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Learning about multiplication: An interpretation of a transition

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Abstract

This thesis examines how children learn multiplication, a new concept in mathematics. Multiplication is an area of concern in primary schools because it is the first abstraction point from counting and addition and because it underpins many subsequent concepts. Understanding more about how children learn multiplication is important for improving teaching and learning in mathematics.

An initial interview study with twenty-eight Year 5 and 6 students explored how we can see their multiplicative thinking using a range of questions and tasks. The children’s responses appeared to fit the pattern suggested by the literature. Further analysis revealed important individual variability and instability. The role of transition between different ways of thinking appeared important.

In order to look at transition and change in six children’s thinking the second study used a moderate hermeneutic viewpoint. The nature of the transitions occurring over eight lessons was analysed and found to have certain characteristics. The transition was idiosyncratic, recursive, shaped by fore-conceptions and tradition processes, influenced by the teacher’s interpretations and by the use of language. These characteristics add depth to our understanding of how children learn multiplication.

The thesis concludes that multiplicative thinking is an ongoing transition through increasingly sophisticated ideas rather than an end-point or definable state. Further it suggests that moderate hermeneutics is a theoretical framework with explanatory value in classroom settings, allowing researchers to view learning in new and useful ways.
Dedication

Fleet foot on the correi,
Sage counsel in cumber,
Red hand in the foray,
How sound is thy slumber!
Like the dew on the mountain,
Like the foam on the river,
Like the bubble on the fountain,
Thou art gone, and for ever!

From 'The Lady of the Lake' by Walter Scott

To the memory of my husband, Ross Jonathan Grant,
this thesis is lovingly dedicated.
Acknowledgements

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The staff and students of the schools used in this study, who were such enthusiastic participants, made this study possible. In particular I would like to acknowledge the teacher and the six children who form the core of this thesis. Their willingness to share their thoughts and ideas with me made this study fascinating.

A Bright Futures Top Achiever Doctoral Scholarship supported me while undertaking this thesis. It would not have been possible for me to study for this degree without this support.

This thesis has provided a bridge for me from a time of extreme grief to a more peaceful place. That I could do this is due to the support of my family and my friends. In particular I would like to acknowledge my mother, Ruth Ell. I also thank with gratitude my mother-in-law, Val Grant. Jen, Gordon, Kelvin, Sarah …and my sons, Angus and Lachlan, who have played quietly in the early mornings. I thank Matthew for the inspiration to finish.
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