in a building foundation during an earthquake, and a large effort was made on the day of testing to mop up the excess water on the ground surface. Nevertheless water still found its way down to the underside of the foundation with each pull back, and little rocking occurred during these tests.

Figure 4.44 gives the normalised rotation against the impact number for all the tests. The rotation was normalised to \( \alpha \); the angle between the corner of the foundation to the centre of mass of the structure. Housner created a relationship between the energy reduction value, \( r \), the number of impacts, and the amplitude of rotation. The equation takes on the form:

\[
\theta_n = 1 - \sqrt{1 - r^n[1 - (1 - \theta_{n0})^2]}
\]  

(4.11)

where \( \theta_n = \) normalised amplitude of rotation; \( r = \) energy impact reduction value; \( n = \) number of impacts; and \( \theta_{n0} = \) initial normalised amplitude of rotation.

Figure 4.45 gives \( \theta_n \) plotted against the number of impacts, \( n \), for some snap-backs in test 7. Equation 4.11 above is also plotted for each initial amplitude in black. The \( r \) value used in the equation was set to 0.4. The figure shows that a constant \( r \) value of 0.4 can capture the reduction of rotation after each impact. This suggests that there is relatively constant damping per impact. This is interesting because damping and energy dissipation have been thought of as being highly nonlinear – as the motion is – however it is shown here that it remains relatively constant.

To obtain an equivalent viscous damping ratio we can use the logarithmic decrement that was used for the experiments to obtain a rotation at subsequent impacts. This method was outlined in Priestley et al. (1978). For a comparison the \( r \) value of 0.4 was around 28% of equivalent viscous damping. This is relatively independent on the size of the initial rotation, \( \phi_0 \). The method was to use Equation 4.11 above to calculate the rotation after subsequent impacts. Then these rotations would be fed back into the damping calculation of Equations 4.9 and 4.10.
Chapter 4 - Experimental Results

Figure 4.44 The normalised rotation against the number of impacts during all the tests

Figure 4.45 The normalised rotation against the number of impacts during test 7
4.3.16 Natural Rocking Frequency

The mechanics of a rocking body is such that the frequency it rocks at is dependent on the amplitude of rotation (Chopra and Yim 1985, Housner 1963). Housner first came up with an equation that predicts the quarter period (the period between maximum rotation to zero rotation) for a rocking body on a rigid surface – Equation 2.13. Figure 4.46 shows a plot of the equation, it plots the period against rotation normalised with respect to $\alpha$ (the angle at which overturning would occur). This was calculated at 0.3 radians for the structure. The equation follows a hyperbolic cosine function with an x axis asymptote at 1 – when $\theta/\alpha$ is close to unity the period is long, and when $\theta/\alpha$ is close to zero the period is short (Housner 1963).

Figure 4.47 shows an enlarged modified section of the previous plot, individual points from all the snap back tests are plotted as well. The Housner equation is slightly altered by adjusting the y intercept from 0 to 0.15. As the figure shows, there is a good relationship between the equation and the experimental results, up to a normalised rotation of 0.12. Also given on the graph were results of an analysis of the structure done in SAP2000 (CSI 2004) as a fixed base system. Figure 4.48 shows a schematic of the SAP2000 model that was created. The SAP2000 model was a fixed base model created as close to the experiment as possible. Steel members were modelled exactly as fabricated, and the mass was modelled as steel sections (like the billets in the snap-back tests) and fixed to the top of the structure. The half period calculations were; $T/2 = 0.096$ for test 6, and $T/2 = 0.106$ for test 7. The shortest half period recorded was around twice that of the fixed base half period. No half periods below 0.2 seconds were recorded during the tests because these low values were damped out; it only took two cycles for the motion to be completely damped out and thus the low amplitude cycles, and consequently high frequency, were minimal. From this data it could be concluded that rocking foundations increase the period by at least twice on any cycles that have recordable amplitude. Again this is positive when considering forced based design of rocking foundations as an increase in period results in a decrease of the spectral acceleration value, hence foundations can be made for smaller forces and are more economical.
Chapter 4 - Experimental Results

Figure 4.46 Housner’s function for the period of a rocking structure on a rigid surface

Figure 4.47 An enlarged section of Housner’s function, with periods from all the snap-back tests, plus fixed base periods from the SAP2000 analysis
4.3.17 Settlement

The settlement throughout the testing program can be broken down into static settlement and dynamic settlement, as was the case for the forced-vibration tests. There were three sets of static readings taken from the snap back testing; one from pre and post loading of test 6 (around 200 kN of vertical load); one from pre and post loading of test 7 (around 260 kN of vertical load); and one from pre and post loading of test 9 (around 260 kN of vertical load). The settlement of test 6 that occurred because of the vertical load was around 12 mm. The settlement of test 7 that occurred because of the extra 60 kN of vertical load (this test was performed on the same set of foundations as test 6) was around 2 mm. The settlement of test 9 under the vertical load was around 10 mm. The small differences can be attributed to the high variability of the residual soil. Figures 4.49 and 4.50 plot both the settlement time history and the settlement-rotation for two snap-back tests; snap-back 5 from test 7 and snap-back 6 from test 9. Both the settlements and the rotations were calculated about the centre of the foundation. There is no evidence of any settlement during the dynamic part of the tests, shown in the time history plots. The settlement-rotation plots show again how little rotation occurred on the second half cycle compared to the initial pull back because of high amounts of damping, mentioned above.
4.3.18 Energy Analysis

Figures 4.51 through 4.56 show phase plots of test 6, test 7, and test 9 respectively. Figures 4.51, 4.53, and 4.55 show selected phase plots from those tests to give an indication of the motion across the whole test. Figures 4.52, 4.54, and 4.56 give the phase plots from the largest snap-back in each test; snap-back 4 from test 6; snap-back 6 from test 7; and snap-back 6 from test 9. The rotational displacement was taken from the LVDT readings and the rotational velocity was calculated by integrating the rotational displacement.
4.3 - Snap-Back Test Results

Figure 4.51 Selected phase plots of test 6

Figure 4.52 Phase plot of snap-back 4 in test 6; blips from higher mode effects are circled in black
Figure 4.53 Selected phase plots of test 7

Figure 4.54 Phase plot of snap-back 6 in test 7; blips from higher mode effects are circled in black
4.3 - Snap-Back Test Results

Figure 4.55 Selected phase plots of test 9

Figure 4.56 Phase plot of snap-back 6 in test 9; blips from higher mode effects are circled in black
Across all three tests the point of maximum velocity did not correspond to zero rotational displacement. In a perfectly elastic phase diagram the maximum velocity would occur at the same point as the zero rotation and vice-versa. In all cases it occurs on the left-hand side of the zero line, therefore the maximum velocity occurred when the foundation is still on its first quarter cycle (the path from maximum rotation to impact). This is most prominent in test 9. It suggests that instead of a sudden ‘impact’, the foundation increases soil contact over the whole cycle, and when it gets to zero rotation, the velocity has already been reduced. Again this is evidence of the significant damping within the system.

The blips in the phase plots – circled in Figures 4.52, 4.54 and 4.56 – can be attributed to higher mode affects that were generated during the snap-back tests. This could be due to a few causes: a) the structure may not have been pulled back exactly straight each time, thus instigating some torsion in the motion; b) the billets may have not been perfectly secured to the top of the structure, the movement of billets would induce a mode different to the predominant mode; c) the structure may have higher mode effects from the difference in soil conditions from one foundation to the other – having one foundation stiffer due to a stiffer soil would mean that the natural frequency would not be identical and higher modes may be introduced.

4.4 DISCUSSIONS AND CONCLUSIONS

This chapter presents test results from large scale field experiments performed on a site in Auckland. It presents key findings from the two series of tests; forced-vibration tests and snap-back tests. The chapter before this details the experiment set-up and soil properties that were necessary to make the valid conclusions about the results.

Undrained shear strength’s obtained from the hand shear vanes were variable, and although gave good indication of a ‘ball-park’ figure, more rigorous form of soil testing should be undertaken for rocking shallow foundation design. This could either be insitu testing – such as CPT tests – or laboratory testing – such as triaxial tests. It is very important to be able to predict the moment capacity of a rocking foundation, so a proper
design of the super-structure can take place, and thus undrained shear strength values should be accurate.

Figure 4.12 shows a well defined moment capacity of around 48 kNm, and Figure 4.13 shows a moment capacity of around 100 kNm; although somewhat less well defined. Figures 4.12 and 4.13 also show the moment capacity equation can predict the capacity of a foundation with sufficient accuracy. This is consistent with what past research has found also (Deng et al. 2010, Gajan 2006, Gajan and Kutter 2008a, Gajan and Kutter 2008b, Gajan et al. 2005). The undrained shear strength values that were back calculated from the moment capacities of the snap-back tests were consistent with the hand shear vane data – although as mentioned above the hand shear vane data was highly variable. This equation gives a good benchmark for performing designs of rocking foundations. For example, if specifying a rocking foundation, a designer would have to ensure the super-structure was stronger than the foundation so that rocking occurred before structural yielding. Consequently the capacities of both super-structure and foundation have to be obtained. The ultimate strength of structures and structural elements can be accurately predicted, and the moment capacity equation is a good way to do the same with the foundations, so that a correct design can take place.

Figures 4.29 and 4.30 show that the static and dynamic moment-rotations during the snap-back tests correlate well. The static moment was calculated using load cell forces and converted into moments by multiplying them by the lever arm. The dynamic moment was calculated by Equation 3.20 using the stain recorded in the strain gauge bridges attached to the end frames. The dynamic moment-rotation loops display good energy dissipation and the motion was damped out after two or three cycles. Unfortunately during the final phase of the snap-back tests the strain gauges became somewhat tired and started to produce inconsistent results.

The equation used to fit curves to the static snap-back moment-rotations captured the upper and lower bounds of the behaviour well. In addition this equation (Equation 4.7) is attractive because of its simplicity; two user defined inputs that relate to the initial stiffness and moment capacity. The comparisons to the test results from test 7 and 9 show that the initial stiffness is somewhat reduced from theoretical stiffness values.
predicted by Gazetas. Thus an ‘operational’ stiffness could be introduced if wanting to create static moment-rotation curves. The moment capacities predicted by the curve fitting were similar, reinforcing the need to have accurate soil strengths and moment capacities.

The stiffness reduction due to the foundations rotating was significant for both series of tests. The steady state forced-vibration stiffness reduced to around 3% and 18% for tests 1 and 4 respectively. In the snap-back tests, the static moment-rotation plots given in Figures 4.27 and 4.28 also show significant reduction in stiffness. This stiffness reduction is from two mechanisms; one from contact length reduction as the foundation lifts off and one from material yielding under the edges of the foundation. Stiffness reduction due to uplift can be easily modelled as an elastic system using a bed of springs, and if the factor of safety is sufficiently high this would be the preferred option. If the factor of safety is lower however, the model must incorporate soil nonlinearity. This is further discussed in the next chapter that describes the models developed to predict behaviour of soil-foundation systems.

The shear sliding graph presented in Figure 4.18 shows that no nonlinear behaviour was from a sliding mode. The rocking mode was the predominant mode which nonlinear action occurred. The high shear to moment ratio of around 4, coupled with the large sliding resistance from passive earth pressure of the embedded footings, gives an understanding as to why all the nonlinearity occurred in rocking. In terms of design, most shallow foundation applications will not have an issue for sliding because of the passive earth pressure resistance from embedment. However designers that are specifying rocking foundations must check that this won’t be an issue. In some cases – such as foundations sitting on the ground surface – sliding may be the critical form of motion for a shallow foundation.

The calculated damping for the three forced-vibration tests 1, 2, and 4 were not related to the amount of load on the structure. This may be counter-intuitive, because greater mass would result in greater soil yielding, and hence greater damping. However the test with the largest amount of damping – test 1 – had the smallest vertical load. The moment-rotation plots of the steady state motion, Figures 4.16 and 4.17, show ‘fatter’
loops for test 1 than test 4 – and hence greater damping. Taking an average of the damping values given in Table 4.2 the damping values were: $\zeta = 22.6\%$ for test 1, $\zeta = 14.1\%$ for test 2, and $\zeta = 10.4\%$ for test 4. Figure 4.13 indicates that the moment capacity of test 4 had not quite been reached. It does not display a clear moment capacity like it does in test 1 (Figure 4.12). This could explain the reduced damping – the foundations were not able to rotate enough to get material yielding and hence greater damping. The lesser rotation experienced by the foundations in tests 2 and 4 were due to the eccentric mass shaker not being powerful enough to generate the required rotation with the amount of vertical load on the structure (around 200 kN). The original proposal was to perform these forced-vibration tests with two, in-synced, shakers running that would deliver twice the capacity. However they were not able to be run together properly, and after countless attempts to get them working, that idea was abandoned. It was one of the major factors in deciding to perform snap-back tests as well; to achieve more material yielding and greater rotations.

Figure 4.41 shows that the damping is not necessarily related to the amount of rotation. This is consistent with what some past research has found. Damping appears to be significant when rotation is above 0.6 milliradians and should be considered for all rocking foundation applications. Figure 4.42 shows that the damping was lower on even impacts than on odd impacts. This was the result of soil damage occurring on one edge during the slow pull back part of the tests.

Figure 4.45 shows that the experiments produced a constant energy dissipation value ($r$ value) of around 0.4. Although Housner created this $r$ value for impact damping only – a rigid block rocking on a rigid surface – it appears to still hold true for the experiment damping. This again could point to what is mentioned above; the damping appears to be unrelated to amplitude of rotation. The damping remains constant even though on each impact the rotation is reducing, which would point to constant energy dissipation. This $r$ of 0.4 was shown to equate to around 28% damping using Housner’s equation as well as the logarithmic decrement method to calculate damping.

Figure 4.21 shows that reduction in the natural frequency due to a large excitation was not large. The natural frequency reduced from 6.1 Hz before the large shake to 5.8 Hz.
after the large shake. Using Equation 4.12 below the reduction in stiffness that occurred during test 4 was around 10%.

\[ \omega = \sqrt{\frac{K}{m}} \]  

(4.12)

where \( \omega \) = natural frequency of the structure; \( K \) = stiffness of the structure; and \( m \) = mass of the structure.

Excessive settlement did not occur during either the forced-vibration or snap-back tests. This is evident by the settlement time history and settlement-rotation plots in Figures 4.22 and 4.23 for the forced-vibration, and Figures 4.49 and 4.50 for the snap-back tests. In both cases the static settlement, that occurred onsite from the vertical load applied to the foundations, was greater than settlements recorded dynamically during the tests themselves.

The phase plots in Figures 4.51 to 4.56 show some insight into the system behaviour as well. They show the point of maximum velocity did not correspond to zero displacement. This indicates the impact of the foundation on the ground surface occurred over a time, and was not instant like might be expected for rocking on a rigid base. The figures also show some interesting higher order affects occurring during the tests. This could potentially be from the imperfect nature of the pull backs, or how the mass was held to the structure, or it could be a result of how highly variable the soil was.

### 4.4.19 Limitations

There were several limitations on what was being recorded during the tests because of the instruments. During the forced-vibration tests the pressure sensors introduced produced poor results and were discarded. During the snap-back tests there were problems with the load cell and the strain gauges. During test 6 the load cell began producing faulty results; however this fault was fixed for subsequent tests. The strain gauges started to become unreliable and began to produce erratic results due to the long
and extensive exposure to the elements. In some of the later snap-back tests they started to produce faulty results also.

A limitation to the experimental testing was the frequency content of the dynamic shaking, both in the forced-vibration and in the snap-back tests. The eccentric mass shaker’s input forces were purely sinusoidal and the amplitude increased relatively slowly. The snap back-tests released the structure to rock at its natural frequency, and although gave very good insights, it did not cover a range of frequencies that would be expected in an earthquake. Kutter (1995) says that varying frequency content is important for discovering response characteristics of soil. Performing experiments on shake tables using real earthquake records is one way to address this; however this is impossible to do in large scale field tests.
Chapter 4 - Experimental Results
This chapter covers the second part to this thesis; numerical modelling to predict the behaviour of rocking shallow foundations during earthquakes. It outlines the development of two numerical models, the first is a finite element model using the software program *Abaqus* (Simulia 2010), and the second is a spring bed model using the open source platform *OpenSEES* (PEER 2009).

The chapter begins with a look at some of the past numerical models developed from other researchers. It describes the different ways foundations can be modelled: uncoupled springs, spring beds, macro-elements, and finite element models. A number of numerical models from different researchers have been developed, a few of which are described.

The development of the *Abaqus* model is described in detail, from initial validation on simple elastic models right through to calibration of experiment pushover curves. The constitutive soil model is a nonlinear model with a von Mises failure criterion, and soil
properties were set to the values obtained by the experimental testing. The model only considers static pushover curves and does not perform time history analysis. This is comparative to the pullback parts of the snap-back tests. It is acknowledged that dynamic time history analysis would be the next logical step in developing this finite element model, and is recommended for future research.

The development of the *OpenSEES* model is described. This model is a nonlinear spring bed model and is considered much simpler than the finite element model. The validation of an elastic model is still undertaken, and subsequently turned into a nonlinear model. The model is also calibrated against the experimental data and spring properties are altered to achieve best-fit results. Unlike the *Abaqus* model, this model is capable of performing dynamic time history analysis, and such analysis is done in the next chapter.

The chapter concludes with a discussion about the two models developed. It suggests future research that could be undertaken in the area of modelling shallow foundation behaviour during earthquakes.

### 5.1 PAST NUMERICAL MODELS OF SHALLOW FOUNDATIONS

#### 5.1.1 Uncoupled Springs

The simplest way to model shallow foundations is by an uncoupled springs approach. Three springs are used to represent the soil in vertical, horizontal and rotational directions, and the foundation is assumed to be rigid. Figure 5.1 gives a diagram of this model showing all three springs.

The FEMA 356 document prescribes this method and calculates constant stiffness values in all three directions based on theoretical elastic solutions (ATC 2000). If foundation response has little effect on the response of the total system, the soil-foundation behaviour may be modelled sufficiently by the use of three uncoupled springs. However this technique does not model the real mechanism that occurs during cyclic or dynamic loading of a footing. Gajan (2006) stated that experimental evidence has suggested that:
5.1 - Past Numerical Models of Shallow Foundations

a) The stiffness values cannot be assumed as constants, and there can be considerable reduction in stiffness depending on the amount of strain, this point was strongly enforced by the experiment evidence given in the previous chapter.

b) The stiffness in all three directions cannot be uncoupled – there exists strong coupling between vertical and moment, vertical and shear, and shear and moment loading.

c) Uncoupled springs do not take into account uplift of a foundation – when uplift, or rocking, occurs then the geometry of the problem has changed completely and uncoupled springs cannot model this change.

Although simplest, uncoupled springs are also the most unrealistic representation of shallow foundation behaviour. Models of this type, as mentioned above, should only be used when the foundation response has little effect on the response of the system.

5.1.2 Bed of Springs

The bed of springs approach to modelling foundation behaviour was originally proposed by Winkler (1867). Figure 5.2 shows the set up of a Winkler model; a set of closely spaced springs is used to model the soil-footing behaviour. The figure shows the foundation is modelled by a bed of vertical springs that captures the vertical and rotational behaviour, and one horizontal spring that captures the horizontal behaviour.
There have been numerous Winkler models developed to predict the seismic behaviour of shallow foundations (Bartlett 1976, Wiessing 1979, Taylor et al. 1981, Gajan 2006, Ugalde et al. 2007, Allotey and El Naggar 2008, Gajan et al. 2008, Jendoubi et al. 2008, Raychowdhury 2008, Raychowdhury and Hutchinson 2008b, Raychowdhury and Hutchinson 2008a, Raychowdhury and Hutchinson 2009). One of the more widely used applications of this is called the beam-on-nonlinear-Winkler-foundation (BNWF) approach. The BNWF approach assumes each spring has a strength and stiffness based on an equivalent area of foundation.

Raychowdhury and Hutchinson (2008, 2008b, 2008a, 2009) have done comprehensive research in modelling shallow foundation behaviour using a BNWF technique in the OpenSEES platform. They used nonlinear springs for both vertical and horizontal behaviour; qz type springs to model vertical behaviour, and px and tx type springs to model the horizontal behaviour from passive resistance and friction resistance respectively. The nonlinear spring models available in OpenSEES were developed by Boulanger (1999) for analysis of soil-pile interaction. Figure 5.3 gives the backbone curves for clay and sand. The curves in Figure 5.3 were adopted from research done by Reese and O’Neill (1987) for clay and Vijayvergiya (1977) for sand.

*Figure 5.2 A typical bed of spring’s numerical model*
Raychowdhury and Hutchinson (2009) and Boulanger et al. (1999) give descriptions of the spring elements from OpenSEES. The initial part of a qz spring element is a linear elastic region defined by:

\[ q = K_{ini} z \]  

(5.1)

where \( q \) = load; \( K_{ini} \) = initial stiffness; and \( z \) = displacement. The range of the linear elastic region is defined by:

\[ q_0 = C_r q_{ult} \]  

(5.2)

where \( q_0 \) = load at the yield point; \( C_r \) = parameter controlling the range of the linear elastic region; and \( q_{ult} \) = ultimate load capacity. After the linear elastic region, the post-yield nonlinear backbone curve is described as:
where \( z_{50} \) = displacement where 50% of the ultimate load is mobilised; \( z_0^p \) = displacement at the yield point; \( z^p \) = any displacement in the plastic region; and \( c \) and \( n \) are parameters that define the shape of the post yield curve.

The spring functions have a gap part that is a parallel combination of a closure spring and a drag spring. The closure part \( (q^c) \) is a bilinear elastic spring that is relatively rigid in compression and very flexible in tension. The drag part \( (q^d) \) is defined by the following equation:

\[
q^d = C_d \cdot q_{ult} - \left( C_d \cdot q_{ult} - q_0^d \right) \left[ \frac{z_{50}}{z_{50} + 2z^g - z_0^g} \right]
\]

(5.4)

where \( q^d \) = drag force; \( q_0^d = q^d \) at the start of the current loading cycle; \( z^g \) = displacement of the gap spring; \( z_0^g = z^g \) at the start of the current loading cycle; and \( C_d \) = ratio of maximum drag (suction) to the ultimate resistance (Raychowdhury and Hutchinson 2009).

Raychowdhury and Hutchinson used FEMA 356 guidelines for their spring bed development (ATC 2000). Figure 5.4 is taken from FEMA 356 and outlines its procedure for modelling shallow foundations as a bed of springs. It splits the foundation into a central region and an end region, and makes the springs stiffer in the end regions to capture initial rotational stiffness. A bed of springs will under-predict rotational stiffness if the spring stiffness is constant and vertical stiffness is matched to theoretical equations (Pender et al. 2006). Alternatively it will over predict vertical stiffness if the rotation stiffness is matched to theoretical equations. Equations 5.5 and 5.6 give the stiffness per unit length of foundation for the end and central regions respectively.

\[
k_{\text{end}} = \frac{6.83G}{1-\nu}
\]

(5.5)
\[ k_{central} = \frac{0.73G}{1 - \nu} \] (5.6)

where \( k_{end} \) and \( k_{central} \) are the stiffness per unit length for the end and cover regions respectively; \( G \) = shear modulus; and \( \nu \) = Poisson’s ratio. Figure 5.5 presents a schematic of what a spring bed model using FEMA 356 guidelines would represent.

Figure 5.6 presents initial vertical and rotational stiffness from the FEMA 356 equations against theoretical Gazetas equations. The plot shows that the vertical stiffness calculated from FEMA is much stiffer than that calculated by Gazetas – around 2.5
times stiffer; however the rotational stiffness appears to match well – around 90% of the Gazetas value.

OpenSEES spring bed models cannot couple the vertical and rotational stiffness with the horizontal stiffness. The vertical and rotational stiffness is coupled by having vertical springs represent the vertical and rotational behaviour; however uplift of a foundation will not result in reduced horizontal stiffness, contrary to what might be expected because the frictional resistance of foundations is related to contact area.
Wotherspoon (2007) developed a spring bed model in the software platform Ruaumoko (Carr 2004). Ruaumoko has the ability to couple vertical and horizontal springs, allowing uplift of vertical springs to also result in an uplift of horizontal springs. Wotherspoon and Pender (2010) describe the development of this Ruaumoko model and present an example of a soil-/foundation-structure system of an elastic two bay frame.

5.1.3 Macro-Element Model

A macro-element model considers the foundation and the soil as one macro-element where forces act as generalised stress variables and the displacements act as generalised strain variables (Gajan 2006). Figure 5.7 shows visually what the concept of a macro-element is.

Chatzigogos et al. (2009) provide a good summary of some macro-element models that have been created. The paper mentions the need for an extensive validation procedure and calibration of numerical parameters, especially when considering material nonlinearity. It states the need to consider the effect of seismic acceleration on the

![Figure 5.7 Concept of the footing-soil interface macro-element model, taken from Gajan (2006)](image-url)
bearing capacity of shallow foundations. This effect is considered in the bearing strength surfaces for Eurocode 8 that are presented in Chapter 2.

Cremer et al. (2001, 2002) developed a macro-element model around two different nonlinear models – one in plasticity and one in uplift. Coupling between the submodels allowed for the influence of material yielding on uplift and vice versa. The foundation behaviour was characterised based on the elaboration of numerical data, rather than experimental testing. A comparison to a finite element model developed in Dynaflow (Prevost 2011) produced good correlation.

Gajan developed a footing-soil interface macro-element model, named a Contact Interface Model (CIM), that simulates load displacement behaviour during lateral cyclic and dynamic loading (Gajan 2006). The model is described in the paper by Gajan and Kutter (2009a) titled ‘A Contact Interface Model for Nonlinear Cyclic Moment-Rotation Behaviour of Shallow Foundation’.

The model considers the rocking foundation issue as a moving contact problem, because a foundation must always have a certain length of soil in contact with the foundation ($L_c$) to withstand the vertical load. The location of $L_c$ moves along the foundation in the direction of shaking as the foundation rocks. Figure 5.8 gives a diagram of the CIM that was developed.

The Contact Interface Model was implemented in OpenSEES as the function SoilFootingSection2D. Users can apply the model in the OpenSEES platform to predict

![Figure 5.8 Contact interface model (CIM) as outlined in Gajan (2006)]
the behaviour of shallow foundations during earthquakes. A comparison between the CIM, described above, and a BNWF model, developed in OpenSEES at The University of California, Davis, to centrifuge data is made in Gajan et al. (2008) and the benefits and limitations of each model is described. In particular, the paper gives certain applications that would suit either model that include:

- Simulations for structural design of footings, or the footing flexibility is anticipated to contribute to foundation response, the BNWF model should be used.
- If the moment to shear ratio is low, the CIM should be used, as this model can account for the coupling between moment and shear.
- If a designers wishes to use a computer program other than OpenSEES, the BNWF model should be used as this is more easily adaptable to different software platforms.

A further comparison between the CIM and other models was made in Pender et al. (2009). In this paper, the CIM, a spring bed model developed by Wotherspoon (2007), and a bearing strength surface macro-element model developed by Toh (Toh and Pender 2008) are compared to centrifuge tests done on clay (Rosebrook and Kutter 2001c) and satisfactory results are obtained.

### 5.1.4 Finite Element Models

Anastasopoulos et al. (2010) created a finite element model of soil-foundation-structure system in Abaqus – the same software that was used for the finite element modelling in this research. Their research incorporated a nonlinear soil model with a von Mises failure criterion that was calibrated from $G$-$\gamma$ curves produced in Ishibashi and Zhang (1993). The model also had a yielding superstructure – a column and mass designed to represent a bridge deck. The column was modelled as reinforced concrete and was calibrated against reinforced concrete cross-section analysis. The research performed static pushover and dynamic time history analysis using real earthquake records. A system where the foundation yielded was compared with one where the superstructure was the yielding mechanism. They found that in some cases the ductility requirement
for the conventionally designed system was an order of magnitude larger than the capacity – however for the foundation rocking system the displacement ductility was lower and the new philosophy was able to provide larger safety margins (Anastasopoulos et al. 2010).

The finite element modelling in this research, as mentioned above, uses Abaqus to perform pushover curves and calibrate them off the experiments. Some of the modelling techniques were adapted from the research noted above. The nonlinear soil model was taken from what was described in the paper however it was adapted to experimental results that were obtained during the snap-back tests.

5.2 DEVELOPMENT OF THE ABAQUS MODEL

A finite element model was created in the Abaqus software platform (Simulia 2010). The development of the model was done in stages and each was stage checked against theoretical values to ensure accuracy. Titled accordingly to their outcome, the stages of development were: vertical loading and vertical settlement; foundation rotation; elastic soil pushover; and full nonlinear constitutive soil model pushover.

5.2.1 Vertical Loading and Vertical Settlement

The first stage in the development was to create a foundation with vertical loading on an elastic soil, and to check this against settlement predictions using the Gazetas formulas outlined in Chapter 2. Figure 5.9 shows the meshed model for this initial stage; the soil block and foundation were the only parts to the model. The model was created very deep because the effects of strain within soil extends much deeper than the effects of stress. It was created as a ¼ mesh model with lines of symmetry down the centre of the foundation and soil block in both directions, consequently reducing the amount of nodes in the model. The boundary conditions for all models were: fixed in all six degrees of freedom (translation and rotation) on the bottom of the soil block, fixed in horizontal translation on the rear two sides of the soil block, and two axes of symmetry were included in each of the front two sides of the block. The soil was an elastic soil with a Young’s modulus, $E$, of 120 MPa, and a Poisson’s ratio, $\nu$, of 0.49. A Poisson’s ratio of
0.49 was chosen because 0.5 causes computation error in finite element programs. The corresponding shear modulus for the soil was 40 MPa.

For this initial model the foundation was modelled as a rigid element, stopping any deformations that otherwise may occur in the concrete instead of the soil from the vertical pressure load.

Contact between the foundation and soil was both normal and tangential contact. The normal contact was modelled as ‘Hard’ contact – meaning that there was no pressure overclosure. The normal contact parameters also allowed separation between the materials, essential to rocking foundation behaviour. The tangential contact was modelled as ‘rough’, allowing no slip between soil and foundation.
Figure 5.10 The pressure output on the underside of a foundation from a vertical displacement, from Pender et al. (2006) (left) and Abaqus (right)

The model was meshed as linear 8 node elements, with reduced integration and hour glass control. The size of the elements in the vicinity of the foundation (the smallest element size) was 0.1 m.

The model predicted the settlement of the foundation within 97% of Gazetas formulae when a pressure load was applied to the surface of the foundation. The theory behind Gazetas is such that it assumes an infinite half space, accounting for the slight difference between the Abaqus model and the theoretical formulae. The model from Pender et al. (2006) performed the same analysis by applying a displacement and measuring the pressure distribution. The Abaqus model did it a slightly different way because the finite element model failed when a displacement was applied instead of a force. However the overall pressure distribution, settlements, factors of safety and loads were the same. In both models the soil had an $s_u$ of 100 kPa, a factor of safety of 3, an applied or calculated load of 206 kN, and an applied or calculated settlement of 0.68 mm. The calculated vertical stiffness was 302,100 kN/m.

The second verification on this model was to apply a vertical displacement to the foundation (in the form of a boundary condition) and output the pressure distribution on the underside of the foundation. This was checked against past research from Pender et al. (2006). Figure 5.10 presents 3D representations of the pressure distribution obtained from the paper and Abaqus respectively. It shows the two pressure distributions are the
same; the highest pressure occurring towards the edges and corners, and the lowest towards the middle. The reason for high stress at the edges and corners is because those are the areas that generate the highest amount of strain. Following this initial investigation, the model was considered ready to move to the next phase, explained below.

### 5.2.2 Foundation Rotation

The next model development step was to turn the \( \frac{1}{4} \) mesh foundation into a \( \frac{1}{2} \) mesh foundation in order to accommodate moments and rotations. Figure 5.11 shows the setup for this particular model. There was one plane of symmetry through the middle of the foundation and the rest of the boundary conditions were kept the same. The foundation was subject to a rotation and the pressure distribution compared to that in Pender et al. (2006) again. Figure 5.12 shows the pressure distributions from the paper and *Abaqus*. Again it displays good correlation between the two, showing a similar distribution of stress. The numeric values are not the same because the foundation in *Abaqus* was a different size and was subject to different loading than the foundation in the paper. For this comparison however it is the shape that is of most interest. In this model a rotation was applied such that the edge of the foundation was at zero displacement (the vertical settlement from the previous model equalled the upwards rotation). The rotational stiffness of this model was calculated as 7,207,000 kNm/rad.
Figure 5.11 The $\frac{1}{2}$ meshed model set up for the second phase of development

Figure 5.12 The pressure output on the underside of a foundation from a rotation, from Pender et al. (2006) (left) and Abaqus (right)
5.2.3 Elastic Soil Loading-Unloading Curves

The models in Figures 5.9 and 5.10 were for square foundations – this was so comparisons could be made from past experiments on pressure distributions in Pender et al. (2006). However the foundations in the experiments were 2.0 m by 0.4 m in plan. Therefore the ½ mesh model was adjusted so the actual size of foundation used in the tests was modelled. This required changing the foundation size and re-meshing the model. Additionally a wall coupled to the foundation was introduced into the model where horizontal displacements could be applied to – much like the pull back parts of the snap-back tests. Figure 5.13 shows the setup of this final ½ mesh model. The soil was still kept elastic at this stage with a Young’s modulus of 120 MPa and a Poisson’s ratio of 0.49. The foundations in all models rested on the ground surface, and were not embedded as they were in the experiments. Although strictly not precise, effects from sidewall and end pressure on rotational stiffness counts for less than 5% (refer to section 4.1.2), so it was decided to construct the models with the foundations on the ground surface for ease of computation. Although it is recognised that additional modelling with the foundation embedded should be undertaken.

Figure 5.14 gives the static loading-unloading curve on the left, and the settlement-rotation plot on the right, also shown is the theoretical initial rotational stiffness predicted by Gazetas. The figure on the left includes unloading as single red circles. It shows that the initial stiffness of the system is consistent with the Gazetas stiffness; the curve then becomes nonlinear as the foundation uplifts, and reaches a capacity approaching around 125 kNm. The unloading points are evidence that the material remained elastic by the way the curve retraces itself back to zero. It emphasises how much nonlinearity there is in the form of geometric nonlinearity. Geometric nonlinearity occurs from a reduction in rotational stiffness and subsequently bearing capacity from loss of contact between the soil and foundation. The second form of nonlinearity occurs when the soil material yields as the foundation rotates. Material yielding is incorporated into the model next by the introduction of a fully nonlinear constitutive soil model.

The settlement-rotation in Figure 5.14 indicates the large amount of uplift that is occurring – the uplift resulting from the foundation rotation is around 14 mm, whereas
the settlement is less than 1 mm. This suggests that the point of rotation is further towards the edges of the foundation and not in the centre.

Figure 5.13 The ½ mesh setup for the experimental model

Figure 5.14 The load-unload moment-rotation curve (left) and settlement-rotation curve (right) for the experimental model setup
5.2.4 Full Constitutive Soil Model Pushover Curves

The final development of the model was to introduce a fully nonlinear constitutive soil model. The soil is similar to that described in Anastasopoulos et al. (2010) and the description of the soil model from the paper is included below for completeness.

The soil was modelled with a von Mises failure criterion; additionally this model had a combined nonlinear kinematic and isotropic hardening definition and an associated plastic flow rule. According to the failure criterion, the stress evolution is described by:

\[ \sigma = \sigma_0 + \alpha \]  

(5.7)

where \( \alpha \) = the ‘backstress’, this defines the kinematic evolution of the yield surface in the stress space. Gazetas et al. (2010) mentions that the value of \( \sigma_0 \) was set at 1/10 of the ultimate yield stress, \( \sigma_y \), for his research. An associated plastic flow rule is included in the model:

\[ \dot{\varepsilon}^{pl} = \dot{\varepsilon}^{pl} \frac{\partial F}{\partial \sigma} \]  

(5.8)

where \( \dot{\varepsilon}^{pl} \) = plastic flow rate, calculated by equivalent plastic work; \( \dot{\varepsilon}^{pl} \) = equivalent plastic strain rate; and \( F \) = a function that defines the yield surface:

\[ F = f(\sigma - \alpha) - \sigma_0 \]  

(5.9)

The model contains an evolution law that consists of two components; a nonlinear kinematic hardening part and an isotropic hardening component. The nonlinear kinematic hardening defines the translation of the yield surface in the stress space – this is defined through the ‘backstress’, \( \alpha \). The isotropic hardening defines the change of the equivalent stress defining the size of the yield surface, \( \sigma_0 \), as a function of plastic deformation.

The kinematic hardening component is broken up further into two terms; a purely kinematic term (a linear Ziegler hardening law) and a relaxation term (the recall term) that introduces the nonlinearity. The evolution of the kinematic part of the yield stress is defined as:
where $C = \text{initial kinematic hardening modulus}$; and $\gamma = \text{a parameter that defines the rate the kinematic hardening decreases with increasing plastic strain.}$

The evolution law of the kinematic hardening part implies that the backstress, $\alpha$, is contained within a cylinder of radius:

\[
\sqrt{\frac{2}{3}} \alpha^s = \sqrt{\frac{2}{3}} \frac{C}{\gamma}
\]

(5.11)

where $\alpha^s = \text{value of } \alpha \text{ at saturation}$. The model implies that any stress point must lie within a cylinder of radius $\sqrt{2/3} \sigma_y$. The maximum yield stress at saturation is:

\[
\sigma_y = \frac{C}{\gamma} + \sigma_0
\]

(5.12)

And according to the von Mises failure criterion this ultimate stress is:

\[
\sigma_y = \sqrt{3} \cdot s_u
\]

(5.13)

From the above two equations, the value of $\gamma$ can be calculated:

\[
\gamma = \frac{C}{\sqrt{3} \cdot s_u - \sigma_0}
\]

(5.14)

The undrained shear strength used in this model was 200 kPa. The isotropic hardening component defines the evolution of the yield surface, $\sigma^0$, as a function of the equivalent plastic strain, $\varepsilon^{pl}$. The form has a simple exponential law:

\[
\sigma^0 = \sigma^0_0 + Q_\infty \left(1 - e^{-b \varepsilon^{pl}}\right)
\]

(5.15)

where $\sigma^0_0 = \text{yield stress at zero plastic strain } (\sigma_0 \text{ from Equation 5.1})$; $Q_\infty = \text{the maximum change in size of the yield surface}$; and $b = \text{rate the yield surface changes as plastic straining develops}$.
Figures 5.15 shows the one dimensional and three dimensional representation of the hardening of the nonlinear kinematic/isotropic model. Isotropic hardening is not required in simple loading curves, however it is essential when unloading takes place.

Initially the model was not performing the total pushover and the analysis was failing for this particular setup, so was redefined further. First, the mesh was made coarser than the previous elastic models, this was because larger elements are able to cope with nonlinear deformations better than the smaller elements. However there is of course a trade off, because smaller elements are able to predict the geometric nonlinearity more accurately. A few different size meshes were trialled and an element size of 0.2 m around the foundation was settled on (twice as large as the elastic model). Second, the type of element was switched from linear 8 node elements to 20 node quadratic brick elements with reduced integration. This increased the computation expense significantly, however the change of element was necessary in order for the model to solve. All the parameters used in the Abaqus modelling are given in Table 5.1.

Figure 5.16 presents the moment-rotation pushover and the settlement-rotation curves for the nonlinear model. The circles plotted on each of the Abaqus figures represent the step size the analysis is undergoing – a smaller step is the result of the software working harder and the analysis becoming more nonlinear. The moment capacity of the pushover is approaching 100 kNm, comparative to what the experiments displayed. The right-
hand side shows that the static settlement was around 8 mm (indicated by the first point in the plot).

### Table 5.1 Input parameters for the nonlinear Abaqus model

<table>
<thead>
<tr>
<th>Physical Parameters</th>
<th>Model Parameters</th>
<th>Load Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Density</td>
<td>N</td>
</tr>
<tr>
<td>B</td>
<td>Mass Density</td>
<td>V* (measured)</td>
</tr>
<tr>
<td>h_e</td>
<td>Elastic</td>
<td>M* (measured)</td>
</tr>
<tr>
<td>s_d</td>
<td>E</td>
<td>120,000</td>
</tr>
<tr>
<td>ρ</td>
<td>v</td>
<td>0.49</td>
</tr>
<tr>
<td>E</td>
<td>Plastic</td>
<td>7000</td>
</tr>
<tr>
<td>G_max</td>
<td>σ_0</td>
<td>34.6</td>
</tr>
<tr>
<td>ν</td>
<td>C</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Figure 5.17 presents the full cycle moment-rotation and settlement-rotation curves. The initial pushover can be seen as the slightly stiffer curve, then as the foundation rocks back on its second cycle it is less stiff – due to loss of contact between soil and foundation. The moment capacity predicted by the full cycle was around 100 kNm – again comparing well to what the experiments produced. The settlement-rotation indicates that the static settlement was around 10 mm, and the settlement during the rocking cycles was around 5 mm. Once at this stage the model was considered fully developed and comparisons were made to the pushover data gathered from the

![Figure 5.16 Moment-rotation (left) and settlement-rotation (right) pushover curves for the nonlinear Abaqus model](image.png)
5.2 - Development of the Abaqus Model

experiments. These are included below after the development of the OpenSEES model.

Figure 5.18 presents moment-rotation curves from the elastic soil case compared with the nonlinear soil case. The plot shows how different the two models are, and emphasises why nonlinear soil should always be considered, and not assumed to

![Figure 5.17 Moment-rotation (left) and settlement-rotation (right) cyclic curves for the nonlinear Abaqus model](image)

![Figure 5.18 Nonlinear and elastic moment-rotation pushover curves](image)
respond elastically.

The nonlinear model generates the smallest step size around 3 millirads, corresponding to where the elastic model started to uplift and produce nonlinear behaviour. This suggests that the program found the analysis most difficult at the onset of the greatest geometric nonlinear behaviour.

### 5.2.5 Effect of Vertical Load on Pushover Response

Figure 5.19 presents *Abaqus* runs with varying vertical loads. Four pushovers were performed with static factors of safety of 2.8, 6.5, 9.0 and 12.0 – corresponding to vertical loads of 300 kN, 130 kN, 95 kN and 71 kN respectively.

The initial stiffness of all the pushovers was the same except the $FS_v = 2.8$ curve, where the initial stiffness was less. This is discussed in more detail below when comparing the *Abaqus* curves to the moment-rotation equation developed in Chapter 4. The capacities of the pushovers reflected the vertical load on the structure – a higher vertical load corresponding to a higher moment capacity. The higher factor of safety cases reach their

![Figure 5.19 Nonlinear Moment-rotation pushover curves for different vertical loads](image-url)
moment capacity quicker (at a smaller rotation) than those with lower factors of safety.

5.2.6 Effect of Changing the Mesh Size

The sensitivity of the model was studied so that the mesh size chosen could be justified. The labelling of each mesh is according to the size of element in the vicinity of the foundation because the model was meshed such that the size of the elements were smallest around the foundation.

The size of mesh chosen, and presented in Figures 5.16, 5.17 and 5.18 was 0.2 m. It was mentioned before that that a size of 0.1 m did not solve, even when the elements were adjusted to quadratic, additionally a mesh size of 0.14 m and 0.17 m did not solve.

Figure 5.20 presents the pushover curves of the different mesh sizes that did solve – or part solved. The figure shows the only element size that produced a full pushover curve was 0.2 m. Both 0.25 m and 0.33 m element sizes failed part way through the analysis. However up to the point of failure the pushover curves corresponded well with each

![Figure 5.20 Moment-rotation pushover curves comparing the different mesh size in the model](image-url)
The attractive nature of having a 0.2 m mesh size was that, in the model, the foundation was 0.2 m wide and so soil elements and foundation elements lined up. In the larger mesh sizes (both 0.25 m and 0.33 m) the soil elements were larger than the foundation elements.

5.3 DEVELOPMENT OF THE OPENSEES MODEL

The spring bed model developed in the open source finite element platform OpenSEES (PEER 2009) uses nonlinear vertical and elastic horizontal springs to capture foundation behaviour. The FEMA 356 guide mentioned above is a useful starting point to developing a nonlinear model.

The model was developed in stages similar to the Abaqus model: elastic spring bed with constant stiffness; elastic spring bed model after the FEMA 356 guidelines (ATC 2000); a nonlinear spring bed model using the qz spring elements after the FEMA guidelines; and calibration against experiments. This model is also capable of running time history analysis, and this is done in the next chapter – Rocking Foundation Design Guideline.

The model is a similar model to that described in Raychowdhury and Hutchinson (2009), however the horizontal spring used in this model was an elastic spring as it was demonstrated in the experiments that the sliding behaviour was elastic. The structure was modelled as a single degree of freedom elastic lollipop type structure, similar to that presented in Figure 5.2.

Table 5.2 presents the elastic moduli, equivalent spring stiffness, spring ultimate load capacity, and axial loads for all the different spring bed models developed in this section.
Table 5.2 Properties of the OpenSEES spring bed models developed

<table>
<thead>
<tr>
<th>Property</th>
<th>Elastic FEMA</th>
<th>Qz FEMA</th>
<th>Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (MPa)</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_{v, central}$ (kN/m)</td>
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<td>5,840</td>
<td>5,840</td>
</tr>
<tr>
<td>$K_{v, side}$ (kN/m)</td>
<td>54,640</td>
<td>54,640</td>
<td>54,640</td>
</tr>
<tr>
<td>$Q_{v, spring}$ (kN)</td>
<td>$\infty$</td>
<td>30.7</td>
<td>30.7</td>
</tr>
<tr>
<td>$N^*$ (kN)</td>
<td>130</td>
<td>130</td>
<td>130</td>
</tr>
</tbody>
</table>

5.3.1 Elastic Spring Bed with Constant Spring Stiffness

Pushover Curves

Figure 5.21 presents moment-rotation and settlement-rotation curves for the constant elastic spring bed models. The moment-rotation has the Gazetas stiffness plotted as

Figure 5.21 The moment-rotation (top) and settlement-rotation (bottom) pushover curves for a constant elastic spring bed model based on theoretical vertical stiffness (left) and rotational stiffness (right)
well. The plots on the left-hand side have constant elastic springs based on the Gazetas vertical stiffness and the plots on the right have constant elastic springs based on the Gazetas rotational stiffness. It is evident that the moment-rotation behaviour on the left show the rotational stiffness is somewhat reduced from the theoretical value; however it matches the initial stiffness well when based on the rotational stiffness calculation in the plot on the right. The figure on the right however results in a vertical stiffness that is over predicted by around 1.6 times. This is confirmed when comparing the initial settlements of both models. The model based on vertical stiffness has an initial settlement of around 0.6 mm and the model based on rotational stiffness has an initial settlement of around 0.4 mm. Pender et al. (2006) mention this issue of mismatching stiffness values when creating spring bed models.

5.3.2 Elastic Spring Bed after FEMA 356 Pushover Curves

From the initial elastic observations, the model was adjusted according to the guidelines set out in FEMA 356. Figure 5.22 presents the moment-rotation pushover and settlement-rotation behaviour of the elastic spring bed using FEMA 356 guidelines. It shows that the initial rotational stiffness matches the theoretical rotational stiffness well (above it is explained that it is around 90% of this value). The moment-rotation almost follows elastic perfectly plastic behaviour, with a sudden change in stiffness occurring around 100 kNm. This feature was also mentioned in Wotherspoon and Pender (2010).

![Figure 5.22 Moment-rotation (left) and settlement-rotation (right) of an elastic spring bed model after FEMA 356](image)

*Figure 5.22 Moment-rotation (left) and settlement-rotation (right) of an elastic spring bed model after FEMA 356*
The initial settlement was around 0.6 mm, however the model has greater uplift (to around 4.8 mm), and appears to be more of a straight line than the two constant spring bed models presented above. The much stiffer springs in the end regions cause greater concentration of load towards the outside and hence greater uplift.

Although the moment-rotation curve follows the initial small strain stiffness predicted by Gazetas, it was considered not the best representation of the experiments. The next stage in the development was to introduce nonlinear springs into the model, this is presented below.

### 5.3.3 Qz Spring Bed after FEMA 356 Pushover Curves

Figure 5.23 presents the moment-rotation pushover and settlement-rotation curves for the nonlinear qz spring model based on FEMA. It shows that the initial stiffness matched to the Gazetas formula as in the previous model, however the curve became nonlinear earlier – from material nonlinearity as well as geometric nonlinearity. The two input variables to each spring – the ultimate load and the displacement to half the ultimate load – were based on strength and stiffness respectively. A simple bearing capacity calculation was done to calculate the ultimate load of each spring – using an undrained shear strength of 200 kPa. While the displacement to half the ultimate load was based on a reduced Gazetas stiffness.

**Figure 5.23 Moment-rotation (left) and settlement-rotation (right) of a nonlinear spring bed model after FEMA 356**
The moment-rotation shows, again, that the much stiffer springs on the outside of the foundation contribute to a very abrupt change in stiffness during the pushover at around 90 kNm. The settlement-rotation also shows a large amount of uplift – indicating that the soil is still remaining essentially elastic. Further development on this model was needed in order to capture experiment behaviour, this development is discussed below in comparing the OpenSEES pushover to experimental pushover curves.

5.4 COMPARISON TO EXPERIMENT PUSHOVER CURVES

Figure 5.24 presents the pushover curve from Figure 5.16 compared with pullback moment-rotations of test 9. It shows that Abaqus models the upper bound on the snap-backs well. As mentioned in Chapter 4 the experiments degraded because of soil rounding that occurred during the slow pull back, thus the more realistic pushover curves are the initial ones with less soil damage. The Abaqus model was considered accurate when compared to the experiments, and did not need further development.

Figure 5.25 presents the OpenSEES pushover curve compared to the pullback moment-rotations of test 9. It shows the model based on FEMA 356 (in green) and the line of best-fit (in blue). The figure shows that the model based on FEMA guidelines is not accurate compared to experimental data; the initial stiffness is too high, and the nonlinearity did not develop early enough.

The model was adjusted by reducing the size of the end regions from 17% to 7.5% of the foundation length. The length specified for the experiment modelling was 0.15 m, based on a foundation length of 2.0 m, this was reduced from 0.33 m – a reduction of just over half. This reduced the initial stiffness to around 57% of that proposed by FEMA and to introduce more material nonlinearity earlier in the pushover. The rest of the model was kept the same as described in FEMA 356.

5.5 MOMENT-ROTATION EQUATION

Figure 5.26 presents the pushover curves from Abaqus and OpenSEES plotted against the equation generated for nonlinear moment-rotation behaviour in Chapter 4 –
Equation 4.7. The line for the equation (in red) was calculated based on a soil modulus of 60% of $G_{\text{max}}$. The next chapter introduces an operational modulus factor, $c_{\text{op}}$, which reduces the small strain modulus. It was shown in the experiments, and again in the modelling, that this reduction is necessary. The figure shows that the three lines all have around the same initial stiffness, moment capacity and correspond well with each other.

Figure 5.27 presents the same plots – *Abaqus*, *OpenSEES*, and the equation – with a higher vertical load. The load on the foundation was increased from 130 kN (what was loaded on each foundation in the experiments) to 300 kN. This reduced the factor of safety from around 6.5 to 2.8.

The *Abaqus* and *OpenSEES* models both had similar predictions, however the initial stiffness in the equation had to be further reduced from 60% to 30% of $G_{\text{max}}$. This suggests that the value of the operational modulus factor, $c_{\text{op}}$, is related to vertical load, indicating that a higher vertical load will induce more static strain and therefore will be even further away from the small strain stiffness once horizontal loading takes place.

*Figure 5.24 The moment-rotation pushover curve in Abaqus compared with the experimental data from test 9*
Figure 5.25 The moment-rotation pushover curve in OpenSEES based on FEMA 356 guidelines and from the line of best fit compared with the experimental data from test 9.

Figure 5.26 Moment-rotation pushover curves from Abaqus, OpenSEES and Equation 4.7 modelling the experiment setup ($V = 130$ kN, $FS_v = 6.5$).
A correlation of $c_{op}$ to the static factor of safety is recommended, and this should be studied further. In normal design circumstances, the static factor of safety does not usually fall below 3 – so a reduction to 30% of $G_{\text{max}}$ is a good bound for designers to use.

5.6 DISCUSSION AND CONCLUSIONS

This chapter has discussed the development of two different models for predicting the behaviour of a rocking shallow foundation during an earthquake. The first, developed in the software program *Abaqus*, is a finite element model that utilises nonlinear elements with a von Mises failure criterion. The second, developed in the open source software platform *OpenSEES*, is a spring bed model that uses nonlinear springs developed by Boulanger et al. (1999).

The model developed in *Abaqus* began with an elastic soil, and was able to predict settlements and pressure distributions accurately (Figures 5.10 and 5.12). The moment-rotation pushover curves from *Abaqus*, *OpenSEES* and *Equation 4.7* modelling a greater vertical load ($V = 300$ kN, $FS_v = 2.8$) are shown in Figure 5.27.

![Figure 5.27 Moment-rotation pushover curves from Abaqus, OpenSEES and Equation 4.7 modelling a greater vertical load ($V = 300$ kN, $FS_v = 2.8$)](image_url)
rotation pushover of this elastic model (Figure 5.14) demonstrated the geometric nonlinearity of a rocking foundation – nonlinearity that occurs because of loss of contact between foundation and soil.

A nonlinear constitutive soil model was introduced that incorporated a combined kinematic and isotropic hardening model. The model was further refined when trying to model experimental behaviour because of nonlinear deformation occurring within the elements. The original size of the elements (0.1 m) was too small for the amount of deformation they were required to undergo, and the size was increased to 0.2 m. The type of element was altered from a linear 8 node element type to a quadratic 20 node element type.

Figures 5.16 and 5.17 present the moment-rotation and settlement-rotation curves for this model undergoing pushover and cyclic analysis respectively. The cyclic moment-rotation shows good energy dissipation within its loops, and a moment capacity of around 100 kNm – comparative to the experiments. Figure 5.17 indicates the static settlement from the loading was around 10 mm (again comparative to experimental data), and the settlement during the cyclic loading was around 5 mm.

Figure 5.18 indicates how different the response is between a nonlinear model and an elastic model. It shows that nonlinearity within the soil has an effect on the initial stiffness, the moment capacity, and the general shape of the curve.

The OpenSEES model was developed in a similar methodology to the Abaqus model. The elastic models (Figures 5.21 and 5.22) showed once more that there is a considerable amount of geometric nonlinearity within this problem.

The spring bed models based on the FEMA 356 guidelines tended to have a point where most yielding took place, and was not too dissimilar to an elastic perfectly plastic response. The qz spring model displayed a little less of this behaviour, however it was still not comparable to experimental data from the snap-back testing (refer to Figure 5.25).

The model was adjusted so the end zone regions were 7.5% of the foundation length and not 17% as suggested in FEMA. The rest of the parameters were kept the same.
5.6 - Discussion and Conclusions

Figure 5.25 shows the difference between the two *OpenSEES* pushover curves and demonstrates how well the best fit model compares to the experimental data. Reducing the size of the end region had the effect of reducing the initial stiffness of the system to around 57% of the FEMA guidelines. This reduction is similar to the reduction of initial stiffness used in the moment-rotation equation (60%).

The soil modulus operational factor introduced was shown to be dependent on the amount of vertical load on the structure. This was evident in Figure 5.27, where the initial stiffness in the line of the equation had to be reduced to 30% of the initial value. Because this particular setup had a vertical factor of safety towards the lower end of design, a bound of 30% reduction on small strain soil modulus is recommended.

Models that have elastic soil properties – and rely on geometric nonlinearity only – are not sufficient to model rocking foundation behaviour on cohesive soil. Figure 5.18 highlights why geometric nonlinearity should be coupled with material nonlinearity to produce accurate results.

Figures 5.24 and 5.25 show how the two separate models can both be used to predict moment-rotation pushover behaviour of shallow foundations. The models were both able to produce the upper bound pushover curves from the experiments, and to model lower bound pushovers the soil properties, or contact length, would have to be reduced.

Figures 5.26 and 5.27 are encouraging as they demonstrate how model pushover behaviour can be accurately predicted by the equation developed in Chapter 4 – Equation 4.7. Chapter 4 demonstrated the equation can model both the upper and lower bound of the experiments, however for considering realistic earthquake conditions it is the upper bound that is more important. The reduction in initial stiffness for the equation is around 60%, and the reduction of stiffness for the *OpenSEES* model was around 57%. The *Abaqus* curve was produced using the nonlinear model with soil properties determined onsite, and highlights the need for soil modulus reduction when considering rocking shallow foundations.
5.6.4 **Future Work**

Further development of the finite element model in *Abaqus* should be undertaken. Pushover curves from different foundation configurations should be made (for example, square foundations), so that the moment-rotation equation can be further verified. The model should be expanded to accommodate dynamic input motions such as earthquakes, and developed to further predict the experiment behaviour dynamically – both as a forced-vibration setup and a snap-back set up.

Modelling the effect of contact length in *Abaqus* will provide good insight to how much stiffness degradation, in the form of loss of contact, could be expected from seismic shaking. It is a perception by some that this rounding will abate over time as the foundation settles again – an accurate *Abaqus* model able to run a static analysis for a period after an earthquake will prove useful as well.

Additionally the *Abaqus* model could also be extended to include super-structure behaviour as well. The model developed for this research had an elastic super-structure, however structural nonlinearity cannot be ignored, and a holistic approach should be undertaken.

It was shown in this chapter that the FEMA 356 guidelines do not predict the behaviour of shallow foundations compared to the experiments, and a new spring bed approach, based on a smaller end region was proposed. This model needs further development, both against other numerical models and against more experiment testing.

The next chapter is the final main body chapter of this thesis; it incorporates the research performed in the previous two chapters (on experimental results and numerical modelling) into a design guideline. It presents two forms of design, based on displacement and force respectively, and provides design examples of rocking foundations sitting under concrete shear walls.
This chapter considers the engineering design of rocking foundations. The previous chapters on both the experiments and the numerical modelling contribute to the conclusions of this chapter. It begins with an overview of some of the past rocking foundation design guides, concentrating more on New Zealand standards. The older version of the New Zealand earthquake standard (NZS 4203) had an allowance for rocking, which was mentioned in Chapter 1 and is outlined briefly again here. The latest publication by Kelly (2009) is introduced as well as some of the older publications by Priestley et al. (1978) and Taylor and Williams (1979). Additionally current practices in USA, Europe, and Canada are outlined.

Recently there has been a shift in engineering thought to try and develop displacement based designs (DBD’s). Subsequently a methodology for designing rocking foundations using a displacement based method is outlined. The DBD that is described falls into the framework described in: (Priestley et al. 2007, Sullivan et al. 2010, Paolucci et al. 2011). Many of the steps for the design are taken from those references, however the
contribution of this thesis is the nonlinear moment-rotation prediction of foundation behaviour on cohesive soil that has been developed based on experimental and numerical research. Although DBD may be the way forward it is recognised that some designers would prefer a traditional force based design (FBD) method. Thus a FBD is given following the DBD. The guides give a methodology for foundation rocking, different to structural rocking. Figure 6.1 shows the distinction between the two, the wall is rocking on the foundation on the left, whereas the foundation is rocking on the soil on the right. It is the wall on the right that this research is concerned with. Both design guides do not include yielding of the superstructure – all the nonlinear behaviour is occurring in foundation rotation.

Following the two design guides, four examples are given from two rocking shear wall situations. The first wall configuration is a single story shear wall and is designed using the DBD approach, the FBD approach, and the foundation also has been designed to preclude rocking. By comparing the rocking versus non-rocking designs it is evident at how much benefit rocking foundations have both in protecting the superstructure and in economical construction. The second wall configuration, a six story wall, is designed using the DBD approach. The design examples are given as MathCad documents because it is easier to follow.

![Figure 6.1 Different types of rocking](image)

*Figure 6.1 Different types of rocking – the wall on the left shows structural rocking, and the wall on the right shows foundation rocking (what is considered here)*
Time history analyses of two foundation examples are given. The first example is a single story shear wall and the second a six story shear wall. Several earthquake motions scaled to match structure characteristics were used. The time history analyses utilise the fully developed OpenSEES spring bed model for each example.

The chapter highlights the conclusions from the designs, including; some of the differences in the designs, an overview of the time history results, a summary of the issues with FBD, and a consideration to retrofitting already in place foundations as rocking foundation designs. The chapter concludes with some future research suggested on rocking foundation design.

**6.1 PAST ROCKING FOUNDATION DESIGN GUIDES**

New Zealand has had in the past an allowance for rocking structures, however the clause governing rocking foundation design has been left out of the latest version of the code. The older New Zealand design and loadings standard, NZS 4203: 1992, had a provision for rocking foundations (Standards New Zealand 1992), this stated:

*Where dissipation of energy is primarily through rocking of foundations, the structure shall be subject to a special study, provided that this need not apply if the structural ductility factor is equal to or less than 2.0*

The understanding of this clause was that engineers could design a foundation for the reduced forces from a ductility factor, $\mu$, of 2.0 without any extra analysis. Thus the foundation could be sized such that at design forces for $\mu = 2.0$, the overturning moment on a foundation would be equal to the resisting moment and rocking, or uplift, would occur.

The $\mu = 2.0$ part of this clause was removed in the newer New Zealand design and loadings standard, NZS 1170.5 (Standards New Zealand 2004), stating:

*Where energy dissipation is through rocking of structures or structural sub-assemblies... the actions on the structures and parts being supported by the structures shall be determined by special study.*
In New Zealand a special study would require the development of a computer model with time history analysis undertaken. Often design offices will not have the time or resources to carry out such an extensive study, and so rocking foundation design is not a viable option (Kelly 2009).

Kelly (2009) produced a guideline for rocking structures for New Zealand situations. He mentions that a lot of new and existing buildings have insufficient self-weight to resist overturning without uplift. He suggested that instead of having expensive tie down options, a simple design guideline for these situations should be introduced back into New Zealand standards. The paper outlines a method and gives examples of different situations of rocking walls. Kelly, however, considers only elastic properties of the soil and concentrates on the structural response of rocking shear walls. To represent the soil Kelly uses linear elastic springs, but he acknowledges that nonlinear soil behaviour will alter the response and that an appropriate way of designing for this would benefit designs. He goes through four designs in the paper; a single wall, two planar walls, two non-planar walls and a U shaped wall. The first of the design examples given in this chapter below was taken from the single wall example Kelly gives in his paper.

Priestley et al. (1978) make reference to an even older version of the New Zealand standard, NZS 4203: 1976 (Standards New Zealand 1976):

\[ \text{...no foundation system need be designed to resist forces and moments greater than those resulting from a horizontal force corresponding to } S \times M = 2... \text{ (clause 3.3.6.3.1)} \]

The consequences of this load-limitation clause are that some structural systems will be allowed to rock on their foundations under seismic attack (Priestley et al. 1978). This clause, although using different terminology, is essentially the same as the \( \mu = 2.0 \) clause described above for NZS 4203: 1992. The response spectra design approach given in the paper is based on an equivalent single degree of freedom model and considers elastic material behaviour of the soil. It is assumed that the peak response depends only on the equivalent elastic characteristics, and a trial and error response spectrum approach may be used to find the peak displacement. The theory behind the guide is based on Housner’s research (Housner 1963), especially in estimating the energy dissipation factor, \( r \), and in the prediction of the rocking period.
Taylor and Williams (1979) presented a guide to foundations, including rocking shallow foundations, from a more geotechnical point of view. The paper mentions that rocking foundations are not recognised as a distinct structural type, however it would be logical to define a new structural type where foundation rocking forms the principal mode of yielding and energy dissipation. Foundations that include rocking are divided into four subsets: elastic rocking; separation of footing and subsoil; soil yield; and tipping foundations. Elastic rocking occurs if the overturning moments are small enough and there is no loss of contact between the foundation and soil. Separation of footing and subsoil occurs if the overturning moment is greater than the resisting moment. It will result in nonlinear moment-rotation behaviour because of loss of contact between the soil and foundation. Soil yield will occur when the rotation is great enough that the bearing pressures over part of the foundation underside on the soil reach the ultimate bearing pressure, $q_u$. A tipping foundation happens in extreme situations where the underlying soil is very strong and rock-like, and can be considered rigid. Therefore rocking will occur about the corner of the foundation. Taylor and Williams mention that the previous design guide, by Priestley et al. (1978), should be used in this situation.

Taylor and Williams (1979) recommend a spring bed, or Winkler, model in evaluating rocking foundation response. A related paper (Taylor et al. 1981) describes a spring bed model developed using elastic perfectly plastic springs, with the capacity of each spring based on ultimate bearing pressure of the soil.

Chapter 5 presented an outline of the FEMA 356 (ATC 2000) guideline for designing a foundation as a bed of springs. Chapter four of FEMA 356 – Foundations and Geologic Site Hazards – states:

_Buildings may rock on their foundations in an acceptable manner provided the structural components can accommodate the resulting displacements and deformations. Consideration of rocking can be used to limit the force input to a building; however, rocking should not be considered simultaneously with the effects of soil flexibility._

Based on this theory designers can only use the FEMA guideline with elastic springs, and material yielding of the soil should not occur. FEMA 356 presents methods of rocking foundation design using an uncoupled spring approach as well as a spring bed
approach. Both methods are based on Housner’s pioneering research, and for a rigid base, this situation works well. In reality, however, this very rarely occurs, mentioned by Taylor and Williams as an extreme situation (foundation tipping), rocking foundations on a rigid base will only occur if the underlying soil is very strong.

Annex D of Eurocode 8 – Dynamic soil-structure interaction (SSI). General effects and significance – state that due to the SSI effects a foundation rocking on flexible ground will behave differently to one rocking on a rigid base (European Committee for Standardization 1998). The standard mentions that, in most cases, SSI will be beneficial to structures, and so does not have to be considered. In some instances SSI will be detrimental to structures and therefore must be considered. These situations are: structures where $P\cdot\delta$ (second order) effects play a significant role; structures with massive or deep-seated foundations; slender tall structures such as towers or chimneys; and structures supported on very soft soils (European Committee for Standardization 1998). It presents moment-shear-axial load bearing strength surface relationships, these were presented in Chapter 2.

Anderson (2003) performed research on the seismic response of rocking shear walls in high rise structures. His paper was a part of the ‘Proposed Earthquake Design Requirements of the National Building Code of Canada, 2005 edition’. In many cases shear walls will be designed with a hinge mechanism at the base and the foundation should be stronger than the yield strength of the wall. However, many walls with a specified force reduction factor, $R$, of 2 will be stronger than they need to be due to architectural sizing and minimum reinforcement requirements. Thus foundations have to be sized to accommodate forces of $R < 2$ and are over designed. The theory is similar to that proposed in NZS 4203: 1992, where a force reduction factor of 2 is applied, and suggests that footings need not be designed for $R < 2$.

6.2 DISPLACEMENT BASED DESIGN METHOD

The displacement based design guide presented falls into the framework proposed in: (Priestley et al. 2007, Sullivan et al. 2010, Paolucci et al. 2011). Several of these steps below are similar to what is proposed in those papers. However, results from the field
experiments and the numerical modelling on cohesive soils, particularly in developing the nonlinear moment-rotation equation, can be considered the main contribution of this research to further advance the DBD frameworks already in place. It is important to note that this design guide only considers elastic behaviour of the superstructure – it does not include inelastic effects of structural components. The design can be performed in 10 steps and is outlined as follows:

**Step 1**

The first step is to define a design displacement, \( \Delta_d \). This could be a function of drift limits – 2.5% for New Zealand (Standards New Zealand 2004), strain limits within structural components, or proximity to adjacent buildings (to avoid building pounding). Note however that the drift limit of 2.5% may want to be reduced when considering foundation rotation. A limit of 2% drift has been introduced for the design examples given below.

If the structure is a multi degree of freedom structure, the design displacement can be obtained by defining a displacement profile and using the displacement at each mass point. The displacement profile used for rocking structures has been the inelastic first mode shape, and this was what is used in the design examples below. Taking the equation from Priestley et al. (2007) it has the following form:

\[
\Delta_d = \frac{\sum_{i=1}^{n} (m_i \Delta_i^2)}{\sum_{i=1}^{n} (m_i \Delta_i)}
\]

(Priestley et al. (2007) Equation 3.26)    \hspace{1cm} (6.1)

where \( m_i \) = mass at each level; and \( \Delta_i \) = displacement at each level. Subsequently a single degree of freedom representation must be created to find the effective mass, \( m_e \), and the effective height, \( h_e \). This SDOF representation is portrayed in the left-hand side of Figure 6.2 and can be defined by the following relationships, taken from Priestley et al. (2007):
Step 2

An equivalent period for the structure-foundation system, $T_e$, is read off spectral displacement charts. The right-hand side of Figure 6.2 shows this process.

In New Zealand spectral displacement plots can be defined for the five different types of soil classed in NZS 1170.5:

- **Class A** – Strong rock
- **Class B** – Rock
- **Class C** – Shallow soil sites
- **Class D** – Deep or soft soil sites
- **Class E** – Very soft soil sites

Class A and Class B have identical spectral displacement relationships. The report ‘Assessment and Improvement of the Structural Performance of Buildings in Earthquake’ (IPENZ 2005) defines the site hazard spectral displacements, $\delta(T)$, as:

$$\delta(T) = 9800 \cdot C(T) \cdot \frac{T^2}{4\pi^2}$$

where $C(T)$ = elastic site spectra; and $T$ = period. The elastic site spectra are defined as:

$$C(T) = C_h(T) \cdot Z \cdot R \cdot N(T,D)$$
where \( C_h(T) \) = the spectral shape factor; \( Z \) = the hazard factor; \( R \) = the return period; and \( N(T,D) \) = the near fault factor. The spectral shape factor is obtained by reading the \( C_h(T) \) value from Figure 6.3 corresponding to the system period.

![Figure 6.2 An SDOF representation of a MDOF system (left), and the process of reading off the effective system period, \( T_e \), based on the design displacement – taken from Priestley et al. (2007)](image)

![Figure 6.3 The spectral shape factor, \( C_h(T) \), for the different soil types found in New Zealand, redrawn from (Standards New Zealand 2004)](image)
Figure 6.4 presents the spectral displacement, $\delta(T)$, plots for Wellington, Christchurch and Auckland based on 5% damping. The corresponding $Z$ values are 0.4, 0.22 and 0.13 for Wellington, Christchurch and Auckland respectively.

The experiments demonstrated that damping of the soil-foundation-structure system was considerably greater than 5%. As mentioned in Chapter 4 the average damping throughout the tests was between 20-30%, and the ‘$r$’ value of best fit was around 0.4 – or 28% equivalent viscous damping. The tests also showed that the amount of damping was relatively independent of the amplitude or rotation. For this guide an equivalent viscous damping of 20% – towards the lower, more conservative end of the results – is recommended. This value is recommended based on the results of the experiments, however other research on rocking foundations has found similar outcomes (Gajan and Kutter 2008a). A designer may want to specify less damping if wanting to be more conservative.

The total system damping can be broken into structural damping, $\xi_s$, and foundation damping, $\xi_f$ (Sullivan et al. 2010). The recommended value of 20% damping above is for the foundation damping. To calculate system damping, the foundation damping, structural damping, and design displacements are input into Equation 6.6. The designs examples show however, that for a linear shear wall situation, the structural displacement is so small compared to the foundation displacement that the system damping is around the same as the foundation damping – 20%. An initial damping value of 20% is recommended, and, using Equation 6.6, then checked to see system damping is not significantly different.

$$\xi_{sys} = \frac{\xi_f \Delta_f + \xi_s \Delta_s}{\Delta_f + \Delta_s}$$  \hspace{1cm} (6.6)
Figure 6.4 Displacement spectra at 5% damping (and R = 1 and N(T,D) = 1) for Wellington, Christchurch and Auckland
Figure 6.5 Displacement spectra for different damping levels on the site subsoil class C (shallow soil sites) for Wellington, Christchurch, and Auckland
The report ‘Assessment and Improvement of the Structural Performance of Buildings in Earthquake’ mentioned above, has the following equation for adjusting the displacement spectrum to account for higher damping:

\[
K_\xi = \left[ \frac{7}{2 + \xi} \right]^{1/2}
\]

(6.7)

where \( \xi \) = equivalent viscous damping (in %). Displacement spectra for different damping values may be obtained by multiplying the displacement spectra, \( \delta(T) \), by \( K_\xi \).

The research behind this equation looked at various methods for adjusting spectra for damping, however a method similar to that described in Kawashima and Aizawa (1986) was adopted.

Figure 6.5 gives the displacement spectra for a site subsoil class C in Wellington, Christchurch, and Auckland for different damping values. It is evident that at a given design displacement, \( \Delta_d \), the period of the structure is elongated if damping is greater.

**Step 3**

Once the effective period, \( T_e \), is obtained the effective system stiffness, \( K_e \), can be calculated from the relationship in Equation 6.8. This is just a rearrangement of a simple single degree of freedom natural frequency calculation.

\[
K_e = 4\pi^2 \frac{m_i}{T_e^2}
\]

(6.8)

Subsequently the system natural frequency, \( \omega_{sys} \), is:

\[
\omega_{sys} = \frac{2\pi}{T_e}
\]

(6.9)

**Step 4**

The base shear, \( V_b \), and moment, \( M^* \), of the system is calculated by:

\[
V_b = K_e \cdot \Delta_d
\]

(6.10)
\[ M^* = V_b \cdot h_c \]  

(6.11)

**Step 5**

The stiffness of the superstructure, \( K_s \), is calculated through simple elastic principles. For a single degree of freedom cantilever, the equation for stiffness has the form:

\[ K_s = \frac{3EI}{h_v^3} \]  

(6.12)

If wanting to design a yielding structure, this stiffness value could be reduced to account for yielding or plastic hinging – however as mentioned above this is not considered here.

**Step 6**

A determination of the amount of structural deflection, \( \Delta_s \), due to the superstructure stiffness is obtained through:

\[ \Delta_s = \frac{V_b}{K_s} \]  

(6.13)

The structural displacement in this case will be minimal because of the high stiffness of the elastic structure and the reduced base shear. This structural displacement will increase with increasing yielding within structural components, and consequently the foundation rotation will be less.

**Step 7**

Similarly to the structural deflection, the horizontal displacement from sliding behaviour can be calculated by finding the horizontal stiffness, \( K_{f, h} \), using elastic principles from Gazetas outlined in Chapter 2. Shear will usually be low compared to moment because of the typical aspect ratios of rocking walls. Additionally it was shown from the experiment results that having an embedded foundation and a high moment-shear ratio resulted in a linear shear sliding relationship (refer to Figure 4.18). The
equation for horizontal stiffness value defined by Gazetas is sufficient. However the designer should use an operational modulus factor, $c_{op}$, to account for the ‘operational soil modulus’:

$$ G_{op} = c_{op} G_{max} \quad (6.14) $$

where $G_{max} = $ small strain soil modulus. The operational soil modulus should be based on the vertical factor of safety. The recommended $c_{op}$, for factors of safety ranging from 6-10, is 0.5. However lower factors of safety should be more conservative and the recommended lower bound of $c_{op}$ is 0.3. The basis for this was the nonlinear pushover curves from both the experiments and numerical models. The Abaqus runs done at different vertical loads demonstrated how this reduction factor is related to the vertical factor of safety, $FS_v$. Additionally Table 4.1 from EC8 – Part 5: Foundations, retaining structures and geotechnical aspects (European Committee for Standardization 1998) includes soil modulus reduction factors based on ground acceleration. The lower bound given in the table is 0.36 ± 0.2.

**Step 8**

The displacement at the centre of mass for the structure due to foundation movement, $\Delta_f$, is:

$$ \Delta_f = \Delta_d - \Delta_s \quad (6.15) $$

This displacement that has resulted from foundation movement can be broken into foundation sliding and foundation rotation. Figure 6.6 presents a single degree of freedom system showing all the displacement components – structural, foundation horizontal and foundation rotation. The foundation rotation, $\theta_f$, can be obtained by:

$$ \Delta_f = \Delta_H + \theta_f h_x , \quad (6.16) $$

$$ \Delta_H = \frac{V_h}{K_{f-H}} \quad (6.17) $$
Therefore, 

\[
\theta_f = \frac{\Delta_f - \Delta_H}{h_c} 
\]  

(6.18)

where \( \Delta_H \) = displacement from sliding.

Similar to the structural displacement, the displacement from horizontal movement of the foundation, \( \Delta_H \), will be minimal because the stiffness is very high and remains elastic.

Now the effective foundation stiffness, \( K_{f\text{-}eff} \), can be found by dividing the maximum moment, \( M^* \), by the foundation rotation, \( \theta_f \):

\[
K_{f\text{-}eff} = \frac{M^*}{\theta_f} 
\]  

(6.19)

Figure 6.7 shows how these two parameters are represented on a nonlinear moment-rotation relationship.
**Step 9**

The moment-rotation behaviour has been shown, both from experiments and numerical modelling, to follow a hyperbolic shape of the form:

\[ M = \frac{\theta}{a + b\theta} \]  

(6.20)

and the foundation rotational stiffness as:

\[ K_{f,\phi} = \frac{1}{a + b\theta_f} \]  

(6.21)

It was mentioned in previous chapters that the coefficient, \( a \), relates to the initial stiffness of the foundation and the coefficient, \( b \), relates to the moment capacity of the system.

Figure 6.8 shows the coefficients \( a \) and \( b \) graphically on a nonlinear moment-rotation plot. The coefficient, \( a \), is related to the initial stiffness by:

\[ a = 1/K_{ini,\phi} \]  

(6.22)

where \( K_{ini,\phi} = \) initial stiffness of the foundation. This can be equated using the Gazetas equations with the operational soil modulus factor included.

The coefficient, \( b \), is related to the moment capacity by:

\[ b = \frac{1}{M_{ult}} \]  

(6.23)

For this study, we will use the definition for the moment capacity defined in Gajan et al. (2005) – given in Chapter 5 and repeated below:

\[ M_{ult} = \frac{NL}{2} \left[ 1 - \frac{L}{L_c} \right] \]  

(6.24)
Figure 6.7 The moment-rotation of a foundation, showing the design foundation, $\theta_f$, and the effective foundation stiffness, $K_{fe}$.

Figure 6.8 The moment-rotation behaviour of a foundation showing the two coefficients to account for initial stiffness and moment capacity of a foundation.
It is evident that the two coefficients, $a$ and $b$, are related to the soil modulus and the undrained shear strength (for clay) respectively. This emphasises the need for a thorough and accurate geotechnical investigation. Step 9 is the major contribution to the design procedure developed in this research.

The foundations are sized as such that the calculated foundation stiffness, $K_{f,g}$, from Equation 6.19 matches that calculated in Equation 6.21. The design will rarely match 100%, so a margin of ±10% is recommended. This leeway again shows that perhaps the drift limits for rocking foundations should be reduced from 2.5%.

**Step 10**

The final calculation in the design is to check that the strengths of both the wall and the foundation are adequate to accommodate the forces imposed from the earthquake. These could be referred to as ‘strength checks’.

Figure 6.9 gives an overview of the displacement based design of rocking soil-foundation-structure systems.
Figure 6.9 Overview of the DBD procedure for a rocking soil-foundation-structure design
6.3 FORCE BASED DESIGN METHOD

The force based design (FBD) guide for rocking foundations can be divided into 7 steps, described below. It uses several formulae from the previous DBD and takes a lot of reference from NZS 1170.5 (Standards New Zealand 2004).

Step 1

The first step is identical to the DBD – to find equivalent SDOF properties of the structure concerned. The important parameters are the equivalent mass and equivalent height; \( m_c \) and \( h_c \). These are calculated using Equations 6.1 to 6.3.

Step 2

A guess of the foundation dimensions, \( L \) and \( B \) is carried out and the foundation elastic stiffness values, \( K_{f,H} \) and \( K_{f,\phi} \) are calculated. Again here it is adequate to use Gazetas elastic stiffness formulae in conjunction with the operational soil modulus factor.

The structure elastic stiffness, \( K_s \), is also calculated using Equation 6.12 and the system natural frequency, \( \omega_{sys} \), can be found by Wolf’s equation in Chapter 2 – this has the form:

\[
\omega_{sys}^2 = \frac{\omega_s^2}{1 + \frac{K_s}{K_{f,H}} + \frac{K_s h^2}{K_{f,\phi}}} \tag{6.25}
\]

The system period is:

\[
T_c = \frac{2\pi}{\omega_{sys}} \tag{6.26}
\]

Step 3

A guess at the amount of system damping should be made. Again this can be assumed to be 20% (the foundation damping value). This should be checked again at the end of the design to ensure damping is correct.
**Step 4**

Spectral acceleration values, as well as base shear, are obtained through principles set out in NZS 1170.5. The $C_h(T)$ factor is read off Figure 6.2 corresponding to the system period obtained above, and the elastic site spectra, $C(T)$, is calculated. Again this value should be adjusted to account for the higher damping of the system – calculated by Equation 6.6.

According to NZS 1170.5 the base shear, $V_b$, equals:

$$V_b = C_d(T_1) \cdot W_t \quad (6.27)$$

where $C_d(T_1) = \text{the horizontal design action coefficient}$; and $W_t = \text{the seismic weight of the structure – obtained by multiplying the effective mass, } m_e$, by gravity. The horizontal design action coefficient is calculated by:

$$C_d(T_1) = \frac{C(T) \cdot S_p}{k_\mu} \quad (6.28)$$

where $S_p = \text{structural performance factor, defined as:}$

$$S_p = 1.3 - 0.3\mu \quad (6.29)$$

and $k_\mu$ is defined for soil class types A, B, C, and D as:

$$k_\mu = \mu \quad \text{for } T_e > 0.7 \text{ s} \quad (6.30)$$

$$k_\mu = \frac{\mu - 1}{0.7}T_e + 1 \quad \text{for } T_e < 0.7 \text{ s} \quad (6.31)$$

where $\mu = \text{ductility}$.

Past design guides from both New Zealand and Canada have had limiting factors on the force reduction. New Zealand specified a ductility of 2 in NZS 4203: 1992, this equated to a $C_d(T_1)$ of 0.6 in the worst case scenario. For this guide, the recommended ductility, $\mu$, is 2.5; chosen as this is the value that gives the most similar design to the DBD. Additionally this ductility is not very far off the past value of 2 – validating previous
work undertaken in this area. The ductility of 2.5 however does not directly relate to the same force reduction as the older version did. In the old code a constant damping of 5% was assumed for rocking foundations, however this design guide recommends a damping of 20% - resulting in a greater reduction of design forces.

**Step 5**

The maximum overturning moment, $M^*$, exerted on the foundation must be calculated (Equation 6.11) as well as the ultimate resisting moment, $M_{ult}$ (Equation 6.24). The two moments are compared and, if the design is correct, the ultimate resisting moment is greater than the maximum overturning moment, if not the foundation needs to be resized.

**Step 6**

Deflections must be checked to ensure that drift limits or other displacement restrictions are within compliance. NZS 1170.5 states that using response spectrum analysis the deflections can be calculated as the equivalent elastic deflections multiplied by the ductility. However the increase in system damping should be accounted for – as this has the effect of increase spectral displacement. The follow equation is proposed to calculate the equivalent elastic deflection:

$$\Delta_{elastic} = \frac{V_b}{K_e \cdot K_\xi}$$  \hspace{1cm} (6.32)

where $\Delta_{static} = \text{equivalent static deflection}$; and $K_e = \text{system stiffness}$; and $K_\xi = \text{adjustment factor for higher damping}$. This deflection is then multiplied by the ductility and checked against deflection limits:

$$\Delta_{total} = \Delta_{elastic} \mu$$  \hspace{1cm} (6.33)
**Step 7**

The last step is to use Equation 6.6 to calculate the system damping based on the deflections calculated in step 6. If the guessed system damping is similar to the calculated damping then the design is sufficient. If, however, the damping calculated is different from the damping guessed in step 3, a new guess of damping should be made and the design redone.

Figure 6.10 gives an overview of the force based design of rocking soil-foundation-structure systems.
The following sections cover three separate designs of a single story shear shallow foundation. The designs were done in MathCad (PTC 2007) and the MathCad documents are presented below, this is easier to show development of the designs rather than trying to explain it in writing. The situation is similar to that presented in Kelly (2009), with the exception that the soil is a stiff clay (subsoil site class C) and not dense.
gravel. The wall is 3.6m long by 3.6m high, and the thickness is assumed as 150mm. The seismic weight on the wall is 1040 kN, and the axial weight is half that; 520 kN. The soil properties are: a $s_u$ of 100 kPa, a Poisson’s ratio, $\nu$, of 0.5, and a $G_{\text{max}}$ of 40 MPa. The wall is assumed to be situated in Wellington for these examples ($Z = 0.4$), and has an $R = 1.0$ and a $N(T,D) = 1.0$ also.

The wall is designed as a rocking structure using the DBD method, the FBD method and then as a non-rocking foundation. Time history analyses are presented following the DBD design. The benefits of a rocking foundation design are illustrated when comparing the foundation sizes and design forces of the DBD method to those from the design to preclude rocking.

6.4.1 DBD of the Single Shear Wall

**Wall Properties:**

- $L_{\text{wall}} := 3.6\text{m}$
- $h_{\text{wall}} := 3.6\text{m}$
- $t_{\text{wall}} := 150\text{mm}$

**Seismic Properties:**

- $N_{\text{wall}} := 520\text{kN}$
- $W_{\text{total}} := 1040\text{kN}$

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of wall</td>
<td>$L_{\text{wall}}$ = 3.6m</td>
</tr>
<tr>
<td>Height of wall</td>
<td>$h_{\text{wall}}$ = 3.6m</td>
</tr>
<tr>
<td>Thickness of wall</td>
<td>$t_{\text{wall}}$ = 150mm</td>
</tr>
<tr>
<td>Axial load on the wall</td>
<td>$N_{\text{wall}}$ = 520kN</td>
</tr>
<tr>
<td>Total weight</td>
<td>$W_{\text{total}}$ = 1040kN</td>
</tr>
</tbody>
</table>
Effective Mass and Height Calculation:

\[ \gamma_{\text{concrete}} = \frac{24 \text{kN}}{\text{m}^3} \quad \text{Weight of concrete} \]

\[ W_{\text{wall}} := L_{\text{wall}} h_{\text{wall}} t_{\text{wall}} \gamma_{\text{concrete}} \quad W_{\text{wall}} = 46.656 \text{kN} \quad \text{Weight of wall} \]

\[ m_{\text{wall}} := \frac{W_{\text{wall}}}{g} \quad m_{\text{wall}} = 4.758 \times 10^3 \text{ kg} \quad \text{Mass of wall} \]

\[ x_{\text{wall}} := \frac{h_{\text{wall}}}{2} \quad x_{\text{wall}} = 1.8 \text{ m} \quad \text{Height to the centroid of wall} \]

\[ W_{\text{t.non.wall}} := W_{\text{total}} - W_{\text{wall}} \quad W_{\text{t.non.wall}} = 993.344 \text{kN} \quad \text{Weight of 'other' components} \]

\[ m_{\text{non.wall}} := \frac{W_{\text{t.non.wall}}}{g} \quad m_{\text{non.wall}} = 1.013 \times 10^5 \text{ kg} \quad \text{Mass of 'other' components} \]

\[ x_{\text{non.wall}} := 3.6 \text{m} \]

**Note:** We will assume the wall remains elastic, and so \( \Delta \) at 1.8 m height equals to half that at 3.6 m height. A linear mode shape has been assumed for the soil-footing-structure displacement.

Therefore:

\[ \Delta_{i,\text{wall}} := 0.5 \]

\[ \Delta_{i,\text{non.wall}} := 1.0 \]

\[ \Delta_{d,\text{normalised}} := 1.0 \]

\[ m_e := \frac{m_{\text{wall}} \Delta_{i,\text{wall}} + m_{\text{non.wall}} \Delta_{i,\text{non.wall}}}{\Delta_{d,\text{normalised}}} \quad \text{Eq. 6.2} \]

\[ m_e = 1.037 \times 10^5 \text{ kg} \]
Chapter 6 - Rocking Foundation Design Guideline

\[ h_c := \frac{m_{\text{wall}} \Delta \text{wall} + m_{\text{non.wall}} \Delta \text{non.wall}}{m_{\text{wall}} \Delta \text{wall} + m_{\text{non.wall}} \Delta \text{non.wall}} \]  \hspace{1cm} \text{Eq. 6.3} \\

\[ h_c = 3.559 \text{m} \]

\[ W_t := m_c \cdot g \hspace{1cm} W_t = 1.017 \times 10^3 \text{kN} \hspace{1cm} \text{Seismic weight} \]

**Design Displacement:**

Make the design displacement based on 2.0% drift

\[ \Delta_d := \frac{2h_{\text{wall}}}{100} \hspace{1cm} \Delta_d = 72 \text{mm} \hspace{1cm} \text{Design Displacement} \]

Reading off the Displacement Spectra for subsoil class C for 20% damping (Figure 6.5), the system effective period is:

\[ T_e := 1.1s \hspace{1cm} \text{Effective period} \]

\[ K_e := 4 \pi^2 \frac{m_c}{T_e^2} \hspace{1cm} K_e = 3.382 \times 10^3 \text{kN/m} \hspace{1cm} \text{Eq. 6.8} \]

\[ \omega_{\text{sys}} := \frac{2\pi}{T_e} \hspace{1cm} \omega_{\text{sys}} = 5.712 \frac{1}{s} \text{ rad/s} \hspace{1cm} \text{Eq. 6.9} \]

**Base Shear:**

\[ V_b := K_e \Delta_d \hspace{1cm} V_b = 243.538 \text{kN} \hspace{1cm} \text{Eq. 6.10} \]

**Structural Stiffness:**

\[ I_{\text{wall}} := \frac{t_{\text{wall}} L_{\text{wall}}^3}{12} \hspace{1cm} I_{\text{wall}} = 0.583 \text{m}^4 \]

**Note:** We are using here the gross inertia value (without considering reinforcement). In practice this I value should be perhaps the cracked I value - $I_c$. 

- 200 -
We will assume the concrete has an \( f'_{c} \) of 30 MPa

\[
f'_{c} := 30
\]

\[
E_{\text{conc}} := \left(3320 \sqrt{f'_{c}} + 6900\right) \cdot 1 \text{MPa} \quad E_{\text{conc}} = 25.084 \text{GPa}
\]

(From concrete code)

\[
K_{s} := \frac{3E_{\text{conc}} \cdot I_{\text{wall}}}{h_{e}^{3}} \quad K_{s} = 9.738 \times 10^{5} \text{ kN/m}
\]

Eq. 6.12

\[
\omega_{s} := \sqrt{\frac{K_{s}}{m_{e}}} \quad \omega_{s} = 96.918 \frac{1}{s}
\]

Structural natural frequency

**Structural Deformation:**

\[
\Delta_{s} := \frac{V_{b}}{K_{s}} \quad \Delta_{s} = 0.25 \text{mm}
\]

Eq. 6.13

**Note:** It is shown here that the elastic deformation of the wall is almost insignificant when looking at the total system deflection.

\[
\Delta_{f} := \Delta_{d} - \Delta_{s} \quad \Delta_{f} = 71.75 \text{mm}
\]

Eq. 6.15

Horizontal Deflection due to foundation

**Damping Check:**

Assume:
Foundation damping of 20%
Structural damping of 5%

\[
\xi_{f} := 20
\]

\[
\xi_{s} := 5
\]

\[
\xi_{\text{sys}} := \frac{\xi_{f} \Delta_{f} + \xi_{s} \Delta_{s}}{\Delta_{d}} \quad \xi_{\text{sys}} = 19.948
\]

Eq. 6.6

Note: The system damping is essentially unaffected by the structural damping because of the low structural displacement - 20% system damping is OK
Foundation Stiffness:

Foundation Properties:

\[ G_{\text{max}} := 40 \text{MPa} \]
\[ \nu := 0.5 \]
\[ c_{\text{op}} := 0.5 \]

Operational modulus factor

\[ G_{\text{op}} := G_{\text{max}} c_{\text{op}} \quad G_{\text{op}} = 20 \text{MPa} \quad \text{Eq. 6.14} \]

Guess dimensions:

\[ L := 3.7 \text{m} \]  
Foundation length

\[ B := 1 \text{m} \]  
Foundation width

\[ D := 1 \text{m} \]  
Foundation depth

\[ A_b := L \cdot B \]  
Base area

\[ A_s := 2 \cdot L \cdot D \]  
Side wall area

Horizontal Stiffness (Gazetas formulae):

\[ K_{H_{\text{basic}}} := \frac{G_{\text{op}} \cdot L}{2 - \nu} \]

\[ I_{H_{\text{shape}}} := 2 + 2.5 \left( \frac{A_b}{L^2} \right)^{0.85} \]

\[ I_{H_{\text{depth}}} := 1 + 0.15 \left( \frac{2D}{B} \right)^{0.5} \]

\[ I_{H_{\text{side}}} := 1 + 0.52 \left( \frac{8 \left( \frac{D}{2} \right) \cdot A_s}{B \cdot L^2} \right)^{-0.4} \]
\[ K_{f,H} := K_{H, basic} \cdot I_{H, shape} \cdot I_{H, depth} \cdot I_{H, side} \]

\[ K_{f,H} = 2.882 \times 10^5 \text{ kN/m} \]

**Horizontal Movement:**

\[ \Delta_H := \frac{V_b}{K_{f,H}} \]

\[ \Delta_H = 0.845 \text{ mm} \quad \text{Eq. 6.17} \]

**Note:** Again we see here that the deflection from the horizontal movement of the foundation is very small - this is consistent with what the experiments showed as well.

**Foundation Rotation:**

\[ \theta_f := \frac{\Delta_f - \Delta_H}{h_e} \]

\[ \theta_f = 0.02 \text{ rads} \quad \text{Eq. 6.18} \]

**Rotational Stiffness:**

\[ M_{\text{star}} := V_b \cdot h_e \]

\[ M_{\text{star}} = 866.679 \text{ kNm} \quad \text{Eq. 6.11} \]

\[ K_{f,\text{eff}} := \frac{M_{\text{star}}}{\theta_f} \]

\[ K_{f,\text{eff}} = 4.35 \times 10^4 \text{ kNm rad}^{-1} \quad \text{Eq. 6.19} \]

**Moment Rotation Curve:**

**Note:** The form of the moment-rotation curve is \( M = \theta/(a+b\theta) \)

**Rocking stiffness (Gazetas formulae):**

\[ G_{\text{op}} = 20 \text{ MPa} \quad \text{Operational modulus} \]

\[ d := D \]

\[ I_{\text{ind}} := \frac{B L^3}{12} \]
Chapter 6 - Rocking Foundation Design Guideline

\[ K_{\phi\_basic} := \frac{G_{\text{op}} I_{\text{find}}^{0.75}}{1 - \nu} \]

\[ I_{\phi\_shape} := 3 \left( \frac{L}{B} \right)^{0.15} \]

\[ I_{\phi\_depthside} := 1 + 1.84 \left( \frac{d}{L} \right)^{0.6} \left[ 1.5 + \left( \frac{2d}{L} \right)^{1.9} \left( \frac{D}{d} \right)^{0.6} \right] \]

\[ K_{\text{ini} \_\phi} := K_{\phi\_basic} I_{\phi\_shape} I_{\phi\_depthside} \]

\[ K_{\text{ini} \_\phi} = 1.083 \times 10^6 \text{kN m} \quad \text{Rotational foundation stiffness} \]

\[ a := \frac{1}{K_{\text{ini} \_\phi}} \quad \text{Eq. 6.22} \]

**Bearing Capacity:**

\[ s_u := 100 \text{kPa} \quad \text{Undrained shear strength} \]

\[ c := s_u \]

\[ \phi := 0 \]

\[ \gamma := 17 \frac{\text{kN}}{\text{m}^3} \quad \text{Unit weight (kN/m}^3) \]

\[ q := D \gamma \quad \text{Surcharge} \]

**Bearing Capacity Factors:**

\[ N_q := e^{-\pi \tan(\phi) \tan(45 \deg + \frac{\phi}{2})^2} \]
\[ N_c := \begin{cases} (N_q - 1) \cot(\phi) & \text{if } \phi > 0 \\ 5.14 & \text{otherwise} \end{cases} \quad N_c = 5.14 \]

\[ N_q := 2 \left( N_q - 1 \right) \tan(\phi) \]

**Shape Factors:**

\[ \lambda_{cs} := 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) \]

\[ \lambda_{qs} := 1 + \left( \frac{B}{L} \right) \tan(\phi) \]

\[ \lambda_{qs} := 1 - 0.3 \left( \frac{B}{L} \right) \]

**Depth Factors:**

\[ \lambda_{qd} := \begin{cases} 1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \left( \frac{D}{B} \right) & \text{if } \frac{D}{B} \leq 1 \\ 1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \text{atan} \left( \frac{D}{B} \right) & \text{otherwise} \end{cases} \]

\[ \lambda_{cd} := \begin{cases} 1 + 0.4 \left( \frac{D}{B} \right) & \text{if } \frac{D}{B} \leq 1 \\ 1 + 0.4 \text{atan} \left( \frac{D}{B} \right) & \text{otherwise} \end{cases} \]

\[ \lambda_{qd} - \frac{1 - \lambda_{qd}}{N_q \tan(\phi)} & \text{otherwise} \]

\[ \lambda_{xl} := 1 \]
**Inclination Factors:**

**Note:** Assume vertical loading only at this stage

\[ \lambda_{qi} := 1 \]
\[ \lambda_{ci} := 1 \]
\[ \lambda_{\gamma i} := 1 \]

**Bearing Capacity Equation:**

\[
q_u := c \lambda_{cs} \lambda_{cu} \lambda_{ci} N_c + q \lambda_{qs} \lambda_{qd} \lambda_{qj} N_q + 0.5 \gamma B \lambda_{\gamma s} \lambda_{\gamma d} \lambda_{\gamma i} N_{\gamma}
\]

\[ q_u = 7.744 \times 10^5 \text{ Pa} \quad q_u = 774.438 \text{kPa} \quad \text{Ultimate bearing capacity} \]

\[ Q_u := q_u B L \quad Q_u = 2.865 \times 10^3 \text{kN} \quad \text{Ultimate Load} \]

**Factor of Safety:**

\[ N_{\text{fnd}} := \gamma_{\text{concrete}} L \cdot B \cdot D \quad N_{\text{fnd}} = 88.8 \text{kN} \quad \text{Weight of foundation} \]

\[ N := N_{\text{wall}} + N_{\text{fnd}} \quad N = 608.8 \text{kN} \quad \text{Total weight} \]

\[ F_{S_{\text{v}}} := \frac{Q_u}{N} \quad F_{S_{\text{v}}} = 4.707 \quad \text{Factor of Safety} \]

**Moment Capacity**

\[
M_{\text{ult}} := \frac{N L}{2} \left( 1 - \frac{1}{F_{S_{\text{v}}}} \right) \quad M_{\text{ult}} = 886.985 \text{kN m} \quad \text{Eq. 6.24} \]

\[ b := \frac{1}{M_{\text{ult}}} \quad \text{Eq. 6.23} \]
The design shows that a foundation of 3.7 m long (the length of the wall) and 1.0 m wide is adequate for this configuration. Additionally, the structural capacity of the wall must be checked against $M^*$ and $V_b$, however, it was assumed in this situation to have enough capacity.

An important aspect of this design is the very little reserve the foundation appears to have. The ultimate moment capacity of the foundation is 887 kNm and the maximum moment exerted on the foundation is 867 kNm – 98% of capacity. This equates to a traditional factor of safety of 1.02. The hyperbolic nature of the moment-rotation curves are such that the moment approaches the ultimate moment capacity quickly, and does not need much rotation of the foundation for the moment exerted on the foundation to become very close to capacity. The sensitivity to any displacement response appears to be quite vulnerable to any slight change in moment because of where the design lies on the moment-rotation plot. This was the main reason why time history analyses was performed; detailed below.
6.4.1.1 Time History Analyses of the Single Story Shear Wall

Time history analysis was performed on the above single story DBD shear wall design. The methodology for selecting and scaling the earthquake records is described in Oyarzo-Vera et al. (2010), and the seven records used were taken from El Centro (1940), Duzce (1999), Tabas (1978), Hokkaido (2003), La Union (1985), Lucerne (1992), and Arcelik (1999). Figure 6.11 presents the time history records of the seven earthquakes, scaled according to guidelines in the report based on the structure properties, and the geographic location (Wellington). Table 6.1 presents the input properties to the OpenSEES spring bed model.

Table 6.1 Foundation and wall properties of the single story OpenSEES example

<table>
<thead>
<tr>
<th>Foundation and Wall Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3.7 m</td>
</tr>
<tr>
<td>$B$</td>
<td>1.0 m</td>
</tr>
<tr>
<td>$h$</td>
<td>3.6 m</td>
</tr>
<tr>
<td>$E$</td>
<td>40 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_{v_{central}}$</td>
<td>10,051 kN/m</td>
</tr>
<tr>
<td>$K_{v_{side}}$</td>
<td>98,350 kN/m</td>
</tr>
<tr>
<td>$Q_{v_{spring}}$</td>
<td>97.5 kN</td>
</tr>
<tr>
<td>$\xi$</td>
<td>20%</td>
</tr>
<tr>
<td>$N^*$</td>
<td>1000 kN</td>
</tr>
</tbody>
</table>

Figures 6.12 and 6.13 present the spectra of the seven earthquakes, and compare them to the NZS1170.5 spectrum. Figure 6.12 presents the spectra on a normal scale and Figure 6.13 present the data on a log-log scale.

The analysis was carried out with the validated spring bed model in OpenSEES using the same wall configuration as above. The input files for the design can be found in Appendix D, and can be used in guiding the development of other models.

Figures 6.14 through 6.19 presents some example results from Lucerne and El Centro. The response to the Lucerne motion was the most critical, as it generated the greatest displacement and rotation. The El Centro behaviour indicates that the response was significantly less than the Lucerne earthquake. Appendix E contains all the response plots from all seven earthquakes.
Figure 6.11 The time histories (in g) for the seven earthquake records
Figure 6.12 Earthquake spectra for the seven earthquakes with the NZS 1170.5 spectra

Figure 6.13 Earthquake spectra for the seven earthquakes with the NZS 1170.5 spectra plotted in log-log scale
6.4 - Single Story Shear Wall Design Example

**Figure 6.14** Moment-rotation and settlement-rotation plots for Lucerne

**Figure 6.15** Displacement time history of the effective mass for Lucerne

**Figure 6.16** Rotation time history of the foundation mass for Lucerne
Figure 6.17 Moment-rotation and settlement-rotation plots for El Centro

Figure 6.18 Displacement time history of the effective mass for El Centro

Figure 6.19 Rotation time history of the foundation mass for El Centro
Table 6.2 Summarised results from the time history analyses performed on the single story shear wall in example 1

<table>
<thead>
<tr>
<th></th>
<th>Scaled PGA (g)</th>
<th>Max Moment (kNm)</th>
<th>Rotation (millirads)</th>
<th>Displacement (mm)</th>
<th>Dynamic Settlement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max</td>
<td>Residual</td>
<td>Max</td>
</tr>
<tr>
<td>El Centro</td>
<td>0.216</td>
<td>655.5</td>
<td>9.0</td>
<td>0.48</td>
<td>32.4</td>
</tr>
<tr>
<td>Duzce</td>
<td>0.237</td>
<td>652.6</td>
<td>7.0</td>
<td>0.01</td>
<td>26.4</td>
</tr>
<tr>
<td>Tabas</td>
<td>0.448</td>
<td>766.6</td>
<td>19.4</td>
<td>0.28</td>
<td>69.0</td>
</tr>
<tr>
<td>Hokkaido</td>
<td>0.223</td>
<td>738.4</td>
<td>18.3</td>
<td>1.98</td>
<td>67.7</td>
</tr>
<tr>
<td>La Union</td>
<td>0.309</td>
<td>777.5</td>
<td>12.5</td>
<td>1.71</td>
<td>44.7</td>
</tr>
<tr>
<td>Lucerne</td>
<td>0.755</td>
<td>795.1</td>
<td>36.5</td>
<td>4.51</td>
<td>131.1</td>
</tr>
<tr>
<td>Arcelik</td>
<td>0.638</td>
<td>719.8</td>
<td>13.3</td>
<td>1.68</td>
<td>44.5</td>
</tr>
<tr>
<td>Average</td>
<td>0.403</td>
<td>729.4</td>
<td>16.6</td>
<td>1.52</td>
<td>59.4</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.755</td>
<td>795.1</td>
<td>36.5</td>
<td>4.51</td>
<td>131.1</td>
</tr>
</tbody>
</table>

Three earthquakes had strong forward directivity, or pulses, these were Tabas, Lucerne and Arcelik. This is indicated by the much stronger PGA’s across these motions. The strong directivity pulse during the Lucerne earthquake explains why this particular motion had the most critical response. This is evident in both time history plots (Figures 6.16 and 6.15) by the large spike in the data, as well as the moment-rotation and settlement-rotation plots (Figure 6.14).

Table 6.2 summarises the key results from all seven earthquakes. The average maximum displacements and rotations across the seven earthquakes were 59.4 mm and 16.6 millirads respectively. Comparatively the design displacement and rotation for this system was 71.2 mm and 20 millirads. The single story shear wall performed better than the design in all but the Lucerne earthquake, however the analysis showed that even with the extra rotation and displacement demand the wall still performed satisfactorily. It was mentioned above that the stronger demand during the Lucerne earthquake was from the extra forward directivity that this particular record had.

The residual displacements and rotations were minimal apart from those in Lucerne. Figures 6.15 and 6.16 indicate the presence of a strong pulse, mentioned previously, by the spike in the two time history plots. The settlement-rotation plot (Figure 6.14) shows that the foundation had a large uplift brought on by the pulse in the earthquake. For this particular motion the ultimate moment was reached, however the rotation recorded was greater than the design rotation. The foundation is therefore able to withstand greater
rotations than the design rotation and proves there is redundancy in the system. The residual displacement of 16.08 mm after this earthquake corresponds to a drift of 0.45%. Although for this particular earthquake some repair may be necessary to have the building serviceable again, the structure still survived without collapse.

The maximum moment generated during the time history analyses was 795.1 kNm – 93% of the moment capacity. This demonstrates that the system can perform well even though maximum moments generated during the excitation leave little strength reserve compared to the moment capacity.

The average dynamic settlement during the analyses was 6.76 mm, the corresponding static settlement from OpenSEES was 2.59 mm – less than the dynamic settlement. The static settlement predicted by Gazetas equations is 1.60 mm, less than the prediction made by OpenSEES. The settlement recorded during the analyses was greater than static predictions, this should be considered in future research on rocking foundation design.

The static settlements recorded during the forced-vibration experiments were much greater than the dynamic settlements. The clay onsite during the experiments was very stiff, and could be a reason why the dynamic settlements were very low. Two more reasons could be:

- The forced-vibration method (and subsequently the snap-back) method does not generate much settlement during the rocking motion.
- The spring bed model over-predicts settlement.

In any case further research should be undertaken on dynamic settlement of rocking foundations on cohesive soil. It is proposed to perform these experiments as shake table experiments, negating any deficiencies the forced-vibration method has on the settlement.

The frequency content in the time history plots of the two earthquakes presented above was significantly different. This reinforces the statement that the period of a rocking structure is dependent on its amplitude of rotation. The Lucerne earthquake generated a much higher rotation and subsequently the frequency content of the system was a lot less, and vice-versa for El Centro.
The area of the loops in the moment-rotation plots presented above show that the damping within the model is high. Damping within the model was in the form of hysteretic damping from the unload response of the qz springs. Additionally there was 5% Rayleigh damping to account for structural energy dissipation during the motion.

### 6.4.2 FDB of the Single Shear Wall

**Wall Properties:**

\[ L_{wall} := 3.6 \text{m} \]

Length of wall

\[ h_{wall} := 3.6 \text{m} \]

Height of wall

\[ t_{wall} := 150 \text{mm} \]

Thickness of wall

**Seismic Properties:**

\[ N_{wall} := 520 \text{kN} \]

Axial load on the wall

\[ W_{total} := 1040 \text{kN} \]

Total weight

**Effective Mass and Height Calculation:**

\[ \gamma_{concrete} := 24 \frac{\text{kN}}{\text{m}^3} \]

Weight of concrete

\[ W_{wall} := L_{wall} h_{wall} t_{wall} \gamma_{concrete} \]

\[ W_{wall} = 46.656 \text{kN} \]

Weight of wall

\[ m_{wall} := \frac{W_{wall}}{g} \]

\[ m_{wall} = 4.758 \times 10^3 \text{ kg} \]

Mass of wall

\[ x_{wall} := \frac{h_{wall}}{2} \]

\[ x_{wall} = 1.8 \text{m} \]

Height to the centroid of wall
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\[ W_{\text{t,non.wall}} := W_{\text{total}} - W_{\text{wall}} \quad \text{Weight of 'other' components} \]

\[ m_{\text{non.wall}} := \frac{W_{\text{t,non.wall}}}{g} \quad \text{Mass of 'other' components} \]

\[ x_{\text{non.wall}} := 3.6 \text{m} \]

\[ \Delta_{i,\text{wall}} := 0.5 \]

\[ \Delta_{i,\text{non.wall}} := 1.0 \]

\[ \Delta_{d,\text{normalised}} := 1.0 \]

\[ m_e := \frac{m_{\text{wall}} \Delta_{i,\text{wall}} + m_{\text{non.wall}} \Delta_{i,\text{non.wall}}}{\Delta_{d,\text{normalised}}} \quad \text{Eq. 6.2} \]

\[ m_e = 1.037 \times 10^5 \text{kg} \]

\[ h_e := \frac{m_{\text{wall}} x_{\text{wall}} \Delta_{i,\text{wall}} + m_{\text{non.wall}} x_{\text{non.wall}} \Delta_{i,\text{non.wall}}}{m_{\text{wall}} \Delta_{i,\text{wall}} + m_{\text{non.wall}} \Delta_{i,\text{non.wall}}} \quad \text{Eq. 6.3} \]

\[ h_e = 3.559 \text{m} \]

\[ W_t := m_e \cdot g \quad W_t = 1.017 \times 10^3 \text{kN} \quad \text{Seismic weight} \]

**Structural Stiffness:**

\[ I_{\text{wall}} := \frac{t_{\text{wall}} I_{\text{wall}}}{12} \quad I_{\text{wall}} = 0.583 \text{m}^4 \]

\[ f_c := 30 \]

\[ E_{\text{conc}} := \left( 3320 \sqrt{f_c} + 6900 \right) \text{MPa} \quad E_{\text{conc}} = 25.084 \text{GPa} \]

\[ K_s := \frac{3 E_{\text{conc}} I_{\text{wall}}}{h_e^3} \quad K_s = 9.738 \times 10^5 \text{kN/m} \quad \text{Eq. 6.12} \]
6.4 - Single Story Shear Wall Design Example

\[ \omega_s := \sqrt{\frac{K_s}{m_e}} \quad \omega_s = 96.918 \frac{1}{s} \]

Structural natural frequency

Foundation Stiffness:

Foundation Properties:

\[ G_{\text{max}} := 40 \text{MPa} \]

\[ \nu := 0.5 \]

\[ c_{\text{op}} := 0.5 \]

Operational modulus factor

\[ G_{\text{op}} := c_{\text{max}} c_{\text{op}} \]
\[ G_{\text{op}} = 20 \text{MPa} \]

Eq. 6.14

Guess dimensions:

\[ L := 4.5 \text{m} \]

Foundation length

\[ B := 1 \text{m} \]

Foundation width

\[ D := 1 \text{m} \]

Foundation depth

\[ A_b := L \cdot B \]

Base area

\[ A_s := 2 \cdot L \cdot D \]

Side wall area

Horizontal Stiffness:

\[ K_{H_{\text{basic}}} := \frac{G_{\text{op}} L}{2 - \nu} \]

\[ I_{H_{\text{shape}}} := 2 + 2.5 \left( \frac{A_b}{L^2} \right)^{0.85} \]

\[ I_{H_{\text{depth}}} := 1 + 0.15 \left( \frac{2 \cdot D}{B} \right)^{0.5} \]
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\[ I_{H\text{_side}} := 1 + 0.52 \left( \frac{8 \left( \frac{D}{2} \cdot A_s \right)}{B \cdot L^2} \right)^{0.4} \]

\[ K_{f\_H} := K_{H\text{_basic}} \cdot I_{H\text{_shape}} \cdot I_{H\text{_depth}} \cdot I_{H\text{_side}} \]

\[ K_{f\_H} = 3.244 \times 10^5 \text{ kN/m} \]

Horizontal foundation stiffness

Rocking stiffness:

\[ G_{op} = 20\text{MPa} \]

Operational modulus

\[ d := D \]

\[ I_{\text{find}} := \frac{B \cdot L^3}{12} \]

\[ K_{\phi\_basic} := \frac{G_{op} \cdot I_{\text{find}}^{0.75}}{1 - \nu} \]

\[ I_{\phi\_shape} := 3 \left( \frac{L}{B} \right)^{0.15} \]

\[ I_{\phi\_depthside} := 1 + 1.84 \left( \frac{d}{L} \right)^{0.6} \left[ 1.5 + \left( \frac{2 \cdot d}{L} \right)^{1.9} \left( \frac{D}{d} \right)^{0.6} \right] \]

\[ K_{i_{\phi\_\text{init}}} := K_{\phi\_\text{basic}} \cdot I_{\phi\_\text{shape}} \cdot I_{\phi\_\text{depthside}} \]

\[ K_{i_{\phi\_\text{init}}} = 1.568 \times 10^6 \text{ kN\cdotm} \]

Rotational foundation stiffness

System Natural Frequency:

\[ \omega_{\text{sys}} = \frac{\omega_s^2}{\sqrt{\frac{K_s}{K_{f\_H}} + \frac{K_{s\cdot h_e}}{K_{i_{\phi\_\text{init}}}}}} \]

\[ \omega_{\text{sys}} = 28.134 \frac{1}{s} \]

\[ \omega_{\text{sys}} = 28.134 \frac{1}{s} \]

Eq. 6.25
The $Ch(T)$ factor is now read from Figure 6.3 corresponding to the system period above.

**Elastic Site Spectra:**

\[ Ch(T) := 3.0 \]

\[ Z := 0.4 \]

\[ R := 1.0 \]

\[ N_{T,D} := 1.0 \]

\[ C_{T,1} := Ch(T) \cdot Z \cdot R \cdot N_{T,D} \quad C_{T,1} = 1.2 \quad \text{Eq. 6.5} \]

**Calculate the damping adjustment factor:**

\[ \xi_{\text{sys}} := 20 \quad \text{System damping} \]

\[ K_\xi := \left( \frac{7}{2 + \xi_{\text{sys}}} \right)^{0.5} \quad K_\xi = 0.564 \quad \text{Eq. 6.7} \]

\[ C_T := C_{T,1} \cdot K_\xi \]

\[ \mu := 2.5 \quad \text{Ductility} \]

\[ S_p := 1.3 - 0.3 \mu \quad S_p = 0.55 \quad \text{Eq. 6.29} \]

\[ S_p := 0.7 \quad \text{Limiting factor} \]

\[ k_\mu := \frac{(\mu - 1) \cdot \left( \frac{T_e}{s} \right)}{0.7} + 1 \quad k_\mu = 1.479 \quad \text{Eq. 6.31} \]

**Design action coefficient:**

\[ C_{d,T} := \frac{C_T \cdot S_p}{k_\mu} \quad C_{d,T} = 0.32 \quad \text{Eq. 6.28} \]
Chapter 6 - Rocking Foundation Design Guideline

Base Shear:

\[ V_b := W_t \cdot C_{d,T} \]

\[ V_b = 325.803 \text{kN} \quad \text{Eq. 6.27} \]

Note: A similar base shear to the previous DBD

\[ M_{\text{star}} := V_b \cdot h_c \]

\[ M_{\text{star}} = 1.159 \times 10^3 \text{kN-m} \quad \text{Eq. 6.11} \]

Max Resisting Moment:

Bearing Capacity:

\[ s_u := 100 \text{kPa} \quad \text{Undrained shear strength} \]

\[ c := s_u \]

\[ \phi := 0 \]

\[ \gamma := 17 \frac{\text{kN}}{\text{m}^3} \quad \text{Unit weight (kN/m}^3\text{)} \]

\[ q := D \cdot \gamma \quad \text{Surcharge} \]

Bearing Capacity Factors:

\[ N_q := e^{\pi \cdot \tan(\phi)} \cdot \tan\left(45 \text{ deg} + \frac{\phi}{2}\right)^2 \]

\[ N_c := \begin{cases} (N_q - 1) \cdot \cot(\phi) & \text{if } \phi > 0 \\ 5.14 & \text{otherwise} \end{cases} \quad N_c = 5.14 \]

\[ N_y := 2 \cdot (N_q - 1) \cdot \tan(\phi) \]

Shape Factors:

\[ \lambda_{cs} := 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) \]
\[ \lambda_{qs} := 1 + \left( \frac{B}{L} \right) \tan(\phi) \]

\[ \lambda_{xs} := 1 - 0.3 \left( \frac{B}{L} \right) \]

**Depth Factors:**

\[ \lambda_{qd} := \begin{cases} 
1 + 2 \tan(\phi) \cdot \left( 1 - \sin(\phi) \right)^2 \left( \frac{D}{B} \right) & \text{if } \frac{D}{B} \leq 1 \\
1 + 2 \tan(\phi) \cdot \left( 1 - \sin(\phi) \right)^2 \cdot \tan \left( \frac{D}{B} \right) & \text{otherwise} 
\end{cases} \]

\[ \lambda_{cd} := \begin{cases} 
\text{if } \phi = 0 \\
1 + 0.4 \left( \frac{D}{B} \right) & \text{if } \frac{D}{B} \leq 1 \\
1 + 0.4 \cdot \tan \left( \frac{D}{B} \right) & \text{otherwise} 
\end{cases} \]

\[ \lambda_{xd} := 1 \]

**Inclination Factors:**

**Note:** Assume vertical loading only at this stage

\[ \lambda_{qi} := 1 \]

\[ \lambda_{ci} := 1 \]

\[ \lambda_{yi} := 1 \]

**Bearing Capacity Equation:**

\[ q_u := c \cdot \lambda_{cs} \cdot \lambda_{cd} \cdot \lambda_{ci} \cdot N_c + q \cdot \lambda_{qs} \cdot \lambda_{qd} \cdot \lambda_{qi} \cdot N_q + 0.5 \gamma B \lambda_{ys} \cdot \lambda_{yd} \cdot \lambda_{yi} N_y \]
\[ q_u = 7.677 \times 10^5 \text{Pa} \quad q_u = 767.71 \text{kPa} \]  

Ultimate bearing capacity

\[ Q_u := q_u \cdot B \cdot L \quad Q_u = 3.455 \times 10^3 \text{kN} \]  

Ultimate Load

**Factor of Safety:**

\[ N_{\text{fnd}} := \gamma_{\text{concrete}} \cdot L \cdot B \cdot D \quad N_{\text{fnd}} = 108 \text{kN} \]  

Weight of foundation

\[ N := N_{\text{wall}} + N_{\text{fnd}} \quad N = 628 \text{kN} \]  

Total weight

\[ \text{FS}_v := \frac{Q_u}{N} \quad \text{FS}_v = 5.501 \]  

Factor of Safety

**Moment Capacity**

\[ \text{Mult} = \frac{N \cdot L}{2} \left( 1 - \frac{1}{\text{FS}_v} \right) \quad \text{Mult} = 1.156 \times 10^3 \text{kN} \cdot \text{m} \]  

Eq. 6.24

Is Mult > M*?

\[ \text{Mult} - M_{\text{star}} = -3.292 \text{kN} \cdot \text{m} \]  

Positive is OK  
Near Enough

**Displacement Check:**

\[ K_e := \omega_{\text{sys}}^2 \cdot m_e \quad K_e = 8.206 \times 10^4 \frac{\text{kN}}{\text{m}} \]  

System stiffness

\[ \Delta_{\text{static}} := \frac{V_b}{K_e \cdot K_e} \quad \Delta_{\text{static}} = 7.039 \text{mm} \]  

Eq. 6.32

\[ \Delta_{\text{total}} := \Delta_{\text{static}} + \Delta_{\text{mu}} \quad \Delta_{\text{total}} = 17.597 \text{mm} \]  

Eq. 6.33

**Drift:**

\[ \text{drift} := \frac{\Delta_{\text{total}}}{h_e} \cdot 100 \quad \text{drift} = 0.494 \]

The drift is well within the limits
6.4 - Single Story Shear Wall Design Example

Damping Check:

\[ \xi_f := 20 \]
\[ \xi_s := 5 \]

\[ \Delta_s := \frac{V_b}{K_s} \quad \Delta_s = 0.335 \text{mm} \quad \text{Structure deflection} \]

\[ \Delta_F := \Delta_{\text{total}} - \Delta_s \quad \Delta_F = 17.263 \text{mm} \quad \text{Foundation deflection} \]

\[ \xi_{\text{sys}} := \frac{\xi_f \Delta_F + \xi_s \Delta_s}{\Delta_{\text{total}}} \quad \xi_{\text{sys}} = 19.715 \quad \text{Eq. 6.6} \]

Note: Again the system damping is essentially unaffected by the structural damping because of the low structural displacement - 20% system damping is OK

The dimensions for this design were similar to the DBD previous – 4.5 m by 1.0 m. However the calculated deflections in the FBD were less than the DBD – the total calculated deflection in the FBD was 18 mm, whereas the design displacement in the DBD was 70 mm. Ultimately the DBD is more conservative – as it assumes a higher displacement for a similar size foundation – and is the recommended option for rocking foundation design.

6.4.3 Shear Wall to Preclude Rotation

In this design, the same situation as above – a single story shear wall – is designed assuming no foundation rotation. The method used is a traditional bearing capacity approach, where moment is accounted for by eccentricity in the foundation. This eccentricity, \( e \), is described in Chapter 2.
Chapter 6 - Rocking Foundation Design Guideline

Wall Properties:

\[ L\text{wall} = 3.6\text{m} \]  
Length of wall

\[ h\text{wall} = 3.6\text{m} \]  
Height of wall

\[ t\text{wall} = 150\text{mm} \]  
Thickness of wall

Seismic Properties:

\[ N\text{wall} = 520\text{kN} \]  
Axial load on the wall

\[ W_{\text{total}} = 1040\text{kN} \]  
Total weight

Effective Mass and Height Calculation:

\[ \gamma_{\text{concrete}} = \frac{24\text{ kN}}{\text{m}^3} \]  
Weight of concrete

\[ W_{\text{wall}} = L\text{wall} h\text{wall} t\text{wall} \gamma_{\text{concrete}} \]  
Weight of wall

\[ W_{\text{wall}} = 46.656\text{kN} \]

\[ m_{\text{wall}} = \frac{W_{\text{wall}}}{g} \]  
Mass of wall

\[ x_{\text{wall}} = \frac{h_{\text{wall}}}{2} \]  
Height to the centroid of wall

\[ x_{\text{wall}} = 1.8\text{m} \]

\[ W_{\text{t.non.wall}} = W_{\text{total}} - W_{\text{wall}} \]  
Weight of 'other' components

\[ W_{\text{t.non.wall}} = 993.34\text{.34 kN} \]

\[ m_{\text{non.wall}} = \frac{W_{\text{t.non.wall}}}{g} \]  
Mass of 'other' components

\[ m_{\text{non.wall}} = 1.013 \times 10^5 \text{ kg} \]

\[ x_{\text{non.wall}} = 3.6\text{m} \]

\[ \Delta_i.\text{wall} = 0.5 \]

\[ \Delta_i.\text{non.wall} = 1.0 \]

\[ \Delta_d.\text{normalised} = 1.0 \]
\[ m_e := \frac{m_{\text{wall}} \Delta_{i, \text{wall}} + m_{\text{non,wall}} \Delta_{i, \text{non,wall}}}{\Delta_{\text{d,normalised}}} \quad \text{Eq. 6.2} \]

\[ m_e = 1.037 \times 10^5 \text{ kg} \]

\[ h_e := \frac{m_{\text{wall}} x_{\text{wall}} \Delta_{i, \text{wall}} + m_{\text{non,wall}} x_{\text{non,wall}} \Delta_{i, \text{non,wall}}}{m_{\text{wall}} \Delta_{i, \text{wall}} + m_{\text{non,wall}} \Delta_{i, \text{non,wall}}} \quad \text{Eq. 6.3} \]

\[ h_e = 3.559 \text{ m} \]

\[ W_t := m_e g \quad W_t = 1.017 \times 10^3 \text{ kN} \quad \text{Seismic weight} \]

**Structural Stiffness:**

\[ I_{\text{wall}} := \frac{l_{\text{wall}}^3}{12} \quad I_{\text{wall}} = 0.583 \text{ m}^4 \]

\[ f'_{c} = 30 \]

\[ E_{\text{conc}} := \left(3320 \sqrt{f'_{c}} + 6900\right) \cdot 1\text{MPa} \quad E_{\text{conc}} = 25.084 \text{GPa} \]

\[ K_s := \frac{3 E_{\text{conc}} l_{\text{wall}}}{h_e^3} \quad K_s = 9.738 \times 10^5 \frac{\text{kN}}{\text{m}} \quad \text{Eq. 6.12} \]

\[ \omega_s := \sqrt{\frac{K_s}{m_e}} \quad \omega_s = 96.918 \frac{1}{\text{s}} \quad \text{Structural natural frequency} \]

\[ T_s := \frac{2\pi}{\omega_s} \quad T_s = 0.065 \text{s} \quad \text{Structural period} \]

The Ch(T) factor is now read from Figure 6.3 corresponding to the system period above.

**Elastic Site Spectra:**

\[ C_{h,T} := 3.0 \]

\[ Z := 0.4 \]
R := 1.0

N_{T,D} := 1.0

C_T := C_{h,T} \cdot Z \cdot R \cdot N_{T,D} \quad C_T = 1.2 \quad \text{Eq. 6.5}

\mu := 2 \quad \text{Ductility}

S_p := 1.3 - 0.3 \mu \quad S_p = 0.7 \quad \text{Eq. 6.29}

\left( \mu - 1 \right) \left( \frac{T_s}{s} \right) 0.7 + 1 \quad k_\mu = 1.093 \quad \text{Eq. 6.31}

\text{Design action coefficient:}

C_{d,T} := \frac{C_T \cdot S_p}{k_\mu} \quad C_{d,T} = 0.769 \quad \text{Eq. 6.28}

\text{Base Shear:}

V_b := W_t \cdot C_{d,T} \quad V_b = 781.616 \text{kN} \quad \text{Eq. 6.27}

M_{\text{star}} := V_b \cdot h_e \quad M_{\text{star}} = 2.782 \times 10^3 \text{kN} \cdot \text{m} \quad \text{Eq. 6.11}

\text{Effective Foundation Dimensions:}

L := 7.5 \text{m} \quad \text{Foundation length}

B := 2 \text{m} \quad \text{Foundation width}

D := 1 \text{m} \quad \text{Foundation depth}

N_{\text{fnd}} := \gamma_{\text{concrete}} \cdot L \cdot B \cdot D \quad N_{\text{fnd}} = 360 \text{kN} \quad \text{Weight of foundation}

N := N_{\text{wall}} + N_{\text{fnd}} \quad N = 880 \text{kN} \quad \text{Total weight}

\epsilon' := \frac{M_{\text{star}}}{N} \quad \epsilon' = 3.161 \text{m}
6.4 - Single Story Shear Wall Design Example

L' := L - 2e'  

L' = 1.178m  

Effective foundation length

A_b := L' B

Base area

Bearing Capacity:

s_u := 100kPa

Undrained shear strength

c := s_u

\( \phi := 0 \)

\( \gamma := 17 \frac{kN}{m^3} \)

Unit weight (kN/m^3)

q := D \cdot \gamma

Surcharge

Bearing Capacity Factors:

\( N_q := e^{\pi \cdot \tan(\phi) \cdot \tan\left(45 \text{ deg} + \frac{\phi}{2}\right)^2} \)

\( N_c := \begin{cases} (N_q - 1) \cdot \cot(\phi) & \text{if } \phi > 0 \\ 5.14 & \text{otherwise} \end{cases} \)

(\( N_c = 5.14 \))

\( N_q := 2 \cdot (N_q - 1) \cdot \tan(\phi) \)

Shape Factors:

\( \lambda_{cs} := 1 + \left(\frac{B}{L'}\right) \cdot \left(\frac{N_q}{N_c}\right) \)

\( \lambda_{qs} := 1 + \left(\frac{B}{L'}\right) \cdot \tan(\phi) \)

\( \lambda_{qs} := 1 - 0.3 \left(\frac{B}{L'}\right) \)
Depth Factors:

\[
\lambda_{qd} := \begin{cases} 
1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \left( \frac{D}{B} \right) & \text{if} \quad \frac{D}{B} \leq 1 \\
1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \tan\left( \frac{D}{B} \right) & \text{otherwise}
\end{cases}
\]

\[
\lambda_{cd} := \begin{cases} 
\quad 
\quad & \text{if} \quad \phi = 0 \\
1 + 0.4 \left( \frac{D}{B} \right) & \text{if} \quad \frac{D}{B} \leq 1 \\
1 + 0.4 \tan\left( \frac{D}{B} \right) & \text{otherwise}
\end{cases}
\]

\[
\lambda_{yd} := 1
\]

Inclination Factors:

**Note:** Assume vertical loading only at this stage

\[
\lambda_{qi} := 1 \quad \lambda_{ci} := 1 \quad \lambda_{yi} := 1
\]

Bearing Capacity Equation:

\[
qu := c \lambda_{cs} \lambda_{cd} \lambda_{ci} \lambda_{qi} N_c + q \lambda_{qs} \lambda_{qy} N_q + 0.5 \gamma B \lambda_{yd} \lambda_{yi} N_y
\]

\[
qu = 8.375 \times 10^5 \text{ Pa} \quad q_u = 837.478 \text{kPa} \quad \text{Ultimate bearing capacity}
\]

\[
Q_u := q_u B L' \quad Q_u = 1.974 \times 10^3 \text{kN} \quad \text{Ultimate Load}
\]

Factor of Safety:

\[
FS_v := \frac{Q_u}{N} \quad FS_v = 2.243 \quad \text{Factor of Safety}
\]
The design above is done using traditional methods for foundation design and the dimensions are 7.5 m long by 2 m wide – significantly larger than the foundations designed for rocking. The ductility of the wall was set at 2, having the effect of reducing the forces on the structure. The system must undergo structural yielding to satisfy this requirement. The static factor of safety for this design method is 2.2, even though the foundation was significantly larger.

The moment to vertical load ratio, high because of the low vertical load, is the cause as to why the footing length is much longer. This high ratio results in high footing eccentricity and the length specified, 7.5 m, only results in an effective length, L’, of 1.2 m.

This design again did not consider the effects of the base shear on the foundation by assuming the inclination factors were unity. The reason being is that usually a shear wall of this type is ‘locked in’ within a structure, and so has high shear resistance.

6.5 SIX STORY SHEAR WALL DESIGN EXAMPLE

Figure 6.20 presents an outline of the second design example – a six storied shear wall. The design was done using the DBD methodology and is given below as a MathCad document, with time history analyses in OpenSEES following. The height of each story is 3.5 m. The seismic weight at each floor level is 1120 kN and half of that (560 kN) at roof level. The vertical load on the wall was one tenth of the seismic load – 120 kN at each floor and 56 kN at the roof. Again this example assumes the building to be situated in Wellington.
Figure 6.20 Building Elevation showing the six story shear wall being designed for

**Wall Properties:**

- $L_{\text{wall}} := 5\text{m}$  
  Length of wall

- $h_{\text{wall}} := 21\text{m}$  
  Height of wall

- $t_{\text{wall}} := 250\text{mm}$  
  Thickness of wall

**Seismic Properties:**

- $N_{\text{wall}} := 616\text{kN}$  
  Axial load on the wall

- $W_{\text{total}} := 6160\text{kN}$  
  Total weight

- $W_{\text{each}} := 1120\text{kN}$

- $W_{\text{top}} := \frac{W_{\text{each}}}{2}$  
  $W_{\text{top}} = 560\text{kN}$

- $m_{\text{each}} := \frac{W_{\text{each}}}{g}$  
  $m_{\text{each}} = 114208.21\text{kg}$
Effective Mass and Height Calculation:

Therefore:

\[ \Delta_1 := 0.1666, \quad h_1 := 3.5\text{m} \]
\[ \Delta_2 := 0.3333, \quad h_2 := 7.0\text{m} \]
\[ \Delta_3 := 0.5, \quad h_3 := 10.5\text{m} \]
\[ \Delta_4 := 0.6666, \quad h_4 := 14\text{m} \]
\[ \Delta_5 := 0.8333, \quad h_5 := 17.5\text{m} \]
\[ \Delta_6 := 1.0, \quad h_6 := 21\text{m} \]

\[ \Delta_{\text{d.normalised}} := 1.0 \]

\[ m_e := \frac{m_{\text{each}} \cdot \Delta_1 + m_{\text{each}} \cdot \Delta_2 + m_{\text{each}} \cdot \Delta_3 + m_{\text{each}} \cdot \Delta_4 + m_{\text{each}} \cdot \Delta_5 + m_{\text{top}} \cdot \Delta_6}{\Delta_{\text{d.normalised}}} \quad \text{Eq. 6.2} \]

\[ m_e = 3.426 \times 10^4 \text{kg} \]

\[ h_e := \frac{m_{\text{each}} \cdot \Delta_1 \cdot h_1 + m_{\text{each}} \cdot \Delta_2 \cdot h_2 + m_{\text{each}} \cdot \Delta_3 \cdot h_3 + m_{\text{each}} \cdot \Delta_4 \cdot h_4 + m_{\text{each}} \cdot \Delta_5 \cdot h_5 + m_{\text{top}} \cdot \Delta_6 \cdot h_6}{m_{\text{each}} \cdot \Delta_1 + m_{\text{each}} \cdot \Delta_2 + m_{\text{each}} \cdot \Delta_3 + m_{\text{each}} \cdot \Delta_4 + m_{\text{each}} \cdot \Delta_5 + m_{\text{top}} \cdot \Delta_6} \quad \text{Eq. 6.3} \]

\[ h_e = 14.195\text{m} \]

\[ \gamma_{\text{concrete}} := 24 \frac{\text{kN}}{\text{m}^3} \quad \text{Weight of concrete} \]

\[ W_t := m_e \cdot g \]

\[ W_t = 3.36 \times 10^3 \text{kN} \quad \text{Seismic weight} \]
Design Displacement:

Make the design displacement based on 2.0% drift

\[ \Delta_d := \frac{2h_e}{100} \]

\[ \Delta_d = 283.89 \text{ mm} \quad \text{Design Displacement} \]

Note: Reading off the Displacement Spectra for subsoil class C for 20% damping (Figure 6.4), the displacement does not reach so high based on NZS 1170.5 guidelines. Therefore we will specify a design displacement of 200 mm. A linear mode shape has been assumed for the soil-footing-structure displacement.

\[ \Delta_d := 200 \text{ mm} \]

\[ T_e := 2.75 \text{s} \quad \text{Effective period} \]

\[ K_c := 4\pi^2 \frac{m_e}{T_e^2} \]

\[ K_c = 1.789 \times 10^3 \frac{\text{kN}}{\text{m}} \quad \text{Eq. 6.8} \]

\[ \omega_{sys} := \frac{2\pi}{T_e} \]

\[ \omega_{sys} = 2.285 \frac{\text{rads}}{\text{s}} \quad \text{Eq. 6.9} \]

Base Shear:

\[ V_b := K_c \Delta_d \]

\[ V_b = 357.707 \text{kN} \quad \text{Eq. 6.10} \]

Structural Stiffness:

\[ I_{wall} = \frac{t_{wall}^3}{12} \]

\[ I_{wall} = 2.604 \text{m}^4 \]

Note: We are using here the gross inertia value (without considering reinforcement). In practice this I value should be perhaps the cracked I value - \( I_c \).

We will assume the concrete has an \( f_c \) of 30 MPa

\[ f_c := 30 \]

\[ E_{\text{conc}} := \left( 3320 \sqrt{f_c} + 6900 \right) \text{MPa} \]

\[ E_{\text{conc}} = 25.084 \text{GPa} \quad \text{(From concrete code)} \]

\[ K_s := \frac{3E_{\text{conc}}I_{wall}}{h_e^3} \]

\[ K_s = 6.852 \times 10^4 \frac{\text{kN}}{\text{m}} \quad \text{Eq. 6.12} \]
\( \omega_s := \sqrt{\frac{K_s}{m_c}} \)

\( \omega_s = 14.142 \frac{1}{s} \)

**Structural natural frequency**

**Structural Deformation:**

\( \Delta_s := \frac{V_b}{K_s} \)

\( \Delta_s = 5.22 \text{mm} \)  
**Eq. 6.13**

**Note:** It is shown here that the elastic deformation of the wall is almost insignificant when looking at the total system deflection.

\( \Delta_f := \Delta_d - \Delta_s \)

\( \Delta_f = 194.78 \text{mm} \)  
**Eq. 6.15**

**Horizontal Deflection due to foundation**

**Damping Check:**

Assume:
- Foundation damping of 20%
- Structural damping of 5%

\( \xi_f := 20 \)

\( \xi_s := 5 \)

\( \xi_{sys} := \frac{\xi_f \Delta_f + \xi_s \Delta_s}{\Delta_d} \)  

\( \xi_{sys} = 19.608 \)  
**Eq. 6.6**

**Note:** The system damping is essentially unaffected by the structural damping

**Foundation Stiffness:**

**Foundation Properties:**

\( G_{max} := 40 \text{MPa} \)

\( \nu := 0.5 \)

\( c_{op} := 0.5 \)  
**Operational modulus factor**

\( G_{op} := G_{max} c_{op} \)

\( G_{op} = 20 \text{MPa} \)  
**Eq. 6.14**
Guess dimensions:

\[ L := 9 \text{m} \quad \text{Foundation length} \]
\[ B := 2.8 \text{m} \quad \text{Foundation width} \]
\[ D := 1 \text{m} \quad \text{Foundation depth} \]
\[ A_b := L \cdot B \quad \text{Base area} \]
\[ A_s := 2 \cdot L \cdot D \quad \text{Side wall area} \]

**Horizontal Stiffness (Gazetas formulae):**

\[ K_{H,\text{basic}} := \frac{G_{op} \cdot L}{2 - \nu} \]

\[ I_{H,\text{shape}} := 2 + 2.5 \left( \frac{A_b}{L^2} \right)^{0.85} \]

\[ I_{H,\text{depth}} := 1 + 0.15 \left( \frac{2D}{B} \right)^{0.5} \]

\[ I_{H,\text{side}} := 1 + 0.52 \left[ \frac{8 \left( \frac{D}{2} \right)^{0.4}}{A_s B \cdot L^2} \right] \]

\[ K_{f,H} := K_{H,\text{basic}} I_{H,\text{shape}} I_{H,\text{depth}} I_{H,\text{side}} \]

\[ K_{f,H} = 5.258 \times 10^5 \text{ kN/m} \quad \text{Horizontal foundation stiffness} \]

**Horizontal Movement:**

\[ \Delta_H := \frac{V_b}{K_{f,H}} \]

\[ \Delta_H = 0.68 \text{mm} \quad \text{Eq. 6.17} \]

**Note:** Again we see here that the deflection from the horizontal movement of the foundation is very small - this is consistent with what the experiments showed as well.
Foundation Rotation:

\[ \theta_f := \frac{\Delta_f - \Delta_H}{h_e} \quad \theta_f = 0.014 \text{ rads} \quad \text{Eq. 6.18} \]

Rotational Stiffness:

\[ M_{\text{star}} := V_b h_e \quad M_{\text{star}} = 5.077 \times 10^3 \text{ kN m} \quad \text{Eq. 6.11} \]

\[ K_{f,\text{eff}} := \frac{M_{\text{star}}}{\theta_f} \quad K_{f,\text{eff}} = 3.713 \times 10^5 \text{ kN m rad}^{-1} \quad \text{Eq. 6.19} \]

Moment Rotation Curve:

Note: The form of the moment-rotation curve is \( M = \phi(a+b\phi) \)

Rocking stiffness (Gazetas formulae):

\[ G_{\text{op}} = 20\text{MPa} \quad \text{Operational modulus} \]

\[ d := D \]

\[ I_{\text{fnd}} := \frac{B L^3}{12} \]

\[ K_{\phi,\text{basic}} := \frac{G_{\text{op}} I_{\text{fnd}}^{0.75}}{1 - v} \]

\[ I_{\phi,\text{shape}} := 3 \left( \frac{L}{B} \right)^{0.15} \]

\[ I_{\phi,\text{depthside}} := 1 + 1.84 \left( \frac{d}{L} \right)^{0.6} \left[ 1.5 + \left( \frac{2 d}{L} \right)^{1.9} \left( \frac{D}{d} \right)^{0.6} \right] \]

\[ K_{\text{ini},\phi} := K_{\phi,\text{basic}} I_{\phi,\text{shape}} I_{\phi,\text{depthside}} \]
K_{\text{ini,} \phi} = 1.19 \times 10^7 \text{kN-m} \quad \text{Rotational foundation stiffness}

\begin{align*}
a & := \frac{1}{K_{\text{ini,} \phi}} \quad \text{Eq. 6.22}
\end{align*}

**Bearing Capacity:**

\begin{align*}
s_u & := 100 \text{kPa} \quad \text{Undrained shear strength} \\
c & := s_u \\
\phi & := 0
\end{align*}

\begin{align*}
\gamma & := 17 \frac{\text{kN}}{\text{m}^3} \quad \text{Unit weight (kN/m}^3) \\
q & := D \gamma \\
N_q & := e^{\pi \tan(\phi) \tan\left(45 \text{ deg} + \frac{\phi}{2}\right)^2} \\
N_c & := \begin{cases} (N_q - 1) \cot(\phi) & \text{if } \phi > 0 \\ 5.14 & \text{otherwise} \end{cases} \quad N_c = 5.14 \\
N_q & := 2(N_q - 1) \tan(\phi)
\end{align*}

**Shape Factors:**

\begin{align*}
\lambda_{cs} & := 1 + \left(\frac{B}{L}\right) \left(\frac{N_q}{N_c}\right) \\
\lambda_{qs} & := 1 + \left(\frac{B}{L}\right) \tan(\phi)
\end{align*}
\[ \lambda_{ps} := 1 - 0.3 \left( \frac{B}{L} \right) \]

**Depth Factors:**

\[ \lambda_{qd} := \begin{cases} 
1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \frac{D}{B} & \text{if } \frac{D}{B} \leq 1 \\
1 + 2 \tan(\phi) \left( 1 - \sin(\phi) \right)^2 \atan \left( \frac{D}{B} \right) & \text{otherwise} 
\end{cases} \]

\[ \lambda_{cd} := \begin{cases} 
\text{if } \phi = 0 \\
1 + 0.4 \left( \frac{D}{B} \right) & \text{if } \frac{D}{B} \leq 1 \\
1 + 0.4 \atan \left( \frac{D}{B} \right) & \text{otherwise} 
\end{cases} \]

\[ \lambda_{qd} - \frac{1 - \lambda_{qd}}{N_q \tan(\phi)} & \text{otherwise} \]

\[ \lambda_{qd} := 1 \]

**Inclination Factors:**

**Note:** Assume vertical loading only at this stage

\[ \lambda_{qi} := 1 \]

\[ \lambda_{ci} := 1 \]

\[ \lambda_{qi} := 1 \]

**Bearing Capacity Equation:**

\[ q_u := c \lambda_{cs} \lambda_{cd} \lambda_{ci} N_c + q \lambda_{qs} \lambda_{qd} \lambda_{qi} N_q + 0.5 \gamma B \lambda_{ys} \lambda_{yd} \lambda_{yi} N_y \]

\[ q_u = 6.4 \times 10^5 \text{ Pa} \quad q_u = 639.98 \text{ kPa} \quad \text{Ultimate bearing capacity} \]
\[ Q_u := q_u \cdot B \cdot L \quad Q_u = 1.613 \times 10^4 \text{kN} \quad \text{Ultimate Load} \]

**Factor of Safety:**

\[ N_{\text{fnd}} := \gamma_{\text{concrete}} \cdot L \cdot B \cdot D \quad N_{\text{fnd}} = 604.8 \text{kN} \quad \text{Weight of foundation} \]

\[ N := N_{\text{wall}} + N_{\text{fnd}} \quad N = 1.221 \times 10^3 \text{kN} \quad \text{Total weight} \]

\[ FS_v := \frac{Q_u}{N} \quad FS_v = 13.211 \quad \text{Factor of Safety} \]

**Moment Capacity**

\[ M_{\text{ult}} := \frac{N \cdot L}{2} \left( 1 - \frac{1}{FS_v} \right) \quad M_{\text{ult}} = 5.078 \times 10^3 \text{kN} \cdot \text{m} \quad \text{Eq. 6.24} \]

\[ b := \frac{1}{M_{\text{ult}}} \quad \text{Eq. 6.23} \]

\[ K_{f_\phi} := \frac{1}{a + b \cdot \theta_f} \quad K_{f_\phi} = 3.601 \times 10^5 \text{kN} \cdot \text{m} \quad \text{Eq. 6.21} \]

**Check to see the stiffnesses match up**

\[ \frac{K_{f_\phi}}{K_{f_{\text{eff}}}} = 0.97 \]

Is it within 10%? Yes - OK

**Check Moment Capacity of the Foundation:**

\[ M_{\text{star}} = 5.077 \times 10^3 \text{kN} \cdot \text{m} \quad \text{Eq. 6.11} \]

\[ M_{\text{ult}} - M_{\text{star}} = 0.27 \text{kN} \cdot \text{m} \quad \text{Positive is OK} \]
The design calculates the foundation dimensions to be 9.0 m long by 2.8 m wide for the six storied shear wall. The foundation for this example is particularly large because of the reduced rotation that was specified. The design foundation rotation for the example was 14 millirads, the foundation had to be stiffer, and consequently larger, to achieve a rotation of this amount.

An important point is the selection of the design displacement, $\Delta d$, altered from 284 mm to 200 mm. A drift limit of 2% on the shear wall originally calculated a design displacement of around 284 mm, however Figure 6.5 suggests that for a system damping of 20%, 284 mm displacement cannot be achieved on a class C soil. Earthquakes in New Zealand will not shake at low enough frequencies to achieve a spectral displacement of 284 mm. Before a structure in New Zealand displaced that much, the earthquake motion would have changed direction and the structure would be moving back the other way. This is why the spectral displacement plots in Figure 6.5 have upper bounds on the higher period structures. For an example such as this one, building proximity may be critical instead of drift limits, as it is shown the drift limits on taller structures results in larger displacements.

6.5.3.1 Time History Analyses of the Six Story Shear Wall

Time history analysis was performed on this example, using the OpenSEES spring bed model – identical to the first example. The analyses used the same seven earthquake records as the previous, scaled according to the structures period using the same technique as previous. The model was simplified to a single degree of freedom model, and the effective mass, $m_e$, was used at the effective height, $h_e$.

The model includes the nonlinear vertical qz springs as described in Chapter 5. This model correlates well with the equation for the moment-rotation (refer to Figures 5.26 and 5.27). Rotational inertia effects were included by the mass at height that had vertical, horizontal and rotational mass.

Table 6.2 summaries the results of the time history analyses. The average maximum was again under the design value – 11.1 millirads on average and 14 millirads in design. The maximum rotation recorded during the analyses was 15.7 millirads. The average
maximum displacement was higher than the design displacement – 213.0 mm compared with 200.0 mm. Again high directivity of the Lucerne motion causing the greatest displacement – 288.2 mm.

The average maximum moment recorded was greater than the moment capacity the foundation in the design. The moment capacity of the foundation was 5270 kNm, and the average maximum moment in the analysis was 6797 kNm – around 1.3 times greater than the capacity. Comparing the two Lucerne moment-rotation plots for the single story and six storied shear walls (Figures 6.14 and 6.21) gives insight as to why the moment was greater than the design moment but the rotation was less. The shape in the moment-rotation in Figure 6.21 indicates elastic soil behaviour and suggests the nonlinearity comes from geometric nonlinearity, whereas Figure 6.14 indicates more soil nonlinearity. Less soil yielding is occurring in the six storied example because the foundation is rotating much less, hence the moment rotation curve remains more elastic. The increase in moment comes from higher order effects (the foundation rotating one direction but the mass rotating another direction) and should be checked in design. Higher order effects are generated because the foundation remained essentially elastic, and therefore the assumed first order rocking mode did not occur. Additional research looking into higher order effects of taller rocking structures is recommended as this aspect is not researched any further in this thesis.

### Table 6.3 Summarised results from the time history analyses performed on the six story shear wall in example 4

<table>
<thead>
<tr>
<th></th>
<th>PGA (g)</th>
<th>Max Moment (kNm)</th>
<th>Rotation (millirads)</th>
<th>Displacement (mm)</th>
<th>Dynamic Settlement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max</td>
<td>Residual</td>
<td>Max</td>
<td>Residual</td>
</tr>
<tr>
<td>El Centro</td>
<td>0.358</td>
<td>6203.9</td>
<td>6.7</td>
<td>0.09</td>
<td>201.9</td>
</tr>
<tr>
<td>Duzce</td>
<td>0.247</td>
<td>7031.2</td>
<td>13.6</td>
<td>0.26</td>
<td>175.5</td>
</tr>
<tr>
<td>Tabas</td>
<td>0.584</td>
<td>7344.2</td>
<td>15.7</td>
<td>0.27</td>
<td>222.4</td>
</tr>
<tr>
<td>Hokkaido</td>
<td>0.261</td>
<td>6035.7</td>
<td>5.9</td>
<td>0.40</td>
<td>214.6</td>
</tr>
<tr>
<td>La Union</td>
<td>0.409</td>
<td>7177.0</td>
<td>13.4</td>
<td>0.31</td>
<td>167.1</td>
</tr>
<tr>
<td>Lucerne</td>
<td>0.515</td>
<td>7026.9</td>
<td>12.4</td>
<td>0.05</td>
<td>288.2</td>
</tr>
<tr>
<td>Arcelik</td>
<td>0.525</td>
<td>6764.6</td>
<td>10.2</td>
<td>0.21</td>
<td>221.3</td>
</tr>
<tr>
<td>Average</td>
<td>0.414</td>
<td>6797.6</td>
<td>11.1</td>
<td>0.23</td>
<td>213.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.584</td>
<td>7344.2</td>
<td>15.7</td>
<td>0.40</td>
<td>288.2</td>
</tr>
</tbody>
</table>
Figure 6.21 Moment-rotation and settlement-rotation plots for Lucerne

Figure 6.22 Displacement time history of the effective mass for Lucerne

Figure 6.23 Rotation time history of the foundation mass for Lucerne
Figure 6.24 Moment-rotation and settlement-rotation plots for Duzce

Figure 6.25 Displacement time history of the effective mass for Duzce

Figure 6.26 Rotation time history of the foundation mass for Duzce
The settlement calculated during the earthquakes was, on average, less than the first example. The static settlement for this example was around 0.50 mm, and the average settlement recorded during the dynamic motion was 2.86 mm. The settlement predicted by Gazetas equations was 0.73 mm – greater than what OpenSEES calculated.

The average residual displacements were 3.57 mm from all seven earthquakes, corresponding to a drift of 0.03%. The maximum residual displacement that occurred was during the Tabas motion – 5.76 mm – corresponding to 0.04% drift.

Figures 6.21 through 6.26 present the moment-rotation, settlement-rotation, displacement time history, and rotation time history of the Lucerne and Duzce motions respectively. The results of all seven earthquakes can be found in Appendix E following the first example.

The moment-rotation plots from the six story example display somewhat less damping than the single story example. Damping within the system comes from hysteretic behaviour of the qz spring element, and the magnitude of damping is dependent on displacement. The six story wall had a foundation rotation less than the single story wall (14 millirads compared to 20 millirads) and this was evident in the time history plots as well, therefore the six story example produced less damping during the rocking motion.

The displacement and rotation time history plots (Figures 6.22, 6.23, 6.25 and 6.26) indicate higher order effects occurring during the motion. The high frequency peaks suggest the structural wall response influencing the system response as much as the foundation. The higher mode frequencies of the structure are close to the frequencies during the earthquake, and hence account for the moment-rotation plots reaching larger magnitudes than the design suggested. The wall for the OpenSEES analyses was considered elastic, however in such cases accurate structural modelling, in addition to foundation modelling, should be undertaken.

6.6 DISCUSSIONS AND CONCLUSIONS

This chapter has presented guides for rocking foundation design both as a displacement based philosophy and a force based philosophy. The steps for each are outlined and two
examples of shear walls were given – one single story shear wall and one six story shear wall. The single story shear wall was designed using the DBD and the FBD, and the six story shear wall was designed with the DBD. Additionally the single story example was designed to preclude rocking, it was shown that a significantly larger foundation was required to satisfy requirements.

The foundation dimensions for the DBD was 3.6 m long by 1.0 m wide and 4.0 m long and 1.0 m wide for the FBD. In contrast the foundation required to preclude rocking was 7.5 m long by 2 m wide – the extra length required was a result of the large moment to vertical load ratio. The basis of the DBD was taken from past researchers: (Priestley et al. 2007, Sullivan et al. 2010, Paolucci et al. 2011), however a significant part of this research was the development of the curve for the nonlinear moment-rotation behaviour that was experienced in the experiments, and the equations predicting this. This nonlinear moment-rotation behaviour prediction contributes to the framework for DBD already in place and adds the rocking foundation dimension to it.

![Figure 6.27 Moment-rotation behaviour from snap-back 3 – test 9, Abaqus, and Equation 6.18](image_url)

Figure 6.27 Moment-rotation behaviour from snap-back 3 – test 9, Abaqus, and Equation 6.18
Figure 6.27 presents the pushover moment-rotation behaviour from snap-back 3 in test 9, the pushover output from Abaqus, the pushover curve from the *OpenSEES* model, and the curve from Equation 6.19. The curve for the equation was based on a modulus of $0.6G_{\text{max}}$ – close to the modulus factor of 0.5 used in the design. The figure shows that the three systems – experiment, finite element model, and equation – correlate well. However, more research needs to be undertaken to correlate the soil modulus factor with vertical load. Additionally, the operational modulus factor will change depending on soil type, and further experiments should be undertaken on different soil conditions to assess what acceptable reduction factors are.

Time history analyses were undertaken on both examples, to ensure the designs were not too un-conservative. The results showed that the foundation performance was satisfactory, and in most cases the displacements and rotations calculated were less than the design values. Tables 6.2 and 6.3 summarise the response of example 1 and example 2 to seven earthquake time histories respectively. For the few times where the response exceeded the design, the results show that the foundation was capable of handling the extra rotation and displacement demands.

### 6.6.1 Issues with FBD

Priestley et al. (2007) outline some disadvantages to force based design in the book: ‘Displacement-Based Seismic Design of Structures’. Some of the areas of deficiency in force based design include:

- The interdependency of strength and stiffness. FBD relies on estimates of initial stiffness, period, and distribution of design forces. Since stiffness is dependent on strength, this cannot be known until the end of a design process.
- Allocating seismic force between elements based on initial stiffness (even if this is known) is illogical for many structures, because it incorrectly assumes that the different elements can be forced to yield simultaneously.
- FBD is based on the assumption that unique force-reduction factors (based on ductility) are appropriate for a given structural type and material.
The two design methods given in this chapter produce similar foundation dimensions. However the ductility specified in the FBD was set for this purpose. The calculated rotations were different between the two design methodologies, however the spring bed model predicted rotations that were similar to the DBD, and therefore it is the recommended option for designing rocking foundations.

6.6.2 Considerations Related to Retrofitting Foundations

The applications for rocking foundation design can be extended to retrofitting foundations – not only new designs like those given in the design examples. To perform a rocking foundation design on an existing building a thorough knowledge of the soil and foundations is required, as well as knowledge of the structural capacity. In New Zealand many unreinforced masonry buildings built pre-1965 are inadequate for current practice. Buildings identified that need retrofitting have to be done so at the expense of the owner and can result in considerable cost. Rocking foundations offer an attractive option as, in some cases, structural components may not need to be strengthened.

For example – A DBD can be undertaken on a building that is in need of retrofitting provided the foundation dimensions and soil strength are known. Once the design is finished and the base shear, $V_b$, and maximum overturning moment, $M^*$, is calculated, these can be checked against existing structural capacities. The structure itself may then be determined not to need retrofitting because the current capacity satisfies the reduced forces associated from the rocking foundation design.

6.6.3 Serviceability Limit State and Ultimate Limit State

The design examples only consider ultimate limit state design, however consideration to serviceability limit state has to be made as well. Serviceability limit state design will have lower demands on the structure, and design displacements (and hence foundation rotations) should accommodate those lower intensities. NZS 1170.5 reduces the demands by a quarter if considering an elastic structure from ultimate to serviceability limit state in the force based approach. It is suggested that a similar approach is taken here – reduce the recommended drift of 2% by a quarter to 0.5% and subsequently
proceed with the design. The question on what is acceptable performance of rocking foundations under serviceability earthquakes should be considered in future research.

### 6.6.4 Future Research

The operational modulus factor should be further verified against different soil conditions. This should be done with experimental verification, similar to the experiments performed for this research. One advantage is that the frame used for the field tests was designed to be transportable and therefore similar tests on different soils do not require as much development. This coupled with a testing methodology already in place, the continuation of large scale experimental testing of rocking foundations will, in theory, not be too difficult. The other primary soil type is sand, and areas suitable for testing around New Zealand should be identified.

One limitation in the design guides was to assume that horizontal stiffness of the foundation remained elastic. The basis for this was experimental evidence in the tests that suggested the shear-sliding relationship was elastic. The moment to shear ratio for the experiments was around 4:1, contributing to the relatively low shear generated, and in addition the experiments had embedded foundations that raised the horizontal capacity due to passive earth pressure at the ends. Typical building shallow foundations are usually part of a larger foundation system, and so any sliding that would occur is accommodated for by other foundation components. Further research however should be undertaken on walls with low aspect ratios (low moment to shear). These more squat walls are perhaps more vulnerable to the shear-sliding deformation than a moment-rotation deformation. The particular section of the design guide may need to be revised for walls of this type to account for nonlinear horizontal behaviour.

Another limitation to the design guides only consider elastic behaviour of the superstructure, however some designers may want to include structural as well as foundation nonlinearity, if so reference to Sullivan et al. (2010) should be made as structural yielding is included. However if wanting to perform a design with structural yielding as well as foundation yielding a designer must be cautious, because two independent systems very rarely yield at the same occasion. More often than not one system will...
yield completely and one will remain elastic – this phenomenon is mentioned above in the issues with FBD and is stated in Priestley et al. (2007). Research of yielding foundations coupled with a yielding structure has been performed in centrifuge experiments on bridge foundations – outlined in Chapter 4 – however experiments on a large scale such as those in this research should be undertaken.

Settlement of rocking foundations should be investigated more and numerical models validated against them. Shake table experiments are suggested to further research settlement effects as more comprehensive earthquake records can be studied. Behaviours from the design earthquakes used in the examples can be submitted and settlements compared. Although shake table experiments require models to be scaled, it will be a good platform to compare to the numerical models and design procedure presented. Scaling effects could be compensated for by performing numerical modelling on identical structure that are being tested, or alternatively, more centrifuge experiments could be undertaken. The design guides assumed settlement was not a problem, because settlements recorded during the forced-vibration tests were much less than static settlement from self weight. However the spring bed model indicates that settlement during the dynamic part is greater than the static part. The report ‘Numerical Models for Analysis and Performance-Based Design of Shallow Foundations Subjected to Seismic Loading’ written for the Pacific Earthquake Engineering Research Center (PEER) developed a similar spring bed model and made predictions to centrifuge experiments (Gajan et al. 2008). The report mentions that for lower intensity earthquakes the settlement is predicted well, however in the more intensive motions the model over-predicts settlement. If worried about settlement on a particular rocking foundation, a designer may want to develop a spring bed model and check the settlements recorded against static settlement, then decide if dynamic settlement is satisfactory. A model such as one used in this chapter, and given in Appendix D, could be used.

This chapter is the final major chapter of this thesis, it gave design guides for rocking foundations and provided examples. The following chapter is a summary of all the discussions and conclusions made from this research.
CONCLUSIONS

The overall aims for this research, stated in the Chapter 1, were to produce good quality experimental data, develop numerical models calibrated to that data, and devise design guides for rocking shallow foundations. In terms of those three particular aims the objectives were achieved. Large scale field tests were performed, numerical models developed, and two separate design methodologies presented. The contribution of this research should help to address the latest New Zealand design and loadings standard, thus making foundation design in New Zealand more economical and better performing under seismic attack. Many numerical models have been developed, including spring bed, macro-element and finite element models, however there is a need for experimental data to assist in validating them. The experiments performed will also contribute to this gap between numerical and experimental knowledge of rocking foundations.

The literature review presented broader topics of shallow foundations, including bearing capacity, stiffness and an outline on soil-structure-interaction. A history of rocking
foundations is given, tracking the development from the pioneering research by Housner to the most recent study.

An experimental program was devised – undertaken onsite on an Auckland residual soil – and several geotechnical and geophysical tests were performed to understand soil properties. These tests included; cone penetration tests, seismic cone penetration tests, wave activated stiffness tests, spectral analysis of surface wave’s tests, and hand shear vane tests. Additionally laboratory tests were performed to measure Atterberg limits and water content.

A frame made from structural steel was designed and fabricated for the testing. It was designed to be demountable and transportable, making future research on different sites possible. The frame was instrumented with up to 42 instruments to capture system behaviour.

The experiments were done in two sets – forced-vibration tests and snap-back tests – and were performed over an extensive testing period. The forced-vibration tests excited the foundations to rock, however were deemed not the most realistic representation of an earthquake. The snap-back tests allowed the structure to behave in a similar manner to that expected during ground shaking.

Two numerical models were developed – a finite element model in Abaqus and a spring bed model in OpenSEES. The Abaqus model was developed in stages to ensure accuracy, and pushover curves compared well with experiment data. The OpenSEES model was developed in a similar way, and FEMA 356 guidelines for spring bed modelling were used as a starting point. The model was adjusted according to what produced the best fit results when compared to experimental pushover curves.

The two design methodologies utilised a displacement based philosophy and a force based philosophy respectively. Two design examples, one of a single story shear wall and one of a six story shear wall, were given to illustrate the design methodologies, and to highlight benefits compared to the same design without rocking. The design displacements and forces from the examples compared well to dynamic time history
analyses performed using the *OpenSEES* model – submitting the structure to several different earthquake motions.

Although the objectives of this study were achieved, there is still scope for research to advance the knowledge of rocking foundation systems.

### 7.1 MAIN CONCLUSIONS

1. Chapter 4 introduces a relatively simple equation to predict the nonlinear moment-rotation pushover behaviour of shallow foundations on cohesive soil. It was based on a hyperbolic stress-strain relationship by Konder (1963). There are two inputs to the equation – one based on moment capacity, and one based on initial stiffness. The equation was shown to match the experiment pushover curves well, and was verified against numerical models in Chapter 5, and introduced into the design guide in Chapter 6.

2. The field experiments undertaken on an Auckland residual soil – predominately clay – showed that initial foundation stiffness reduction occurred rapidly, and a foundation rotational stiffness using an ‘operational modulus’ should be considered. The operational modulus factor, $c_{op}$, was around 0.6 for the experiments, and the numerical modelling later showed this to be dependent on vertical factor of safety. A soil operational modulus factor is necessary for rocking shallow foundation design.

3. The two numerical models developed, one in *Abaqus* and one in *OpenSEES*, both correlated well with experimental pushover curves. The nonlinear *Abaqus* model, developed in stages, was able to model experimental behaviour well. The *OpenSEES* model was originally based on FEMA 356 recommendations, however it was shown that the pushover behaviour was not accurate. A new spring bed setup was proposed, based on end regions of 7.5% instead of 17%, and this predicted experimental curves satisfactorily. The validated numerical models
present a platform where comprehensive shallow foundation research may be undertaken.

4. The numerical models developed in Chapter 5 also show the two forms of nonlinearity occurring in shallow foundation rotation – geometric nonlinearity and material nonlinearity. This ‘double nonlinearity’ was mentioned in the Introduction, and it was shown that both forms of nonlinearity should be considered in shallow foundation behaviour.

5. The field experiments demonstrated a large amount of damping occurring during the rocking motion. The average damping during the experiments was around 30%, and the damping recommended in the design guide for rocking foundations was 20%. High damping can be beneficial to a structure by reducing seismic forces.

6. The two design examples presented in Chapter 6 had similar predictions of displacements and rotations to the time history analyses performed using the OpenSEES model. The guide shows how a quick design may be performed, however the time history model may also be used to undertake more comprehensive analysis as a final design check.

7. Overall this thesis presents a strong argument for the use of rocking shallow foundations as a viable option for earthquake resistant design. Although this research has contributed to understanding nonlinear rotational behaviour of shallow foundations on cohesive soil, it is acknowledged that more research is required for a full understanding of this topic.

### 7.1.5 Major Future Research Subjects

1. Additional field experiments are required, on different soil types, to further help fully understand nonlinear shallow foundation behaviour. Experiments on different sites around New Zealand are necessary, and the snap-back method
developed should be the chosen form of testing. Different foundation sizes/shapes/configurations should be tested also, to get a broad representation of potential rocking foundations for New Zealand.

2. The Abaqus numerical model should be further developed to consider the dynamic response of rocking shallow foundations. Coupled with the experiments mentioned above, this model could be extended to different soil types and foundation configurations.

7.2 CONCLUDING REMARKS FROM EACH CHAPTER

7.2.6 Experimental Configuration – Chapter 3

- This chapter described the soil properties onsite through a variety of tests performed – both onsite and in the lab. The Auckland residual soil found was predominately a very stiff Waitemata Clay. The soil had high undrained shear strengths – between 100 to 240 kPa – and a small strain shear modulus between 30-40 MPa.

- The CPT data gathered suggested that the soil was more sandy than silty/clayey, however onsite observation, coupled with laboratory tests indicated the soil being more clay than silt or sand.

- One main conclusion from the soil testing was the variability of residual soil. Thus the need for thorough ground investigation and the proper understanding of soil properties for rocking foundation response to be predicted accurately.

7.2.7 Experimental Results – Chapter 4

- Chapter 4 presented the experimental results from the field tests. It covers major areas in foundation and structural response from both the forced-vibration and snap-back tests.
The moment-rotation behaviour during the forced-vibration tests showed well defined moment capacities. Test 1 displayed a moment capacity of around 48 kNm and test 4 around 100 kNm, however test 4 was less well defined. The number of loops presented in the moment-rotation curves indicates how many cycles the foundations had to endure as the eccentric mass shaker was ramped up through the frequency range. This is not a realistic phenomenon for an earthquake and was one of the reasons why the form of excitation was changed. Another reason was the excitation of the shaker perhaps did not generate as much rotation as necessary (10 millirads for test 1 and 3 millirads for test 4), and the foundations could have been yielded further.

The pushover curves from the snap-back tests displayed nonlinear moment-rotation behaviour with a moment capacity of around 100 kNm for tests 7 and 9. This correlated well with the moment capacity equation for a rocking foundation. The curves displayed a degradation of initial rotation stiffness throughout the tests. This was from soil damage and rounding that occurred during the slow pullback part of the cycle. Test 7 shows the amount of initial stiffness degradation over the 9 snaps was equivalent to a loss of contact from 2.0m to 1.5m – or around 50% stiffness. This behaviour would not occur in such capacity during an earthquake, however a foundation will still display some form of soil rounding.

Moment-rotation curves also showed how quickly nonlinear action occurred. Enlarged sections of the moment-rotation curves throughout test 7 show how behaviour matched a linear representation for a very short while only, and started becoming nonlinear at a very small rotation.

The equation proposed for predicting the nonlinear moment-rotation pushover behaviour was based on a hyperbolic stress-strain equation for soil mechanics. It correlated well with the experiments and was able to predict upper and lower bounds of the pushover curves. The equation has two inputs; an initial stiffness
and a moment capacity. The initial stiffness was calculated on the initial rotational stiffness of the system with a reduction factor – the soil operational modulus factor, $c_{op}$.

- The shear sliding relationship during the forced-vibration tests displayed linear elastic behaviour. Large horizontal resistance is provided through embedment, and, in most cases, building foundations will be embedded. The aspect ratio of the structure (around 4:1) provided a high moment to shear ratio that also contributed to the elastic behaviour. In more squat walls the shear sliding relationship cannot be assumed to behave elastically, and care needs to be taken in designing rocking foundations for these situations.

- Damping calculated throughout the snap-back tests was significant; damping ratios ranged from around 0 to 50%. Damping was calculated by the logarithmic decrement method and was shown to alter depending on the impact number of the foundation. The even numbered cycles produced much higher damping than the odd numbered. Be this as it may, the average value of damping was still around 30%.

- The normalised rotation against impact number was compared to Housner’s equations, and an ‘$r$’ value of 0.4 was produced. This corresponded to an equivalent viscous damping of 28% - close to what the average value of damping across the snap-back tests were.

- The periods recorded after each half cycle were also compared to Housner’s equations. The half periods of the snap-back tests against normalised rotation show that the equation of the curve can be adjusted to correlate well with the test data.

- Excessive settlement did not occur during either the forced-vibration or snap-back tests. This is evident by the settlement time history and settlement-rotation plots.
In both forced-vibration and snap-back tests the static settlement that occurred onsite from the vertical load applied to the foundations was greater than settlements recorded dynamically during the tests.

### 7.2.8 Numerical Modelling – Chapter 5

- Chapter 5 discussed the development of two different numerical models for predicting rotational behaviour of rocking shallow foundations. The first, developed in the software program *Abaqus*, is a finite element model that utilises nonlinear elements with a von Mises failure criterion. The second is a nonlinear spring bed model developed in the open source software platform *OpenSEES*.

- The elastic pushover response of both models demonstrated how much geometric nonlinearity there is to a rocking foundation problem. It was discovered, through comparing elastic behaviour to the experimental data, that elastic models do not accurately predict shallow foundation response although the soil was very stiff. Figure 5.18 presents the difference between the elastic soil model and nonlinear soil model in *Abaqus*. For analysis of rocking shallow foundations, both geometric and material nonlinearity must be considered.

- The nonlinear constitutive soil model incorporated combined kinematic and isotropic hardening. The model was further refined because of nonlinear deformation occurring within the elements. The original size of the elements (0.10 m) was too small for the amount of deformation they were required to undergo, and the size was increased to 0.20 m. The type of element was altered from a linear 8 node element type to a quadratic 20 node element type. The sensitivity of the mesh size was further investigated by attempting several different mesh sizes: 0.1 m, 0.14 m, 0.17 m, 0.20 m, 0.25 m, 0.33 m. From these only the 0.20 m mesh size completed the pushover analysis.

- Static settlement predicted by the *Abaqus* model was around 10 mm, whereas settlement from cyclic motion was around 5 mm. Static settlements were
comparable to what was recorded in the experiments. Dynamic settlements recorded during the tests were less than the 5mm predicted by *Abaqus*, however full cyclic loading was not performed in the experiments, as was the case in *Abaqus*.

- Pushover analyses for different vertical loads were performed. The vertical loads corresponded to factors of safety of 2.8, 6.5 (experiment value), 9.0 and 12.0. The moment-capacity was shown to be more dependent on vertical load, and less so on the factor of safety.

- The *Abaqus* moment-rotation pushover was compared with pushovers from experiment 9. The finite element model can capture the system behaviour of the upper bound of the results – the pushover response before any soil rounding effects from previous snaps occurred. The *Abaqus* model has the capacity to model soil rounding effects. The size of the elements within the model will determine how accurately soil-rounding can be measured as rounding can only be captured between elements. The validating of numerical models with experimental results was one of the main aims of this research.

- FEMA 356 guidelines for modelling a shallow foundation as a bed of springs did not accurately model experiment behaviour. Instead a curve of best fit was adopted where the length of end regions (the region of increased stiffness) was reduced from 17% of the foundation length to 7.5%.

- Both numerical models were compared to the moment-rotation equation for two vertical loads. The greater vertical load was shown to have a lower operational modulus factor – a reduction from 0.6 to 0.3 was necessary to ensure the curve fitted well. A soil operational modulus factor 0.3 represents the lower bound of this factor, as this particular configuration corresponded to a factor of safety of 2.8 – around the lowest for shallow foundations design.
7.2.9 **Design Guideline – Chapter 6**

- This chapter presented guides for rocking foundation design both as a displacement based philosophy and a force based philosophy. The steps for each are outlined and two examples, using the DBD approach, of shear walls were given. The first example was a single story shear wall, and the second a six story shear wall. Additionally the single story shear wall example was designed using the FDB and also to preclude rotation. It was shown that a significantly larger foundation was required to satisfy requirements.

- The soil modulus factor used in the design was 0.5; however, as is discussed above, this value was shown to be dependent on vertical factor of safety. A designer may wish to be conservative and use a low soil operational modulus factor for rocking foundation design.

- Time history analysis using the *OpenSEES* spring bed model showed promising results when compared to the DBD examples. The input motions were from the earthquakes: El Centro, Duzce, Tabas, Hokkaido, La Union, Lucerne and Arcelik. The Tabas, Lucerne and Arcelik motions had high directivity, and the Lucerne earthquake generated the greatest response for both examples.

- The maximum rotations and displacements were less than the design values (20 millirads and 71.2 mm respectively) in all but the Lucerne earthquake for the first example. In that case, however, the model indicated that the performance of the foundation was satisfactory despite exceeding the design values. The maximum residual displacement corresponded to a drift of 0.45%, however the rest of the six earthquakes were much less. The average dynamic settlement predicted for example 1 was 6.76 mm. The static settlement calculated by *OpenSEES* was only 2.59 mm and the Gazetas formulae predicted 1.60 mm.

- The FBD produced similar a similar size foundation to the DBD using a ductility of 2.5. However some issues with FBD were outlined at the end of the chapter,
and the method of calculating displacements in the FBD is inaccurate when compared to time history outcomes. The common perception is that DBD is the way forward and thus out of the two guidelines presented the DBD is the recommended option.

- The *OpenSEES* analysis of the six story shear wall produced positive results also. The structure was modelled as an equivalent single degree of freedom model and submitted to the same earthquakes as the previous example, scaled accordingly. Again the maximum rotations and displacements correlated well with the design values. The moment-rotation curves presented show less damping (indicated by the size of the loops) than the single story wall example. Damping within the model was in the form of hysteretic damping from the$qz$ spring elements, and so is related to displacements. The second example had a foundation rotation less than the first example, and this explains why not as much damping is occurring. The average settlement predictions were; 2.86 mm during the dynamic motion, and 0.50 mm during the static part. Gazetas formulae predicted the settlement as 0.73 mm. The maximum residual displacement at the centre of mass was 5.76 mm, corresponding to a drift of 0.04%. This amount of drift is very low and suggests foundation rocking would not generate any permanent damage, or necessary repair, on this particular example.

- The applications for rocking foundation design can be extended to retrofitting foundations – not only new designs like those given in the design examples. For example – A DBD can be undertaken on a building that is in need of retrofitting provided the foundation dimensions and soil strength are known. Once the design is finished and the base shear,$V_b$, and maximum overturning moment,$M^*$, is calculated, these can be checked against existing structural capacities. The structure itself may then be determined not to need retrofitting because the current capacity satisfies the reduced forces associated from the rocking foundation design.
• The design examples only considered ultimate limit state design; however consideration to serviceability limit state has to be made as well. Serviceability limit state design will have lower demands on the structure, and design displacements (and hence foundation rotations) should accommodate those lower intensities. If taking a force based approach, NZS 1170.5 reduces the demands from ultimate to serviceability limit state by a quarter if considering an elastic structure. It is suggested that a similar approach is taken here – reduce the recommended drift of 2% by a quarter to 0.5% and proceed with the design.

7.2.10 Future Research

• To try and assist the need for more experimental data on rocking foundations the frame that was designed and built should be tested at as many different sites around New Zealand as possible. Testing of this nature will provide good insight on how shallow foundations perform on natural ground – and highlight any variability in performance. The method of determining of testing that was found best describes system behaviour was the snap-back method – greater rotations were able to be induced, and the testing method is a lot simpler.

• Substantial research on the soil operational modulus factor needs to be undertaken. The sensitivity of this modulus to different soil types, and different foundation configurations must be considered. Ideally this research should be done with both more field experiments, and comprehensive numerical modelling.

• The experiments performed in this study did not induce much settlement, due to the high strength and stiffness of the residual soil. Testing on softer ground trying to generate greater settlement will give a better understanding of how foundations settle during earthquakes. From good quality settlement data, acceptable performance criteria, with respect to settlement may be achieved.

• Performing experiments with super-structure yielding as well as foundation yielding is also important because it will will reveal the coupling and sensitivity
of each mechanism. This however is difficult to do in large scale because of cost and safety issues. Slow cyclic tests could be undertaken on a large scale similar to the experiments performed for this research. However for this situation a new frame and test methodology needs to be developed.

- Further research should be undertaken on walls/structures with low aspect ratios (low moment to shear). These more squat walls are perhaps more vulnerable to the shear-sliding deformation than a moment-rotation deformation. The nonlinear shear-sliding relationship of squat walls should be addressed. The particular section of the design guide may need to be revised for walls of this type to account for nonlinear horizontal behaviour.

- The finite element model is Abaqus can be developed further. The model should be expanded to accommodate dynamic input motions such as earthquakes. Additionally the model can study foundation behaviour of different foundation sizes and soil types.

- The proposed spring bed model for this research in OpenSEES should be further verified. The model was changed from FEMA guidelines to include an end region of 7.5% compared to 17%. This subsequently reduced the initial stiffness of the system, and while comparing well to the experiments for this study, should be checked against other experimental data as well.

- Questions on what is acceptable performance of rocking foundations also need to be addressed. For ultimate limit state design, the drift limit recommended is 2% - slightly less than the current drift limit for structural components in the New Zealand code. However consideration must be made to serviceability limit state, and what are acceptable drift limits/performance indicators for such earthquakes.
Figure A-1 CPT results from CPT01, the y axis is depth below the ground in metres
Figure A-2 CPT results from CPT02, the y axis is depth below the ground in metres.
Figure A-3 CPT results from CPT03, the y axis is depth below the ground in metres
CPT Results (Chapter 3)

CPT 04

Figure A-4 CPT results from CPT04, the y axis is depth below the ground in metres

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**CPT 05**

*Figure A-5 CPT results from CPT05, the y axis is depth below the ground in metres*

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Figure A-6 CPT results from CPT06, the y axis is depth below the ground in metres
Figure A-7 CPT results from CPT07, the y axis is depth below the ground in metres
Figure A-8 CPT results from CPT08, the y axis is depth below the ground in metres.
Figure A-9 CPT results from CPT09, the y axis is depth below the ground in metres
Figure A-10 CPT results from CPT10, the y axis is depth below the ground in metres
Figure A-11 CPT results from CPT11, the y axis is depth below the ground in metres.
CPT 12

Figure A-12 CPT results from CPT12, the y axis is depth below the ground in metres

Figure A-12 CPT results from CPT12, the y axis is depth below the ground in metres
Figure A-13 CPT results from CPT13, the y axis is depth below the ground in metres
CPT 14

Figure A-14 CPT results from CPT14, the y axis is depth below the ground in metres

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CPT 15

Figure A-15 CPT results from CPT15, the y axis is depth below the ground in metres

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**Figure A-16** CPT results from CPT16, the y axis is depth below the ground in metres
APPENDIX B

CRITICAL PLOTS FROM THE FORCED-VIBRATION TESTS (CHAPTER 4)
TEST 1

Test 1 – Frequency Response

Figure B-1 Frequency Response: top left = test 1-A; top right = test 1-B; bottom = test 1-C
**Test 1 – Input Force Time History**

*Figure B-2 Time history input force: top = test 1-B; bottom = test 1-C*
Test 1 – Moment-Rotation

Figure B-3 Moment-Rotation: top left = test 1-B (west footing); top right = test 1-B (east footing); bottom left = test 1-C (west footing); bottom right = test 1-C (east footing)
Test 1 – Settlement-Rotation

Figure B-4 Settlement-Rotation: top left = test 1-B (west footing); top right = test 1-B (east footing); bottom left = test 1-C (west footing); bottom right = test 1-C (east footing)
Test 1 – Settlement Time History

Figure B-5 Settlement time history: top left = test 1-B (west footing); top right = test 1-B (east footing); bottom left = test 1-C (west footing); bottom right = test 1-C (east footing)
TEST 2

Test 2 – Frequency Response

Figure B-6 Frequency Response: top left = test 2-A; top right = test 2-B; bottom = test 2-C
Appendix B

Test 2 – Input Force Time History

Figure B-7 Time history input force test 2-B

Test 2 – Moment-Rotation

Figure B-8 Moment-Rotation: left = test 2-B (west footing); right = test 2-B (east footing)
Test 2 – Shear-Sliding

Figure B-9 Shear-sliding: left = test 2-B (west footing); right = test 2-B (east footing)

Test 2 – Settlement-Rotation

Figure B-10 Settlement-rotation: left = test 2-B (west footing); right = test 2-B (east footing)
Test 2 – Settlement Time History

*Figure B-11* Settlement time history: left = test 2-B (west footing); right = test 2-B (east footing)
TEST 4

Test 4 – Frequency Response

Figure B-12 Frequency Response: top left = test 4-A; top right = test 4-B; bottom = test 4-C
Appendix B

Test 4 – Input Force Time History

![Graph of Input Force Time History](image)

*Figure B-13 Time history input force test 4-B*

Test 2 – Moment-Rotation

![Graph of Moment-Rotation](image)

*Figure B-14 Moment-Rotation: left = test 4-B (west footing); right = test 4-B (east footing)*
Test 4 – Shear-Sliding

![Graph showing shear-sliding plots for test 4-B (west and east footings).]

*Figure B-15 Shear-sliding: left = test 4-B (west footing); right = test 4-B (east footing)*

Test 4 – Settlement-Rotation

![Graph showing settlement-rotation plots for test 4-B (west and east footings).]

*Figure B-16 Settlement-rotation: left = test 4-B (west footing); right = test 4-B (east footing)*
Test 4 – Settlement Time History

Figure B-17 Settlement time history: left = test 4-B (west footing); right = test 4-B (east footing)
APPENDIX C

CRITICAL PLOTS FROM THE SNAP-BACK TESTS (CHAPTER 4)
It was mentioned in Chapter 4 that during the snap-back tests some instruments became faulty or strained from the extensive testing program. Not all the results from every test are included because of this.

**TEST 5**

**Test 5 – Moment-Rotation**

![Moment-rotation: snap 1](image1)

**Test 5 – Rotation Time History**

![Rotation time history: snap 1; snap 3](image2)
Test 5 – Phase

Appendix C-3 Phase: snap 1 (left); snap 3 (right)

TEST 6

Test 6 – Pushover

Appendix C-4 Pushover: snap 1 (left); snap 2 (right)
Appendix C-5 Moment-rotation: snap 1 (top left); snap 2 (top right); snap 3 (middle left); snap 4 (middle right); snap 5 (bottom left); snap 6 (bottom right)
Critical Plots from the Snap-Back Tests (Chapter 4)

Appendix C-6 Moment-rotation: snap 7

Test 6 – Rotation Time History

Appendix C-7 Rotation time history: snap 1 (top left); snap 2 (top right); snap 3 (bottom left); snap 4 (bottom right)
Appendix C

Appendix C-8 Rotation time history: snap 5 (top left); snap 5 (top right); snap 7 (bottom)

Test 6 – Phase

Appendix C-9 Phase: snap 1 (left); snap 2 (right)
Critical Plots from the Snap-Back Tests (Chapter 4)

Appendix C-10 Phase: snap 3 (top left); snap 4 (top right); snap 5 (middle left); snap 6 (middle right); snap 7 (bottom)
TEST 7

Test 7 – Pushover

Appendix C-11 Pushover: snap 1 (top left); snap 2 (top right); snap 3 (middle left); snap 4 (middle right); snap 5 (bottom left); snap 6 (bottom right)
Critical Plots from the Snap-Back Tests (Chapter 4)

Appendix C-12 Pushover: snap 7 (top left); snap 8 (top right); snap 9 (bottom)

Test 7 – Moment-Rotation

Appendix C-13 Moment-rotation: snap 1 (left); snap 2 (right)
Appendix C-14 Moment-rotation: snap 3 (top left); snap 4 (top right); snap 5 (middle left); snap 6 (middle right); snap 7 (bottom)
Test 7 – Rotation Time History

Appendix C-15 Rotation time history: snap 1 (top left); snap 2 (top right); snap 3 (middle left); snap 4 (middle right); snap 5 (bottom left); snap 6 (bottom right)
Appendix C-16 Rotation time history: snap 7 (top left); snap 8 (top right); snap 9 (bottom)

Test 7 – Phase

Appendix C-17 Phase: snap 1 (left); snap 2 (right)
Appendix C-18 Phase: snap 3 (top left); snap 4 (top right); snap 5 (middle left); snap 6 (middle right); snap 7 (bottom left); snap 8 (bottom right)
Appendix C-19 Phase: snap 9

TEST 8

Test 8 plots are not included here as the foundations did not rock because water onsite (refer to Chapter 4). The results generated for test 8 do not show any relevant behaviour.

TEST 9

Test 9 – Pushover

Appendix C-20 Pushover: snap 1 (left); snap 2 (right)
Critical Plots from the Snap-Back Tests (Chapter 4)

Appendix C-21 Pushover: snap 3 (top left); snap 4 (top right); snap 5 (middle left); snap 6 (middle right); snap 7 (bottom left); snap 8 (bottom right)
Test 9 – Rotation Time History

Appendix C-22 Rotation time history: snap 1 (top left); snap 2 (top right); snap 3 (middle left); snap 4 (middle right); snap 5 (bottom left); snap 6 (bottom right)
Critical Plots from the Snap-Back Tests (Chapter 4)

Appendix C-23 Rotation time history: snap 7 (left); snap 8 (right)

Test 9 – Phase

Appendix C-24 Phase: snap 1 (top left); snap 2 (top right); snap 3 (bottom left); snap 4 (bottom right)
Appendix C-25 Phase: snap 5 (top left); snap 6 (top right); snap 7 (bottom left); snap 8 (bottom right)

TEST 10

Test 10 plots are not included here as the foundations did not rock because water onsite (refer to Chapter 4). The results generated for test 8 do not show any relevant behaviour.
APPENDIX D

OPENSEES CODE FOR EXAMPLE 1 (CHAPTER 6)
INPUT

run.tcl

# MODEL SET UP ----------------------------------------------------
wipe; # clear openses model
model basic -ndm 2 -ndf 3; # 2 dimensions, 3 dof per node
file mkdir Data; # create data directory

# define GEOMETRY -----------------------------------------------
# nodal coordinates:
node 2 0 3.559 # node#, X Y

set mass1 103671.692
set Iwall 2388324.038
set vert 61832.58

# nodal masses:
mass 2 $mass1 0 $Iwall; # node#, Mx My Mz Mass=Weight/g.

# Define ELEMENTS -----------------------------------------------
# define geometric transformation: performs a linear geometric
# transformation of beam stiffness and resisting force from the basic
# system to the global-coordinate system
geomTransf Linear 1; # associate a tag to transformation

source footing.tcl #Source the footing file

element elasticBeamColumn 1 $ColNode 2 0.54 2508400000 0.583 1;
#element elasticBeamColumn $eleTag $iNode $jNode $A $E $Iz $transfTag
# define GRAVITY -----------------------------------------------

pattern Plain 1 Linear {
    load 2 0. [expr -9.81*$vert] 0.;
}

# node#, FX FY MZ -- superstructure-weight
constraints Plain;
# how it handles boundary conditions

numberer Plain;
# renumber dof's to minimize band-width (optimization)

system BandGeneral;
# how to store and solve the system of equations in the analysis

test NormDispIncr 1.0e-8 6 ;
#determine if convergence has been achieved at the end of an
iteration step

algorithm Newton;
# use Newton's solution algorithm: updates tangent stiffness at
every iteration

integrator LoadControl 0.1;
# determine the next time step for an analysis,

analysis Static
#define type of analysis static or transient

analyze 10;
# perform gravity analysis in 10 steps

loadConst -time 0.0;
# hold gravity constant and restart time

#Set the Rayleigh Damping to 5% for Nodes and Elements

set alphaM [expr 2*0.05*pow([eigen 1],0.5)]
puts [eigen 1] #Puts the eigen value of the first mode

rayleigh $alphaM 0.0 0.0 0.0
#rayleigh $alphaM $betaK $betaKinit $betaKcomm
# Define RECORDERS ------------------------------------------------
recorder Node -file Data/DFree.out -time -node 2 -dof 1 2 3 disp;
    # displacements of free nodes

recorder Node -file Data/DBase.out -time -node 1011 -dof 1 2 3 disp;
    # displacements of support nodes

recorder Node -file Data/RBase.out -time -node 1011 -dof 1 2 3 reaction;
    # support reaction

recorder Element -file Data/FCol.out -time -ele 1 globalForce;
    # element forces - column

recorder Node -file Data/massacc.out -time -node 2 -dof 1 accel
    # Acceleration of the mass

set xFile elcentro.txt  #Source Earthquake
set nSteps 4000        #Number of steps
set dt 0.02            #Time step
source DynamicAnalysis.tcl  #Source the dynamic analysis file

#proc Dynamic {File nSteps dt alphaM}
set conv 9.81          #Gravity
set SF 1.2             #Scale factor
set GMfatt [expr $conv*$SF]
puts $GMfatt
Dynamic $xFile $nSteps $dt $alphaM $GMfatt
footing.tcl

# Input Parameters
set bTot 3.6;  # Footing Dimension in X Direction
set E 2500000000;  # Young's modulus of concrete
set A 10000;  # Area of each truss element representing the footing
set ntot 21  # Number of springs

set spc [expr $bTot/($ntot-1)]  # Spring spacing
set nCentNode [expr ($ntot-1)/2]  # Central node spring
set k [expr 1001+$nCentNode]  # Counter

set G 40000000  # Soil shear modulus
set nu 0.5  # Poisson's ratio

set kcent_length [expr 1*0.73*$G/(1-$nu)]  # FEMA 356
set kcov_length [expr 1*6.83*$G/(1-$nu)]  # FEMA 356

set kcent [expr $kcent_length*0.18*1.0]
set kcov [expr $kcov_length*0.18*1.0]
set kcov_2 [expr $kcov_length*0.09*1.0]
# Tributary areas of the springs

set Q 1950400
set qult [expr $Q/20]
set qult_2 [expr $qult/2]

set z50_cent [expr 0.7*$qult/$kcent]
set z50_cov [expr 0.7*$qult/$kcov]

# uniaxialMaterial QzSimple1 $matTag $qzType $qult $z50 <$suction $c>
uniaxialMaterial QzSimple2 1 1 $qult $z50_cent 0.0 0.0
uniaxialMaterial QzSimple2 2 1 $qult $z50_cov 0.0 0.0
uniaxialMaterial QzSimple2 3 1 $qult_2 $z50_cov 0.0 0.0

uniaxialMaterial Elastic 12 242000000000000000000

node 1001 1.8 0
node 1002 1.62 0
node 1003 1.44 0
node 1004 1.26 0
node 1005 1.08 0
node 1006 0.9 0
data 1007 0.72 0
data 1008 0.54 0
data 1009 0.36 0
data 1010 0.18 0
data 1011 0 0
data 1012 -0.18 0
data 1013 -0.36 0
data 1014 -0.54 0
data 1015 -0.72 0
data 1016 -0.9 0
data 1017 -1.08 0
data 1018 -1.26 0
data 1019 -1.44 0
data 1020 -1.62 0
data 1021 -1.8 0

node 2001 1.8 0
node 2002 1.62 0
node 2003 1.44 0
node 2004 1.26 0
node 2005 1.08 0
node 2006 0.9 0
node 2007 0.72 0
node 2008 0.54 0
node 2009 0.36 0
node 2010 0.18 0
node 2011 0 0
node 2012 -0.18 0
node 2013 -0.36 0
node 2014 -0.54 0
node 2015 -0.72 0
node 2016 -0.9 0
node 2017 -1.08 0
node 2018 -1.26 0
node 2019 -1.44 0
node 2020 -1.62 0
node 2021 -1.8 0

fix 2001 1 1 1
fix 2002 1 1 1
fix 2003 1 1 1
fix 2004 1 1 1
fix 2005 1 1 1
fix 2006 1 1 1
fix 2007 1 1 1
fix 2008 1 1 1
fix 2009 1 1 1
fix 2010 1 1 1
fix 2011 1 1 1
fix 2012 1 1 1
fix 2013 1 1 1
fix 2014 1 1 1
fix 2015 1 1 1
fix 2016 1 1 1
fix 2017 1 1 1
fix 2018 1 1 1
fix 2019 1 1 1
fix 2020 1 1 1
fix 2021 1 1 1

set ColNode 1011
puts "Foundation nodes are created..."

element zeroLength 8011 2011 1011 -mat 12 -dir 1

element zeroLength 1001 2001 1001 -mat 3 -dir 2
element zeroLength 1002 2002 1002 -mat 2 -dir 2
element zeroLength 1003 2003 1003 -mat 1 -dir 2
element zeroLength 1004 2004 1004 -mat 1 -dir 2
element zeroLength 1005 2005 1005 -mat 1 -dir 2
element zeroLength 1006 2006 1006 -mat 1 -dir 2
element zeroLength 1007 2007 1007 -mat 1 -dir 2
element zeroLength 1008 2008 1008 -mat 1 -dir 2
element zeroLength 1009 2009 1009 -mat 1 -dir 2
element zeroLength 1010 2010 1010 -mat 1 -dir 2
element zeroLength 1011 2011 1011 -mat 1 -dir 2
element zeroLength 1012 2012 1012 -mat 1 -dir 2
element zeroLength 1013 2013 1013 -mat 1 -dir 2
element zeroLength 1014 2014 1014 -mat 1 -dir 2
element zeroLength 1015 2015 1015 -mat 1 -dir 2
element zeroLength 1016 2016 1016 -mat 1 -dir 2
element zeroLength 1017 2017 1017 -mat 1 -dir 2
element zeroLength 1018 2018 1018 -mat 1 -dir 2
element zeroLength 1019 2019 1019 -mat 1 -dir 2
element zeroLength 1020 2020 1020 -mat 2 -dir 2
element zeroLength 1021 2021 1021 -mat 3 -dir 2

puts "Foundation springs are created..."

#Create the truss elements
for {set i 1001} {$i<[expr 1000+$ntot]} {incr i 1} {
    element elasticBeamColumn [expr 4000+$i] $i [expr $i+1] 10000 $E 10000 $Iz
}

puts "Done"
DynamicAnalysis.tcl

proc Dynamic {xFile nSteps dt alphaM GMfatt} {
    system UmfPack
    constraints Plain
    #set GMfatt 0.001; #overwrite the scaling factors in the data structure
    #test EnergyIncr 1.0e-5 1000; #1000 iterations with a tolerance of 1.0e-5
    test NormDispIncr 1.0e-3 1000
    set gamma 0.5
    set beta 0.25
    algorithm Newton
    numberer RCM
    #integrator Newmark $gamma $beta <$alphaM $betaK $betaKinit $betaKcomm>
    integrator Newmark 0.5 0.25
    analysis VariableTransient
    set comp1 "Series -dt $dt -filePath $xFile -factor $GMfatt"
    pattern UniformExcitation 2 1 -accel $comp1
    analyze $nSteps $dt
}
APPENDIX E

OPENSEES TIME HISTORY PLOTS FROM THE DESIGN EXAMPLES (CHAPTER 6)
Single Story Shear Wall Example – El Centro

Figure E-1 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – Duzce

Figure E-2 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – Tabas

Figure E-3 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – Hokkaido

Figure E-4 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – La Union

Figure E-5 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – Lucerne

Figure E-6 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Single Story Shear Wall Example – Arcelik

Figure E 7 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – El Centro

![Graphs showing moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation.](image)

*Figure E-8 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation*
Six Story Shear Wall Example – Duzce

Figure E-9 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – Tabas

Figure E-10 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – Hokkaido

Figure E-11 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – La Union

Figure E-12 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – Lucerne

Figure E-13 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation
Six Story Shear Wall Example – Arcelik

Figure E-14 Moment-rotation, settlement-rotation, displacement time history of the mass, and rotation time history of the foundation


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